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# Dynamic scheduling of multiproduct pipelines with multiple delivery due dates

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**Abstract**

Scheduling product batches in pipelines is a very complex task with many constraints to be considered. Several papers have been published on the subject during the last decade. Most of them are based on large-size MILP discrete time scheduling models whose computational efficiency greatly diminishes for rather long time horizons. Recently, an MILP continuous problem representation in both time and volume providing better schedules at much lower computational cost has been published. However, all model-based scheduling techniques were applied to examples assuming a static market environment, a short single-period time horizon and a unique due-date for all deliveries at the horizon end. In contrast, pipeline operators generally use a monthly planning horizon divided into a number of equal-length periods and a cyclic scheduling strategy to fulfill terminal demands at period ends. Moreover, the rerouting of shipments and time-dependent product requirements at distribution terminals force the scheduler to continuously update pipeline operations. To address such big challenges facing the pipeline industry, this work presents an efficient MILP continuous-time framework for the dynamic scheduling of pipelines over a multiperiod moving horizon. At the completion time of the current period, the planning horizon moves forward and the re-scheduling process based on updated problem data is triggered again over the new horizon. Pumping runs may extend over two or more periods and a different sequence of batches may be injected at each one. The approach has successfully solved a real-world pipeline scheduling problem involving the transportation of four products to five destinations over a rolling horizon always comprising four 1-week periods.


*Keywords:* Multiproduct pipeline; Dynamic scheduling; Multiple due dates; MILP approach

## 1. Introduction

Pipelines are the safest and least expensive way to deliver large quantities of energy products from refineries to distribution terminals but at the same time the slowest form of transportation with speeds of 3–8 mph. Pipeline low costs mostly result from little product damage along the trip, substantial economies of scale, no need for containers moving with the cargo and no backhauls (Trench, 2001). In addition, the cargo movement is less affected by traffic and weather conditions compared with other modes of transportation. Nearly 68% of the intercity ton-miles of crude oil and refined products in the US are handled by pipelines. The transportation of refined petroleum products generally combines a long-distance delivery by pipeline from the refinery to distribution terminals followed by a truck journey to local markets. Moreover, a single delivery from a refinery to a distant distribution terminal may require multiple pipeline carriers. Liquid products are propelled through pipelines by centrifugal pumps which are sited at pumping stations, one at the origin and the others distributed along the pipeline separated by a distance varying from 20 to 100 miles, depending on the topography and the capacity requirement. Petroleum derivatives are inserted in the line one after another without any separation device between batches. If two consecutive products are dissimilar, such as gasoline and jet fuel, a hybrid product called transmix is created by intermixing at the interface. The transmix must be separated and stored in a small holding tank before sending back to the refinery for reprocessing (Hull, 2005). Pipelines are generally owned by a number of companies and most of them are common carriers transporting petroleum products from different refiners. A pipeline network can have several entry and exit points and the interchange of refined products between two common carrier pipelines may occur at shared terminals. In this paper, the multiperiod scheduling of a single unidirectional pipeline system involving a unique entry point at the origin and several exit points as many as the number of distribution terminals along the line is studied.

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**Nomenclature**

*Sets*

- $I$ chronologically arranged batches ($I^{old} \cup I^{new}$)
- $I^{new}$ new batches to be injected during the time horizon
- $I^{old}$ old batches inside the pipeline at the start of the time horizon
- $J$ distribution terminals along the pipeline
- $J_p$ distribution terminals demanding product $p$
- $P$ refined petroleum products
- $R$ scheduled production runs at the oil refinery
- $T$ time periods of the planning horizon
- $T_{HF}$ hard frozen periods on the planning horizon
- $T_{SF}$ soft frozen periods on the planning horizon

*Parameters*

- $a_r$, $b_r$ starting/finishing time of the refinery production run $r$
- $\mathrm{cb}_{p,j,t}$ unit backorder penalty cost to tardily meet a requirement due at period $t$
- $\mathrm{cf}_{p,p'}$ unit reprocessing cost of interface material involving products $p$ and $p'$
- $\mathrm{cid}_{p,j}$ unit inventory holding cost for product $p$ at depot $j$
- $\mathrm{cir}_p$ unit inventory holding cost for product $p$ in refinery tanks
- $\mathrm{cp}_{p,j}$ unit normal pumping cost to deliver product $p$ from the refinery to depot $j$
- $\mathrm{dd}_t$ upper extreme of period $t$
- $\mathrm{dem}_{p,j,t}$ overall demand of product $p$ to be satisfied at depot $j$ before due date $\mathrm{dd}_t$
- $D_{\max}$ maximum delivery size from a batch to a distribution terminal
- $F_i^{\mathrm{o}}$ current upper pipeline coordinate of old batch $i$
- hf number of hard-frozen time periods
- $h_{\max}$ horizon length
- $h_t$ time period length
- $\mathrm{hw}_{\max}$ maximum working time
- $(\mathrm{ID}_{\max})_{p,j}$ maximum allowed inventory level for product $p$ at depot $j$
- $(\mathrm{ID}_{\min})_{p,j}$ minimum allowed inventory level for product $p$ at depot $j$
- $\mathrm{IF}_{p,p'}$ volume of interface between batches containing products $p$ and $p'$
- $\mathrm{IR}_p^{\mathrm{o}}$ initial inventory of product $p$ in refinery tanks
- $(\mathrm{IR}_{\max})_p$ maximum allowed refinery inventory level for product $p$
- $(\mathrm{IR}_{\min})_p$ minimum allowed refinery inventory level for product $p$
- $k$ current time period
- $l_{\min}, l_{\max}$ minimum/maximum length of a new batch injection
- $N$ number of time periods in the rolling horizon
- $\mathrm{NS/CS}_{p,j,t}$ sizes of new/cancelled nominations for product $p$ due at period $t$ in terminal $j$
- $\mathrm{PH}_{\max}$ accumulated daily peak hours
- $Q_{\max}$ maximum injection size
- $s_{\mathrm{r}}$ size of the refinery production run $r$
- sf number of soft-frozen time periods
- vb pumping rates
- $\mathrm{vm}_{p,j}$ maximum supply rate of product $p$ to the local market from depot $j$
- $\mathrm{vp}_{\mathrm{r}}$ production rate for run $r$
- $W_i^{\mathrm{o}}$ current volume of old batch $i$
- $\rho$ unit-time penalty cost for operating during peak-hour intervals
- $\sigma_j$ volumetric coordinate of depot $j$ from the head terminal
- $\tau_{p,p'}$ changeover time between injections of products $p$ and $p'$

*Continuous variables*

- $B_{p,j,t}$ backorder of product $p$ for depot $j$ due at period $t$ to meet at period $t+1$
- $C_i$, $L_i$ completion time/length of pumping run $i \in I^{new}$
- $D_{i,j}^{(i')}$ volume of batch $i$ diverted to depot $j$ while injecting batch $i'$
- $\mathrm{DM}_{p,j}^{(i')}$ amount of product $p$ sent to local market $j$ during the time interval $[C_{i'-1}, C_{i'}]$
- $\mathrm{DP}_{i,p,j}^{(i')}$ amount of product $p$ supplied by batch $i$ to depot $j \in J_p$ during $[C_{i'} - L_{i'}, C_{i'}]$
- $F_i^{(i')}$ upper coordinate of batch $i$ from the origin at time $C_{i'}$
- $\mathrm{ID}_{p,j}^{(i')}$ inventory of product $p$ in depot $j$ at the end of pumping run $i'$
- $\mathrm{IRF}_p^{(i')}$ inventory of product $p$ in refinery at the end of pumping run $i'$
- $\mathrm{IRS}_p^{(i')}$ inventory of product $p$ in refinery at the start of pumping run $i'$
- PH peak-hour usage
- $Q_i$ initial size of the new batch $i$
- $\mathrm{QP}_{i,p}$ volume of product $p$ injected in the pipeline while pumping batch $i$
- $\mathrm{SL}_{i,r}$ production output from run $r \in R$ available in refinery tanks at time $C_i$
- $\mathrm{SU}_{i,r}$ production output from run $r \in R$ available in refinery tanks at time $(C_i - L_i)$
- $W_i^{(i')}$ size of batch $i$ at time $C_{i'}$
- $\mathrm{WIF}_{i,p,p'}$ interface volume between batches $i$ and $(i-1)$ containing products $p$ and $p'$

*Binary variables*

- $w_{i,t}$ denoting that the injection of batch $i$ ends within time period $t$
- $x_{i,j}^{(i')}$ denoting that a portion of batch $i$ can be transferred to depot $j$ while injecting $i'$
- $y_{i,p}$ denoting that batch $i$ contains product $p$
- $\mathrm{zl}_{i,r}$ denoting that injection $i$ ends after the refinery production run $r$ has started

| | |
|---|---|
| $zu_{i,r}$ | denoting that injection $i$ begins after the refinery production run $r$ has ended |

### 1.1. Batch scheduling and dispatching in multiproduct pipeline systems

The scheduling of pipelines transporting petroleum products from a single refinery to multiple destinations has received increasing attention among researchers in the last decade (see Fig. 1). Usually, customers contact the pipeline carrier to place their transport orders or "nominations" for the next month. Once a customer nominates to a particular pipeline and the nomination has been accepted, the customer must make the batch to be shipped available in the pipeline origin at the right time and have sufficient storage capacity to receive it at the destination. Generally, batch movements in a particular month must be nominated by the 25th of the previous month. Afterwards, the pipeline scheduler develops a detailed hourly schedule of pipeline activities over a monthly horizon. To do that, the scheduling horizon is first divided into a number of cycles or periods with a typical cycle length of 7, 10 or 14 days, i.e. a multiperiod horizon. Moreover, a customer nomination is partitioned into as many portions of equal size as the number of cycles or periods per month, and each portion is due at the end of a cycle. In other words, a cyclic scheduling approach is usually applied by assuming the same product demand profile at every period. Once a complete product sequence has been shipped during a cycle, a second identical one is started (Sheppard, 1984). If a pipeline operates on a 14-day cycle, the shipper must provide storage capacity to receive a batch that will cover his demands for 14 days. With a 7-day cycle, the customer only needs half as much tankage, but the interface volume will duplicate. Therefore, the storage capacity to be provided by the customer is reduced at the expense of increased interface reprocessing costs. If nominations exceed pumping capacity, schedulers must decide which nominations to reduce through the so-called "apportionment" process by using apportionment rules (Hull, 2005).

The pipeline schedule is executed by dispatchers who remotely perform loading, transportation and unloading operations in a fully automated way through computers all from the control room. By using the Supervisory Control and Data Acquisition system (SCADA) estimating batch arrival times to terminals and batch sizes from the information given by interface detectors, dispatchers can trace batches along the line and divert them to one or more terminal tankages. In this manner, they continually monitor each batch to ensure that the physical connection to tankage or other pipelines is open when the batch arrives to the stated terminal (Rabinow, 2004). The entire line must be stopped when a customer cannot receive his shipment at the stated destination because of insufficient tankage capacity or any other practical inconvenience. Similarly to real-life terminal operations, a good problem representation should be capable of tracing batches while flowing inside the pipeline in order to precisely establish (i) the earliest time at which the delivery of a batch to the stated terminal can be started and (ii) the time interval during which the batch has access to the terminal tankage. Moreover, terminals usually have a few tanks just used to facilitate loading/unloading operations rather than being employed for long-term storage. Therefore, a key issue for efficient terminal operations is the coordination among incoming and outgoing product flows to/from every tank. Product stock-outs at the head terminal or overloading conditions at other depots oblige the dispatcher to temporarily stop the line.

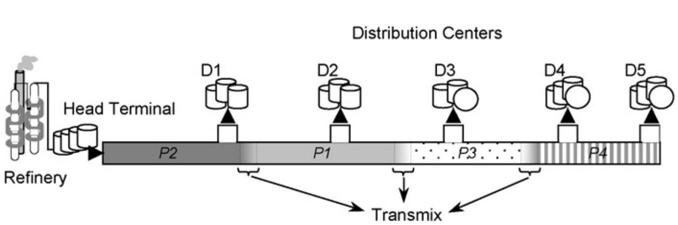

Fig. 1. A single unidirectional multiproduct pipeline system.

### 1.2. Previous contributions and new challenges

Two different types of scheduling methodologies have been proposed in the literature: knowledge-based search techniques (Sasikumar, Prakash, Patil, & Ramani, 1997) and mixed-integer linear mathematical programming (MILP) formulations. All approaches assume a single-period planning horizon and the specification of a unique due-date for all product demands at the different depots, i.e. just at the horizon end. Depending on whether or not the pipeline volume and the time horizon are both discretized, model-based scheduling methods can be grouped into two classes: discrete and continuous MILP approaches. Most of the proposed optimization models not only partitioned the horizon into time intervals of equal or unequal sizes but also the pipeline volume is divided into a significant number of single-product packs (Magatão, Arruda, & Neves, 2004; Neiro & Pinto, 2004; Rejowski & Pinto, 2003, 2004). In contrast, Cafaro and Cerdá (2004) developed a novel MILP continuous formulation that requires neither time discretization nor pipeline division. In comparison with heuristic search techniques, one of the major drawbacks of the optimization approaches is the use of rather short time horizons comprising just a few days to limit the size of the mathematical model. In this way, the solution time remains reasonable and the optimal solution can be efficiently found.

One of the challenges on pipeline operation is to meet large product demands from every depot along the pipeline at different due-dates over rather long planning horizons. Since new transportation requests are placed by customers as time proceeds, the information on the problem is indeed time-dependent and the pipeline schedule should be periodically updated. Additional "nominations" may arrive at any time within the month. Furthermore, some old nominations can be cancelled or their destinations could be changed by the shippers while the batches are already in transit. The pipeline scheduler is not only a planner but also a revisor of plans since it is necessary to update schedules to meet shipper requirements in a profitable way for the carrier. Rerouting of shipments is said to be a fact of life in pipeline operation (Sheppard, 1984). Another complicating factor is the shipment time delay. There is a significant time delay ranging from 3 to 10 days or more between the injection of a batch in the line and its delivery to the stated terminal, depending on the pipeline length and the depot location. If the scheduling horizon is shorter than the average delivery lead-time, most of market demands are to be fulfilled through inventories already available at depot tanks or in pipeline transit. This is why the insertion of new batches in the line for short time-horizon problems has the only purpose of pushing batches ahead along the pipeline from their current locations to the nominated terminals. This is usually called “the end-of-horizon effect”. As a result, most of the late planned batch injections over the scheduling horizon have nothing to do neither with product demands to meet during the current horizon nor with still unknown future requirements. As time passes and new product requests from shippers are considered, the updating process of the current pipeline schedule would surely yield a completely different sequence of batch injections at the head terminal.

As mentioned, pipeline operators generally develop a cyclic delivery schedule every month and product deliveries to local markets feature multiple due dates generally fixed at period ends. Daily or weekly time periods are usually chosen. On the contrary, current pipeline scheduling techniques neither handle multiperiod time horizons nor consider multiple due dates for the product shipments. To overcome such limitations and, at the same time, account for other pipeline operation challenges, this paper presents a new MILP multiperiod continuous-time formulation for the so-called dynamic pipeline scheduling problem (DPSP). In the DPSP, pipeline operations are scheduled over a fixed-length multiperiod rolling horizon. The pipeline schedule should be viewed as a dynamic timetable rather than a static one where only the scheduling decisions for the first or current period of the rolling horizon need to be implemented immediately. In contrast to the usual practice in the oil pipeline industry, the proposed approach accounts for nominated shipments with different promised dates always occurring at period ends. Moreover, the partitioning of customer nominations is no longer required and the sequence of product injections usually changes with the time period, i.e. a non-cyclic scheduling strategy. In the DPSP problem, the information on new booked shipper requests, cancellations of old nominations, changes on batch destinations, updated refinery production planning and on-hand product inventories at refinery and depot tankage becomes available as the time horizon rolls and a new period is started. Since the multiperiod horizon has a fixed length (in terms of number of periods), another time interval is incorporated at the horizon end to replace the old first period just vanished whenever a new period begins. The horizon length should be high enough to prevent the appended product demands from affecting the first-period scheduling decisions.

Based on the new problem data, pipeline operations are optimally rescheduled through solving the proposed DPSP model. In this way, the dynamic pipeline schedule finally executed by the dispatcher is generated. Results provided by the DPSP approach include (a) the updated sequence and timing of the pumping runs inserting new batches in the pipeline over the current multiperiod rolling horizon, (b) the product deliveries to distribution terminals taking place while executing a batch injection (batch source, destination, quantity of product being transferred), (c) the location and size of every batch inside the pipeline immediately before and after a pumping run, (d) the updated projected inventories in refinery and depot tanks immediately before and after every new batch injection. Surprisingly, major changes on late planned batch injections may usually arise because of (1) new shipper requests for the time period recently incorporated in the planning horizon, and (2) anticipated product insertion in the pipeline due to rather long transport lead-times. As a result, the final schedule executed by the dispatcher shows major differences with the one provided by static pipeline scheduling approaches. The proposed method can be extended to schedule pipeline networks with multiple exits not only for delivery of products to depot tankage but also for interchanging shipments with other outgoing pipelines at common terminals.

## 2. Problem statement

Given:

(a) A unidirectional multiproduct pipeline connecting a single origin to multiple distribution terminals, with pipeline segments of equal or different cross-sectional areas.
(b) The available tanks for refined products at every terminal.
(c) A multiperiod rolling horizon comprising $N$ time periods of equal or unequal specified lengths.
(d) A set of shipment requests, usually called “nominations”, each one involving a given volume of a refined product available in the head terminal tankage to be delivered by pipeline to a certain downstream depot.
(e) Due-dates for product deliveries to distribution terminals always occurring at period ends. Different “nominations” may involve the delivery of a particular grade or product to the same distribution terminal but at distinct due dates.
(f) The sequence of “old” batches already inside the pipeline as well as their contents and locations at the present time.
(g) The scheduled production runs at the refinery or, alternatively, scheduled incoming product flows to tankage at the origin.
(h) Inventory levels in refinery and terminal tankage at the present time.
(i) Maximum/minimum pipeline pumping rate, maximum supply rate from the pipeline to terminals and maximum delivery rate from pipeline terminals to marketing terminals from which products are sent to local markets by truck.
(j) The regeneration frequency of the pipeline schedule based on updated information as the multiperiod horizon rolls with time. The pipeline scheduling system is usually rerun at the start of a new period.

The problem goal is to dynamically update the sequence and volumes of new product batches to be pumped in the pipeline throughout a multiperiod rolling horizon in order to: (1) meet every product demand at each terminal in a timely fashion; (2) maintain the inventory level in refinery and terminal tankage within the permissible ranges; (3) trace the size and location of every batch in pipeline transit; (4) minimize the sum of pumping, transition, down-time, backorder and inventory carrying costs. The pipeline schedule should indicate the amount and type of product to be pumped, the batch pumping rate as well as the starting and completion time of every batch injection.

## 3. Model assumptions

(1) A single multiproduct transmission pipeline with unidirectional flow, conveying refined petroleum products from a refinery to several downstream terminals is considered.
(2) The pipeline remains completely full of products at any time. By assuming liquid incompressibility, the only way to get a volume of product out of the line at a downstream terminal is by injecting an equal volume at the origin.
(3) The pipeline operates in fungible mode. If individual batches of the same grade or product from different shippers meet common specifications, they can be mixed into a consolidated or fungible batch and sent through the pipeline as a single batch.
(4) Each fungible batch can be allocated to two or more terminals. As new product batches are injected, a portion of a batch flowing through the pipeline can be diverted to the assigned terminal while the remainder will continue moving to more distant points, i.e. the so-called batch "stripping" operation.
(5) The individual batches flowing together on a fungible batch can be dynamically allocated to distribution terminals; i.e. allocation of batches to terminals can be modified during the rescheduling process. Dynamic allocation is required because a batch while in transit along the line can be traded to another shipper at a different destination.
(6) A product request at some distribution terminal can be satisfied by diverting material from more than one fungible batch.
(7) Product batches are sequentially pumped into the pipeline at turbulent flow to retard mixing.
(8) The transmix or contamination volume between a particular pair of refined products is supposed to be a known constant, independent of the scheduled batch movements. The transmix is kept into the line until it reaches the farthest terminal where it is stored and rerouted to the refinery. Otherwise, the interface would be automatically regenerated, thus increasing transition costs.
(9) A portion of a batch can be delivered to a terminal only if (a) the batch has arrived to the point on the line where the physical connection to the terminal tankage is available and (b) a tank in the terminal is ready to store the batch. If a terminal cannot receive a shipment of product because of insufficient capacity in the assigned tank, then the entire line must be stopped until the problem is solved.
(10) The unit pumping cost is a known constant that varies with the product and the stated destination but it is independent of the pump rate.
(11) The maximum supply rate of refined products to refinery tanks from scheduled production runs is always lesser than the lowest pipeline pumping rate. If several refiners make use of the pipeline, the product batches to be shipped are assumed to be available at the head terminal tankage at the start time of the batch injections. In the examples involving a single refinery, the maximum production rate is about 500 $m^3$/h, whereas the minimum pump rate into a 20 in. pipeline is over 800 $m^3$/h.
(12) A non-cycling pipeline schedule strategy over a multiperiod rolling horizon is applied. Therefore, the sequence of product shipments to be executed by the dispatcher may vary from one to the next period.
(13) The present time is the beginning of the most immediate period of the current rolling horizon, i.e. the first period. The planned product shipments for the first period of the time horizon (the action period) are not subject to changes during the periodic scheduling review. New transportation requests can be accepted just for late periods. First-period shipments are the only ones executed by the dispatcher. The implementation of planned shipments for a later period must wait until it becomes the first period of the rolling horizon.
(14) Since it may take over 1 or 2 weeks to move a batch from the origin to the assigned terminal (the delivery lead-time), the horizon length must exceed the largest delivery lead-time. Otherwise, batches will be put in the pipeline during the action period without knowing their exact destinations.

## 4. Major model variables and constraints

The mathematical formulation for the dynamic multiproduct pipeline scheduling problem (DPSP) is defined in terms of four major sets: (a) the old and new fungible batches ($i \in I = I^{old} \cup I^{new}$), (b) the pipeline distribution terminals ($j \in J$), (c) the refined petroleum products to be delivered ($p \in P$) from the refinery to terminals along the line and (d) the time periods taking part of the multiperiod rolling horizon ($t \in T$). Old batches $i \in I^{old}$ are those already in transit along the line at the present time, while new fungibles batches $i \in I^{new}$ are planned to be pumped in the pipeline at future periods. Moreover, the problem formulation will assume that the set $I$ has been chronologically arranged beforehand with the old batches $i \in I^{old}$ preceding the new batches $i \in I^{new}$. Therefore, the first entry in $I^{old}$ is the farthest old batch from the origin while the last entry is the batch put in the pipeline more recently. On the other hand, the first element of $I^{new}$ corresponds to the first batch to be injected during the current horizon while the last one is the latest pumping run being planned. Then, the insertion of a new batch $i$ in the line should start after ending the injection of batch $(i-1)$. Since the number of pumping runs to be executed throughout the time horizon is unknown beforehand but lower than $|I^{new}|$, some of the later entries of $I^{new}$ are never executed, i.e. they stand for fictitious new batches. Some criteria for choosing $|I^{new}|$ are given in Section 6.

### 4.1. Batch features

A new batch $i \in I^{\text{new}}$ that is planned to be injected in the pipeline is characterized by the following properties:

(a) Allocated product (binary $y_{i,p}$).
(b) Initial batch size ($Q_i$).
(c) Initial injection time ($C_i - L_i$).
(d) Final injection time ($C_i$).
(e) Pumping run duration ($L_i$).
(f) Completion time period (binary $w_{i,t}$), i.e. the period at which the pumping of batch $i$ ends.

They can be regarded as static properties since their values do not change with the pipeline activity, i.e. with the injection of new batches. The set of equations defining the static properties of a new batch to be injected will be called *batch-defining constraints*. Since batch $(i-1)$ precedes batch $i$ (predefined batch sequence) and the allocated products are given by $y_{i,p}$ and $y_{i-1,p'}$, then the interface volume between any pair of consecutive new batches and the feasibility of the batch subsequence $(i-1, i)$ can be easily determined. Then, these additional equations will also be considered together with the batch-defining constraints. In summary, such constraints include two different sets of binary variables denoted by $y_{i,p}$ and $w_{i,t}$, respectively. The assignment variable $y_{i,p}$ indicates that the new batch $i \in I^{\text{new}}$ contains product $p$ whenever $y_{i,p} = 1$. Obviously, a single batch can contain at most one refined product and therefore $\sum_p y_{i,p} \leq 1$ for any $i \in I$. Furthermore, the binary variable $w_{i,t}$ is an assignment variable indicating that the pumping of the new batch $i \in I^{\text{new}}$ is completed in period $t$ whenever $w_{i,t} = 1$. Nonetheless, the pumping run may have begun at an earlier period $t' < t$. Such a definition of $w_{i,t}$ permits to handle a unique set of new batches $I^{\text{new}}$ for the whole multiperiod rolling horizon rather than a different one for each period. In this manner, the increase in the number of potential new batch injections can be effectively bounded and the problem size remains quite reasonable.

### 4.2. Batch tracing and stripping operations

Some other batch properties are pipeline activity-dependent and their values change along the rolling horizon whenever a new batch is injected in the line. They will be referred to as the batch dynamic properties. Therefore, the final batch pumping times can be regarded as the major event points at which the dynamic batch properties are to be determined. For instance, the pipeline coordinate and the size of an old/new batch in pipeline transit both generally change while executing a pumping run. As the shipment moves along the pipeline, some material can also be diverted from the batch to accessible depots through stripping operations causing variations in such dynamic properties. To know when a batch will arrive to a stated destination and what amount of product is to be diverted, the batch movement along the pipeline and the stripping operations to be executed while injecting a new product should be established. Batch tracing then requires to track the dynamic properties of batch $i$ with time, i.e. at time points $C_{i'}$ $(i' \geq i)$. In addition, pipeline dispatchers need to know the stripping operations to carry out on batches in pipeline transit during the time interval $[C_{i'} - L_{i'}, C_{i'}]$. The problem constraints that are aimed to tracing batches and defining stripping operations will be called *batch-tracing constraints*. They involve the following new variables:

(a) Pipeline volumetric coordinate of batch $i \in I$ at time point $C_{i'}(F_i^{(i')})$.
(b) Batch size at time point $C_{i'}(W_i^{(i')})$.
(c) Amount of material diverted from batch $i$ to depot $j$ during the time interval $[C_{i'} - L_{i'}, C_{i'}](D_{i,j}^{(i')})$.
(d) Accessibility at the interconnection between the line and depot $j$ from batch $i$ during the time interval $[C_{i'} - L_{i'}, C_{i'}]$(binary $x_{i,j}^{(i')}$).

Batch-tracing constraints just involve a single set of binary variables $x_{i,j}^{(i')}$ through which the model can establish whether diverting batch $i \in I$ to depot $j$ while pumping a new batch $i' \in I^{\text{new}}$ $(i' \geq i)$ is or is not a feasible action. It will be feasible only if batch $i$ has arrived at (but not surpassed) depot $j$ before or during the time interval $[C_{i'} - L_{i'}, C_{i'}]$ and, consequently, $x_{i,j}^{(i')} = 1$. In turn, the volume-scaled variable $F_i^{(i')}$ stands for the location of the farthest extreme end of batch $i$ from the origin, i.e. the upper coordinate of batch $i$, while $W_i^{(i')}$ represents its volume content, both at time point $C_{i'}(i' \geq i)$. The interface between batches $i$ and $(i+1)$ is just a small volume at the upper edge of batch $(i+1)$ that must be discarded and separated at the farthest distribution terminal.

Fig. 2 shows a sequence of four "old" batches $I = \{\text{B4–B3–B2–B1}\}$ containing products $\{\text{P1–P3–P4–P2}\}$, respectively, already in the line at the start of pumping a new batch $i' = \{\text{B5}\}$. Values for the model variables $(x_{i,j}^{(i')}, W_i^{(i')})$ before and after pumping batch B5 are all shown in Fig. 2. Though some amount of product P3 can be diverted from B3 to depot D2 while injecting B5 because $x_{\text{B3,D2}}^{(\text{B5})} = 1$, no material is really transferred. As a result, there is no change in the size of batch B3 and, therefore, $W_{\text{B3}}^{(\text{B4})} = W_{\text{B3}}^{(\text{B5})} = 200$. A similar situation can also be described for batches B2 and B4 both keeping the same size while pumping B5. However, a portion of batch B1 (containing product P2) similar to the injected volume of B5 has been diverted to the tankage at depot D4. Since the volume of B5 is equal to 60 volumetric units, the size of B1 is decreased by the same amount, i.e. $W_{\text{B1}}^{(\text{B4})} - W_{\text{B1}}^{(\text{B5})} = 60$.

### 4.3. Monitoring depot inventories and product deliveries to local markets

The entire line must be stopped if there is insufficient storage capacity at some depot to receive the specified amount of product from a batch in transit. Then, a pipeline scheduling model should be capable of monitoring depot inventory levels to prevent from defining: (a) batch stripping operations causing tank overloading, and (b) product shipments from depots to neighboring markets that cannot be afforded due to lack of inventory. Tracking product inventories at depots over time requires to

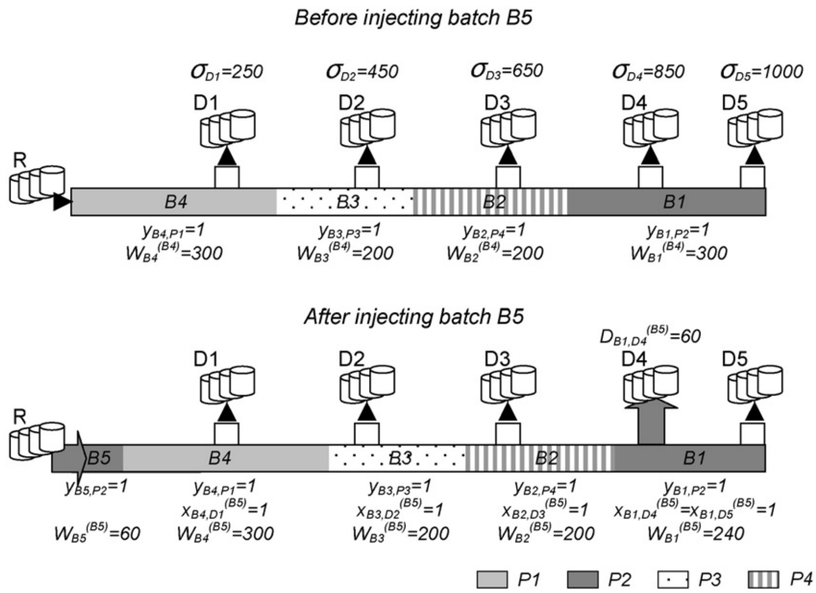


Fig. 2. A simple example illustrating the meaning of major model variables.

establish their values at the time points $C_i$, $i \in I^{\text{new}}$. Moreover, product supplies to local markets must be scheduled in such a way that the specified demands at the end of each period $t$ be timely satisfied to minimize backorder costs. Problem constraints dealing with these issues will be referred to as *depot inventory management constraints*. They involve the following additional variables:

(a) Inventory level of product $p$ in depot $j$ at time point $C_{i'}$ ($\text{ID}_{p,j}^{(i')}$).
(b) Amount of product $p$ shipped through stripping operations to depot $j$ during the time interval $[C_{i'} - L_{i'}, C_{i'}]$ ($\text{DP}_{p,j}^{(i')}$).
(c) Supply of product $p$ from depot $j$ to local markets over the time interval $[C_{i'-1}, C_{i'}]$ ($\text{DM}_{p,j}^{(i')}$).
(d) Backorder of product $p$ destined to a local market supplied from depot $j$ in time period $t$ ($B_{p,j,t}$).

The binary variable $w_{i,t}$ permits to establish the period $t$ to which the time point $C_i$ belongs. In this way, the overall amount of product $p$ sent from depot $j$ to a local market up to the end of time period $t$ can be computed in terms of the variable $\text{DM}_{p,j}^{(i)}$. In turn, the continuous variable $B_{p,j,t}$ represents the unsatisfied demand of product $p$ in depot $j$ at period $t$ that will be fulfilled at later periods.

If petroleum products from a single refinery are carried by the pipeline, then product inventories at refinery tanks must also be monitored. To this aim, the so-called *refinery inventory management constraints* are to be included in the pipeline scheduling model to align the planned batch injections with the specified refinery production schedule. The section devoted to refinery inventory management constraints has been included in Appendix A. In addition, there is a small group of constraints defining the size and location of old batches already in the pipeline at $t = 0$. They are referred to as the *initial conditions*.

## 5. Mathematical framework for the dynamic pipeline scheduling problem (DPSP)

### 5.1. Batch-defining constraints

#### 5.1.1. Product allocation

A batch to be pumped in the pipeline contains at most one single refined petroleum product. Then,

$$\sum_{p \in P} y_{i,p} \leq 1, \quad \forall i \in I^{\text{new}} \tag{1}$$

For fictitious batches never pumped in the pipeline $y_{i,p} = 0$, $\forall p \in P$.

#### 5.1.2. Batch sequencing

The injection of a new batch $i \in I^{\text{new}}$ in the pipeline should start after dispatching the previous one $(i-1)$ and performing the subsequent changeover operation.

$$C_i - L_i \geq C_{i-1} + \tau_{p,p'}(y_{i-1,p'} + y_{i,p} - 1), \quad \forall i \in I^{\text{new}};\ p, p' \in P \tag{2}$$

$$L_i \leq C_i \leq h_{\max}, \quad \forall i \in I^{\text{new}} \tag{3}$$

where $C_i$ is the completion time for the pumping run of batch $i \in I^{\text{new}}$, $L_i$ the related duration and $h_{\max}$ is the overall length of the scheduling horizon. $h_{\max}$ is computed by adding the equal or unequal lengths of all time periods included in the multi-period rolling horizon. Constraint (2) becomes active whenever the new batches $(i-1)$ and $i$ contain products $p'$ and $p$, respectively. For a pair of non-fictitious batches $(i-1, i)$, only one of the constraints (2) will become binding at the optimum. Fig. 3 depicts a time horizon comprising just a single period with a length of 168 h. At time 0, it begins the pumping of batch B1 up to time $C_1 = 24$, i.e. a run duration equal to $L_1 = 24$. Six hours


Fig. 3. Batch Sequencing.

later, it follows the injection of batch B2 up to time $C_2 = 58$ with $L_2 = 28$.

*5.1.3. Initial batch size and pumping run duration*

If $Q_i$ is the initial size of the new batch $i$ injected in the pipeline, the duration of the related pumping run ($L_i$) should satisfy the following condition:

$$\mathrm{vb}_{\min} L_i \leq Q_i \leq \mathrm{vb}_{\max} L_i, \quad \forall i \in I^{\mathrm{new}} \tag{4}$$

to ensure that the pump rate will belong to the feasible range defined by the minimum ($\mathrm{vb}_{\min}$) and maximum ($\mathrm{vb}_{\max}$) permissible values. Moreover, $L_i$ must be neither higher than the specified maximum length $l_{\max,p}$ nor lower than the minimum one $l_{\min,p}$, just in case the batch $i$ is a non-fictitious one ($\sum_p y_{i,p} = 1$).

$$\sum_{p \in P} y_{i,p} l_{\min,p} \leq L_i \leq \sum_{p \in P} y_{i,p} l_{\max,p}, \quad \forall i \in I^{\mathrm{new}} \tag{5}$$

Fig. 4 illustrates a pipeline schedule involving the pumping of four product batches B1–B4, in that order.

Each line in the diagram represents the pipeline condition at the completion time of a pumping run, assuming that the pump rate should pertain to the range: $2.5 \leq \mathrm{vb}_i \leq 8$. The first injected batch B1 containing 150 volumetric units ($10^2\,\mathrm{m}^3$) of product P2 is pumped from time 0 to time 24. The pumping rate is about 6.25 units per hour. The second batch consisting of 80 units of product P4 is pumped from time 30 to 58 at a rate of 2.86 units per hour. B3 contains 180 units of product P1 and is pumped from time 70 to 93 at a rate of 7.83 units per hour, whereas B4 is injected at a pump rate of 2.61 units per hour, from time 102 to 125, conveying 60 units of product P3.

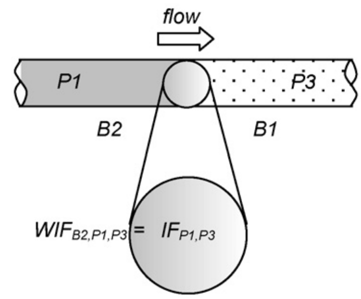


Fig. 5. Interface material between batches B2 and B1.

In order to accelerate the branch-and-bound search for the optimal schedule, fictitious batches $i \in I^{\mathrm{new}}$ featuring $\sum_p y_{i,p} = 0$ and obviously $L_i = 0$ at the optimum should be left at the end of the batch sequence. If NR is the number of pumping runs being executed, the last elements $\{|I^{\mathrm{new}}| - \mathrm{NR}\}$ of the set $I^{\mathrm{new}}$ should be reserved for fictitious batches never injected in the pipeline. Therefore, the following constraint should be added to the problem formulation:

$$\sum_{p \in P} y_{i,p} \leq \sum_{p \in P} y_{i-1,p}, \quad \forall i \in I^{\mathrm{new}} \tag{6}$$

*5.1.4. Interface volume between consecutive batches*

By convention, batch $(i-1) \in I$ has been pumped in the line just before batch $i \in I$. Then the volume of the interface between such adjacent batches will never be lower than the parameter $\mathrm{IF}_{p,p'}$ denoting the size of the transmix between products $p$ and $p'$, just in case batches $(i-1)$ and $i$ contain products $p'$ and $p$, respectively (see Fig. 5). Otherwise, the constraint will become redundant. Likewise previous approaches, the value of $\mathrm{IF}_{p,p'}$ for any ordered pair of products $(p, p')$ is assumed to be known and independent of the pump rate. In contrast to discrete representations, the proposed continuous model is able to account and trace transmix volumes from the origin to the last distribution terminal.

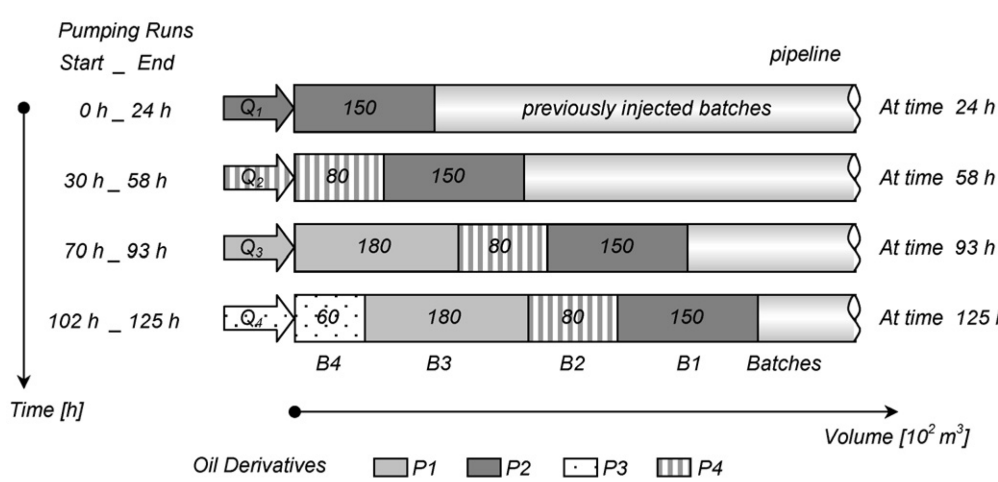


Fig. 4. A simple representation of the pipeline operations schedule.

$$\mathrm{WIF}_{i,p,p'} \geq \mathrm{IF}_{p,p'}(y_{i-1,p'} + y_{i,p} - 1),$$
$$\forall i \in I,\ i > 1,\ p, p' \in P \tag{7}$$

By adding the values of $\mathrm{WIF}_{i,p,p'}$ for any pair of products $(p, p')$ with $p \neq p'$, one can determine the volume of the transmix $\mathrm{WIF}_i$ between batches $i$ and $i-1$. If the amount of transmix rather than the transmix reprocessing cost is to be minimized, then product subscripts can be ignored and the single-subscript variable $\mathrm{WIF}_i$ replaces $\mathrm{WIF}_{i,p,p'}$ in constraint (7). As already mentioned, the interface material is never transferred to intermediate depots. Therefore, it will remain in the pipeline until reaching the final depot where it is withdrawn and reprocessed (Rejowski & Pinto, 2003). Otherwise, a new interface will be permanently generated, thus leading to higher product losses. Moreover, it will act as a plug between incompatible products when the batch separating them vanishes through stripping operations.

#### 5.1.5. Forbidden product sequences

Because of product contamination, some sequences of products in the pipeline are forbidden. If $(p, p')$ represents a forbidden sequence of products, a pair of batches containing products $p$ and $p'$ must not be consecutively pumped in the pipeline. Then, the following constraint is added to the problem formulation,

$$y_{i-1,p} + y_{i,p'} \leq 1, \quad \forall i \in I^{\mathrm{new}} \tag{8}$$

#### 5.1.6. Daily peak hours usage

Usually, oil pipeline operators avoid running pump stations at daily peak periods because a much higher energy price must be paid for the electrical power consumption. Though the total pipeline capacity can be calculated by multiplying the maximum pump rate ($\mathrm{vb}_{\max}$) by the total length of the planning horizon ($h_{\max}$), some loss of transportation capacity results from stopping pipeline activity during peak-hour intervals. Therefore, the pumping stations should never run beyond a maximum working time $\mathrm{hw}_{\max}$ given by the horizon length $h_{\max}$ reduced by the accumulated daily peak hours ($\mathrm{PH}_{\max}$), i.e. $\mathrm{hw}_{\max} = h_{\max} - \mathrm{PH}_{\max} \leq h_{\max}$, except for cases where the peak-hour usage measured by the variable PH ($\leq \mathrm{PH}_{\max}$) is necessary to meet critical due-dates. Thus, the overall pipeline usage should never exceed the effective pipeline transportation capacity:

$$\sum_{i \in I^{\mathrm{new}}} Q_i \leq \mathrm{vb}_{\max}(\mathrm{hw}_{\max} + \mathrm{PH}) \tag{9}$$

#### 5.1.7. Completion time period for the pumping run of a new batch $i \in I^{new}$

The proposed MILP formulation is capable of dealing with multiple delivery due-dates. Let us assume that the rolling horizon is composed by several time periods ($t \in T$) of equal or unequal length such that delivery due dates always occur at period ends. For example, $\mathrm{dd}_t$ will denote the upper extreme of period $t$. In order to establish whether or not some product delivery to a particular distribution terminal has been completed before the specified due date, it is important to know the time period $t$ at which it finishes. As already mentioned, the model variable $D_{i,j}^{(i')}$ provides the amount of material diverted from batch $i$ to depot $j$ while injecting the new batch $i'$. More precisely, the amount $D_{i,j}^{(i')}$ is entirely stored in depot $j$ at time point $C_{i'}$ if the pumping of batch $i'$ finishes at period $t$, i.e. the completion time $C_{i'}$ belongs to the range $\mathrm{dd}_{t-1} \leq C_{i'} \leq \mathrm{dd}_t$. Therefore, such a quantity $D_{i,j}^{(i')}$ will be available at terminal $j$ to meet a product demand with due date $\mathrm{dd}_{t'} \geq \mathrm{dd}_t$. In particular, it can be allocated to meet a requirement of product $p$ from depot $j$ at time $\mathrm{dd}_{t'} \geq \mathrm{dd}_t$ just in case the batch $i$ contains product $p$ ($y_{i,p} = 1$).

As already mentioned, the binary variable $w_{i,t}$ is introduced to indicate that the injection of batch $i \in I^{\mathrm{new}}$ in the pipeline is completed within the period $t$ whenever $w_{i,t} = 1$. Consequently, the last product delivery from the pipeline to depots while pumping batch $i$ will finish at period $t$. The value of $w_{i,t}$ should satisfy the subset of constraints (10)–(12). Constraint (10) states that the dispatching of a non-fictitious batch $i \in I^{\mathrm{new}}$ allocated to some product $p$ ($\sum_p y_{i,p} = 1$) must be completed at some period $t$ of the planning horizon. Then,

$$\sum_{t \in T} w_{i,t} = \sum_{p \in P} y_{i,p}, \quad \forall i \in I^{\mathrm{new}} \tag{10}$$

For a fictitious batch $i \in I^{\mathrm{new}}$, $\sum_t w_{i,t} = 0$. If the run $i \in I^{\mathrm{new}}$ is completed at period $t$, i.e. $\mathrm{dd}_{t-1} \leq C_{i'} \leq \mathrm{dd}_t$, then the following conditions must be fulfilled:

$$C_i \geq \mathrm{dd}_{t-1} w_{i,t} \tag{11}$$

$$C_i \leq \mathrm{dd}_t + (1 - w_{i,t})(h_{\max} - \mathrm{dd}_t), \quad \forall i \in I^{\mathrm{new}},\ t \in T \tag{12}$$

Otherwise, constraints (11) and (12) both become redundant. Note that run $i \in I^{\mathrm{new}}$ can be started at some period $t' < t$ and finished at period $t$ since nothing is said about the time at which the pumping of batch $i$ begins.

Fig. 6 shows a pipeline schedule over a multiperiod time horizon comprising 6 days (144 h). It is divided into 4 time periods of unequal length: T1 (2-day length), T2 (1-day length), T3 (1-day length), T4 (2-day length). Therefore, there are four delivery due-dates occurring at the end of a time period ($\mathrm{dd}_1$: 48; $\mathrm{dd}_2$: 72; $\mathrm{dd}_3$: 96; $\mathrm{dd}_4$: 144). On the other hand, the schedule includes 4 pumping runs: B1 from time 0 to 23, B2 from time 35 to 65, B3 from time 70 to 93 and B4 from time 102 to 125. By definition, the first pumping run is completed at time period T1 ($w_{\mathrm{B1,T1}} = 1$) and the second ends at T2 ($w_{\mathrm{B2,T2}} = 1$), although it has begun in the previous one. The third run finishes at time period T3 ($w_{\mathrm{B3,T3}} = 1$) and the last one is completed in T4 ($w_{\mathrm{B4,T4}} = 1$).

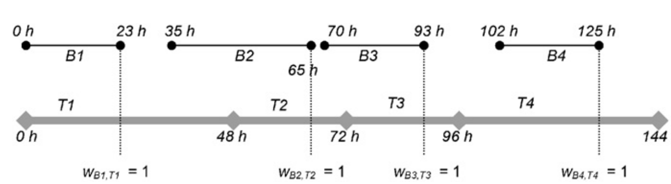


Fig. 6. A simple pumping run schedule over a multiperiod planning horizon.

### 5.2. Batch-tracing constraints

#### 5.2.1. Pipeline coordinates of batch $i \in I^{new}$ at time point $C_{i'}$

Let $F_i^{(i')}$ denote the upper volumetric coordinate of batch $i \in I$ in pipeline transit at the final pumping time of batch $i'$, i.e. $C_{i'}(i' \in I^{\text{new}}, i' \geq i)$. In other words, it is the pipeline volume between the origin and the interface separating batches $i$ and $(i-1)$ at time $C_{i'}$. Therefore, the value of $F_i^{(i')}$ is equal to the sum of the upper coordinate for the next batch $(i+1)$, $[F_{i+1}^{(i')}]$, plus the content of batch $i$, $[W_i^{(i')}]$, both at time $C_{i'}$. Note that the batch $(i+1)$ travels just behind batch $i$ and the interface volume between batches $(i+1)$ and $i$ is included in the size of batch $(i+1)$.

$$F_{i+1}^{(i')} + W_i^{(i')} = F_i^{(i')}, \quad \forall i \in I, \forall i' \in I^{\text{new}}, i' \geq i \tag{13}$$

Moreover, the lower coordinate for batch $i \in I$ at time $C_{i'}$ is $F_{i+1}^{(i')}$. Therefore, batch coordinates, batch contents and batch deliveries to distribution terminals are all traced at the problem time points, i.e. at the pumping run completion times $C_{i'}$, $i' \in I^{\text{new}}$. By definition, changes in pipeline diameter are automatically taken into account by the batch volumetric coordinate $F_i^{(i')}$ and the batch size $W_i^{(i')}$.

Fig. 7 describes the pipeline status after injecting B4 (at time $C_4$). Since the pipeline always remains completely full of products, the lower volumetric coordinate of batch $i$ corresponds to the upper coordinate of the following batch $(i+1)$. For instance, $F_{\text{B1}}^{(\text{B4})} - W_{\text{B1}}^{(\text{B4})} = F_{\text{B2}}^{(\text{B4})}$, $F_{\text{B2}}^{(\text{B4})} - W_{\text{B2}}^{(\text{B4})} = F_{\text{B3}}^{(\text{B4})}$ and so on.

#### 5.2.2. Material diverted from a new batch $i \in I^{new}$ to depots while being injected

Let $W_i^{(i)}$ be the volume of batch $i \in I^{\text{new}}$ in the pipeline at the completion time of its own pumping run $C_i$. If $Q_i$ is the original size of batch $i$, then $[Q_i - W_i^{(i)}]$ is the volume of material transferred from batch $i$ to depots while being injected in the line, i.e. during the time interval $[C_i - L_i, C_i]$. Obviously, $Q_i \geq W_i^{(i)}$ and the lower coordinate of batch $i$ at time $C_i$ is equal to zero.

$$Q_i = W_i^{(i)} + \sum_{j \in J} D_{i,j}^{(i)}; \qquad F_i^{(i)} - W_i^{(i)} = 0, \quad \forall i \in I^{\text{new}} \tag{14}$$

Fig. 8 depicts the pumping run of batch B4. It goes from time $C_4 - L_4$ to $C_4$ to put a volume $Q_4 = 250$ units of product P3 in the line. However, not all of the pumped product remains in the pipeline at time $C_4$. Part of B4 has been supplied to the nearest depot D1 ($D_{\text{B4,D1}}^{(\text{B4})} = 50$ units) while pumping batch B4 itself. As a result, the final content is $W_{\text{B4}}^{(\text{B4})} = 200$ units. Note that the upper coordinate of batch B4 equals its volume ($F_{\text{B4}}^{(\text{B4})} = W_{\text{B4}}^{(\text{B4})}$)

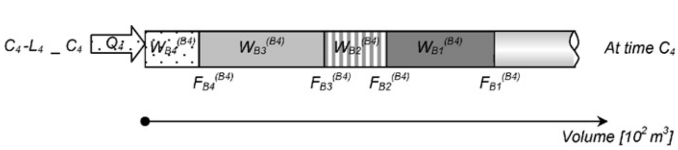


Fig. 7. Positioning of batches in the pipeline.

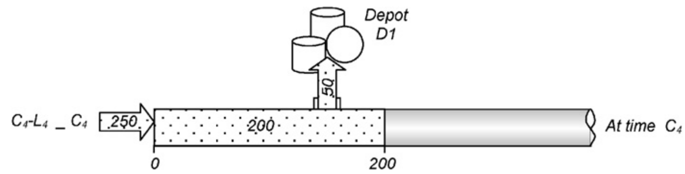


Fig. 8. Injecting batch B4 in the line and diverting material to depot D1.

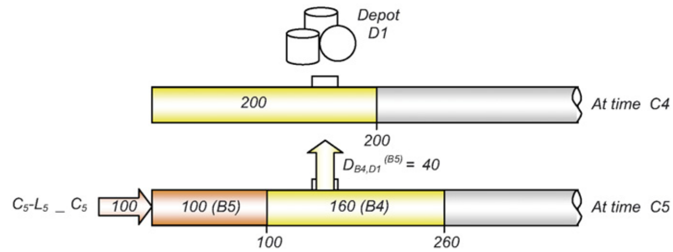


Fig. 9. Batch movement and product delivery while pumping batch B5.

and is beyond the location of depot D1 ($F_{\text{B4}}^{(\text{B4})} > \sigma_{\text{D1}}$) at time $C_4$. In other words, the transfer of material from B4 to D1 is a feasible action.

#### 5.2.3. Material diverted from batch $i \in I$ to depots while pumping a later batch $i' \in I^{new}$

By definition, $C_{i'}$ is the time at which the pumping of a new batch $i' \in I^{\text{new}}$ has been completed. Let us assume that batch $i \in I$ $(i < i')$ is in the pipeline before injecting $i'$. Then, the volume of batch $i$ at time $C_{i'}$ is given by the difference between its size at time $C_{i'-1}$ and the total volume transferred from batch $i$ to depots while injecting batch $i'$.

$$W_i^{(i')} = W_i^{(i'-1)} - \sum_{j \in J} D_{i,j}^{(i')}, \quad \forall i \in I, \forall i' \in I^{\text{new}}, i' > i \tag{15}$$

In multiproduct pipeline operation, the injection of a new batch has a double purpose: (1) to push shipments forward through the pipeline and (2) to deliver products to distribution terminals. Fig. 9 shows the location of batch B4 at time $C_4$ and the events taking place while pumping B5 from time $C_5 - L_5$ to $C_5$. Batch B5 contains 100 units of product P1. Before injecting B5, the size of B4 just dispatched through the line was $W_{\text{B4}}^{(\text{B4})} = 200$ units of product P3. While pumping the next batch B5, 40 units of product P3 from B4 have been supplied to depot D1. Therefore, the size of B4 at time $C_5$ has been reduced to: $W_{\text{B4}}^{(\text{B5})} = W_{\text{B4}}^{(\text{B4})} - D_{\text{B4,D1}}^{(\text{B5})} = 200 - 40 = 160$. The remaining 60 units of B5 push forward batch B4 from the location $F_{\text{B4}}^{(\text{B4})} = 200$ (at time $C_4$) to $F_{\text{B4}}^{(\text{B5})} = 260$ (at time $C_5$).

#### 5.2.4. Feasibility conditions for diverting material from batches in transit to depots

The transfer of material from batch $i \in I$ conveying product $p$ to depot $j \in J_p$ is feasible only if the physical connection to depot $j$ is reachable from batch $i$. The set $J_p$ includes all depots demanding product $p$. Fulfillment of such a feasibility condition while pumping a later batch $i' \in I^{\text{new}}(i' \geq i)$ requires that:

(a) the upper coordinate of batch $i$ at time $C_{i'}$ decreased by the volume of the interface material ($\sum_p \sum_{p'} \mathrm{WIF}_{i,p,p'}$), should never be lower than the $j$th terminal coordinate $\sigma_j$ (except for the farthest depot, where interface material is removed). The feasibility condition for the farthest depot $|J|$ is achieved when $F_i^{(i')} = \sigma_{|J|}$;

(b) the lower coordinate of batch $i$ at time $C_{i'-1}$ must be less than the depot coordinate $\sigma_j$ by at least a certain volume $\varphi$. The value of $\varphi$ represents the total volume of product to be transferred from batch $i$ to distribution terminals along the pipeline up to depot $j$ (including $j$) while pumping batch $i'$.

Let $x_{i,j}^{(i')}$ be a binary variable denoting that the $j$th-terminal tankage is reachable from batch $i$ while injecting batch $i'$ ($x_{i,j}^{(i')} = 1$). Otherwise, $x_{i,j}^{(i')} = 0$ and no material can be transferred from batch $i$ to depot $j$. Therefore,

$$D_{i,j}^{(i')} \le D_{\max} x_{i,j}^{(i')}, \quad \forall i \in I,\ \forall i' \in I^{\text{new}},\ i' \ge i,\ \forall j \in J \tag{16}$$

where $D_{\max}$ is an upper bound on the amount of material that can be transferred from batch $i$ to depot $j$. Moreover, constraints (17) and (18) stand for the feasibility conditions (a) and (b), respectively.

$$\begin{aligned} & F_i^{(i')} - \sum_{p \in P} \sum_{p' \in P, p' \ne p} \mathrm{WIF}_{i,p,p'} \ge \sigma_j x_{i,j}^{(i')}, \\ & \forall i \in I,\ \forall i' \in I^{\text{new}},\ i' \ge i,\ \forall j < |J| \\ & F_i^{(i')} \ge \sigma_j x_{i,j}^{(i')}, \quad \forall i \in I,\ \forall i' \in I^{\text{new}},\ i' \ge i,\ j = |J| \end{aligned} \tag{17}$$

$$\begin{aligned} & F_i^{(i'-1)} - W_i^{(i'-1)} + \sum_{k=1}^{j} D_{i,k}^{(i')} \le \sigma_j + (\sigma_{|J|} - \sigma_j)(1 - x_{i,j}^{(i')}), \\ & \forall i \in I,\ \forall i' \in I^{\text{new}},\ i' \ge i,\ \forall j \in J \end{aligned} \tag{18}$$

Fig. 10 revisits the shipment of batch B5 through the pipeline. It can be noted that the upper coordinate of the prior batch B4 in the line at time $C_5$ ($F_{\text{B4}}^{(\text{B5})} = 260$), even deducting the interface material, is beyond the location of depot D1 ($\sigma_{\text{D1}} = 160$). Condition (b) is also satisfied because the LHS of Eq. (18) is equal to $40 < \sigma_{\text{D1}}$. Then, B4 has reached depot D1 ($x_{\text{B4,D1}}^{(\text{B5})} = 1$) and some material from B4 can be diverted to D1 while pumping B5. In contrast, the batch B5 at time $C_5$ ($F_{\text{B5}}^{(\text{B5})} = 100 < 260$)

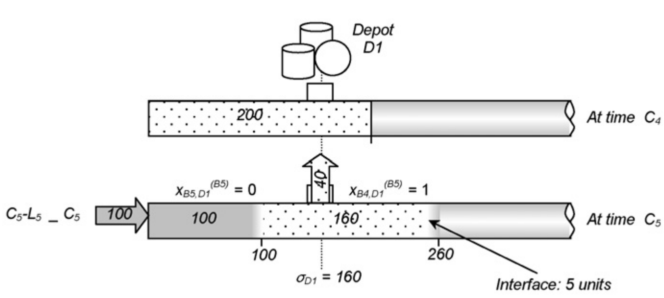


Fig. 10. Feasibility condition for diverting material from batches to depot tankage.

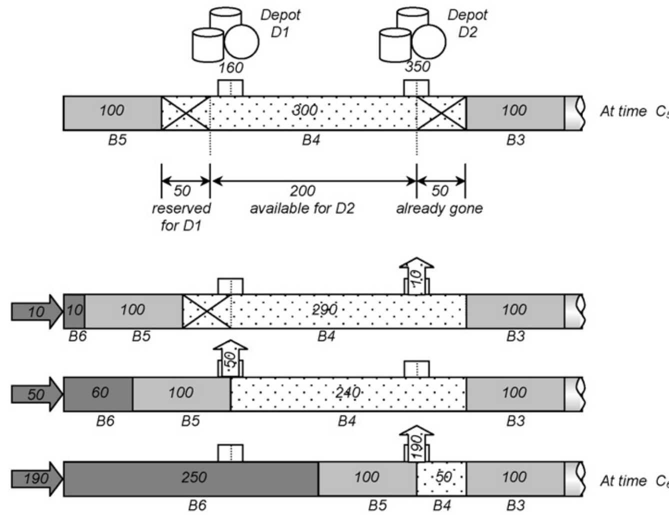


Fig. 11. Successive "stripping" operations on batch B4.

has not arrived at depot D1 yet. Therefore, $x_{\text{B5,D1}}^{(\text{B5})} = 0$, and no product can be delivered from B5 to depot D1 during the time interval $[C_5 - L_5, C_5]$.

Fig. 11 shows multiple product deliveries from batch B4 containing P3 to depots D1 and D2. At time $C_5$ (first line), the batch B4 contains 300 units of product P3 and its location ($F_{\text{B4}}^{(\text{B5})} = 400$) is already beyond the location of depot D2 ($\sigma_2 = 350$). Let us analyze how much product can be diverted from B4 to D2 while pumping B6. Accounting for the flow direction, the portion of batch B4 that can no longer be transferred to D2 and flows to more distant terminals along the line amounts to 50 units. Moreover, some material from batch B4 has been reserved for depot D1 (50 units). Hence, only $(300 - 50 - 50) = 200$ units of product P3 can at most be delivered to depot D2.

The following lines in Fig. 11 depict the evolution of B4 in size and location during the injection of batch B6 comprising 250 volumetric units of product P2. First, the inlet valve to the tank storing P3 at terminal D2 is open to discharge 10 units of product P3. After pumping 10 units of B6 in the pipeline, the reserved portion of B4 reaches the location of terminal D1. At that time, the line supplying product P3 to terminal D2 must be closed and simultaneously the valve to the tankage at depot D1 must be opened. By this operation, usually called "making a cut", 50 units of P3 are diverted from B4 to D1 through pumping additional 50 units of batch B6 in the pipeline. The cut must occur precisely at the time the reserved portion of B4 begins to arrive at D1. After that, the dispatcher closes the valve to D1 and reopens the inlet pipe to depot D2 so as to deliver the remaining 190 units of product P3 from B4 by inserting the rest of batch B6 in the line at the head terminal.

### 5.2.5. *Bound on the amount of material diverted from batch $i \in I$ to depots $j \in J$*

The total volume transferred from batch $i \in I$ to depots $j \in J$ while pumping a new batch $i' \in I^{\text{new}}$ ($i' \ge i$) over the time interval $[C_{i'} - L_{i'}, C_{i'}]$ must never exceed the saleable content of batch $i$ available before pumping batch $i'$, i.e. at time point $C_{i'-1}$.

$$\sum_{j<|J|} D_{i,j}^{(i')} \leq W_i^{(i'-1)} - \sum_{p \in P} \sum_{p' \in P, p' \neq p} \mathrm{WIF}_{i,p,p'},$$

$$\forall i \in I,\ \forall i' \in I^{\text{new}},\ i' > i \qquad (19)$$

$$\sum_{j \in J} D_{i,j}^{(i')} \leq W_i^{(i'-1)}, \quad \forall i \in I,\ \forall i' \in I^{\text{new}},\ i' > i$$

A model improvement with regards to previous approaches is the fact that just saleable material can be transferred from the pipeline to depots, except for the farthest one where the transmix is usually removed for reprocessing.

*5.2.6. Overall pipeline balance during the shipment of batch $i' \in I^{new}$*

Because of the liquid incompressibility condition, the overall volume transferred from batches in transit along the pipeline to depots $j \in J$ while dispatching the new batch $i' \in I^{\text{new}}$ must be equal to $Q_{i'}$, i.e. the initial volume of $i'$.

$$\sum_{i \in I, i \leq i'} \sum_{j \in J} D_{i,j}^{(i')} = Q_{i'}, \quad \forall i' \in I^{\text{new}} \qquad (20)$$

Fig. 12 describes the pipeline status at times $C_4$ (before pumping batch B5) and $C_5$ (after completing the pumping of B5), respectively.

As already pointed out, the pipeline remains full of oil derivatives at any time. At time $C_4$ there are 200 units of product P2 in batch B1, 180 units of product P4 in B2, 190 units of product P1 in B3 and 200 units of product P3 in B4. The total pipeline content amounts to 770 volumetric units. Since the new batch B5 being sent through the pipeline comprises 150 units of product P1, then 150 units of different products must be sequentially delivered to depots, thus preserving the mass overall balance. Depot D1 picks up 50 units of product P3 from B4, depot D2 receives 50 units of product P4 from B2 and 30 units of P1 from B3, whereas 20 units of product P2 are diverted from batch B1 to terminal D3. Such stripping operations can take place as long as the related feasibility conditions (17) and (18) have been satisfied. However, there are indeed several ways to accomplish the above product deliveries to distribution terminals. Systematic procedures for generating more detailed pipeline schedules will be presented in a future publication (Cafaro & Cerdá, 2006).

*5.3. Depot inventory management constraints*

*5.3.1. Product deliveries from distribution terminals to neighboring markets*

Let us define the variable $\mathrm{DM}_{p,j}^{(i')}$ denoting the amount of product $p \in P$ delivered from depot $j \in J_p$ to neighboring markets demanding $p$ while injecting the new batch $i'$. Such a quantity $\mathrm{DM}_{p,j}^{(i')}$ is supplied to the market during the time interval $[C_{i'-1}, C_{i'}]$ at a permissible flow rate. Indeed, the refined products available at pipeline terminals are first sent to marketing terminals where truck load operations take place. If $\mathrm{vm}_{p,j}$ stands for the maximum feed rate of product $p$ from the pipeline terminal $j$ to the related marketing terminal, then:

$$\mathrm{DM}_{p,j}^{(i')} \leq (C_{i'} - C_{i'-1})\ \mathrm{vm}_{p,j}, \quad \forall p \in P,\ \forall j \in J_p,\ \forall i' \in I^{\text{new}} \qquad (21)$$

*5.3.2. Delivery time requirements*

Let us assume that the pumping run $i \in I^{\text{new}}$ is the last one completed at period $t$. Then $w_{i,t} = 1$ and the pumping of the next batch $(i+1)$ is completed at a later period, i.e. $w_{i+1,t} = 0$. Consequently, the amount of product $p$ already transferred from depot $j$ to the related marketing terminal during the injection of new batches $\{1, 2, 3, \ldots, i-1, i\}$ must be large enough to meet $p$th-product demands up to period $t$, i.e. from time zero to $\mathrm{dd}_t$. However, the last pumping run $i$ completed at period $t$ is not known beforehand. Consequently, the following conditional constraint must be incorporated in the problem formulation to timely meet terminal requirements:

$$\sum_{\ell=1}^{i} \sum_{\ell \in I^{\text{new}}} \mathrm{DM}_{p,j}^{(\ell)} \geq \left(\sum_{k=1}^{t} \mathrm{dem}_{p,j,k}\right)(w_{i,t} - w_{i+1,t}) - B_{p,j,t} + B_{p,j,(t-1)},$$

$$\forall p \in P,\ j \in J_p,\ t \in T,\ i \in I^{\text{new}} \qquad (22)$$

where the LHS of Eq. (22) provides the total amount of product $p$ sent to neighboring markets or the marketing terminal from depot $j$ during the pumping of new batches up to batch $i$, i.e.

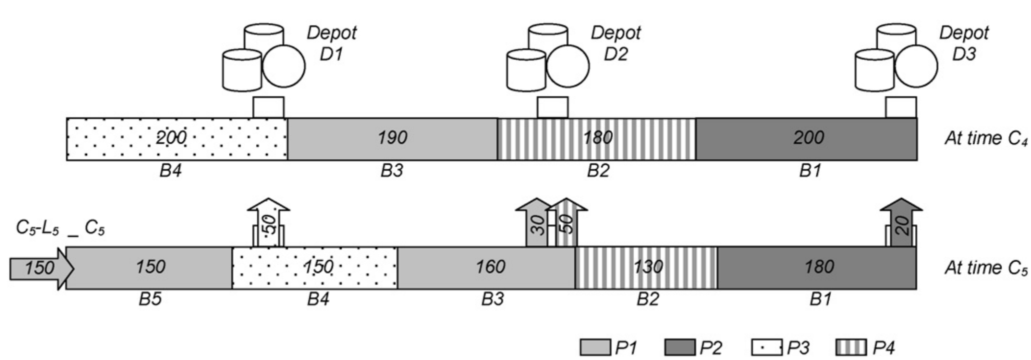


Fig. 12. Overall pipeline balance while pumping batch B5.

while injecting $\{1, 2, 3, \ldots, i\} \in I^{new}$. On the RHS of Eq. (22), the parameter $\text{dem}_{p,j,k}$ denotes the demand of product $p$ with due date $\text{dd}_k$ at terminal $j$. Moreover, the summation stands for the overall $p$th-product demand to be satisfied at terminal $j$ during the time interval [0, $\text{dd}_t$]. However, not every market request is necessarily satisfied on time. Some product shipments may tardily arrive at the desired destination. The variable $B_{p,j,t}$ represents the portion of the $p$th-product requirement from depot $j$ due at time $\text{dd}_t$ left as a backorder to be tardily fulfilled during period $t+1$. In turn, $B_{p,j,(t-1)}$ denotes a backorder of product $p$ from the prior period $(t-1)$ to be tardily met at period $t$.

In case the pumping run $i \in I^{new}$ is the last one completed at period $t$, then $w_{i,t} = 1$, $w_{(i+1),t} = 0$ and $w_{i,t} - w_{(i+1),t} = 1$. If so, the constraint (22) states that the total amount of product $p$ dispatched from terminal $j$ to neighboring markets from time zero to $C_i$ must be high enough to meet the $p$th-demand from period $k=1$ to period $k=t$, except for the backorder $B_{p,j,t}$. For any other pumping run $i' \neq i$, constraint (22) becomes redundant (see Fig. 13). To guarantee that the constraint (22) works properly, a single pumping run must at least be completed at each time period $t$ though it may not necessarily start at the same time period. To this end, the following inequality is incorporated in the problem formulation.

$$\sum_{i \in I^{new}} w_{i,t} \geq 1, \quad \forall t \in T \tag{23}$$

Fig. 13 shows the way constraint (22) is forced to be satisfied. It includes a multiperiod horizon divided into daily periods (T1, T2 and so on). Consequently, there is a pair of due-dates ($\text{dd}_1 = 24$, $\text{dd}_2 = 48$) in the first 48 h. Moreover, there are four planned pumping runs ending at time points: $C_1 = 10$, $C_2 = 24$, $C_3 = 44$ and $C_4 = 52$, respectively. The first two pumping runs both finish inside time period T1 ($w_{\text{B1,T1}} = 1$, $w_{\text{B2,T1}} = 1$) whereas the third one ends during T2 ($w_{\text{B3,T2}} = 1$, and obviously $w_{\text{B3,T1}} = 0$). Finally, the fourth planned run does not end within period T2 but later, so $w_{\text{B4,T2}} = 0$. The injection of batch B2 is the last one completed in period T1 ($C_1 < C_2 \leq \text{dd}_1 < C_3$). Therefore, all material supplied from depots to markets up to time $C_2$ should be high enough to meet all product demands with due-date $\text{dd}_1$. Similarly, batch B3 is the only one whose pumping run is completed in period T2 ($C_2 < C_3 \leq \text{dd}_2 < C_4$) and, consequently, the product supplies up to time $C_3$ should satisfy all the market requirements at periods T1 and T2. None of the pumping runs is indeed forced to finish at the end of a time period. If necessary, however, the length of the last pumping run within a time period $t$ will be automatically prolonged to the end of period $t$ so as to meet all the market requests up to $\text{dd}_t$.

### 5.3.3. Monitoring product inventories in depot tanks

An efficient coordination among incoming flows from the line and outgoing flows to neighboring markets is a key operational issue since every terminal has limited storage. A lack of coordination may force to shut down the pipeline until the problem is solved. Outgoing product flows from depot tanks whose values are bounded by constraints (21) and (22) were already considered. Next, we will introduce the equations defining the depot input streams coming from the line and the product inventories in every terminal at the event points $C_i$, $i \in I^{new}$. The proposed multiperiod pipeline schedule must allow to fulfilling market demands on time while permanently keeping product inventory levels within the feasible range. In this way, neither unforeseen pipeline stops nor product backorders will arise.

(a) *Amount of product p transferred from batch $i \in I$ to depot $j \in J$ while injecting batch $i' \in I^{new}$*. Batch $i \in I$ will be conveying product $p$ only if $y_{i,p} = 1$. Let $\text{DP}_{i,p,j}^{(i')}$ be the amount of product $p$ supplied by batch $i$ to depot $j \in J_p$ during the time interval $[C_{i'} - L_{i'}, C_{i'}]$. Therefore, $\text{DP}_{i,p,j}^{(i')}$ will be equal to zero whenever $y_{i,p} = 0$. If instead $y_{i,p} = 1$, then $\text{DP}_{i,p,j}^{(i')} = D_{i,j}^{(i')}$.

(a1) Product supplies from new batches $i \in I^{new}$:

$$\text{DP}_{i,p,j}^{(i')} \leq D_{\max} y_{i,p}, \quad \forall i \in I,\ p \in P,\ j \in J_p,\ i' \in I^{new} \tag{24}$$

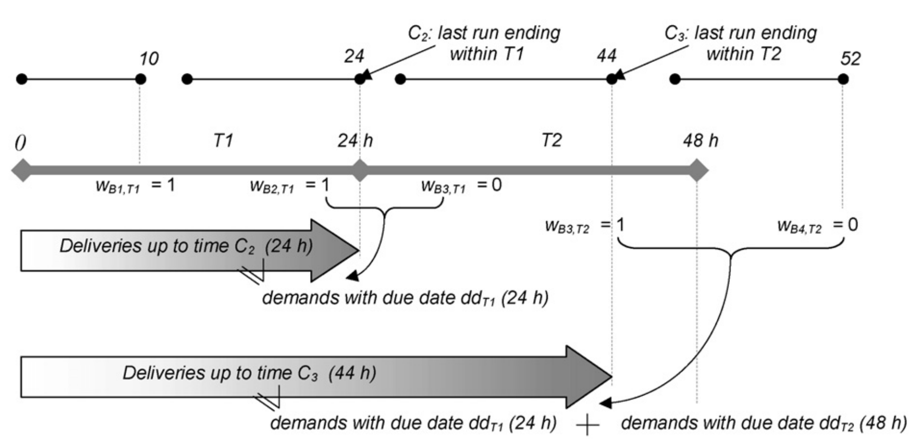


Fig. 13. Illustrating the conditional constraint (22).

$$\sum_{p \in P} \mathrm{DP}_{i,p,j}^{(i')} = D_{i,j}^{(i')}, \quad \forall i \in I,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{25}$$

(a2) Product supplies from old batches $i \in I^{\mathrm{old}}$:

$$\mathrm{DP}_{i,p,j}^{(i')} = D_{i,j}^{(i')}, \quad \forall i \in I_p^{\mathrm{old}},\ p \in P,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{26}$$

where $I_p^{\mathrm{old}}$ comprises every "old" batch involving product $p$.

(b) *Inventory feasible range*. The inventory level of product $p$ in depot $j \in J_p$ at time point $C_{i'}$ is computed through Eq. (27) by adding the stock available at time $C_{i'-1}$ to the amount ($\sum_i \mathrm{DP}_{i,p,j}^{(i')}$) provided by batches $i \in I$ conveying product $p$, and simultaneously subtracting deliveries of product $p$ from depot $j$ to local markets or the related marketing terminal ($\mathrm{DM}_{p,j}^{(i')}$). Since the value of $\mathrm{ID}_{p,j}^{(i')}$ should always remain within the feasible range defined by the specified maximum and minimum inventory levels, then the constraints (28) should also be satisfied.

$$\mathrm{ID}_{p,j}^{(i')} = \mathrm{ID}_{p,j}^{(i'-1)} + \sum_{i \in I, i \le i'} \mathrm{DP}_{i,p,j}^{(i')} - \mathrm{DM}_{p,j}^{(i')},$$
$$\forall p \in P,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{27}$$

$$(\mathrm{ID_{min}})_{p,j} \le \mathrm{ID}_{p,j}^{(i')} \le (\mathrm{ID_{max}})_{p,j},$$
$$\forall p \in P,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{28}$$

### 5.4. Initial conditions

Old batches $i \in I^{\mathrm{old}}$ already in the pipeline at the start of the scheduling horizon have been chronologically arranged by decreasing $F_i^{\mathrm{o}}$, where $F_i^{\mathrm{o}}$ stands for the upper pipeline coordinate of batch $i \in I^{\mathrm{old}}$ at the initial time. Since the old batch $(i-1)$ has been injected right before the old batch $i$, then it will be farther from the origin: $F_{i-1}^{\mathrm{o}} > F_i^{\mathrm{o}}$. Moreover, the current volume of any old batch $i$ ($W_i^{\mathrm{o}}, i \in I^{\mathrm{old}}$) and the product to which each one was assigned are all problem data, generally given by the SCADA remote system. Thus,

$$F_i^{(i'-1)} = F_i^{\mathrm{o}}, \quad \forall i \in I^{\mathrm{old}},\ i' = \mathrm{first}(I^{\mathrm{new}}) \tag{29}$$

$$W_i^{(i'-1)} = W_i^{\mathrm{o}}, \quad \forall i \in I^{\mathrm{old}},\ i' = \mathrm{first}(I^{\mathrm{new}}) \tag{30}$$

### 5.5. Problem objective function

The problem goal is to minimize the total pipeline operating cost including (i) the pumping cost, at daily normal and peak hours, (ii) the cost of reprocessing the interface material between consecutive batches, (iii) the cost of product backorders being tardily delivered to their destinations, (iv) the cost of underutilizing pipeline transportation capacity and (v) the cost of holding product inventory in refinery and depot tanks.

$$\begin{aligned}
\mathrm{Min}\ z = {} & \sum_{p \in P}\sum_{j \in J}\left(\mathrm{cp}_{p,j}\sum_{i \in I}\sum_{i' \in I^{\mathrm{new}}} \mathrm{DP}_{i,p,j}^{(i')}\right) + \rho\,\mathrm{PH} \\
& + \sum_{p' \in P, p' \ne p}\sum_{i \in I, i>1} \mathrm{cf}_{p,p'}\mathrm{WIF}_{i,p,p'} \\
& + \sum_{p \in P}\sum_{j \in J}\sum_{t \in T} \mathrm{cb}_{p,j,t} B_{p,j,t} \\
& + \mathrm{cu}\left(\mathrm{hw_{max}} + \mathrm{PH} - \sum_{i \in I^{\mathrm{new}}} L_i\right) \\
& + \frac{1}{|I^{\mathrm{new}}|}\sum_{p \in P}\left[\sum_{j \in J_p} \mathrm{cid}_{p,j}\left(\sum_{i' \in I^{\mathrm{new}}} \mathrm{ID}_{p,j}^{(i')}\right)\right. \\
& \left. + \mathrm{cir}_p\left(\sum_{i' \in I^{\mathrm{new}}} \mathrm{IRS}_p^{(i')}\right)\right]
\end{aligned} \tag{31}$$

where $\mathrm{cp}_{p,j}$ stands for the cost of pumping a unit volume of product $p$ from the oil refinery to destination $j$ during normal-hour intervals. The parameter $\mathrm{cf}_{p,p'}$ is the cost for reprocessing a unit amount of interface $p-p'$. In turn, $\rho$ is the unit-time penalty cost to be paid for operating the pipeline during peak-hour intervals. Since the pipeline usually remains idle during high-energy cost time intervals, the energy penalty cost term is often zero at the optimum. Furthermore, the parameter $\mathrm{cb}_{p,j,t}$ corresponds to the unit backorder penalty cost to tardily meet some product requirement due at period $t$ during the next time period $(t+1)$. The unit cost cu penalizes the pipeline underutilization capacity given in terms of the pipeline idle time.

Moreover, the last RHS term provides an approximate value for the inventory carrying cost at distribution centers and refinery tanks based on an estimation of the average inventory for each product. A characteristic value of the $p$th-product inventory in depot $j$ over the time interval $[C_{i'-1}, C_{i'}]$ is the one available at the end time $C_{i'}$, i.e. $\mathrm{ID}_{p,j}^{(i')}$. An average $p$th-product inventory in depot $j$ over the whole scheduling horizon can be approximated by adding the product stock estimates at the end of every potential batch injection $i' \in I^{\mathrm{new}}$ and dividing the result by $|I^{\mathrm{new}}|$. When no element of $I^{\mathrm{new}}$ stands for a fictitious batch, a good average inventory estimation is found. The inventory carrying cost for each product $p \in P$ is approximated by multiplying the average inventory at every depot $j \in J_p$ demanding product $p$ by the inventory unit cost $\mathrm{cid}_{p,j}$, and summing the results for all depots. Finally, an estimation of the overall depot inventory cost is obtained by adding the inventory cost for every product. A similar computational scheme is followed to estimate the refinery inventory carrying costs.

## 6. Updating the multiperiod pipeline schedule

There are two major reasons for a periodical review of the pipeline operations schedule:

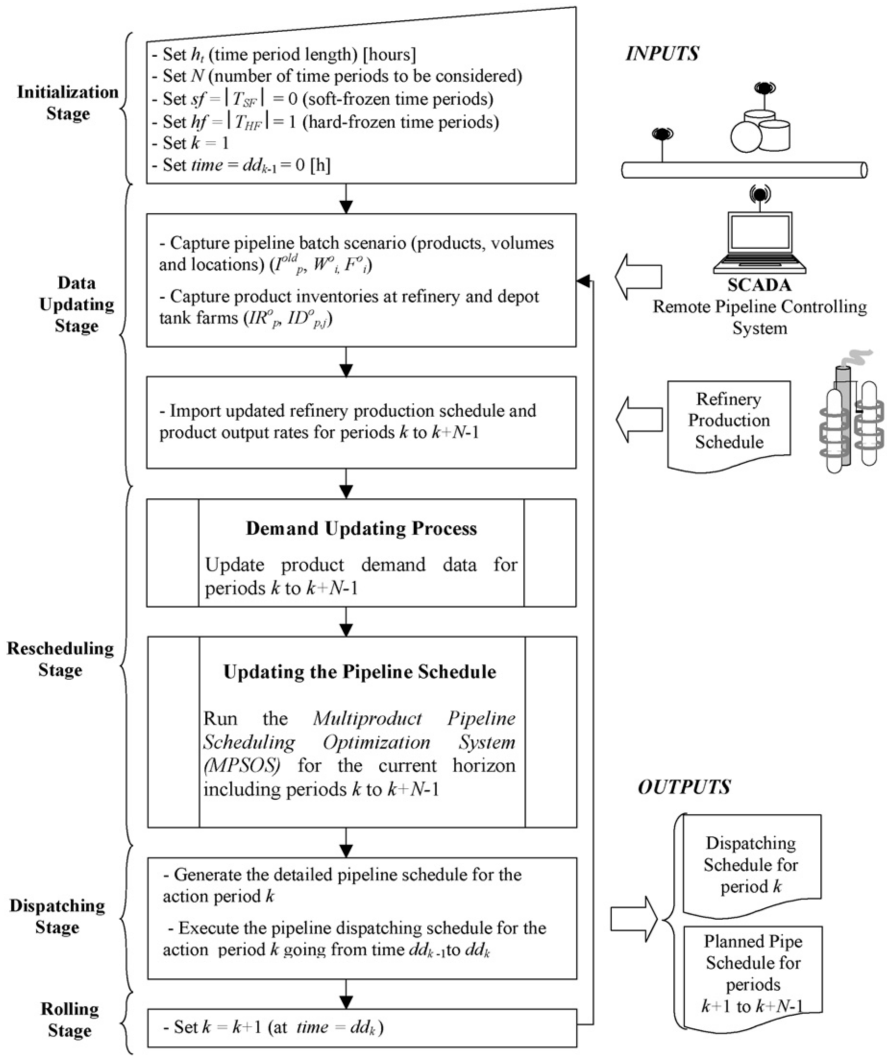


Fig. 14. Pipeline schedule update algorithm.

(1) New shipper nominations are received during the dispatching of scheduled shipments. Such further nominations must usually be delivered to the stated terminals at later periods of the current planning horizon, and they shall be inserted in the pipeline with some anticipation.

(2) A significant batch transportation lead-time, especially for shipments destined to the farthest distribution terminals. As a result, some consolidated batches scheduled for pumping at later periods of the current rolling horizon have the only purpose of pushing forward the batches already in the pipeline towards their stated destinations. Since they are required to meet yet unknown product demands due at time periods beyond the current horizon, the material inserted in the pipeline by those planned batches has nothing to do with future terminal requirements. Generally, long pumping runs are last scheduled. As the time horizon rolls, those large batches are gradually replaced by a sequence of shorter pumping runs through the periodic rescheduling process. Such smaller planned batches are mostly aimed at fulfilling recent shipper requests due at the last period of the new time horizon.

The algorithm for the periodic update of the pipeline operations schedule is described in Fig. 14. It comprises five major stages: (a) initialization, (b) problem data update, (c) pipeline schedule update, (d) batch dispatching and (e) horizon rolling and new instance generation.

### 6.1. Initialization stage

During the initialization stage, the DPSP parameters are set by the scheduler. They include:

(A) The number of time periods ($N$) into which the rolling horizon $T = \{t\}$ is divided, and the length $h_t$ (=$\mathrm{dd}_t - \mathrm{dd}_{t-1}$) of every time period $t$, expressed in hours. In the examples solved in the next section, it has been adopted: $N = 4$ and $h = 168$ h (1 week) for every period $t$. Delivery due dates just occur at period ends. Therefore, the fixed length of the scheduling horizon is $h_{\max} = N \times h = 672$ h and the due-dates over the initial horizon are: $\{\mathrm{dd}_1 = 168,\ \mathrm{dd}_2 = 336,\ \mathrm{dd}_3 = 504,\ \mathrm{dd}_4 = 672\}$.

(B) The number of different refined petroleum products to be shipped from the refinery to the stated destinations, i.e. $|P|$.
(C) The number of consolidated new batches $i \in I^{\text{new}}$ to be pumped in the pipeline along the multiperiod time horizon, i.e. the cardinality of the set $I^{\text{new}}$. The value of $|I^{\text{new}}|$ is usually set equal to: $|I^{\text{new}}| = (N \times |P|)/n$, where $n = 2.0 - 3.5$. If the adopted value for $|I^{\text{new}}|$ is not large enough, the DPSP feasible region may not include the true optimal schedule or, at worst, may be empty. Whenever the number of non-fictitious pumping runs NR at the optimum is equal to $|I^{\text{new}}|$ or the DPSP is infeasible, the value of $|I^{\text{new}}|$ must be increased by one. After that, the DPSP is to be solved again until no improvement in the value of the objective function is achieved.
(D) The permissible ranges for product inventories at refinery and depot tankage ($\text{IR}_{\min}/\text{IR}_{\max}$, $\text{ID}_{\min}/\text{ID}_{\max}$), pipeline pump rates ($\text{vb}_{\min}/\text{vb}_{\max}$) and sent-to-market delivery rates ($\text{vm}_{\max}$).
(E) The different types of pipeline operating unit costs arising in the objective function as well as the product–product interface size matrix.
(F) The time interval between two consecutive reviews of the pipeline schedule ($t_{\text{RS}}$). This schedule regeneration frequency is expressed in time periods. In the examples solved in this paper, $t_{\text{RS}} = 1$ and the pipeline rescheduling process is executed at the start of every time period. Fig. 14 also assumes $t_{\text{RS}} = 1$.
(G) The subset of hard-frozen time periods $T_{\text{HF}} \subset T$, usually including the first-period of the new rolling horizon, where the planned pipeline operations must remain unchanged even during the periodic pipeline rescheduling process. In practice, the regeneration frequency is generally equal to the number of hard frozen periods ($t_{\text{RS}} = |T_{\text{HF}}|$). The illustrative examples solved in the next section and Fig. 14 assume $t_{\text{RS}} = |T_{\text{HF}}| = 1$.
(H) The subset of soft-frozen time periods $T_{\text{SF}} \subset T$, usually including one or two periods immediately after the first one, over which the sequence of planned product injections cannot be modified. However, their pumping run lengths may be changed. In the examples solved in the next section: $T_{\text{SF}} = \emptyset$.
(I) The subset of non-frozen time periods $T_{\text{NF}} = T - \text{T}_{\text{HF}} - \text{T}_{\text{SF}}$, where the pipeline schedule can be completely reviewed.
(J) The first-period of the current moving horizon. Let us call it period $k$. The action period $k$ will be used to identify the corresponding instance of the moving horizon as it rolls over time. Set $k = 1$ for the initial horizon.

### 6.2. Data updating stage

When the rescheduling process is activated or the pipeline schedule for the initial horizon is to be generated, the next stage is to update the input data for the current horizon $k$. Usually, the pipeline schedule for the previous time horizon $k-1$ is available. This stage involves the following steps:

(A) Capture the pipeline current status from the SCADA remote system to establish the sequence of batches in transit ($I^{\text{old}}$), i.e. batch naming ($i$), product ($p_i$), size ($W_i^{\text{o}}$) and location ($F_i^{\text{o}}$). The SCADA remote system is usually available in every multiproduct pipeline network.
(B) Pick up product inventory levels at refinery and terminal tank farms ($\text{IR}_p^{\text{o}}$, $\text{ID}_{p,j}^{\text{o}}$) at the start of the current horizon $k$ from the SCADA system, i.e. at time $\text{dd}_{k-1}$.
(C) Import the updated refinery production schedule and product output rates for periods $k$ to $k+N-1$, i.e. from time $= \text{dd}_{k-1}$ to time $= \text{dd}_{k-1} + h_{\max}$. In most cases, the refinery production schedule is previously defined based on crude oil inventories, product expected demands and available production capacity.
(D) Update product demands at distribution terminals, including old demands not yet satisfied and new/cancelled shipments received while executing the pipeline schedule for the action period of the previous horizon $(k-1)$. To update terminal demands $\text{dem}_{p,j,t}$ it must be taken into account:
  (1) product deliveries to terminals accomplished during period $(k-1)$ in advance of the promised time period $t > k-1$, $\text{AD}_{p,j,t}$;
  (2) product deliveries with due date $\text{dd}_{k-1}$ that were not satisfied during period $k-1$ (backorders) and must be fulfilled on the next action period $k$, $B_{p,j,(k-1)}$.

Therefore, the updated terminal demands $\text{dem}_{p,j,t}$ are given by:

- For time period $t = k$,

$$\text{dem}_{p,j,t} = (\text{dem}_{p,j,t})^{\text{old}} + \text{NS}_{p,j,t} - \text{CS}_{p,j,t} + B_{p,j,t-1} - \text{AD}_{p,j,t}, \quad \forall p \in P,\ j \in J_p$$

- For time periods $k+1 \le t \le k+N-2$,

$$\text{dem}_{p,j,t} = (\text{dem}_{p,j,t})^{\text{old}} + \text{NS}_{p,j,t} - \text{CS}_{p,j,t} - \text{AD}_{p,j,t}, \quad \forall p \in P,\ j \in J_p$$

- For time period $t = k+N-1$ just incorporated in the rolling horizon,

$$\text{dem}_{p,j,t} = \text{NS}_{p,j,t}, \quad \forall p \in P,\ j \in J_p$$

where $(\text{dem}_{p,j,t})^{\text{old}}$ denotes terminal demand data available at time $\text{dd}_{k-1}$ and the parameters $\text{NS}_{p,j,t}/\text{CS}_{p,j,t}$ stand for the sizes of new/cancelled $p$th-product shipment nominations for terminal $j$ and period $t$ received during period $k-1$. Moreover, the sizes of anticipated product deliveries $\text{AD}_{p,j,t}$ and backorders $B_{p,j,k-1}$ can be computed from the batch dispatching schedule for period $k-1$ through the following equations,

$$\text{AD}_{p,j,t} = \max\left[0, \sum_{\ell=1}^{i_{k-1}} \text{DM}_{p,j}^{(\ell)} - \sum_{n=k-1}^{t} (\text{dem}_{p,j,n})^{\text{old}}\right], \quad \forall p \in P,\ j \in J_p,\ t = k, \ldots, k+N-1$$

$$B_{p,j(k-1)} = \max\left[0, (\mathrm{dem}_{p,j,(k-1)})^{\mathrm{old}} - \sum_{\ell=1}^{i_{k-1}} \mathrm{DM}_{p,j}^{(\ell)}\right], \quad \forall p \in P,\ j \in J_p$$

where $i_{k-1}$ is the last pumping run executed during the action period $(k-1)$, and $\mathrm{DM}_{p,j}^{(\ell)}$ represents the $p$th-product delivery to terminal $j$ while injecting batch $\ell$ at period $k-1$.

### 6.3. Pipeline rescheduling stage

This stage is the core step of the algorithm. It provides the pipeline master planning over the current rolling horizon $k$ by running the *Multiproduct Pipeline Scheduling Optimization System* (MPSOS). Its major goal is to optimize the pipeline pumping run and terminal delivery schedule based on the updated input data. Just the proposed schedule for the first period $k$ is subsequently implemented while the pipeline planning for later periods helps schedulers achieve a better coordination of the entire supply system.

### 6.4. Dispatching stage

The next step aims to generate the detailed pipeline schedule for the action period $k$ based on the pipeline master planning found in Section 6.3. In particular, the dispatching stage should account for the set of batch injections and batch stripping operations to be carried out from time $\mathrm{dd}_{k-1}$ to $\mathrm{dd}_k$. Compared with the pipeline master schedule for period $k$, some additional information is provided by the batch dispatching schedule. For instance, the sequence and timing of the planned stripping operations to be performed during the execution of any pumping run scheduled for period $k$. The pipeline master planning guarantees the existence of at least, a feasible sequence of stripping operations for each planned batch injection. Since there are usually several alternative operational schemes, some additional criteria for choosing one of them are to be considered. Algorithmic and heuristic procedures for developing the pipeline schedule at the operational level for the action period $k$ will be discussed in a future paper (Cafaro & Cerdá, 2006). In this work, we are just focused on the pipeline master planning for the action period $k$. The last planned pumping run $i_k$ to be executed in period $k$ is considered up to time $\mathrm{dd}_k$ though it can be extended over period $k+1$. If the run $i_k$ goes beyond period $k$ in the pipeline master schedule, some product deliveries from the line to depots that are planned to carry out during the last run $i_k$ must be decreased or postponed for the next period $k+1$. In the illustrative examples solved in the next section, the execution of stripping operations delivering refined products to the most distant terminals are prioritized. In other words, they are favored to be performed within period $k$.

### 6.5. Horizon rolling and new instance generation

Whenever the time interval $t_{\mathrm{RS}}$ is completed and the pipeline schedule for the first $t_{\mathrm{RS}}$ periods has already been executed (i.e., at time $\mathrm{dd}_{k-t\mathrm{RS}} + h\,t_{\mathrm{RS}}$), the time horizon rolls ahead $t_{\mathrm{RS}}$ periods. If $t_{\mathrm{RS}} = 1$, the new action period will be $k = k+1$ and the new instance $k+1$ of the moving horizon is thus generated. To update the pipeline master schedule for the new horizon, the rescheduling process should be activated. Therefore, the execution of stages in Sections 6.2–6.4 is to be restarted.

## 7. Results and discussion

### 7.1. Case study: a real-world multiproduct pipeline system

To illustrate the advantages of the proposed dynamic pipeline scheduling approach, a modified version of the single-period real-world case study introduced by Rejowski and Pinto (2003), now involving a much longer multiperiod time horizon and multiple delivery due-dates, has been tackled. It considers the distribution of four refined petroleum products (P1, gasoline; P2, diesel; P3, LPG; P4, jet fuel) through a single pipeline of 955 km to five terminals (D1–D5) over a rolling time horizon steadily comprising four 1-week periods. Product demands at depots D1–D5 nominated for periods $t_1$–$t_4$ are given in Table 1. They should be delivered to local markets (or marketing terminals) before period ends. Such terminal requirements may be updated at the start of any new instance of the rolling horizon.

Demand data for the subsequent time periods $t_5$–$t_7$ still unknown at the time of solving the pipeline schedule problem (PSP) for the initial horizon $\{t_1–t_4\}$ become gradually available as the four-period horizon rolls with time. If the pipeline operations schedule is weekly revised, then the first updating process will be made at the start of the new rolling horizon $\{t_2–t_5\}$, i.e. at time $t = 168$ h. The schedule review has a two-fold purpose: (1) to regenerate the pipeline schedule previously proposed for periods $\{t_2–t_4\}$ still taking part of the new rolling horizon, and (2) to schedule the pipeline operations to be accomplished within

Table 1
Product demands for periods $t_1$–$t_4$ at distrubution terminals ($\mathrm{dem}_{p,j,t}$)

| | Product requirements | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | D1 | | | | D2 | | | | D3 | | | | D4 | | | | D5 | | | |
| | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ |
| P1 | 40 | 30 | 50 | 50 | 100 | 100 | 150 | 120 | 90 | 120 | 100 | 110 | 140 | 180 | 170 | 150 | 100 | 120 | 90 | 100 |
| P2 | 100 | 120 | 100 | 120 | 100 | 100 | 100 | 110 | 70 | 80 | 70 | 60 | 200 | 200 | 200 | 220 | 220 | 210 | 250 | 220 |
| P3 | 30 | 40 | 30 | 20 | 0 | 0 | 0 | 0 | 20 | 30 | 20 | 30 | 50 | 60 | 50 | 40 | 30 | 20 | 20 | 40 |
| P4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 60 | 80 | 60 | 70 | 70 | 80 | 60 | 90 |

Table 2
Depot demands due at periods $t_5$–$t_7$

| | Product requirements | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | D1 | | | D2 | | | D3 | | | D4 | | | D5 | | |
| | $t_5$ | $t_6$ | $t_7$ | $t_5$ | $t_6$ | $t_7$ | $t_5$ | $t_6$ | $t_7$ | $t_5$ | $t_6$ | $t_7$ | $t_5$ | $t_6$ | $t_7$ |
| P1 | 40 | 50 | 60 | 100 | 120 | 120 | 90 | 100 | 90 | 140 | 120 | 120 | 100 | 100 | 100 |
| P2 | 100 | 120 | 100 | 100 | 100 | 110 | 70 | 60 | 70 | 200 | 200 | 220 | 220 | 200 | 200 |
| P3 | 30 | 30 | 30 | 0 | 0 | 0 | 30 | 30 | 30 | 50 | 40 | 50 | 30 | 40 | 30 |
| P4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 60 | 70 | 70 | 60 | 70 | 70 |

Table 3
Depot locations, product inventories and pumping costs

| Prod. | Level | Refinery | Depots | | | | |
|---|---|---|---|---|---|---|---|
| | | | D1 | D2 | D3 | D4 | D5 |
| P1 | Min | 400 | 50 | 30 | 20 | 50 | 50 |
| | Max | 2300 | 190 | 90 | 90 | 190 | 180 |
| | Initial | 1000 | 120 | 40 | 50 | 110 | 100 |
| P2 | Min | 400 | 90 | 50 | 90 | 150 | 150 |
| | Max | 2300 | 270 | 190 | 270 | 720 | 720 |
| | Initial | 1200 | 230 | 150 | 180 | 350 | 330 |
| P3 | Min | 50 | 20 | 0 | 20 | 20 | 20 |
| | Max | 600 | 120 | 0 | 120 | 180 | 92 |
| | Initial | 100 | 90 | 0 | 90 | 60 | 60 |
| P4 | Min | 150 | 0 | 0 | 0 | 30 | 25 |
| | Max | 1500 | 0 | 0 | 0 | 140 | 136 |
| | Initial | 315 | 0 | 0 | 0 | 90 | 110 |
| Location from Refinery [$10^2$ $m^3$] | | | 400 | 650 | 900 | 1500 | 1635 |

| Prod. | | Depots | | | | |
|---|---|---|---|---|---|---|
| | | D1 | D2 | D3 | D4 | D5 |
| P1 | Pumping Cost [US\$/$m^3$] | 3.5 | 4.5 | 5.5 | 6.0 | 6.9 |
| P2 | Pumping Cost [US\$/$m^3$] | 3.6 | 4.6 | 5.6 | 6.2 | 7.3 |
| P3 | Pumping Cost [US\$/$m^3$] | 4.8 | 5.7 | 6.8 | 7.9 | 8.9 |
| P4 | Pumping Cost [US\$/$m^3$] | 3.7 | 4.7 | 5.7 | 6.1 | 7.0 |

the new period $t_5$. Since the problem environment is dynamic in nature, some changes in terminal demands may occur while the pipeline schedule for period $t_1$ is being executed. Such changes may arise because of additional or cancelled terminal requests due at periods $\{t_2–t_4\}$ or new terminal requirements to meet at period $t_5$ just added to the end of the rolling horizon. In the proposed case study, some adjustment in "old" terminal requests will be considered at the start of the new rolling horizon $\{t_2–t_5\}$. They arise because of a reduction in the size of a pair of shipments to depots D1 and D5 due at the end of period $t_3$. One of the shipments was directly cancelled.

Table 2 shows the product terminal demands at periods $t_5$–$t_7$ to be gradually known as the planning horizon rolls, and the batch dispatching schedule for the successive action periods $\{t_1, t_2, t_3\}$ has been executed.

Additional problem data for this case study are given in Tables 3–5. Distances from the refinery to every depot (in volumetric units), initial stocks, minimum/maximum inventory levels at refinery and depot tanks and unit pumping costs are all included in Table 3. In turn, Table 4 provides the volume and reprocessing cost of the transmix, together with the changeover time between every ordered pair of products, as well as the product inventory holding costs at refinery and terminals. Forbidden product sequences are denoted with an "×". From Table 4 it follows that inventory holding costs at refinery and depot tanks constitute a large fraction of the total pipeline operational costs over the time horizon. As a result, product inventories at depot tanks are rapidly depleted through early shipments to local markets, one or two periods in advance of the specified due dates.

Table 4
Inventory costs and interface volumes and costs

| | Interface cost [$10^2$ US\$]/volume [$10^2$ $m^3$]/changeover time [h] | | | | Inventory costs [US\$/$m^3$ h] | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | P1 | P2 | P3 | P4 | REF | D1 | D2 | D3 | D4 | D5 |
| P1 | | 30/0.30/2.5 | 37/0.37/2.0 | 35/0.35/1.0 | 0.020 | 0.100 | 0.100 | 0.100 | 0.100 | 0.100 |
| P2 | 30/0.30/4.0 | | × | 38/0.38/5.0 | 0.023 | 0.155 | 0.155 | 0.155 | 0.155 | 0.155 |
| P3 | 37/0.37/1.5 | × | | × | 0.070 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 |
| P4 | 35/0.35/1.0 | 38/0.38/3.0 | × | | 0.025 | 0.170 | 0.170 | 0.170 | 0.170 | 0.170 |

Table 5
Scheduled production runs at the oil refinery for Example 1

| Production run | Product | Volume [$10^2$ $m^3$] | Production rate [$10^2$ $m^3$/h] | Time interval [h] |
|---|---|---|---|---|
| R1 | P2 | 2520 | 5 | 0–504 |
| R2 | P4 | 600 | 5 | 0–120 |
| R3 | P1 | 2520 | 5 | 168–672 |
| R4 | P3 | 500 | 5 | 336–436 |
| R5 | P4 | 1180 | 5 | 436–672 |
| R6 | P4 | 160 | 5 | 672–704 |
| R7 | P1 | 160 | 5 | 672–704 |
| R8 | P2 | 1000 | 5 | 704–904 |
| R9 | P3 | 500 | 5 | 804–904 |

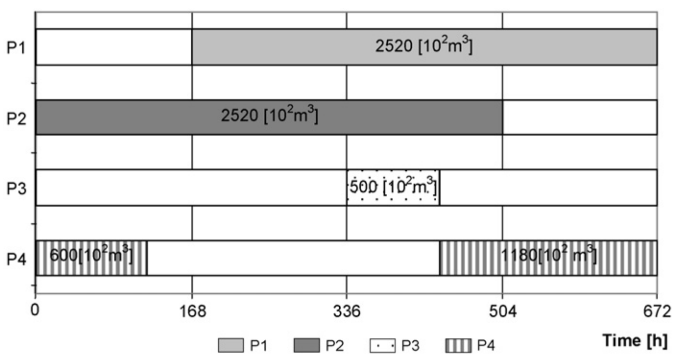


Fig. 15. Refinery production schedule for the initial horizon.

Table 5 and Fig. 15 both describe the scheduled production runs that are effectively executed at the oil refinery over the initial horizon $\{t_1–t_4\}$. Table 5 also includes the planned production runs for subsequent periods $t_5–t_7$.

The pipeline status at the start of the initial horizon $\{t_1–t_4\}$ is given on the first line of Fig. 16. Five batches {B5(P2), B4(P1), B3(P3), B2(P1), B1(P2)} containing the products indicated between brackets are inside the pipeline at $t = 0$ and their initial volumes are 400/700/200/200/135, respectively. The pumping unit cost is assumed to be independent of the injection rate and invariant with time, i.e. no daily peak-hours intervals are considered. In addition, the pumping rate must be within the range 8–12 [$10^2$ $m^3$/h].

Two instances of the proposed case study will be analyzed. The first instance assumes that the refinery production schedule will remain unchanged with time (Example 1). On the other instance, the scheduled production runs to be accomplished over periods $\{t_2–t_4\}$ are slightly modified at the start of the new rolling horizon $\{t_2–t_5\}$ because of crude oil supply adjustments (Example 2). Since the rescheduling procedure is iteratively performed at the beginning of a new rolling horizon, Example 2 is aimed at showing the capability of the proposed DPSP approach for properly reacting against input variations.

## 7.2. Example 1

In Example 1, the refinery production schedule over time periods $t_1–t_4$ is assumed to remain fixed as the scheduling horizon rolls with time. Such production runs indeed represent an important piece of information for the dynamic pipeline scheduling

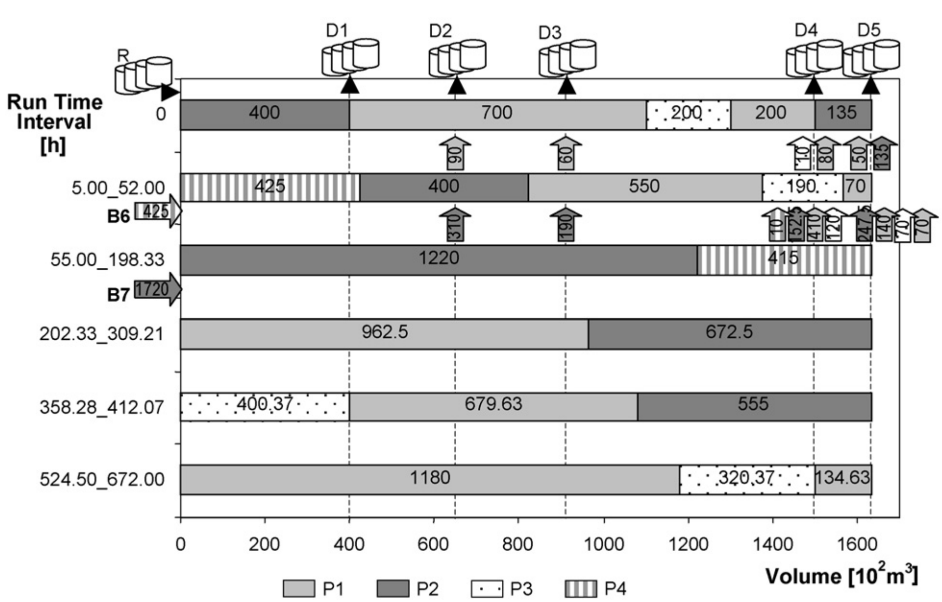


Fig. 16. Optimal static pipeline schedule for the initial horizon $\{t_1–t_4\}$.

problem. The soundness of the best pipeline operations strategy is highly dependent on fairly good decisions taken by the refinery scheduler. Often, he merely considers crude oil availability to define the refinery production schedule and ignore when and what quantities of products have to be delivered to distribution terminals over the planning horizon. The cardinality of the set $I^{new}$, i.e. the number of new batches put in the line, is initially assumed to be equal to: $(|P| \times |T|/3) = (4 \times 4)/3 \approx 5$.

*7.2.1. Static pipeline operations schedule for the initial horizon* $\{t_1–t_4\}$

At time $t = 0$, the optimal static schedule for the initial horizon $\{t_1–t_4\}$ is to be determined. To do that, the MPSOS system is run at time $dd_{k-1} = 0$. The best static pipeline schedule for $\{t_1–t_4\}$ is shown in Fig. 16. It is called the static schedule meaning the opposite of the dynamic schedule for $\{t_i–t_4\}$ to be gradually generated as the planning horizon rolls with time. The proposed pumping run schedule over $\{t_1–t_4\}$ includes a sequence of five batches $\{B6, B7, B8, B9, B10\}$ involving the following products and volumes: $P4^{425}/P2^{1720}/P1^{1282.5}/P3^{430.37}/P1^{1180}$, where the superscripts stand for the batch volumes. Detailed information on batch pumping runs and product deliveries from the pipeline to distribution terminals are also reported in Fig. 16 but only from time 0 to 198.33 h. For the rest of the initial horizon, the timing of the pumping runs and the size and location of every batch flowing inside the pipeline are just given. For instance, the batch B6 initially features a volume of 425 units and is pumped from time 5 to 52 h. While B6 is being injected in the pipeline, some product deliveries from batches {B5, B4, B3, B2, B1} already in the pipeline take place. Such delivery sizes are the following: (i) batch $B1^{135}$ containing product P2 is entirely transferred to depot $D5^{135}$, where the original batch size ($W_{B1}^{(B5)} = 135$) and the product delivery ($D_{B1,D5}^{(B6)} = 135$) are given as superscripts of the batch name and the destination name, respectively; (ii) a large fraction of batch $B2^{200}$ is diverted to depots $D4^{80}$ and $D5^{50}$; (iii) a little fraction of batch $B3^{200}$ is transferred to depot $D4^{10}$ and (iv) material from $B4^{700}$ is diverted to depots $D2^{90}$ and $D3^{60}$. In addition, the train of batches in the pipeline moves forth along the line. In particular, the upper volumetric coordinate of batch B4 changes from 1100 units to 1375 units while injecting B6.

The MILP mathematical model was solved on a Pentium IV 2 GHz processor with CPLEX by using ILOG OPL Studio 3.7 (Ilog, 2004). A relative MIP gap tolerance equal to $1 \times 10^{-4}$ and an integrity tolerance of $1 \times 10^{-5}$ were adopted in both examples. After solving the MILP formulation the cardinality of $I^{new}$ is increased by one and the model is to be solved again. The procedure is repeated until no further decrease in the pipeline operational costs is achieved at the optimum. The best solution for the initial horizon was obtained in the first iteration and the optimal value for $|I^{new}|$ was 5. The size of the MILP model and the required computer time to find the best pipeline operations schedule for the horizon $\{t_1–t_4\}$ are both summarized in the first row of Table 6.

The pumping run of batch B6 scheduled for shipping in period $t_1$ will be executed as originally planned. In contrast, the injection of batch B7 within period $t_1$ will end at time 168. Therefore, it will last $(168 - 55) = 113$ h. Since the injection rate remains constant throughout the whole pumping run, the initial size of batch B7, $Q_{B7}$, put in the line at period $t_1$ will be $(113/143.33) \times 1720 = 1356$ instead of 1720 units. Despite that, all the prescribed product deliveries from batches B2–B6 to the more distant terminals D4–D5 while injecting B7 can be accomplished. However, the amount of product P2 diverted from batch B7 to depot D3 within period $t_1$ should be decreased from 190 to 136 units. In addition, the product supply from B7 to D2 will be postponed for the next period $t_2$. The remaining pipeline schedule comprises planned batch injections that may be modified or cancelled by the *MPSOS* system as the time horizon rolls. The dynamic pipeline schedule for Example 1 finally executed over $\{t_1–t_4\}$ will be later analyzed in this section.

*7.2.2. Updated pipeline schedule for the next rolling horizon* $\{t_2–t_5\}$

The pipeline schedule should be updated at the start of week $t_2$ when terminal request data for period $t_5$ become available. The dispatcher has already executed the pipeline operations scheduled for the action period of the initial horizon. Just two batches B6(P4) and B7(P2) with volumes 425 and 1356 units, respectively, have been injected during period $t_1$. The question is whether or not to continue the injection of product P2 at the start of period $t_2$ as suggested in Fig. 16. With $k = 2$ and time = 168 h, the rescheduling procedure is activated again to find the pipeline schedule for the next horizon $\{t_2–t_5\}$.

Table 7 includes the updated product requirements for "old" periods $t_2–t_4$ at terminals D1–D5. The first line on the P1-row indicates the original "old" demand of P1, the second one provides the "old" P1-requirement already satisfied during period $t_1$, the third includes the residual "old" demand of P1 still to be satisfied, the fourth row shows the updated demand of P1 for periods $t_2–t_4$, including residual "old" demands and new/cancelled nominations, and the fifth one gives the updated refinery inventory

Table 6
Model sizes and time requirements for each instance of the rolling horizon

| Horizon | #/old | #/new | #/ | Binary variables | Continuous variables | Equations | CPU time [s] | Optimal solution [$10^2$US$/month] |
|---|---|---|---|---|---|---|---|---|
| $t_1–t_4$ | 5 | 5 | 10 | 240 | 2223 | 3380 | 15.63 | 175951.68 |
| $t_2–t_5$ | 2 | 6 | 8 | 213 | 1958 | 3228 | 124.41 | 164681.95 |
| $t_3–t_6$ | 4 | 6 | 10 | 273 | 2660 | 3882 | 216.33 | 181538.22 |
| $t_4–t_7$ | 7 | 6 | 13 | 363 | 3418 | 4757 | 330.30 | 189873.39 |

Table 7
Updating product demands for periods $t_2$–$t_4$ at the end of period $t_1$

| | Product requirements | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | D1 | | | | D2 | | | | D3 | | | | D4 | | | | D5 | | | |
| | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ |
| P1 | | | | | | | | | | | | | | | | | | | | |
| TD | 40 | 30 | 50 | 50 | 100 | 100 | 150 | 120 | 90 | 120 | 100 | 110 | 140 | 180 | 170 | 150 | 100 | 120 | 90 | 100 |
| SD | 40 | 30 | 0 | 0 | 100 | 0 | 0 | 0 | 90 | 0 | 0 | 0 | 140 | 180 | 170 | 60 | 100 | 120 | 90 | 0 |
| RD | 0 | 0 | 50 | 50 | 0 | 100 | 150 | 120 | 0 | 120 | 100 | 110 | 0 | 0 | 0 | 90 | 0 | 0 | 0 | 100 |
| UD | | 0 | 0[a] | 50 | | 100 | 150 | 120 | | 120 | 100 | 110 | | 0 | 0 | 90 | | 0 | 0 | 100 |
| UI | | 50 | | | | 30 | | | | 20 | | | | 50 | | | | 50 | | |
| P2 | | | | | | | | | | | | | | | | | | | | |
| TD | 100 | 120 | 100 | 120 | 100 | 100 | 100 | 110 | 70 | 80 | 70 | 60 | 200 | 200 | 200 | 220 | 220 | 210 | 250 | 220 |
| SD | 100 | 40 | 0 | 0 | 100 | 0 | 0 | 0 | 70 | 80 | 70 | 6 | 200 | 152.5 | 0 | 0 | 220 | 210 | 132.5 | 0 |
| RD | 0 | 80 | 100 | 120 | 0 | 100 | 100 | 110 | 0 | 0 | 0 | 54 | 0 | 47.5 | 200 | 220 | 0 | 0 | 117.5 | 220 |
| UD | | 80 | 100 | 120 | | 100 | 100 | 110 | | 0 | 0 | 54 | | 47.5 | 200 | 220 | | 0 | 107.5[a] | 220 |
| UI | | 90 | | | | 50 | | | | 90 | | | | 150 | | | | 150 | | |
| P3 | | | | | | | | | | | | | | | | | | | | |
| TD | 30 | 40 | 30 | 20 | | – | | | 20 | 30 | 20 | 30 | 50 | 60 | 50 | 40 | 30 | 20 | 20 | 40 |
| SD | 30 | 40 | 0 | 0 | | – | | | 20 | 30 | 20 | 0 | 50 | 60 | 50 | 10 | 30 | 20 | 20 | 40 |
| RD | 0 | 0 | 30 | 20 | | – | | | 0 | 0 | 0 | 30 | 0 | 0 | 0 | 30 | 0 | 0 | 0 | 0 |
| UD | | 0 | 30 | 20 | | – | | | | 0 | 0 | 30 | | 0 | 0 | 30 | | 0 | 0 | 0 |
| UI | | 20 | | | | – | | | | 20 | | | | 20 | | | | 20 | | |
| P4 | | | | | | | | | | | | | | | | | | | | |
| TD | | – | | | | – | | | – | | | | 60 | 80 | 60 | 70 | 70 | 80 | 60 | 90 |
| SD | | – | | | | – | | | – | | | | 60 | 10 | 0 | 0 | 70 | 15 | 0 | 0 |
| RD | | – | | | | – | | | – | | | | 0 | 70 | 60 | 70 | 0 | 65 | 60 | 90 |
| UD | | – | | | | – | | | – | | | | | 70 | 60 | 70 | | 65 | 60 | 90 |
| UI | | – | | | | – | | | – | | | | | 30 | | | | 25 | | |

TD: Total depot demands for periods $t_1$–$t_4$ at the start of time period $t_1$ (0 h), SD: Satisfied demand using initial inventories and diverting material from batches during period $t_1$, RD: Residual old demand at the end of time period $t_1$ ($t$ = 168 h), UI: Updated product inventory at the start of period $t_2$.
[a] UD: Updated demand due to new/cancelled transport orders at the start of period $t_2$.

record for P1. Similar information is given for the other refined products.

Fig. 17 shows the best pipeline schedule found for the next planning horizon. It can be observed that the injection of P2 last shipped in period $t_1$ is interrupted to start pumping product P4 after completing the required changeover operation. The updated pipeline schedule now includes a sequence of six pumping runs {B8, B9, B10, B11, B12,

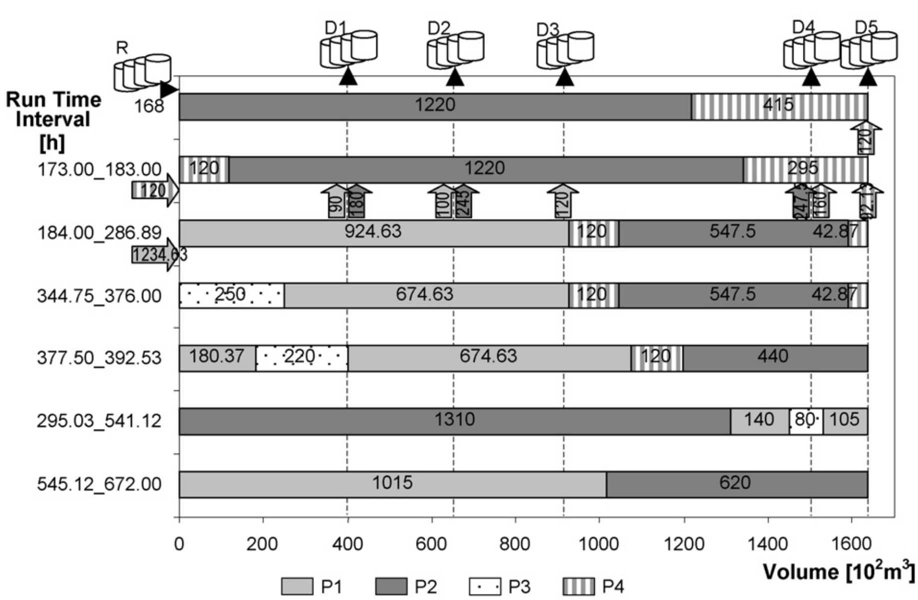


Fig. 17. Optimal pipeline schedule for time periods $t_2$–$t_5$.

B13} involving the following products and volumes (given as superscripts): $P4^{120}/P1^{1234.63}/P3^{250}/P1^{180.37}/P2^{1549}/P1^{1015}$. In addition to stopping the injection of product P2, there are quite significant changes in the pumping run sequence and the batch sizes, especially for the last planned shipments of the previous horizon (see Figs. 16 and 17). The product sequence $P1^{1282.5}/P3^{430.37}/P1^{1180}$ has been modified by: (i) introducing $P4^{120}$ on first place, (ii) largely reducing the length of the last injection of product $P1^{1180}$ from 1180 to 180.37 volumetric units and (iii) injecting at last another large batch containing $P2^{1549}$. The new batch B8 conveying product P4 is inserted at the beginning of the time horizon $\{t_2–t_5\}$ to meet new demands from depots D4 and D5 due at the end of the new period $t_5$. Otherwise, product P4 could not reach such depots (the farthest ones) at the proper time. Variations in the size of the last two batches to be pumped over the initial horizon $\{t_1–t_4\}$, i.e. $P3^{250}/P1^{180.37}$ instead of $P3^{430.37}/P1^{1180}$, can be easily justified. At the initial horizon $\{t_1–t_4\}$, the last two batches containing products P3 and P1 (especially the later one) are pumped just to push forward the batches flowing in the line towards their stated terminals. However, their own destinations were still undefined since terminal requirements at period $t_5$ were unknown. As new demands of products (P1, P2, P3, P4) for period $t_5$ arise at the more distant terminals D4 and D5, batches of P1 and P3 are reduced just to the required volumes and a new batch of P4 is first inserted. Similar to the previous rolling horizon $\{t_1–t_4\}$, two large batches ($P2^{1549}/P1^{1015}$) are last placed in the line to "sweep" previous shipments towards their destinations.

### 7.2.3. *Dynamic pipeline schedule finally executed by the dispatcher during periods $t_1$–$t_4$*

At the end of the second week (time = 336 h) only the pumping runs scheduled for the action period $t_2$ have been performed, i.e. $P4^{120}/P1^{1234.63}$. The next step is to capture the new pipeline system scenario and to refresh the schedule again. As the four-period scheduling horizon has rolled from $\{t_1–t_4\}$ to $\{t_4–t_7\}$ and new demand data were periodically considered, the pipeline schedule undergoes significant changes. The sequence of pumping runs finally performed and the amounts of products delivered from the pipeline to terminals during the action periods present some major differences with regards to the static schedule for $t_1–t_4$ (see Fig. 18).

Fig. 19 shows the multiperiod pipeline schedule finally executed by the dispatcher over periods $t_1–t_4$. It comprises a sequence of 10 pumping runs: [$P4^{425}/P2^{1356}/P4^{120}/P1^{1234.63}/P3^{390}/P1^{665.37}/P4^{259.62}/P2^{449.04}/P2^{1513.96}/P4^{290.38}$] with the superscripts indicating the initial batch sizes, against 5 batch injections suggested by the static pipeline schedule for the initial horizon $\{t_1–t_4\}$ (see Figs. 16 and 19). Note that the first three batches finally put in the line at period $t_3$, [$P3^{390}/P1^{665.3}/P4^{259.62}$], differ from the ones initially proposed $P3^{250}/P1^{180.37}/P2^{1549}$ in Fig. 16 as the terminal demands for period $t_6$ were unveiled at the start of the horizon $\{t_3–t_6\}$.

To meet customer demands, the pipeline remains operative from time 0 to 659.44 with a temporary stop during the time interval [286.89–336.00] (see Fig. 18). The refinery production schedule for the initial horizon $\{t_1–t_4\}$ that is supposed to remain unchanged with time foresees a supply of product P3 to refinery tanks not before time 336. Therefore, the injection of product P3 is delayed until the start of the action period $t_3$ when new production of P3 becomes available at the refinery tankage. Though the significant earliness of some product deliveries to terminals, the pipeline system will still feature a total idle time of 61.67 h over a time horizon length of 672 h, i.e. a pipeline time usage over 90%. As already mentioned, Fig. 19 presents the multiperiod pipeline schedule finally performed throughout the first four periods $t_1–t_4$. It also depicts the evolution of volumes and coordinates for new/old batches as they move along the pipeline. Variations of product inventories at refinery tanks are illustrated in Fig. 20. It shows how the proposed DPSP approach coordinates the pipeline distribution planning, including pumping runs and material deliveries to terminals, and the refinery production schedule so that inventory levels stay within their permissible ranges. Once the initial stock of refined products has been delivered to local markets, product inventories at depot tanks remain at their minimum values throughout the scheduling horizon because of the significant inventory holding costs. Computational requirements and DPSP model sizes for the successive planning horizons are given in Table 6. As the number of pumping runs rises, the required CPU time also increases. If initial stocks are reduced by 25%, the sequence of pipeline pumping runs to be performed remains the same but their lengths increase and the average earliness of product deliveries diminishes.

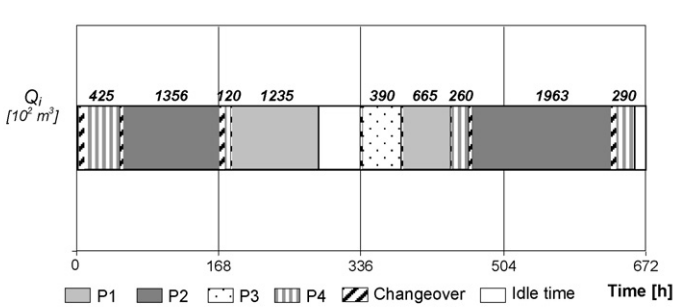


Fig. 18. Gantt chart for the optimal multiperiod pumping run schedule.

## 7.3. *Example 2*

Example 2 deals with the same real-world problem but in this case the refinery production schedule available at time zero experiences some variations as the planning horizon rolls with time. This type of events frequently occurs in actual practice and may have a profound impact on the soundness of the proposed multiperiod pipeline schedule. The DPSP approach is able to cope with such changes introduced by the refinery scheduler to still get a perfect coordination between refinery and pipeline operations. Otherwise, the entire line must be temporarily stopped during some time interval to wait for new refinery supplies of the next product to be injected. The resulting loss of productive time brings about a reduction in the usage of the pipeline transportation capacity.

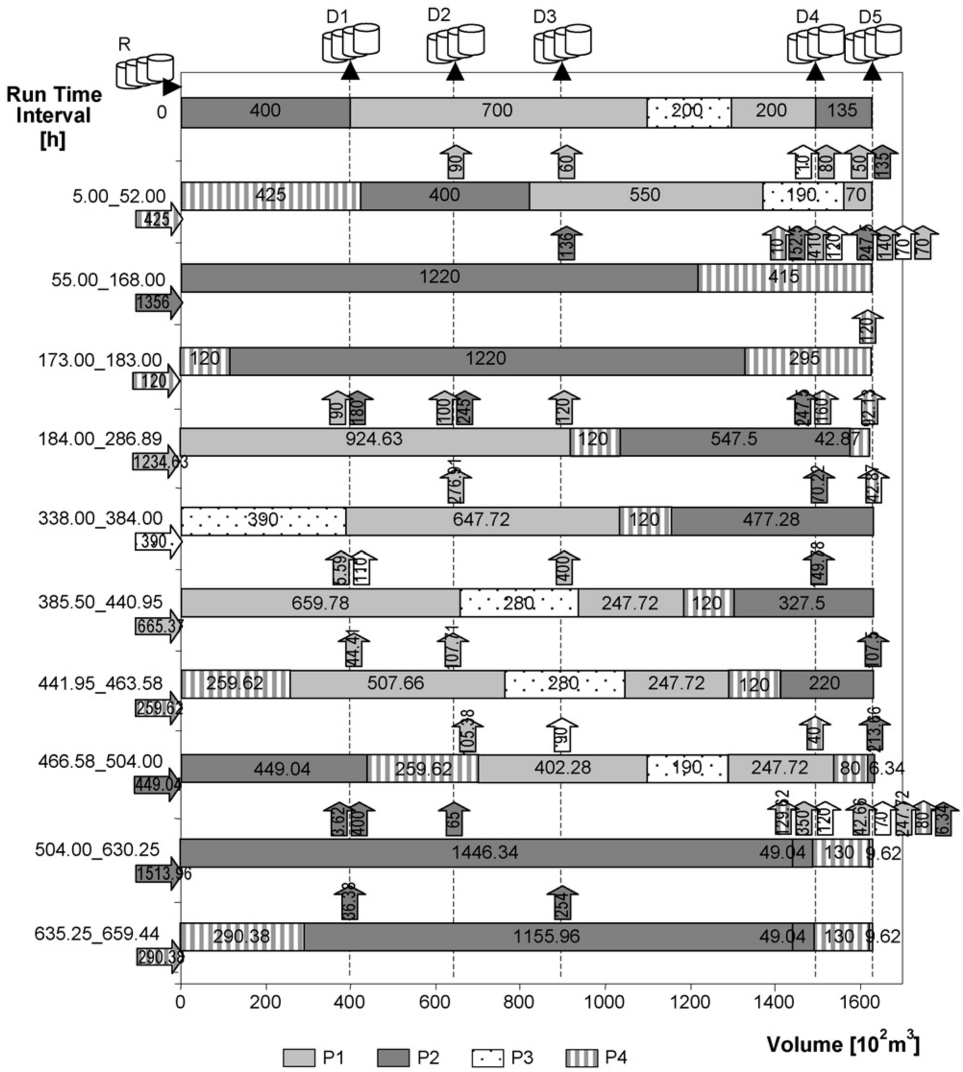


Fig. 19. Optimal dynamic pipeline schedule for time periods $t_1$–$t_4$.

Let us assume that the refinery production run yielding product P3 initially expected to start at time 336 h is anticipated by the refinery scheduler to the request of the pipeline carrier by 168 h (see Fig. 21). The new start time of the production run is taken into account by the pipeline scheduler during the first update of the work schedule carried out at the end of period $t_1$, i.e. at time 168 h. Nonetheless, the size and the extent of the production run remains unchanged (see Fig. 21). The new problem

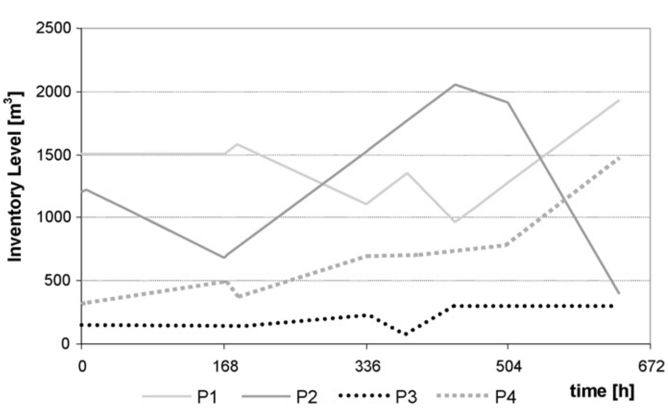


Fig. 20. Projected inventory levels in refinery tanks for time periods $t_1$–$t_4$.

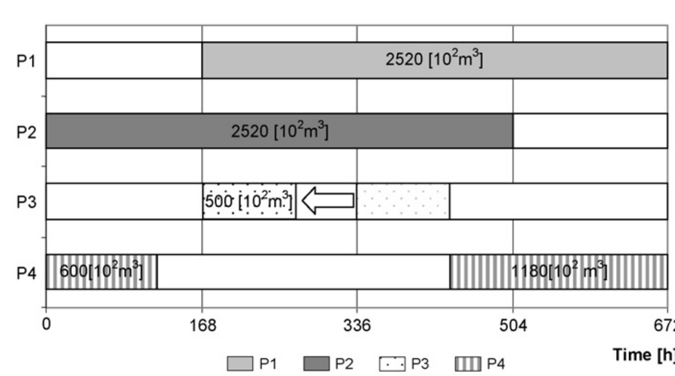


Fig. 21. Modified production schedule at the refinery (Example 2).

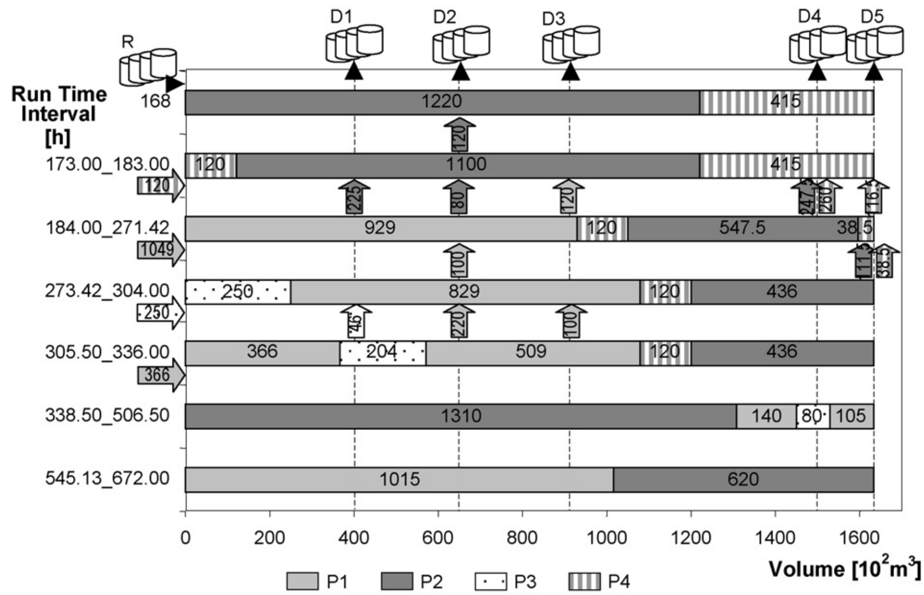


Fig. 22. Reformulated pipeline schedule for time periods $t_2$–$t_5$ (Example 2).

scenario is captured and the MPSOS system is applied to properly reschedule pipeline operations. In contrast to Example 1, the batch of P3 is timely introduced in the pipeline at the origin without stopping the line to wait for the production of P3 in the refinery.

The new optimal pipeline schedule for periods $\{t_2–t_5\}$ is depicted in Fig. 22. Note that the pipeline configuration and the batch evolution are rather similar to the previous example (see Figs. 17 and 22). However, the pipeline remains operative during the entire “action” period $t_2$ (from time 168 to 336), except for the changeover tasks, due to the earlier P3-availability. As a result, the pipeline idle time is reduced by 49.11 h and a better use of the pipeline transport capacity is achieved. To get such a time saving, four new batches instead of only two are inserted in the pipeline during period $t_2$.

An optimal multiproduct pipeline schedule aims to get a better use of the pipeline transport capacity by properly integrating both production and distribution activities along the complex oil derivatives supply chain. So far, it was assumed that the refinery production schedule is given ‘a priori’ mainly based on crude oil availabilities, with little attention focused on the pipeline shipper nominations. In this way, a precise matching in time between refinery and pipeline operations can be hardly achieved. The novel DPSP approach presented in this article helps planners to adjust the refinery production schedule to the size and due dates of future product demands at distribution terminals. As shown in Example 2, a better integration of refinery and pipeline operations brings about significant benefits to the refined products supply chain by strongly increasing the pipeline capacity usage.

## 8. Conclusions

A new MILP continuous-time framework for periodically updating the work schedule of a single unidirectional multiproduct pipeline over a multiperiod rolling horizon has been developed. The proposed formulation for the Dynamic Pipeline Scheduling Problem (DPSP) allows to consider multiple due-dates at period ends. Results show that the sequence of pumping runs finally executed by the pipeline dispatcher along the time horizon looks quite different from the one found through static pipeline scheduling techniques recently published. Pumping runs become shorter and its number is significantly increased. In Example 1, the number of new batches inserted in the line increases from 5 to 10. Such changes arise because the planned pumping runs for later periods found through a static scheduling approach have the only purpose of pushing in-transit batches to their destinations. Due to the new features of the proposed DPSP approach, no batch is finally dispatched just for interface compatibility convenience but mostly to satisfy specific terminal requests due at some future periods. In this way, the scheduled pipeline idle time practically vanishes and the pipeline utilization shows a 21% increase. Computational requirements grow as the time horizon rolls and the number of pumping runs increases, but in any case it remains quite reasonable varying from 16 to 330 CPU seconds. The approach can be easily implemented in enterprise scheduling systems, even incorporating other concepts like hard/soft frozen time periods along the rolling horizon to restrict the kind of changes introduced during the periodic schedule review. Moreover, the DPSP model is flexible enough to dynamically adapt the pumping run schedule to account for changes in terminal demands, refinery production runs or new batch destinations.

## Acknowledgments

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## Appendix A. Control of refinery inventories at the origin

Let $R_p \subset R$ be the set of refinery production runs involving product $p$ to be executed over the current rolling horizon. A pair of binary variables must be introduced to ensure that: (i) enough product $p$ is available in refinery tanks ($\mathrm{IR}_p$) at the time of injecting a batch of product $p$ in the line, i.e. $\mathrm{IR}_p \geq (\mathrm{IR}_{\min})_p$; (ii) the maximum inventory level $(\mathrm{IR}_{\max})_p$ of product $p$ in the assigned refinery tank is never exceeded, i.e. $\mathrm{IR}_p \leq (\mathrm{IR}_{\max})_p$. Since the refinery production rate is always lower than the pipeline pumping rate (see assumptions), then the two worst conditions to meet the constraints $(\mathrm{IR}_{\min})_p \leq \mathrm{IR}_p \leq (\mathrm{IR}_{\max})_p$ in refinery tanks arise at either the start or the completion time of a pumping run involving product $p$, respectively. The worst condition for stock-outs of product $p$ occurs at the completion time of a pumping run $i \in I^{\mathrm{new}}$ injecting $p$ in the line. In other words, the pumping of a batch $i$ containing product $p$ can be executed if at least the specified minimum inventory $(\mathrm{IR}_{\min})_p$ is still available at the completion time $C_i$. Otherwise, it should be delayed. Similarly, the worst condition for $p$th-product overloading in a refinery tank arises at the start of a pumping run injecting that product in the line. Product $p$ will never spill from the assigned tank if a permissible inventory level $\mathrm{IR}_p \leq (\mathrm{IR}_{\max})_p$ is on hand just before injecting a new batch $i$ containing $p$, i.e. at time $(C_i - L_i)$.

### A.1. Definition of new binary variables $zl_{i,r}$ and $zu_{i,r}$

If the inventory $\mathrm{IR}_p$ at the completion time $C_i$ is to be determined, then the refinery runs $r \in R_p$ partially and/or completely executed before $C_i$ must be taken into account. Let us define the binary variable $\mathrm{zl}_{i,r}$ to indicate that the injection of a new batch $i \in I^{\mathrm{new}}$ has been completed before ($\mathrm{zl}_{i,r} = 0$) or after ($\mathrm{zl}_{i,r} = 1$) the production run $r \in R_p$ has begun. Therefore, refinery runs $r \in R_p$ with $\mathrm{zl}_{i,r} = 0$ do not contribute at all to the value of $\mathrm{IR}_p$ at time $C_i$ and, therefore, they must be ignored. At the same time, some runs with $\mathrm{zl}_{i,r} = 1$ have been partially executed and the production outputs already loaded in the assigned tank up to time $C_i$ should be determined. Therefore,

$$a_r \mathrm{zl}_{i,r} \leq C_i \leq a_r + h_{\max} \mathrm{zl}_{i,r}, \quad \forall i \in I^{\mathrm{new}},\ r \in R \qquad (A.1)$$

On the other hand, we are also interested in the value of $\mathrm{IR}_p$ at the initial pumping time of a new batch $i$ containing product $p$. To this end, it will be introduced the binary variable $\mathrm{zu}_{i,r}$ to denote that the injection of batch $i \in I^{\mathrm{new}}$ has begun before ($\mathrm{zu}_{i,r} = 0$) or after ($\mathrm{zu}_{i,r} = 1$) the completion of the refinery run $r \in R_p$. A production run $r$ with $\mathrm{zu}_{i,r} = 0$ has been either partially executed or not executed at all at time $(C_i - L_i)$. Instead, refinery runs featuring $\mathrm{zu}_{i,r} = 1$ have already finished and their total production outputs were already loaded in the assigned tank at time $(C_i - L_i)$.

$$b_r \mathrm{zu}_{i,r} \leq C_i - L_i \leq b_r + h_{\max} \mathrm{zu}_{i,r}, \quad \forall i \in I^{\mathrm{new}},\ r \in R \qquad (A.2)$$

Fig. A1 depicts a simple Gantt chart including both pipeline pumping runs and refinery production campaigns to illustrate the meaning of variables $\mathrm{zl}_{i,r}$ and $\mathrm{zu}_{i,r}$. The pumping of batch B1 starts at time $C_1 - L_1 = 15$, 18 h before the end of the first production campaign ($b_{\mathrm{R1}} = 33$). Therefore, $\mathrm{zu}_{\mathrm{B1,R1}} = 0$. It implies that a fraction of the production output expected from run R1 may be loaded in the refinery tank at time $C_1 - L_1 = 15$. On the other hand, the dispatching of B1 finishes at $C_1 = 25$, i.e. 6 h after the starting time of run R1 ($a_{\mathrm{R1}} = 19$). Therefore, $\mathrm{zl}_{\mathrm{B1,R1}} = 1$. It implies that some product from campaign R1 has already been loaded in the refinery tank at time 25. In contrast, the pumping of batch B2 starts after the end of run R1 ($\mathrm{zu}_{\mathrm{B2,R1}} = 1$). Therefore, the production from run R1 is entirely stored in inventory at time $C_2 - L_2 = 35$. Moreover, the injection of batch B2 finishes before starting R2 ($\mathrm{zl}_{\mathrm{B2,R2}} = 0$), and therefore no material from this production run is available in the refinery tank at time $C_2 = 45$.

### A.2. Production output from run $r \in R_p$ already stored in the refinery tank at time $C_i$

Assume that $s_r$ is the expected $p$th-production output from run $r \in R_p$, $\mathrm{vp}_r$ is the rate of production and $[a_r, b_r]$ denotes the time interval during which the production output from run $r$ is stored in the assigned refinery tank. Let $\mathrm{SL}_{i,r}$ ($\leq s_r$) be the portion of the production output from run $r \in R_p$ already loaded in the refinery tank at time $C_i$. Three cases can be considered:

(I) $C_i \geq b_r$, then $\mathrm{zl}_{i,r} = 1$ and the full run $r$ has been loaded in the assigned tank;
(II) $C_i \leq a_r$, then $\mathrm{zl}_{i,r} = 0$ and the production run $r$ has not yet begun at time $C_i$;
(III) $a_r \leq C_i \leq b_r$, then $\mathrm{zl}_{i,r} = 1$ and a portion of the $r$th-run production output has already been loaded in the tank during the interval $[a_r, C_i]$.

Therefore,

$$\mathrm{SL}_{i,r} \leq s_r \mathrm{zl}_{i,r} \qquad (A.3)$$

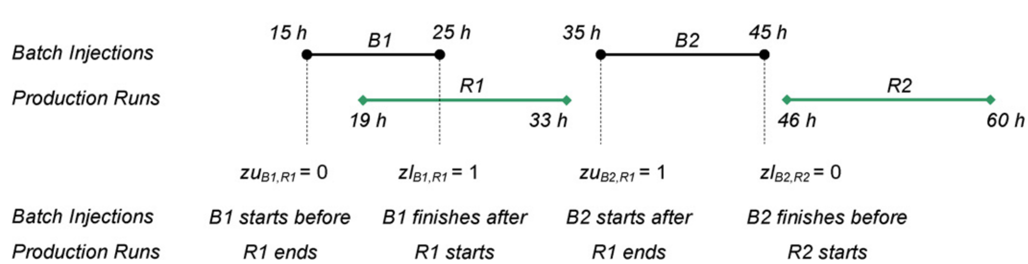


Fig. A1. Coordinating batch injections and refinery production runs.

$$\mathrm{SL}_{i,r} \leq \mathrm{vp}_r(C_i - a_r \mathrm{zl}_{i,r}), \quad \forall i \in I^{\mathrm{new}},\ r \in R \tag{A.4}$$

Constraint (A.3) is binding for cases (I) and (II). In this way, $\mathrm{SL}_{i,r} = s_r$ for case (I) and $\mathrm{SL}_{i,r} = 0$ for case (II). In turn, the equation (A.4) becomes active for case (III) and $\mathrm{SL}_{i,r} = \mathrm{vp}_r(C_i - a_r)$.

### *A.3. Production output from run $r \in R_p$ already loaded in the refinery tank at time $(C_i - L_i)$*

Let $\mathrm{SU}_{i,r}$ denote the amount of product $p$ from run $r \in R_p$ already loaded in the refinery tank at the starting time $(C_i - L_i)$ of a new pumping run $i \in I^{\mathrm{new}}$. Two cases can be considered:

(I) $(C_i - L_i) \geq b_r$, then $\mathrm{zu}_{i,r} = 1$ and the whole production run $r$ is already stored in the refinery tank at time $(C_i - L_i)$;
(II) $(C_i - L_i) < b_r$, then $\mathrm{zu}_{i,r} = 0$ and either a portion of or the whole production run $r$ is still to be loaded in the assigned tank at time $(C_i - L_i)$.

Therefore,

$$\mathrm{SU}_{i,r} \geq s_r \mathrm{zu}_{i,r} \tag{A.5}$$

$$\mathrm{SU}_{i,r} \geq \mathrm{vp}_r[(C_i - L_i) - a_r - h_{\max} \mathrm{zu}_{i,r}), \quad \forall i \in I^{\mathrm{new}},\ r \in R \tag{A.6}$$

Since the worst condition for overloading in refinery tanks occurs at the start of a pumping run and the amount $\mathrm{SU}_{i,r}$ contributes to the value of $\mathrm{IR}_p$, the model will tend to make $\mathrm{SU}_{i,r}$ as small as possible. This is why lower bounds are defined for the value of $\mathrm{SU}_{i,r}$ through constraints (A.5) and (A.6). If $\mathrm{zu}_{i,r} = 1$ and run $r$ has been entirely loaded in the refinery tank at time $(C_i - L_i)$, then Eq. (A.5) prevents from reducing $\mathrm{SU}_{i,r}$ below $s_r$. Otherwise, constraint (A.5) becomes redundant. In case $\mathrm{zu}_{i,r} = 0$ and $a_r < C_i - L_i < b_r$, constraint (A.6) would force $\mathrm{SU}_{i,r}$ to never drop below $\mathrm{vp}_r[(C_i - L_i) - a_r]$. If $\mathrm{zu}_{i,r} = 1$, constraint (A.6) turns to be redundant.

### *A.4. Testing worst conditions for product shortages and overloadings*

As explained before, the inventory level for product $p$ is forced to never falling below the minimum level $(\mathrm{IR}_{\min})_p$ at the end of every pumping run $i \in I^{\mathrm{new}}$, i.e. $\mathrm{IRF}_p^{(i)}$. In addition, the $p$th-inventory level is required to never exceed the maximum permissible level $(\mathrm{IR}_{\max})_p$ at the starting time of a new pumping run $i \in I^{\mathrm{new}}$, i.e. $\mathrm{IRS}_p^{(i)}$. Therefore, the following constraints must be included in the problem formulation to guarantee that the $p$th-product inventory level in the assigned refinery tank always remains within the specified feasible range $[(\mathrm{IR}_{\min})_p, (\mathrm{IR}_{\max})_p]$.

$$\mathrm{IRF}_p^{(i)} = \mathrm{IR}_p^{\mathrm{o}} + \sum_{r \in R_p} \mathrm{SL}_{i,r} - \sum_{i' \in I^{\mathrm{new}}, i' \leq i} \mathrm{QP}_{i',p} \geq (\mathrm{IR}_{\min})_p, \quad \forall i \in I^{\mathrm{new}},\ p \in P \tag{A.7}$$

$$\mathrm{IRS}_p^{(i)} = \mathrm{IR}_p^{\mathrm{o}} + \sum_{r \in R_p} \mathrm{SU}_{i,r} - \sum_{i' \in I^{\mathrm{new}}, i' < i} \mathrm{QP}_{i',p} \leq (\mathrm{IR}_{\max})_p, \quad \forall i \in I^{\mathrm{new}},\ p \in P \tag{A.8}$$

where $\mathrm{IR}_p^{\mathrm{o}}$ stands for the initial inventory of product $p$. Constraint (A.7) accounts for $p$th-product supplies to inventory from refinery runs $r \in R_p$ starting before time $C_i$. In addition, it considers the product withdrawals from refinery tanks related to batches of product $p$ injected in the pipeline up to $C_i$. Constraint (A.8) is similar to (A.7) but the referenced time point is now the start point of a pumping run $(C_i - L_i)$ rather than $C_i$. Note that the amount of product $p$ injected in the line $\mathrm{QP}_{i',p}$ equals zero if batch $i'$ does not contain $p$. Otherwise, it is equal to the initial size of batch $i'$ $(Q_{i'})$. Therefore,

$$\mathrm{QP}_{i,p} \leq Q_{\max} y_{i,p}, \quad \forall i \in I^{\mathrm{new}},\ p \in P \tag{A.9}$$

$$\sum_{p \in P} \mathrm{QP}_{i,p} = Q_i, \quad \forall i \in I^{\mathrm{new}} \tag{A.10}$$

where $Q_{\max}$ stands for the maximum permissible injection size.

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