Available online at www.sciencedirect.com
SCIENCE DIRECT®
Computers and Chemical Engineering 28 (2004) 2053–2068
Computers & Chemical Engineering
www.elsevier.com/locate/compchemeng

Optimal scheduling of multiproduct pipeline systems using a non-discrete MILP formulation

Diego C. Cafaro, Jaime Cerdá*

INTEC (UNL-CONICET), Güemes 3450, 3000 Santa Fe, Argentina

Received 21 October 2003; received in revised form 13 February 2004; accepted 26 March 2004

Available online 6 May 2004

Abstract

Multiproduct pipelines permit to transport large volumes of a wide range of refined petroleum products from major supply sources to distribution centers near market areas. Batches of refined products and grades are pumped back-to-back in the same pipeline, often without any separation device between batches. The sequence and lengths of such pumping runs should be carefully selected in order to meet market demands at the promised dates while satisfying many pipeline operational constraints. This paper deals with the scheduling of a multiproduct pipeline system receiving a number of liquid products from a single refinery source to distribute them among several depots. A novel MILP continuous mathematical formulation that neither uses time discretization nor division of the pipeline into a number of single-product packs is presented. By developing a more rigorous problem representation, the quality of the pipeline schedule is significantly improved. Moreover, a severe reduction in binary variables and CPU time with regards to previous approaches is also achieved. To illustrate the proposed approach, a pair of real-world case studies was solved. Both involve the scheduling of a single pipeline carrying four oil derivatives from an oil refinery to five distribution depots. Higher pumping costs at daily peak periods were also considered. Compared with previous work, better solutions were found at much lower computational time.

Keywords: Multiproduct pipeline; Scheduling; MILP formulation; Continuous-time approach

1. Introduction

Products pipelines are capable of delivering high volumes of refined petroleum products from major supply sources such as refineries or bulk terminals to industry-owned distribution centers near market areas (see Fig. 1). They represent the most reliable and cost-effective way of transporting large quantities of oil derivatives over long distances. Products are then conveyed from depots to the market mostly by tanker trucks. Pipelines carry a wide range of products, including various grades of motor gasoline, jet fuel, diesel fuel, heating oil and domestic kerosene on behalf of major oil companies. A measure of the importance of oil pipelines is the fact that nearly two-thirds of all petroleum products in the US are carried by them. Usually, crude oil and refined petroleum products are not transported in the same pipeline.

Batches of different refined products and grades are pumped back-to-back in the pipeline, usually without any device separating them (see Fig. 1). Mechanical separators called *pigs* are seldom used. At the interface of two adjacent batches, therefore, some mixing occurs. Interface material resulting from pumping batches of different grades of the same product one after the other, such as premium and regular gasoline, is typically mixed with the lower grade batch, thus reducing the batch size of the higher quality product. In turn, the interface between two different products, like gasoline and a distillate, produces a mixture called *transmix*. In this case, the transmix is cut out and sent to separate tanks, and subsequently reprocessed in full-scale refineries or special purpose-built facilities. The actual volume of mixed material generated depends on some physical parameters such as pipeline diameter, flow regime, traveled distance, topography and types of adjacent products.

At the oil refinery, each tank is usually dedicated to holding a single petroleum product to avoid purge and cleaning operations during the routine unloading and loading cycle.

* Corresponding author. Tel.: +54-342-4530028; fax: +54-342-4550944.
E-mail address: jcerda@intec.unl.edu.ar (J. Cerdá).

0098-1354/$ – see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compchemeng.2004.03.010

Nomenclature

Sets

I set of chronologically ordered slugs ($I^{\mathrm{old}} \cup I^{\mathrm{new}}$)
I^{old} subset of old slugs inside the pipeline at the start of the time horizon
I^{new} subset of new slugs to be potentially injected during the time horizon
J set of distribution terminals along the pipeline
P set of refined petroleum products
R set of scheduled production runs at the oil refinery

Parameters

a_r, b_r starting/finishing time of the refinery production run r
B_r size of the refinery production run r
$\mathrm{cid}_{p,j}$ unit inventory cost for product p at depot j
cir_p unit inventory cost for product p at refinery tanks
$\mathrm{cf}_{p,p'}$ unit reprocessing cost of interface material involving products p and p'
$\mathrm{cp}_{p,j}$ unit pumping cost to deliver product p from the refinery to depot j
FPH_k upper limit of the peak-hour period k
$h_{\max}$ horizon length
$\mathrm{ID}^0_{p,j}$ initial inventory of product p at depot j
$\mathrm{IF}_{p,p'}$ volume of interface between slugs containing products p and p'
IPH_k lower limit of the peak-hour period k
IR^0_p initial inventory of product p at refinery tanks
IRmin_p minimum allowed refinery inventory level for product p
IRmax_p maximum allowed refinery inventory level for product p
$l_{\min}/l_{\max}$ minimum/maximum length of a new slug
$\mathrm{qd}_{p,j}$ overall demand of product p to be satisfied by depot j
v_{m} maximum supply rate to the local market
vb pumping rate
W^0_i initial size of the old slug $i \in I^{\mathrm{old}}$
ρ_k unit penalty cost for pipeline operation during the peak-hour period k
σ_j volumetric coordinate of depot j from the origin terminal
$\tau_{p,p'}$ changeover time between injections of products p and p'

Variables

$A_{i,p}$ volume of product p injected in the pipeline while pumping run i
C_i/L_i completion time/length of pumping run $i \in I^{\mathrm{new}}$
$D^{(i')}_{i,j}$ volume of slug i transferred from the pipeline to depot j while injecting slug i'
$\mathrm{DV}^{(i')}_{i,p,j}$ volume of product p transferred from slug i to depot j while injecting slug i'
$F^{(i')}_i$ upper coordinate of slug i from the refinery at time $C_{i'}$
$H_{i,k}$ denoting the portion of the new slug i pumped within the time interval k
$\mathrm{ID}^{(i')}_{p,j}$ inventory of product p in depot j at the completion time of pumping run i'
$\mathrm{IRF}^{(i')}_p$ inventory of product p in refinery at the completion time of pumping run i'
$\mathrm{IRS}^{(i')}_p$ inventory of product p in refinery at the starting time of pumping run i'
$\mathrm{ql}_{i,r}$ volume of production run $r \in R$ available at time $(C_i - L_i)$
$\mathrm{qm}^{(i)}_{p,j}$ amount of product p transferred to depot j during the time interval (C_{i-1}, C_i)
$\mathrm{qu}_{i,r}$ volume of production run $r \in R$ available at time C_i
Q_i original volume of the new slug i
$u_{i,k}$ denoting that the injection of slug i starts not earlier than the time interval k
$v_{i,k}$ denoting that the injection of slug i ends before the time interval k is closed
$W^{(i')}_i$ volume of slug i at time $C_{i'}$
$\mathrm{WIF}_{i,p,p'}$ interface volume between slugs i and $(i-1)$ if they contain products p and p'

$x_{i,j}^{(i')}$	denoting that a portion of slug i can be transferred to depot j while injecting slug i'
$y_{i,p}$	denoting that product p is contained in slug i whenever $y_{i,p} = 1$
$\mathrm{zl}_{i,r}$	denoting that injection i begins after the refinery production run r has ended
$\mathrm{zu}_{i,r}$	denoting that injection i ends after the refinery production run r has started

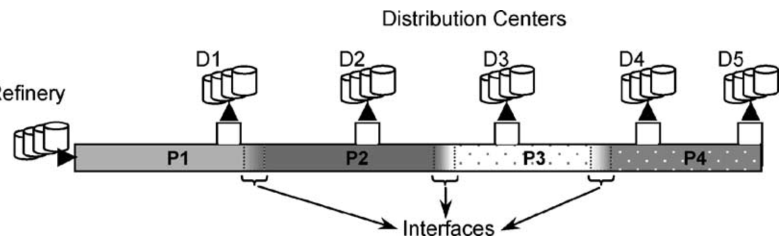

Fig. 1. A single multiproduct pipeline system.

Moreover, different oil derivatives are delivered into distinct containers at nominated depots. Because of liquid incompressibility, such transfers of material from the pipeline to depots necessarily occur simultaneously with the injection of new batches of products into the pipeline. In addition to tanks at origin and destination terminals, working or breakout tanks are usually available over the pipeline route. They are regularly used to interrupt the flow of pipeline material in transit because of pipeline branching, variation in pipeline size or capacity, change in the pipeline operation mode, etc.

Petroleum products pipelines differ themselves by the way they are operated. They can work on either a batch or fungible basis. In batch operations, a specific volume of refined petroleum product is accepted for shipment to a particular destination and the identity of the material shipped is maintained throughout the pipeline. Then, the same material that was accepted for shipment at the origin is delivered to the nominated destination. In fungible operations, the pipeline operator does not deliver exactly the same batch of material received at the origin for shipment. Instead, the carrier may supply material different from the original one but having the same product specifications at the destination. This is possible only if the pipeline carries generic products. Fungible operation has several advantages: (i) tend to minimize the generation of interface material; (ii) permit split-stream operations; and (iii) allow a more efficient utilization of storage tanks. In a split-stream operation, material originating at site 1 and destined to terminals 2 and 3 can be shipped together. Part of the material can continue on to terminal 3 while delivery is still underway at terminal 2. In a batch mode, a delivery operation to terminal 2 means that the batch no longer exists beyond terminal 2. In fungible operations, large storage tanks are used to accumulate or deliver multiple shipments of identical refined products. In batch mode, only one shipment of material is typically held in each tank and, consequently, storage tanks tend to be smaller and more numerous.

Scheduling product batches in pipelines is a complex task with many constraints. Producers' production schedules and market demands together with operational constraints forbidding some products to be pumped one after another are all to be considered. In addition, the scheduling task should also account for actual inventories available in storage tanks at origin and distribution terminals as well as product batches already in pipeline transit to the nominated destinations. Pipeline scheduling aims to: (1) minimize the cost of pipeline operations and keep the pipeline running as close as possible to maximum capacity; (2) enhance shipper information about the status of product movements; and (3) take advantage of time-varying energy costs for pump power. Pipeline operators require sophisticated tools to properly scheduling batch injections in the pipeline and batch removals from the pipeline to depots as well as managing tank loading/unloading operations at origin, intermediate and destination terminals. In addition, batch positions in the pipeline must be tracked with time to provide information about the status of the product movements to shippers. Often, the pipeline operator can help shippers to optimize their production schedules at refineries and petrochemical plants.

Any request from an oil company for transportation service on the pipeline during the next monthly period is a transaction called *nomination*. A single nomination is a unique combination of shipper, product, volume, origin and destination. Nominations initiate the scheduling process. The normal procedure followed at pipeline operator companies to develop the pipeline schedule is next described. A cyclic scheduling process involving three stages is usually performed: the batch plan, the slug plan and the products pipeline schedule. Generally, pipeline operators use a recurring monthly schedule involving cycles of 6–10-day length. The batch plan for each cycle is found by dividing each nomination evenly over the number of cycles per month. In other words, a nomination will usually give rise to as many batches of equal size as the number of monthly cycles. After completing the batch plan, the scheduler is free to merge (or even split) batches provided they contain similar products. The slug plan starts as a one-to-one relationship between batches and slugs. A slug identifies a continuous stream of a single homogeneous product within the pipeline. The major attributes of a slug are both the product it contains and its volume at the time it enters the pipeline. The scheduler sequences the slugs and can subsequently merge and split them or insert buffer batches ("plugs") to develop the slug plan. At any time, one or more batches or even one or more partial batches may be contained in a slug as it moves along the pipeline. The slug plan establishes the sequence and sizes of slugs, but not their entry times and pumping rates. Finally, the pipeline schedule is generated for one or several cycles, depending on the cycle duration. The basic information for the scheduling process includes the slug sequence and the specified pumping rates. Variations in pump rates originate flow rate changes in the pipeline stream.

A few papers on the scheduling of a multiproduct pipeline transporting refined petroleum products from a single origin to multiple destinations have been published. Sasikumar, Prakash, Patil, & Ramani (1997) presented a heuristic search technique that generates good monthly pumping schedules for a single-source multiple-destinations oil pipeline carrying a range of products. The knowledge-based procedure takes into account product availability and requirements while satisfying a wide variety of problem constraints like permissible inventory levels, product sequencing and delivery constraints. In turn, Rejowski and Pinto (2001, 2003) developed a pair of large-size MILP discrete time scheduling models by also dividing the pipeline into a number of single-product packs of equal and unequal sizes, respectively. Key model decisions are those related to the pumping of new products in the pipeline, and to loading and unloading operations at refinery and depot tanks. The approach is able to satisfy many operational constraints such as mass balances, distribution constraints, product demands and sequencing constraints. Neiro and Pinto (2004) proposed a general framework for modeling petroleum supply chains comprising multiple pipelines. Nodes of the chain stand for crude oil suppliers, distribution centers and oil refineries interconnected by intermediate and final product streams. Distribution through pipelines is defined from petroleum terminals to refineries and from refineries to either intermediate terminals or distribution centers. However, the approach assumed that different products are never mixed when transported in pipelines and, in addition, used simple equations to model the pipeline operation. Consequently, the inventory level tracking at refinery and depot storage tanks becomes rather poor to truly guarantee the feasibility of the proposed operation policy. In this way, a large-scale MINLP multiperiod model was derived and applied to a real-world case study under different scenarios. Recently, Magatão, Arruda and Neves (2004) developed an optimization framework for scheduling an oil pipeline transporting different types of oil derivatives from a harbor to an inland refinery over a limited time horizon. The approach is based on mixed integer linear programming (MILP) with uniform time discretization, but the computational burden was avoided through a decomposition strategy involving two MILP models, a time computation auxiliary routine and a database.

This work presents a novel MILP continuous-time approach for the scheduling of a single pipeline transporting refined petroleum products from a unique oil refinery (a single origin) to several distribution terminals. Product batches to be delivered are given by providing the product demands to meet at each depot over the scheduling horizon. The approach assumes that the pipeline is operated on a fungible basis and accounts for slug sequencing constraints, forbidden slug sequences, mass balances, pipeline and tank loading and unloading operations, tank permissible levels and feasibility conditions for transferring material from pipeline slugs to depots. Solution of the proposed MILP problem formulation permits to optimally determine not only the set of slug injections and their volumes but also the sequencing and scheduling of such slugs. In addition, the changes in slug volumes and locations along the pipeline over the time horizon can be tracked. If necessary, the model can even insert plugs to avoid undesirable interfaces. By using this approach, therefore, the slug plan and the products pipeline schedule are both found at the same time. The problem objective is to minimize pumping, inventory and transition costs while satisfying all problem constraints. The latter cost accounts for material losses and interface reprocessing costs at depots due to product contamination between consecutive slugs. Moreover, the objective function is able to account for higher pumping costs at daily peak hours. To illustrate the computational performance of the proposed approach, a pair of real-world case studies has been successfully solved.

2. Problem definition

Given:

(a) The refined petroleum products pipeline configuration, including the number of depots, the pipeline segment diameters and the distance between every depot and the oil refinery.
(b) The available tanks at every depot, including the capacity and the product assigned to each one.
(c) The product demands to be satisfied at every distribution terminal at the end of the scheduling horizon.
(d) The sequence of slugs in transit along the pipeline and their actual volumes at the initial time.
(e) The scheduled production runs (product, amount and time interval) to be performed at the refinery during the scheduling horizon.
(f) Initial product inventories available in refinery and depot tanks.
(g) Maximum/minimum allowed inventory levels at refinery and depot tanks.

(h) Maximum values for the slug pump rate, the product supply rate from the pipeline to depots and the product delivery rate from depots to local markets.
(i) The length of the scheduling horizon.

The problem goal is to establish the optimal sequence of new slugs injections in the pipeline, their initial volumes and the product assigned to each one in order to: (1) meet product demands at each depot in a timely fashion; (2) keep inventory levels in refinery and depot tanks within the permissible range all the time; and (3) minimize the sum of all pumping, transition and inventory carrying costs. At the same time, variations in sizes and coordinates of new/old slugs as they move along the pipeline as well as the evolution of inventory levels in refinery and depot tanks are tracked over the time horizon.

3. Mathematical formulation

The proposed mathematical formulation has been defined in terms of three major elements: the set of old and new slugs ($i \in I = I^{\text{new}} \cup I^{\text{old}}$), the set of depots ($j \in J$) and the set of petroleum products ($p \in P$). The model includes two different types of binary variables denoted by $y_{i,p}$ and $x_{i,j}^{(i')}$, respectively. The assignment variable $y_{i,p}$ indicates that the new slug $i \in I^{\text{new}}$ contains the product p whenever $y_{i,p} = 1$. In turn, the decision variable $x_{i,j}^{(i')}$ specifies that material from slug $i \in I$ in transit along the pipeline can be transferred to depot j while injecting the new slug $i' \in I^{\text{new}}$ ($i' > i$). In addition, four important continuous variables standing for different attributes of a slug $i \in I$ denoted by C_i, L_i, $F_i^{(i')}$ and $W_i^{(i')}$, have been incorporated in the mathematical formulation. Continuous variables C_i and L_i represent the completion time and the length of the new pumping run or slug $i \in I^{\text{new}}$. Furthermore, $F_i^{(i')}$ and $W_i^{(i')}$ indicate the pipeline location and the size of the (new/old) slug $i \in I$ at the completion time of the new pumping run $i' \in I^{\text{new}}$. These continuous variables permit to track the slug movement and the slug size change over time.

Fig. 2 depicts a simple example involving a multi-product pipeline conveying oil derivatives $p \in P = \{P1, P2, P3, P4\}$ from a single oil refinery to several depots $j \in J = \{D1, D2, D3, D4, D5\}$. A sequence of four slugs $I = \{S4–S3–S2–S1\}$ is inside the pipeline containing products $\{P1–P3–P4–P2\}$, respectively, at the starting time of the pumping run $i' = \{S5\}$. Values for the model variables ($y_{i,p}$, $W_i^{(i')}$, $F_i^{(i')}$) before and after injecting slug $i' = \{S5\}$ in the pipeline, and for the binary variable $x_{i,j}^{(i')}$ while pumping slug $i' = \{S5\}$ are all shown in Fig. 2. Though some amount of product $P3$ can be delivered from $S3$ to depot $D2$ when injecting $S5$, i.e. $x_{S3,D2}^{(S5)} = 1$, no material transfer is really performed. As a result, there is no change in the size of slug $S3$ and, therefore, $W_{S3}^{(S4)} = W_{S3}^{(S5)} = 200$, since slug $S4$ was injected in the pipeline right before $S5$. A similar situation can also be described for slugs $S2$ and $S4$. However, a portion of slug $S1$ containing product $P2$ has been transferred to depot $D3$ while pumping $S5$. The amount of $P2$ delivered from $S1$ to $D3$ is as large as the volume of the new slug $S5$. Therefore, the size of $S1$ shows a decrease of 60 volumetric units, i.e. $[W_{S1}^{(S4)} - W_{S1}^{(S5)}] = 60$.

3.1. Slug sequencing constraints

The injection of a new slug $i \in I^{\text{new}}$ in the pipeline should never start before completing the pumping of the previous

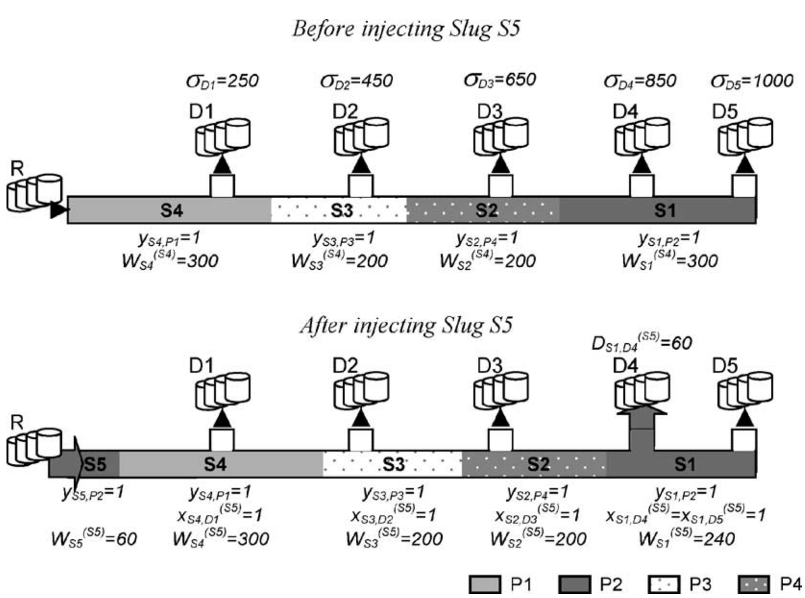

Fig. 2. A simple example illustrating the meaning of major model variables.

one and the subsequent changeover operation:

$$C_i - L_i \geq C_{i-1} + \tau_{p,p'}(y_{i-1,p'} + y_{i,p} - 1),\quad \forall i \in I^{\text{new}};\ p, p' \in P \tag{1}$$

$$L_i \leq C_i \leq h_{\max},\quad \forall i \in I^{\text{new}} \tag{2}$$

where C_i is the completion time for the pumping run of slug $i \in I^{\text{new}}$, L_i the related pumping run duration and $h_{\max}$ the specified length of the scheduling horizon. The changeover time $\tau_{p,p'}$ was ignored in previous approaches. Constraint (1) becomes active whenever the slugs $(i-1)$ and i contain products p' and p, respectively. From the chronological viewpoint, slug $(i-1) \in I$ is a direct predecessor of slug $i \in I^{\text{new}}$, since it has been pumped immediately before i. In the pipeline, therefore, slug $(i-1)$ is located farther from the origin than slug i.

3.2. Relationship between the volume and the length of a new slug

The volume of a new slug injected in the pipeline $i \in I^{\text{new}}$, Q_i, should belong to the feasible range:

$$\text{vb}_{\min} L_i \leq Q_i \leq \text{vb}_{\max} L_i,\quad \forall i \in I^{\text{new}} \tag{3}$$

where $\text{vb}_{\min}$ and $\text{vb}_{\max}$ denote the minimum and maximum pump rates. Moreover, the length of slug $i \in I^{\text{new}}$ must never be higher than the specified maximum length $l_{\max}$ neither lower than the minimum one $l_{\min}$, whenever it is really injected in the pipeline ($\sum_{p\in P} y_{i,p} = 1$):

$$\left(\sum_{p\in P} y_{i,p}\right) l_{\min} \leq L_i \leq \left(\sum_{p\in P} y_{i,p}\right) l_{\max},\quad \forall i \in I^{\text{new}} \tag{4}$$

In order to get better results, fictitious slugs $i \in I^{\text{new}}$ featuring $L_i = 0$ at the optimum should be left at the end of the slug sequence. Therefore, the following constraint should be added to the problem formulation:

$$\sum_{p\in P} y_{i,p} \leq \sum_{p\in P} y_{i-1,p},\quad \forall i \in I^{\text{new}} \tag{5}$$

3.3. Segments of new slugs pumped in the pipeline during daily peak periods

Let us define the binary variable $u_{i,k}$ denoting that the pumping of a new slug $i \in I^{\text{new}}$ starts not earlier than the lower limit of the high-energy cost interval k (IPH_k), whenever $u_{i,k} = 1$. In turn, the variable $v_{i,k}$ is required to indicate that the pumping of the new slug i is completed before the high-energy cost interval k has closed (FPH_k).

$$C_i - L_i \leq \text{IPH}_k + u_{i,k}\text{MT} \tag{6a}$$

$$C_i \geq \text{FPH}_k(1 - v_{i,k}) \tag{6b}$$

$$C_i - L_i \geq \text{IPH}_k u_{i,k} \tag{6c}$$

$$C_i \leq \text{FPH}_k + (1 - v_{i,k})\text{MT},\quad \forall i \in I^{\text{new}},\ k \in K \tag{6d}$$

In order to establish the portion of a new slug $i \in I^{\text{new}}$ pumped into the pipeline within a high-energy cost interval k, $H_{i,k}$, four cases described through Eqs. (7a)–(7d) should be considered:

(a) The start time $(C_i - L_i)$ and the completion time (C_i) for the pumping of slug i, i.e. the pumping run i, both belong to interval k ($u_{i,k} = 1$, $v_{i,k} = 1$). Then, the constraints (7a) is enforced and the pumping run i is completely inside the high-energy cost interval k. Since pipeline energy costs are to be minimized, then $H_{i,k} = L_i$ at the optimum.

(b) In this case, just the end time of run i is inside the interval $k(u_{i,k} = 0, v_{i,k} = 1)$. Case (b) has two different instances depending on the value of C_i. If C_i is smaller than IPH_k, then the pumping of slug i entirely occurs outside the interval k and $H_{i,k}$ is equal to zero. As a result, constraint (7b) becomes redundant. Otherwise, the pumping run i is partially performed inside the interval k and, therefore, $H_{i,k} = C_i - \text{IPH}_k$ at the optimum.

(c) Only the start time of run i is inside the interval $k(u_{i,k} = 1, v_{i,k} = 0)$. Two instances can arise depending on the time at which the pumping of slug i begins. If $(C_i - L_i)$ is higher than FPH_k, then the pumping run i is completely outside the interval k and $H_{i,k} = 0$. As a result, constraint (7c) turns to be redundant. Otherwise, $H_{i,k} = \text{FPH}_k - (C_i - L_i)$ at the optimum.

(d) Start and completion times of run i are both outside the interval k, one at each side ($u_{i,k} = 0$, $v_{i,k} = 0$) and constraint (7d) will hold. Then, slug i is partially pumped inside the interval k and $H_{i,k} = \text{FPH}_k - \text{IPH}_k$ at the optimum:

$$H_{i,k} \geq L_i + (u_{i,k} + v_{i,k} - 2)\text{MT} \tag{7a}$$

$$H_{i,k} \geq C_i - \text{IPH}_k - (1 - v_{i,k})\text{MT} - u_{i,k}\text{MT} \tag{7b}$$

$$H_{i,k} \geq \text{FPH}_k - (C_i - L_i) - (1 - u_{i,k})\text{MT} - v_{i,k}\text{MT} \tag{7c}$$

$$H_{i,k} \geq \text{FPH}_k - \text{IPH}_k - u_{i,k}\text{MT} - v_{i,k}\text{MT},\quad \forall i \in I^{\text{new}},\ k \in K \tag{7d}$$

MT is a relatively large number. It is recommended to choose $\text{MT} = (1.0\text{–}1.1)h_{\max}$.

3.4. Interface material between consecutive slugs

By convention, slug $(i-1) \in I$ has been pumped into the pipeline just before slug $i \in I$. Then, the volume of the interface between such adjacent slugs will never be lower than $\text{IF}_{p,p'}$ (i.e. the size of the interface between products p and p') in case slug $(i-1)$ and slug i contain products p' and p, respectively. Otherwise, the constraint will become redundant. Likewise previous approaches, the value of $\mathrm{IF}_{p,p'}$ for any ordered pair of products (p, p') is assumed to be known and independent of the pump rate:

$$\mathrm{WIF}_{i,p,p'} \geq \mathrm{IF}_{p,p'}(y_{i-1,p'} + y_{i,p} - 1), \quad \forall i \in I, i > 1, \; p, p' \in P \tag{8}$$

If the amount of interface material rather than the interface cost is to be minimized, then product subindices can be ignored and the variable $\mathrm{WIF}_{i,p,p'}$ can be substituted by WIF_i.

3.5. Forbidden sequences

Because of product contamination, some sequences of products in the pipeline are forbidden. Let (p, p') represent a forbidden sequence of products. Then, the following constraint is added to the problem formulation:

$$y_{i-1,p} + y_{i,p'} \leq 1, \quad \forall i \in I^{\mathrm{new}} \tag{9}$$

It will be supposed that the interface material is never transferred to depots. On the contrary, the interface will remain in the pipeline until reaching the final depot where it is withdrawn and reprocessed (Rejowski & Pinto, 2001). Otherwise, a new interface will be permanently generated, thus leading to higher product losses.

3.6. Upper and lower pipeline coordinates for slug $I \in I$ at time $C_{i'}$

Let $F_i^{(i')}$ be the upper volumetric coordinate of slug $i \in I$ in pipeline transit at the completion time $C_{i'}$ of the later pumping run $i' \in I^{\mathrm{new}}$, i.e. the pipeline volume delimited by both the origin and the interface between slugs i and $(i-1)$, at time $C_{i'}$. The value of $F_i^{(i')}$ is equal to the sum of the upper coordinate for the next slug $(i+1) \in I$, $F_{i+1}^{(i')}$, and the volume of slug i, $W_i^{(i')}$, both at time $C_{i'}$:

$$F_{i+1}{}^{(i')} + W_i{}^{(i')} = F_i{}^{(i')}, \quad \forall i \in I, \; \forall i' \in I^{\mathrm{new}}, \; i' > i \tag{10}$$

The lower coordinate for slug $i \in I$ at time $C_{i'}$ is $F_{i+1}^{(i')}$.

3.7. Volume transferred from slug $I \in I^{new}$ to depots, while pumping slug i itself

Let $W_i^{(i)}$ be the volume of slug $i \in I^{\mathrm{new}}$ in the pipeline at time C_i. If Q_i is the total amount of product injected in the pipeline while pumping slug i, then $(Q_i - W_i^{(i)})$ is the volume of material transferred from slug i to depots during the interval $(C_i - L_i, C_i)$. Obviously, the lower coordinate of slug i at time C_i is equal to zero:

$$Q_i = W_i{}^{(i)} + \sum_{j \in J} D_{i,j}{}^{(i)}, \qquad F_i{}^{(i)} = W_i{}^{(i)}, \quad \forall i \in I^{\mathrm{new}} \tag{11}$$

3.8. Volume transferred from slug $i \in I$ to depots while injecting a later slug $i' \in I^{new}$

The volume of slug $i \in I$ in pipeline transit at time $C_{i'}$ is given by the difference between the amount of slug i available at time $C_{i'-1}$ and the total volume transferred to depots along the pipeline while injecting the slug $i' \in I^{\mathrm{new}}$ $(i' > i)$:

$$W_i{}^{(i')} = W_i{}^{(i'-1)} - \sum_{j \in J} D_{i,j}{}^{(i')}, \quad \forall i \in I, \; \forall i' \in I^{\mathrm{new}}, \; i' > i \tag{12}$$

3.9. Feasibility conditions for transferring material from slugs in pipeline transit to depots

The transfer of material from slug $i \in I$ conveying product p to depot $j \in J_p$ requiring p is feasible only if the pipeline outlet to depot j is reachable from slug i. Fulfillment of such feasibility conditions while injecting a later slug $i' \in I^{\mathrm{new}}$ requires that: (a) the lower pipeline coordinate of slug i at time $C_{i'-1}$ must be less than the depot coordinate σ_j; and (b) the upper pipeline coordinate of slug i at time $C_{i'}$, decreased by the volume of the interface material $(\sum_p \sum_{p'} \mathrm{WIF}_{i,p,p'})$, should never be lower than σ_j. Let the binary variable $x_{i,j}^{(i')}$ denote that depot j is reachable from slug i while pumping slug i' whenever $x_{i,j}^{(i')} = 1$. By definition:

$$D_{i,j}{}^{(i')} \leq D_{\max} x_{i,j}{}^{(i')}, \quad \forall i \in I, \; i' \in I^{\mathrm{new}}, \; i' \geq i, \; \forall j \in J \tag{13}$$

where $D_{\max}$ is an upper bound on the amount of material that can be transferred from slug i to depot j. To guarantee that depot j is reachable from slug i, the upper coordinate of the saleable slug i, excluding the interface material, must not be lower than the depot coordinate σ_j:

$$F_i{}^{(i')} - \sum_{p \in P} \sum_{p' \in P, \, p' \neq p} \mathrm{WIF}_{i,p,p'} \geq \sigma_j x_{i,j}^{(i')}, \quad \forall i \in I, \; \forall i' \in I^{\mathrm{new}}, \; i' \geq i, \; \forall j \in J \tag{14}$$

Moreover, an upper bound on the volume of material transferred from slug $i \in I$ containing product p to depot $j \in J_p$ demanding product p is given by:

$$D_{i,j}^{(i')} \leq \sigma_j - F_{i+1}^{(i'-1)} - \left(\sum_{k=1}^{j-1} D_{i,k}^{(i')} \right) + (1 - x_{i,j}^{(i')})M, \quad \forall i \in I, \; \forall i' \in I^{\mathrm{new}}, \; i' > i, \; \forall j \in J \tag{15}$$

Eq. (15) can also be written as follows:

$$F_{i+1}^{(i'-1)} \leq \sigma_j - \left(\sum_{k=1}^{j} D_{i,k}^{(i')} \right) + (1 - x_{i,j}^{(i')})M, \quad \forall i \in I, \; \forall i' \in I^{\mathrm{new}}, \; i' > i, \; \forall j \in J \tag{15'}$$

Since, $D_{i,j}^{(i')} \geq 0$ $(\forall i, k, i')$, then $F_{i+1}^{(i'-1)} \leq \sigma_j + (1 - x_{i,j}^{(i')})M$.Therefore, the condition (a) specifying that the lower pipeline coordinate of slug i at time $C_{i'-1}$ must be less than the depot coordinate σ_j is also enforced by constraint (15).

3.10. A bound on the volume supplied by slug $i \in I$ to depots $j \in J$ while injecting slug $i' \in I^{new}$

The total volume transferred from slug $i \in I$ to depots $j \in J$ while pumping the new slug $i' \in I^{\text{new}}$ during the time interval $(C_{I'} - L_{i'}, C_{i'})$, must never exceed the saleable content of slug i at time $C_{i'-1}$:

$$\sum_{j\in J} D_{i,j}^{(i')} \leq W_i^{(i'-1)} - \sum_{p\in P} \sum_{p'\in P,\, p'\neq p} \text{WIF}_{i,p,p'}, \quad \forall i \in I,\ \forall i' \in I^{\text{new}},\ i' > i \tag{16}$$

Constraint (16) indicates that the material transferred from slug i to depots while injecting slug $i' \in I^{\text{new}}$ should never be greater than the saleable volume of slug i at the start time of the pumping run i' given by $(C_{i'} - L_{i'})$. A model improvement with regards to previous approaches is the fact that just saleable material can be transferred from the pipeline to depots.

3.11. Overall balance around the pipeline during the pumping of the new slug $i' \in I^{new}$

Because of the liquid incompressibility condition, the overall volume transferred from slugs in transit along the pipeline to depots $j \in J$ while injecting the new slug $i' \in I^{\text{new}}$ must be equal to $Q_{i'}$, i.e. the volume of slug i' injected in the pipeline:

$$\sum_{i\in I,\, i\leq i'} \sum_{j\in J} D_{i,j}^{(i')} = Q_{i'}, \quad \forall i' \in I^{\text{new}} \tag{17}$$

3.12. Speed-up constraints

The following set of redundant constraints has been incorporated in the model to speed up the branch-and-bound solution algorithm. They account for the fact that every slug i in transit moves along the pipeline when a new slug $i' \in I^{\text{new}}$ is injected. As a result, the lower and upper slug coordinates both increase with time. Moreover, the volume of a slug i in pipeline transit is always a lower bound on the value of its upper volumetric coordinate $F_i^{(i')}$:

$$F_i^{(i')} \geq F_i^{(i'-1)}, \quad \forall i \in I,\ \forall i' \in I^{\text{new}},\ i' > i \tag{18}$$

$$F_i^{(i')} - W_i^{(i')} \geq F_i^{(i'-1)} - W_i^{(i'-1)}, \quad \forall i \in I,\ \forall i' \in I^{\text{new}},\ i' > i \tag{19}$$

$$F_i^{(i')} \geq W_i^{(i')}, \quad \forall i \in I,\ \forall i' \in I^{\text{new}},\ i' \geq i \tag{20}$$

Speed-up constraints (18)–(20) often reduce computational requirements by half.

3.13. Product allocation constraint

A slug flowing inside the pipeline just contains a single refinery product. Then,

$$\sum_{p\in P} y_{i,p} \leq 1, \quad \forall i \in I^{\text{new}} \tag{21}$$

3.14. Fulfillment of market demands

The amount of product $p \in P$ delivered from depot $j \in J_p$ to a local market demanding p, during the time interval $(C_{i'-1}, C_{i'})$, i.e. $\text{qm}_{p,j}^{(i')}$, must be supplied at the specified flow rate v_{m}. Moreover, the total volume of product p transferred from depot $j \in J_p$ to the local market throughout the time horizon should be high enough to meet its overall demand $\text{qd}_{p,j}$:

$$\text{qm}_{p,j}^{(i)} \leq (C_i - C_{i-1})v_{\text{m}}, \quad \forall p \in P,\ \forall j \in J_p,\ \forall i \in I^{\text{new}} \tag{22}$$

$$\sum_{i\in I^{\text{new}}} \text{qm}_{p,j}^{(i)} = \text{qd}_{p,j}, \quad \forall p \in P,\ j \in J_p \tag{23}$$

3.15. Control of inventories in refinery tanks

Let $R_p \subset R$ be the set of refinery production campaigns involving product p to be run during the current scheduling horizon. Assume that B_r is the size of production run $r \in R_p$, vp_r is its rate of production and (a_r, b_r) denotes the loading time interval of run r in the assigned refinery tank. Let us also define the binary variable $\text{zu}_{r,i}$ to indicate that the injection of a new slug $i \in I^{\text{new}}$ has been completed before $(\text{zu}_{i,r} = 0)$ or after $(\text{zu}_{i,r} = 1)$ beginning the loading of production run $r \in R_p$ in the dedicated refinery tank. It is also introduced the binary variable $\text{zl}_{i,r}$ to denote that the injection of the new slug $i \in I^{\text{new}}$ has begun before $(\text{zl}_{i,r} = 0)$ or after $(\text{zl}_{i,r} = 1)$ completing the loading of run $r \in R_p$ in the refinery tank.

3.15.1. Definition of binary variables $\text{zu}_{i,r}$ and $\text{zl}_{i,r}$

$$a_r\text{zu}_{i,r} \leq C_i \leq a_r + h_{\max}\text{zu}_{i,r}, \quad \forall i \in I^{\text{new}},\ r \in R \tag{24}$$

$$b_r\text{zl}_{i,r} \leq C_i - L_i \leq b_r + h_{\max}\text{zl}_{i,r}, \quad \forall i \in I^{\text{new}},\ r \in R \tag{25}$$

3.15.2. Volume of production run $r \in R_p$ already loaded in the assigned refinery tank at time C_i

Let $\text{qu}_{i,r}$ denote the volume of order $r \in R_p$ already discharged in the refinery tank at time C_i, i.e. the time at which the injection of a new slug $i \in I^{\text{new}}$ has just been completed. Three cases can be considered:

(i) $C_i \geq b_r$, then $\mathrm{zu}_{i,r} = 1$ and the full run r has been loaded in the assigned tank;
(ii) $C_i \leq a_r$, then $\mathrm{zu}_{i,r} = 0$ and the production run r has not yet begun;
(iii) $a_r \leq C_i \leq b_r$, then $\mathrm{zu}_{i,r} = 1$ and a portion of the production run r has already been loaded over the interval (a_r, C_i).

Therefore,

$$\mathrm{qu}_{i,r} \leq B_r \mathrm{zu}_{i,r} \tag{26}$$

$$\mathrm{qu}_{i,r} \leq \mathrm{vp}_r(C_i - a_r \mathrm{zu}_{i,r}), \quad \forall i \in I^{\mathrm{new}},\ r \in R \tag{27}$$

3.15.3. Volume of production run $r \in R_p$ already loaded in the assigned refinery tank at time $(C_i - L_i)$

Let $\mathrm{ql}_{i,r}$ denote the total volume of production run $r \in R_p$ already loaded in the refinery tank at the starting time $(C_i - L_i)$ of the new pumping run $i \in I^{\mathrm{new}}$. Two cases can be considered:

(i) $(C_i - L_i) \geq b_r$, then $\mathrm{zl}_{i,r} = 1$ and the whole production run is already loaded in the refinery tank at time $(C_i - L_i)$;
(ii) $(C_i - L_i) < b_r$, then $\mathrm{zl}_{i,r} = 0$ and a portion of the production run r is still to be loaded in the assigned tank over the interval $(C_i - L_i, b_r)$.

Therefore,

$$\mathrm{ql}_{i,r} \geq B_r \mathrm{zl}_{i,r} \tag{28}$$

$$\mathrm{ql}_{i,r} \geq \mathrm{vp}_r[(C_i - L_i) - a_r] - M\mathrm{zl}_{i,r}, \quad \forall i \in I^{\mathrm{new}},\ r \in R \tag{29}$$

3.15.4. Amount of product p injected in the pipeline while pumping the slug $i \in I^{new}$

No material is withdrawn from the refinery tank containing product p while pumping slug $i \in I^{\mathrm{new}}$ if such a slug has not been assigned to product $p(y_{i,p} = 0)$. Otherwise, the volume of product p injected in the pipeline from the refinery tank will be equal to Q_i, i.e. the original volume of the new slug I:

$$A_{i,p} \leq M y_{i,p}, \quad \forall i \in I^{\mathrm{new}},\ p \in P \tag{30}$$

$$\sum_{p \in P} A_{i,p} = Q_i, \quad \forall i \in I^{\mathrm{new}} \tag{31}$$

3.15.5. Maximum and minimum allowed inventories in refinery tanks

It will be assumed that the pipeline pump rate is greater than or equal to the processing rate of any production run r. Based on this assumption, the worst condition for running off product p in the assigned tank occurs at the completion time of a pumping run $i \in I^{\mathrm{new}}$ containing product $p \in P$. In turn, the worst condition for overloading the refinery tank devoted to product p arises at the start of pumping a new slug containing p. For simplicity, the inventory level is forced to be greater than the minimum level IRmin at the end of the new pumping run $i \in I^{\mathrm{new}}$ ($\mathrm{IRF}_p^{(i)}$). In addition, the inventory level is required to be lower than the maximum level IRmax at the starting time of each new pumping run ($\mathrm{IRS}_p^{(i)}$). Therefore, the following constraints must be included in the problem formulation to guarantee that the pth product inventory level in the assigned refinery tank stays always within the specified feasible range (IRmin_p, IRmax_p).

$$\mathrm{IRF}_p^{(i)} = \mathrm{IR}_p^0 + \sum_{r \in R_p} \mathrm{qu}_{i,r} - \sum_{i' \in I^{\mathrm{new}},\, i' \leq i} A_{i',p} \geq \mathrm{IRmin}_p, \quad \forall i \in I^{\mathrm{new}},\ p \in P \tag{32}$$

$$\mathrm{IRS}_p^{(i)} = IR_p^0 + \sum_{r \in R_p} \mathrm{ql}_{i,r} - \sum_{i' \in I^{\mathrm{new}},\, i' < i} A_{i',p} \leq \mathrm{IRmax}_p, \quad \forall i \in I^{\mathrm{new}},\ p \in P \tag{33}$$

3.16. Control of inventory levels in depot tanks

3.16.1. Amount of product p transferred from slug $i \in I$ to depot $j \in J_s$ while injecting slug $i' \in I^{new}$

Run $i \in I$ will be conveying product p only if $y_{i,p} = 1$. Let $\mathrm{DV}_{i,p,j}^{(i')}$ be the amount of product p supplied by slug i to depot $j \in J_p$ during the time interval $(C_{i'} - L_{i'}, C_{i'})$. Therefore, $\mathrm{DV}_{i,p,j}^{(i')}$ will be equal to zero whenever $y_{i,p} = 0$. If instead $y_{i,p} = 1$, then $\mathrm{DV}_{i,p,j}^{(i')} = D_{i,j}^{(i')}$.

(i) For new slugs $i \in I^{\mathrm{new}}$:

$$\mathrm{DV}_{i,p,j}^{(i')} \leq D_{\max} y_{i,p}, \forall i \in I^{\mathrm{new}},\ p \in P,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{34}$$

$$\sum_{p \in P} \mathrm{DV}_{i,p,j}^{(i')} = D_{i,j}^{(i')}, \quad \forall i \in I^{\mathrm{new}},\ j \in J,\ i' \in I^{\mathrm{new}} \tag{35}$$

(ii) For old slugs $i \in I^{\mathrm{old}}$:

$$\mathrm{DV}_{i,p,j}^{(i')} = D_{i,j}^{(i')}, \quad \forall i \in I_p^{\mathrm{old}},\ p \in P,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{36}$$

where I_p^{old} comprises every old run involving product p.

3.16.2. Inventory feasible range

Inventory level of product p in depot $j \in J_p$ at time $C_{i'}$ is adding the one available at time $C_{i'-1}$ to the amount $(\sum_i \mathrm{DV}_{i,p,j}^{(i')})$ supplied by slugs $i \in I$ conveying product p, and simultaneously subtracting deliveries of product p to local markets. The value of $\mathrm{ID}_{p,j}^{(i')}$ should always remain within the feasible range defined by the specified maximum and minimum inventory levels.

$$\mathrm{ID}_{p,j}^{(i')} = \mathrm{ID}_{p,j}^{(i'-1)} + \sum_{i \in I, i \leq i'} \mathrm{DV}_{i,p,j}^{(i')} - \mathrm{qm}_{p,j}^{(i')}, \quad \forall p \in P,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{37}$$

$$\mathrm{IDmin}_{p,j} \leq \mathrm{ID}_{p,j}^{(i')} \leq \mathrm{IDmax}_{p,j}, \quad \forall p \in P,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{38}$$

3.17. Initial conditions

Old slugs $i \in I^{\mathrm{old}}$ already in the pipeline at the start of the scheduling horizon have been chronologically arranged by decreasing F_i^0, where F_i^0 stands for the upper pipeline coordinate of slug $i \in I^{\mathrm{old}}$ at time $t = 0$. In other words, the old slug $(i-1)$ has been injected right before the old slug i but it is located farther from the origin terminal than old slug i. Moreover, the initial volumes of old slugs (WO_i, $i \in I^{\mathrm{old}}$) and the product to which each one was assigned are all problem data. Then,

$$W_i^{(I'-1)} = \mathrm{Wo}_i, \quad \forall i \in I^{\mathrm{old}},\ i' = \mathrm{first}(I^{\mathrm{new}}) \tag{39}$$

3.18. Problem objective function

The problem goal is to minimize the total pipeline operating cost including the pumping cost, at daily normal and peak hours, the cost of reprocessing the interface material between consecutive slugs and the cost of holding product inventory in refinery and depot tanks. The estimate of the inventory cost is based on an average value of each product inventory over the time horizon.

$$\begin{aligned}\min z = {} & \sum_{p\in P}\sum_{j\in J}\left(\mathrm{cp}_{p,j}\sum_{i\in I}\sum_{i'\in I^{\mathrm{new}}}\mathrm{DV}_{p,i,j}^{(i')}\right) \\ & + \sum_{k\in K}\sum_{i\in I^{\mathrm{new}}}\rho_k H_{i,k} \\ & + \sum_{p'\in P,\ p'\neq p}\ \sum_{i\in I,\ i>1}\mathrm{cf}_{p,p'}\mathrm{WIF}_{i,p,p'} \\ & + \frac{1}{\mathrm{card}(I^{\mathrm{new}})}\sum_{p\in P}\left[\sum_{j\in J_p}\mathrm{cid}_{p,j}\left(\sum_{i'\in I^{\mathrm{new}}}\mathrm{ID}_{p,j}{}^{(i')}\right)\right. \\ & \left. + \mathrm{cir}_p\left(\sum_{i'\in I^{\mathrm{new}}}\mathrm{IRS}_p{}^{(i')}\right)\right]\end{aligned} \tag{40}$$

where $\mathrm{cp}_{p,j}$ stands for the unit pumping cost for delivering product p from the oil refinery to destination j. The parameter $\mathrm{cf}_{p,p'}$ is the cost for reprocessing a unit amount of interface $p - p'$. In turn, ρ_k is the unit-time penalty cost to be paid for operating the pipeline during the peak-hour interval k. Since the pipeline remains idle during high-energy cost time intervals, the energy penalty cost term is usually zero at the optimum. The last two terms account for inventory costs at the refinery and distribution centers based on an estimation of the average inventory for each product. An average product inventory is obtained by dividing the sum of the product inventory at the beginning of each pumping run by an estimated number of new slugs given by card(I^{new}).

4. Results and discussion

The proposed MILP approach will be illustrated by solving a pair of large-scale products pipeline scheduling problems first introduced by Rejowski and Pinto (2003). Both examples involve a single pipeline transporting four refined petroleum products ($P1$: gasoline; $P2$: diesel; $P3$: LPG; $P4$: jet fuel) to five distribution terminals ($D1$–$D5$) located along the pipeline. The cardinality of the set I^{new}, i.e. the number of new pumping runs, is initially assumed to be equal to the number of oil derivatives transported by the pipeline. After solving the MILP formulation, the cardinality of I^{new} is increased by one and the model is solved again. The procedure is repeated until no further decrease in the pipeline operational costs at the optimum is achieved. In both examples, the optimal solution was found at the first major iteration. The MILP mathematical formulation was solved on a Pentium IV 2 GHz processor with CPLEX using ILOG OPL Studio 3.6 (ILOG, 2003). A relative MIP gap tolerance equal to 1×10^{-4} and an integrity tolerance of 1×10^{-5} were adopted in both examples.

4.1. Example 1

Data for Example 1 are given in Tables 1–3. Table 1 includes the depot locations with regards to the pipeline origin, in hundred cubic meters, the initial product inventories and the allowable inventory ranges at refinery and depot storage tanks. In addition, Table 1 provides the product demands to be satisfied at each distribution terminal and the pumping unit costs at daily normal hours for each product–depot combination. In this regard, the time horizon featuring a total length of 75 h involves a pair of time intervals (15–25 and 40–50 h) both presenting much higher unit pumping costs. Usually, the pipeline stream is stopped during high-energy cost intervals, unless unsatisfied product demands force to keep the slug sequence moving along the pipeline. The interface material cost and volume for each ordered pair of products as well as product inventory costs at refinery and depots are all given in Table 2. Forbidden product sequences are denoted with an "X" in Table 2. In turn, Table 3 includes the information about the scheduled production runs in the refinery during the pipeline time horizon.

There is initially a sequence of five old slugs ($S5$–$S4$–$S3$–$S2$–$S1$) inside the pipeline containing products ($P1$–$P2$–$P1$–$P2$–$P1$) arranged in this order, and featuring the following volumes (75/175/125/25/75), in hundred cubic meters. Slug $S1$ occupies the farthest position from the refinery plant. The optimal solution that is shown in Fig. 3 was found in 34.98 s (see Table 4). This represents a three-order-of-magnitude time saving with regards to the approach of Rejowski and Pinto (2003) who reported a time requirement of 10,000 s to find the pipeline schedule shown in Fig. 4. At the optimum, pipeline operation costs amount to 32,746.83 compared with 34,178.625 reported by Rejowski and Pinto (2003), both expressed in hundred

Table 1
Depot locations, product inventories and pumping costs for Example 1

Product	Level	Refinery	Depots					Product		Depots				
			*D*1	*D*2	*D*3	*D*4	*D*5			*D*1	*D*2	*D*3	*D*4	*D*5
*P*1	Minimum	270	90	90	90	90	90	*P*1	Demand	100	110	120	120	150
	Maximum	1200	400	400	400	400	400		Pumping cost (US$/m^3)	3.5	4.5	5.5	6.0	6.9
	Initial	500	190	230	200	240	190							
*P*2	Minimum	270	90	90	90	90	90	*P*2	Demand	70	90	100	80	100
	Maximum	1200	400	400	400	400	400		Pumping cost (US$/m^3)	3.6	4.6	5.6	6.2	7.3
	Initial	520	180	210	180	180	180							
*P*3	Minimum	50	10	10	10	10	10	*P*3	Demand	60	40	40	0	20
	Maximum	350	70	70	70	70	70		Pumping cost (US$/m^3)	4.8	5.7	6.8	7.9	8.9
	Initial	210	50	65	60	60	60							
*P*4	Minimum	270	90	90	90	90	90	*P*4	Demand	60	50	50	50	50
	Maximum	1200	400	400	400	400	400		Pumping cost (US$/m^3)	3.7	4.7	5.7	6.1	7.0
	Initial	515	120	140	190	190	170							
Location from refinery ($\times 10^2$ m^3)			100	200	300	400	475							

Table 2
Inventory costs and interface material volumes and costs for Example 1

	Interface cost ($\times 10^2$ US$)/volume ($\times 10^2$ m^3)				Refinery	Inventory costs (US$/(m^3 h))				
	*P*1	*P*2	*P*3	*P*4		*D*1	*D*2	*D*3	*D*4	*D*5
*P*1	0/0	30/0.30	37/0.37	35/0.35	0.070	0.100	0.100	0.100	0.100	0.100
*P*2	30/0.30	0/0	X	38/0.38	0.080	0.155	0.155	0.155	0.155	0.155
*P*3	37/0.37	X	0/0	X	0.095	0.200	0.200	0.200	0.200	0.200
*P*4	35/0.35	38/0.38	X	0/0	0.090	0.170	0.170	0.170	0.170	0.170

Table 3
Scheduled production runs at the oil refinery (Example 1)

Production run	Product	Volume ($\times 10^2$ m^3)	Production rate ($\times 10^2$ m^3/h)	Time interval (h)
1	*P*1	250	5	0–50
2	*P*2	250	5	0–50
3	*P*3	125	5	50–75
4	*P*4	125	5	50–75

dollars. Savings in operation costs attained through the proposed formulation are mostly due to the fact that the amount of material injected in the pipeline or, equivalently, transferred from slugs in transit to depots exactly meets product demands at the distribution centers. On the contrary, a surplus of material is delivered to depots when applying the discrete model of Rejowski and Pinto (2003) since an integer number of 2500 m^3 packs is to be pumped. As a

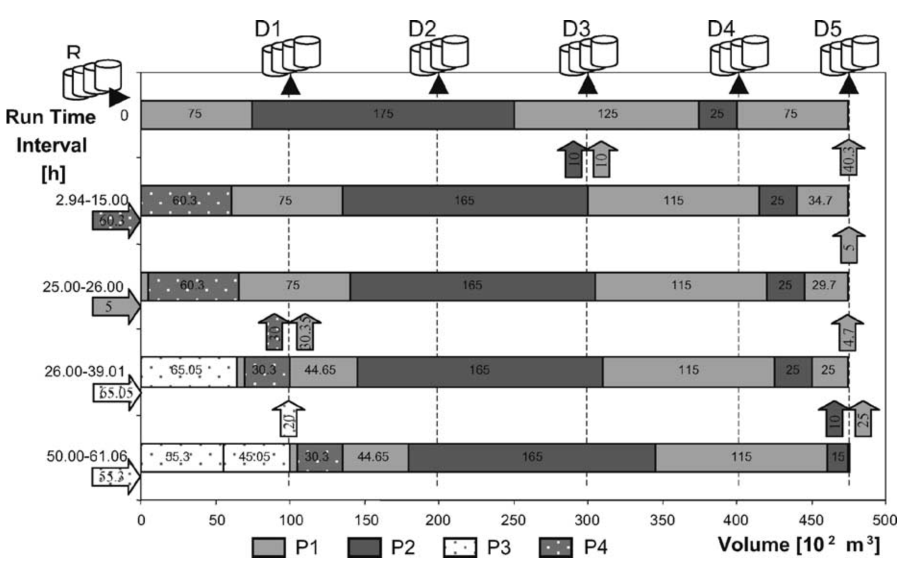

Fig. 3. The optimal pipeline schedule for Example 1.

Table 4
Model sizes and computer time requirements for Examples 1 and 2

	Binary variables	Continuous variables	Equation	CPU time (s)	Optimal solution ($\times 10^2$ US$)
Example 1	214	3000	2349	34.98	32746.83
Example 2	214	3000	2349	14.80	19737.84

result, product requirements at the depots can be satisfied in less time and, consequently, the makespan dropped from 75 to 61 h by using the proposed MILP formulation. However, the comparison of both approaches based on the total operation costs at the optimum may be questionable since our approach estimates inventory holding costs based on average product inventories at refinery and depot tanks over the scheduling horizon. There is also a significant reduction in binary/continuous variables and constraints when compared with those reported by Rejowski and Pinto (2003). Binary variables and constraints are diminished by a factor of 3. When the number of oil derivatives transported by the products pipeline to five different depots over a time horizon of 75 h rises to 8, the number of binary variables will increase: (i) from 214 to 230 if the number of new slugs remains equal to 4, and (ii) from 214 to 375 when more slugs are to be injected during the time horizon and card(I^{new}) is set equal to 6.

To meet customer demands, the pipeline remains operative from time 2.94 to 61.06 h with two temporary stops during the high-energy cost periods going from 15 to 25 h and from 40 to 50 h. In other words, it will be working a total of 48.12 h well below the overall length of the scheduling horizon. Therefore, the pipeline capacity largely exceeds the customer demands to be satisfied by injecting new product slugs into the pipeline. Three new slugs ($S8$–$S7$–$S6$) containing products ($P3$–$P1$–$P4$) in the following volumetric quantities (120.35/5.0/60.3), expressed in hundred cubic meters, are introduced in the pipeline over the time horizon. First, 60.3 volumetric units of product $P4$ is pumped from time 2.94 to 15 h to deliver 40.3 units of product $P1$ from slug $S1$ to depot $D5$, 10 units of $P1$ from slug $S3$ to depot $D3$ and 10 units of $P2$ from slug $S4$ to depot $D3$ (see Fig. 3). Afterwards, a short pumping run of product $P1$ is performed during the time interval (25–26 h) just for generating a buffer between the next slug $S8$ to be injected containing $P3$ and slug $S6$ transporting $P4$. Note that the consecutive pumping of products $P3$ and $P4$ is a forbidden pipeline operation (see Table 2). While pumping slug $S7$, five units of $P1$ is delivered from slug $S1$ to depot $D5$. In this manner, the size of slug $S1$ has been reduced from the initial volume of 75 units to 29.7. Finally, 120.35 units of $P3$ is pumped into the pipeline from time 26.0 to 61.06 h with a temporary stop during the high-energy cost periods going from 40 to 50 h. Pumping of slug $S8$ permits to deliver the remaining 29.7 units of $P1$ from $S1$ to $D5$, 10 units of $P2$ from $S2$ to $D5$, 30.35 units of $P1$ from $S5$ to $D1$, 30 units of $P4$ from $S6$ to $D1$ and 20 units of $P3$ from $S8$ to $D1$. Slug $S1$ no longer exists at the end of the time horizon. The interface material between slugs $S2$ and $S1$ amounting to 0.30 units has already been sent to a separate tank. Fig. 3 also depicts the evolution of volumes and coordinates for new/old slugs as they move along the pipeline.

Variations of product inventories at refinery and depot tanks with time are depicted in Fig. 5. It can be observed that inventory levels remain within the permissible range

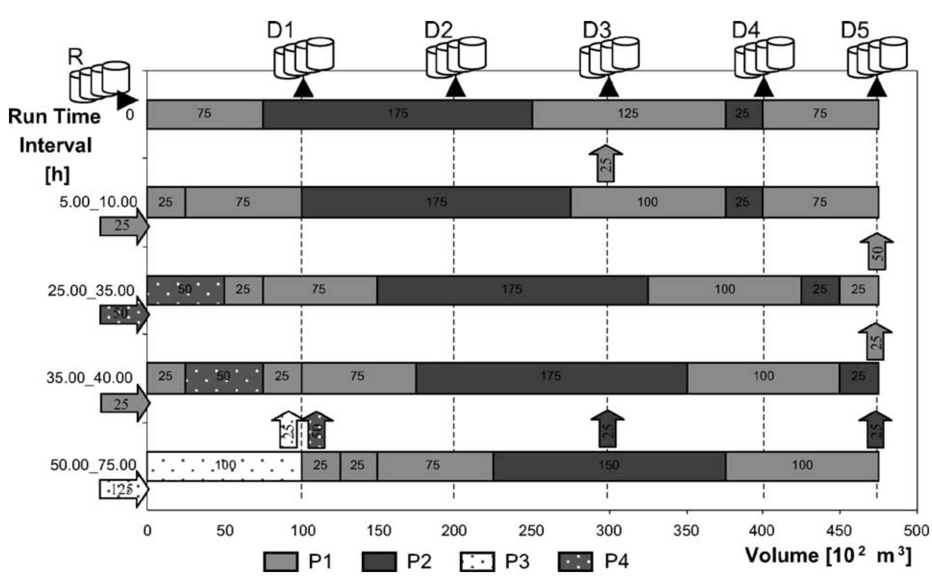

Fig. 4. The best pipeline schedule found by Rejowski and Pinto (2003) for Example 1.

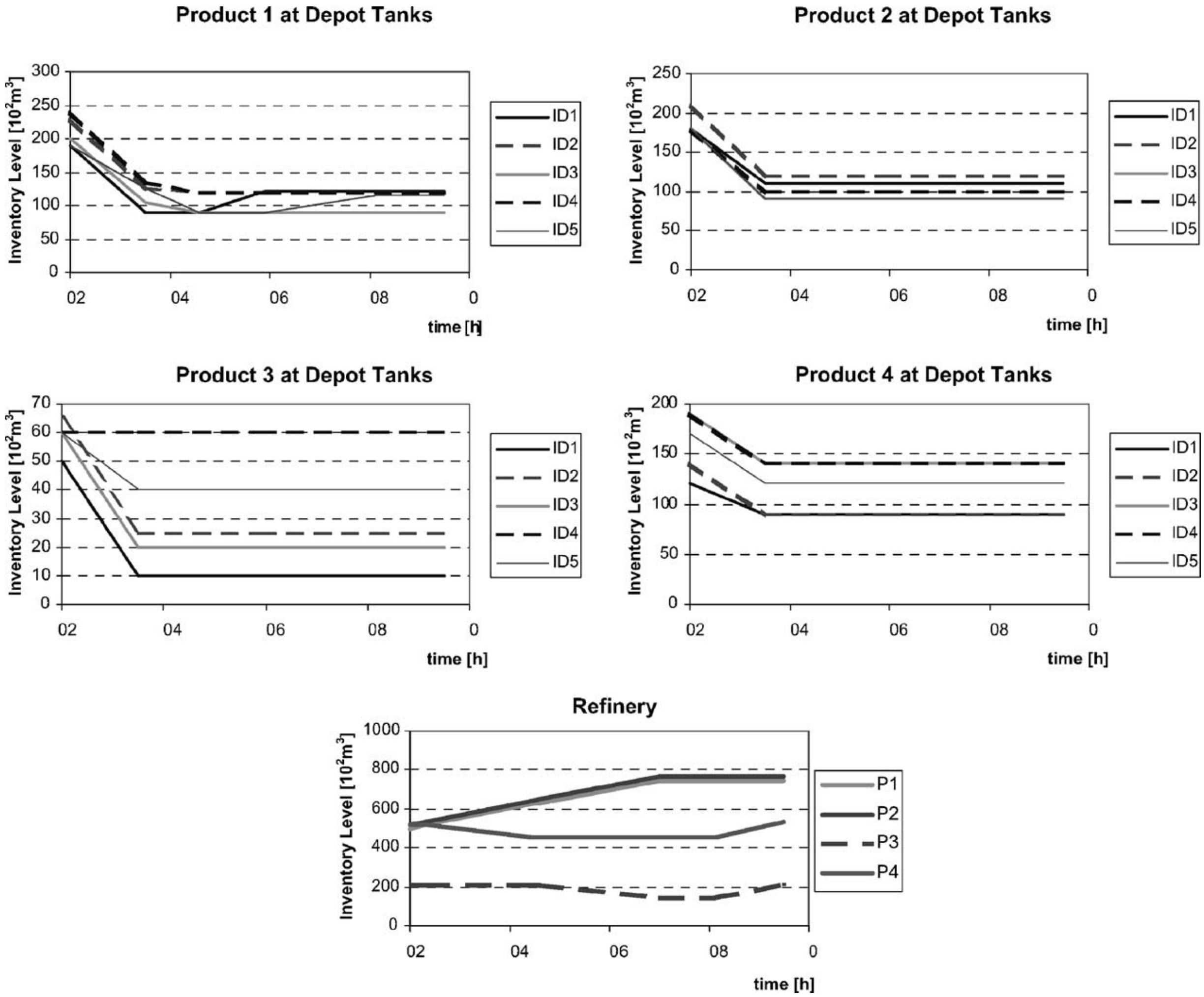

Fig. 5. Evolution of inventories at refinery and depot tanks for Example 1.

Table 5
Depot locations, product inventories and pumping costs for Example 2

Product	Level	Refinery	Depots					Product		Depots				
			*D*1	*D*2	*D*3	*D*4	*D*5			*D*1	*D*2	*D*3	*D*4	*D*5
*P*1	Minimum	400	50	30	20	50	50	*P*1	Demand	0	80	80	130	70
	Maximum	2250	190	90	90	190	180		Pumping cost (US$/m^3)	3.5	4.5	5.5	6.0	6.9
	Initial	1000	100	40	50	110	100							
*P*2	Minimum	400	90	50	90	150	150	*P*2	Demand	100	0	10	200	270
	Maximum	2500	270	190	270	720	720		Pumping cost (US$/m^3)	3.6	4.6	5.6	6.2	7.3
	Initial	1200	180	150	180	350	330							
*P*3	Minimum	50	20	0	20	20	20	*P*3	Demand	20	0	20	50	20
	Maximum	300	120	0	120	180	92		Pumping cost (US$/m^3)	4.8	5.7	6.8	7.9	8.9
	Initial	100	90	0	60	60	60							
*P*4	Minimum	150	0	0	0	30	25	*P*4	Demand	0	0	0	20	50
	Maximum	560	0	0	0	140	136		Pumping cost (US$/m^3)	3.7	4.7	5.7	6.1	7.0
	Initial	315	0	0	0	90	110							
Location from refinery ($\times 10^2$ m^3)			400	650	900	1500	1635							

Table 6
Inventory costs and interface volumes and costs for Example 2

	Interface cost ($\times 10^2$ US\$)/volume ($\times 10^2$ m^3)				Refinery	Inventory costs (US\$/(m^3 h))				
	*P*1	*P*2	*P*3	*P*4		*D*1	*D*2	*D*3	*D*4	*D*5
*P*1	0/0	30/0.30	37/0.37	35/0.35	0.020	0.100	0.100	0.100	0.100	0.100
*P*2	30/0.30	0/0	X	38/0.38	0.023	0.155	0.155	0.155	0.155	0.155
*P*3	37/0.37	X	0/0	X	0.070	0.200	0.200	0.200	0.200	0.200
*P*4	35/0.35	38/0.38	X	0/0	0.025	0.170	0.170	0.170	0.170	0.170

at every tank. Moreover, product inventories at depot tanks tend to remain close to their minimum values throughout the scheduling horizon because of their higher inventory costs. It should be remarked that the injection of very small buffers separating slugs of incompatible products is a unique feature of the proposed formulation. Rejowski and Pinto (2003) divided the pipeline into packs of 25 volumetric units each one involving a single product. Then, the smallest buffer that can be injected in the pipeline must have a volume of 25 units.

4.2. Example 2

Example 2 is a real-world case study first presented by Rejowski and Pinto (2003). It involves the scheduling of a multiproduct pipeline of some 955 km in length transporting gasoline (*P*1), diesel (*P*2), LPG (*P*3) and jet fuel (*P*4) from a single oil refinery to five distribution centers. Data for Example 2 are given in Tables 5–7. Product demands at distribution terminals, depot distances from the refinery (in volumetric units), initial product inventories and maximum/minimum levels at refinery and depot tanks are all included in Table 5. In turn, Table 6 provides the interface costs and volumes for each ordered pair of products as well as the product inventory costs at refinery and depots. Forbidden product sequences are denoted with an “X”. From Table 6, it follows that inventory costs at refinery and depot tanks mostly determine the pipeline operational costs over the time horizon. The scheduled production runs at the oil refinery are described in Table 7. Similarly to Example 1, the time horizon has a total length of 75 h and the pumping unit cost for each product–depot pair changes with time. The slug pump rates range from 800 to 1200 m^3/h. In addition, the flow rates from the pipeline to depots are approximately 150 m^3/h, while flow rates from depots to the markets average 70 m^3/h (Rejowski and Pinto, 2003).

Table 7
Scheduled production runs at the oil refinery (Example 2)

Production run	Product	Volume ($\times 10^2$ m^3)	Production rate ($\times 10^2$ m^3/h)	Time interval (h)
1	*P*1	250	5	0–50
2	*P*2	250	5	0–50
3	*P*3	125	5	50–75
4	*P*4	125	5	50–75

Initial conditions at the pipeline are shown at the top of Fig. 6. At the start of the time horizon, there is a sequence of five old slugs (*S*5–*S*4–*S*3–*S*2–*S*1) inside the pipeline containing products (*P*2–*P*1–*P*3–*P*1–*P*2) in the following amounts (400/700/200/200/135), expressed in hundred cubic meters. Slug *S*1 occupies the farthest position from the oil refinery.

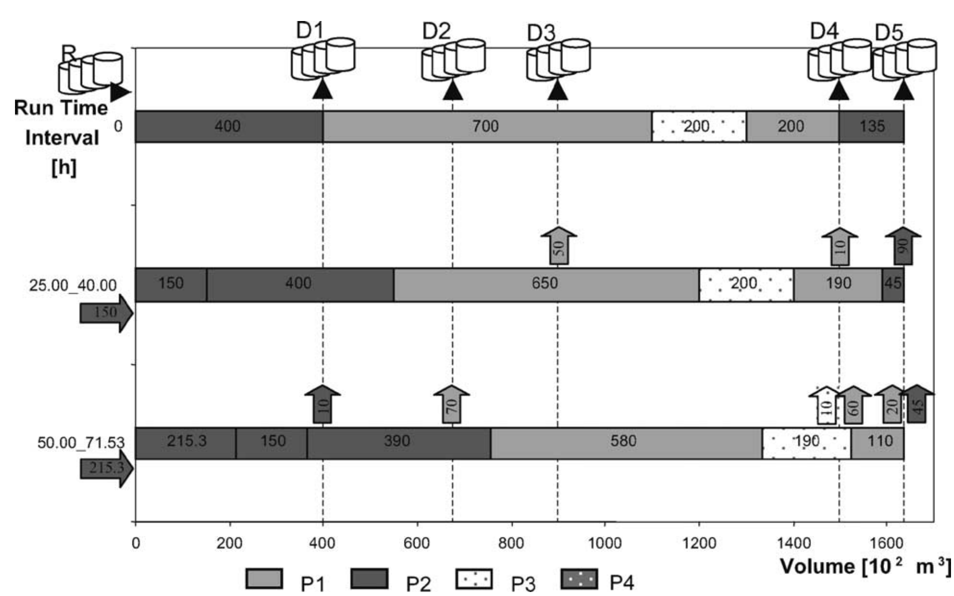

Fig. 6. The optimal pipeline schedule for Example 2.

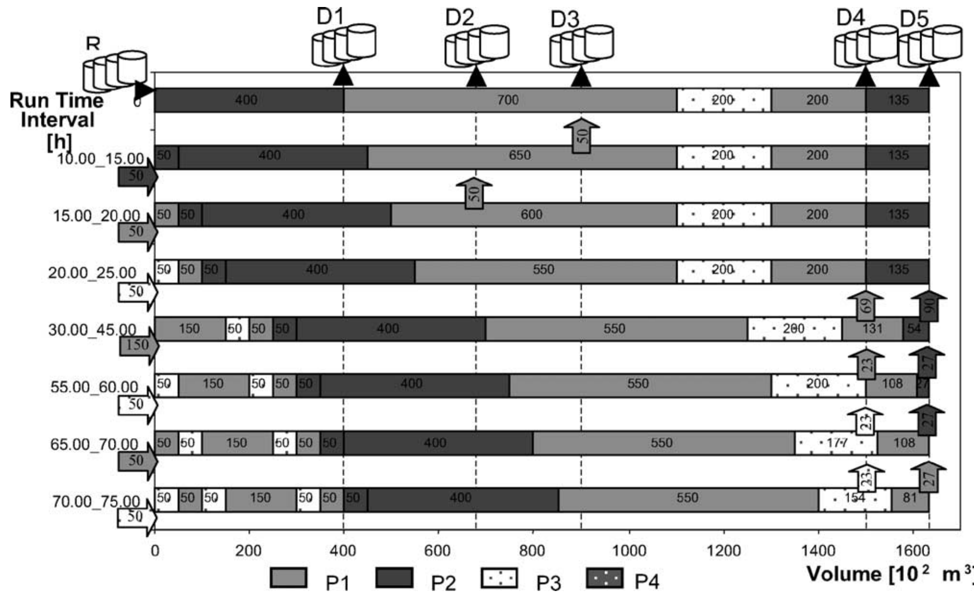

Fig. 7. The best pipeline schedule found by Rejowski and Pinto (2003) for Example 2.

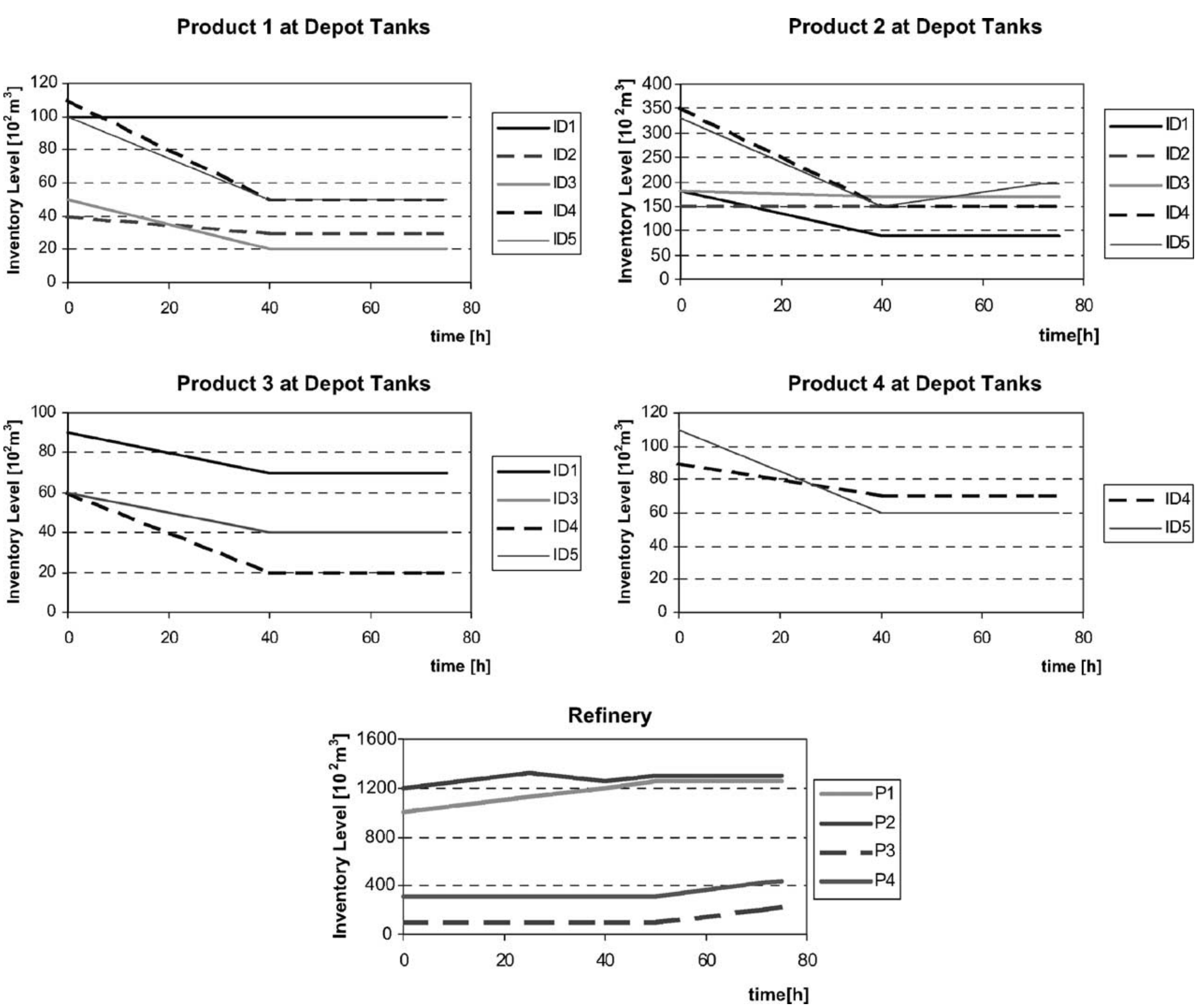

Fig. 8. Evolution of inventories at refinery and depot tanks for Example 2.

The optimal pipeline schedule depicted in Fig. 6 was found in 14.80 s. The model size and the optimal operation cost are given in Table 4. Fig. 7 shows the best solution for Example 2 reported by Rejowski and Pinto (2003). It can be observed that the pipeline schedule provided by the discrete model involves a higher number of short pumping runs; i.e. a total of seven new slugs against only two prescribed by the continuous formulation. Since different products are successively pumped into the pipeline when used the discrete model solution, then more interfaces and a large interface material would be generated. Consequently, the pipeline operational costs at the optimum, in hundred dollars, amount to 19,737.84 for the continuous approach compared with 23,995.925 reported by Rejowski and Pinto (2003). Similarly to Example 1, large savings in all variables, constraints and CPU time have also been achieved (see Table 4). The solution time drops from 10,000 s reported by Rejowski and Pinto (2003) to 14.8 s, i.e. a three-order-of-magnitude time saving. Moreover, the number of binary variables is diminished by a factor of 2 and the number of constraints by a factor of 3. Though the pipeline remains idle during peak hours, the schedule makespan shows a decrease of 4 h.

Product inventories initially available at depot tanks and at the pipeline itself are indeed enough to meet demands at each distribution center. Therefore, the two new slugs ($S7$–$S6$) injected in the pipeline aim to just moving the pipeline stream so as to deliver the required products from old slugs to depot tanks. To avoid the generation of interface material, the new slugs both contain product $P2$ since the pipeline segment connecting the oil refinery to the closest depot $D1$ is completely filled by slug $S5$ also transporting $P2$. Slugs $S6$ and $S7$ are successively pumped in this order into the pipeline during the time intervals 25.0–40.0 and 50.0–71.53 h, respectively. Their volumes are 150×10^2 and 215.3×10^2 m^3, respectively. Again, the pipeline remains operative for only 36.53 h of a total of 75 h, i.e. half of the time available over the scheduling horizon. To balance product demands and inventories at depot tanks, several material transfers from old slugs to depot tanks occur while pumping slugs $S6$ and $S7$ (see Fig. 6). Thus, the injection of slug $S6$ permits to deliver 90 units of $P2$ from $S1$ to $D5$, 10 units of $P1$ from $S2$ to $D4$ and 50 units of $P1$ from $S4$ to $D3$. In turn, the injection of slug $S7$ gives rise to a total of six material transfers from old slugs to depots (see Fig. 3). They are: the remaining 45 units of $P2$ from $S1$ to $D5$, 20 units of $P1$ from $S2$ to $D5$, 60 units of $P1$ from $S2$ to $D4$, 10 units of $P3$ from $S3$ to $D4$, 70 units of $P1$ from $S4$ to $D2$ and 10 units of $P2$ from $S5$ to $D1$. The interface material between slug $S2$ and $S1$ still remains in the pipeline farthest extreme from the refinery. Changes of product inventories at refinery and depot tanks with time are depicted in Fig. 8. Inventories at depot tanks mostly remain at their minimum levels.

5. Conclusions

A new MILP continuous-time approach for the scheduling of a single multiproduct pipeline transporting refined petroleum products from a single oil refinery to several distribution centers has been presented. By adopting a continuous representation in both time and volume, a more rigorous problem description and a severe reduction in binary variables, constraints and CPU time have simultaneously been achieved. The use of a rigorous problem representation brings about several advantages. First, discrepancies between actual pipeline capacities at each pipeline segment and the capacity values provided by discrete models are eliminated. Second, the inflexibility in the selection of slug volumes constrained to be an integer number of packs when using discrete models no longer arises. Moreover, minimum/maximum lengths for the pumping runs can be handled. Third, the computational requirements are reduced by almost three-order-of-magnitude when compared with previous approaches. Fourth, product incompatibility can be easily overcome by introducing small batches of appropriate products acting as buffers. Such small buffers are not constrained to be an integer number of packs. Fifth, by drastically reducing model sizes and CPU requirements, the proposed approach gives the opportunity to schedule pipeline operations over time horizons much longer than merely 3 days. Sixth, interface volumes are explicitly considered by the problem formulation. Their positions along the time horizon can be tracked.

Acknowledgements

The authors acknowledge financial support from FONCYT-ANPCyT under Grant 14-07004, from “Universidad Nacional del Litoral” under CAI+D 28 and from “Fundación Antorchas”.

References

ILOG OPL Studio 3.6 User’s Manual (2003). France: ILOG S.A.

Magatão, L., Arruda, L. V. R., & Neves Jr., F. (2004). *Computers and Chemical Engineering, 28*, 171–185.

Neiro, S. M. S., & Pinto, J. M. (2004). *Computers and Chemical Engineering, 28*, 871–896.

Rejowski, R., & Pinto, J. M. (2001). *Proceedings of Second Pan American Workshop on Process Systems Engineering*, Guaruja, São Paulo, Brazil.

Rejowski, R., & Pinto, J. M. (2003). *Computers and Chemical Engineering, 27*, 1229–1246.

Sasikumar, M., Prakash, P. R., Patil, S. M., & Ramani, S. (1997). *Knowledge-Based Systems, 10*, 169–175.