Pipeline Scheduling and Inventory Management of a Multiproduct Distribution Oil System

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Within the oil supply chain, although refinery operations are extensively studied, distribution center operations are not being explored to their full potential. In this paper, these types of operations are studied. A mixed-integer linear programming (MILP) approach is used to model the problem of oil derivatives pipeline transportation scheduling and supply management. The objective of the model is to attain a high level of operation, satisfying clients and accounting for distribution center restrictions and compulsory tasks. First, a base model is developed, which accounts for product transportation, inventory management, and daily client satisfaction. Later, an extension is presented to account for a settling period for each new lot that arrives at the distribution center. A pumping schedule, including the product sequence, lot volume, and timing issues, is obtained. Also, the inventory management is performed while accounting for daily clients requests and quality control tasks. A continuous representation of both time and pipeline volume is used. The model is applied to a real-world case study of a Portuguese oil distribution company, Companhia Logística de Combustíveis (CLC). Different scenarios are outlined, with the objective of analyzing how the settling period and its minimum duration influence the model performance. Various modeling approaches to the product sequence inside the pipeline have been studied. The results are discussed and compared to the real schedule of a typical monthly plan that has been developed by CLC's schedulers.

1. Introduction

The crude oil supply chain is one of the most important entities in the worldwide industry. It is regulated by an increasingly competitive market where a strict economical environment obliges refinery managers to operate in the sense to raise profits while dealing with a complex structure. Oil exploration, transportation to refineries (either isolated or included in networks), and product distribution are the main nodes within such chain. Each one of these nodes possesses a specific operation, and the transportation between them can be done in a multimodal way, through vessels (more usual for oil), pipelines, trucks, or even trains. Therefore, there should be an effort to develop systematic methods that can help the decision making process along the global chain.[1]

The use of advanced process engineering tools to support the refinery operation has been studied recently.[2] Nevertheless, the majority of the reported works has focused on sub-systems of the chain. Furthermore, although planning tools are well-established among refiners, the scheduling and inventory management levels have a lack of rigorous mathematical approaches to describe the entire refinery operation.[2,3] Therefore, schedulers usually base their work on experience, heuristics, and the use of spreadsheets. One of the main reasons why these systematic methods seem to be inadequate in practice to support production management decisions is because they are used at different levels of the chain with dissimilar underlying models and approaches, different solution algorithms, and, above all, different and large sets of users.[4] One example of combined planning and scheduling models is the work of Pinto et al.[5] For the scheduling case, the complete framework includes crude oil scheduling, production, and distribution, even accounting for a period for brine settling. These authors managed to solve real-world problems that concerned fuel oil/asphalt productions and liquefied propane gas (LPG) scheduling.

Facing the outlined directions and opportunities, several works on the crude oil scheduling and sub-systems problems have appeared over the last years.

One of the first works on this area was from Shah.[6] A mixed-integer linear programming (MILP) model with discrete time representation combined with a decomposition approach was applied to a system that contained a port, a pipeline, and a refinery. Tank heel minimization was defined as the main objective. Almost in parallel is the work of Lee et al.,[7] who also presented a MILP model to address the same problem. Time is again represented in a discrete time fashion. One of the main issues of this work is the elimination of bilinear equations that appear usually in the mixing operations related to the crude distillation units (CDUs). As an objective function, the minimization of the operating costs was considered. Both previous works were tested in industrial-sized problems, proving that large-scale MILP approaches can be applied to this specific problem, where significant economical potential and tradeoffs can be managed and optimized in a positive direction. These works provided solid basis for subsequent works.

In succeeding years, several mathematical programming approaches have appeared in the literature, addressing similar problems. Magalhães and Shah[8] developed a continuous-time MILP model, where the objective was the minimization of the deviation between the planned and the scheduled throughputs

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at the crude units. These authors also noted the importance of developing pipeline models through optimization techniques. An extended review of this approach can be found in Magalhães,[9] where both continuous and discrete formulations were presented, with focus on several features and operational rules optimized under operational criteria. This author included a detailed pipeline model, focusing on the importance of this chain element. To aid the solution-finding process, a decomposition strategy was developed applied to the refinery products' scheduling. A rolling horizon approach was used that considers aggregated information in a short time, reducing the computational complexity.

Neiro and Pinto[1] proposed new particular frameworks for storage tanks and pipelines, following the work of Pinto et al.[5] Jia and Ierapetritou[10,11] presented a spatial decomposition approach to overcome the complexity of the short-term scheduling problem of refinery operations resulting in three main parts: (i) crude oil unloading, mixing, and inventory control; (ii) production unit scheduling (with fractionation and reaction processes); and (iii) finished product blending and the shipping end of the refinery. The MILP formulation is based on a continuous-time representation. The computational effort grew significantly as the number of constraints and variables increased. Results were presented and compared with those of Lee et al.[7] for the first part; meanwhile, real-world scenarios were tested in the remaining part. Más and Pinto[12] worked on a problem that involved tankers, ports, pipelines, substations, and refineries. The corresponding formulation is a large-scale MILP continuous-time model essentially based on events, also involving several issues such as intermediate storage, settling tasks, and crude oil allocation using qualitative characteristics. Because of the high complexity and combinatorial nature of the problem, the authors decided to solve it in a sequential manner, using smaller models for several subsystems, demonstrating that it could be used as a decision-making tool for large logistic systems. Moro and Pinto[13] addressed the problem of short-term inventory management of a real-world refinery, using a continuous-time strategy to compare a mixed-integer nonlinear programming (MINLP) formulation to a MILP formulation. The system accounts for a pipeline that connects tankers to the crude tanks, settling time constraints to separate brine from the oil, interface separation, and charges to the CDU's feeding tanks and the CDU. Reddy et al.[14] developed, in parallel, a continuous-time formulation for the same system. One feature of this work is the use of an iterative approach that reduces the MILP complexity and, consequently, the computational effort. These authors published a comparative study between this formulation with an equivalent that uses a discrete time formulation[15] and have claimed that the discrete-time formulation performs better than the continuous-time formulation, for the given problem.

Within the previously noted published works, the comparison among different approaches has been scarce, because of the fact that each problem considers different restrictions that highly restrict the modeling techniques and their performance. In addition, one major conclusion from the published works is that the use of decomposition approaches or even other type of techniques to overcome problem complexity, such as heuristics,[16] is crucial.

A very important component of the oil supply chain is transportation and distribution. It ensures that crude oil will be available in refineries and that products will be distributed through local markets that are spread throughout the world. Effort is necessary to maintain an efficient transportation network and the correspondent scheduling. This element represents a crucial logistic operation of the crude oil supply chain management, where several resources are used, such as pipelines, tank trucks, or railroad tanks. Cheng and Duran[17] made a revision on oil transportation logistics, where several approaches were analyzed. As a conclusion, pipelines seemed to be a reliable, effective, and low-cost way to transport crude oil and its derivatives. Recently, there have been several attempts to develop techniques that help the scheduling of these entities. One of the first approaches was presented by Sasikumar et al.[18] These authors made use of a knowledge-based heuristic search. The system considers a single pipeline, with one origin and multiple destinations, and the objective is to obtain a pumping schedule. In this work, it is recognized that it is difficult to manage rescheduling situations using this type of method. On the other hand, combinatorial issues that result from the use of binary variables may be overcome.

More-recent works have largely been based on MILP techniques. Rejowski and Pinto[19] have explored discrete representations of both time and pipeline volume to address the problem of oil products distribution from one refinery to several distribution centers. This work was improved later[20] to develop generalizations and include valid cuts. A later work by the same authors[21] explored the continuous-time representation of multiproduct pipeline scheduling and also hydraulic operation, resulting in a MINLP model. Comparisons are made with the previous works, noting that the later approaches showed better results. An alternative work was developed by Cafaro and Cerdá.[22] These authors addressed a continuous-time and pipeline volume representations on a MILP approach and addressed the same problem presented by Rejowski and Pinto.[19] The reported solution times were significantly lower. One of the major drawbacks that has been recognized is the common use of short horizons and unique due dates. For this reason, they apply to this methodology the concept of rolling horizons.[23] In this way, they ensure that the pipeline schedule reflects the client demands, even accounting for the pipeline operational lag.

An alternative approach was presented by Magatão et al.,[24] who developed a different MILP discrete approach, combined with a decomposition strategy, to address the case where a pipeline operates in both flow directions. Later, the same authors[25] expanded their approach by including constraint logic programming (CLP) techniques and concluded that this approach presented a better performance than the separate equivalents on MILP or CLP. All of the previous pipeline models used the minimization of operational costs as the optimization direction.

The list of works is expanding, translating opportunities of work that exist in this area, not only on the global system or subsystems described, but also on the approaches adopted. From the analysis that has been made, it can be observed that the node of the chain less explored is the end, where no effort has been exerted to explore the operation at the distribution centers. This is the focus of the present work.

A mathematical model to account for the pipeline scheduling and inventory management at distribution centers is developed. The model that is developed extends the issues raised by previous work and goes further by considering a detailed supply of client demands where a daily requirement is modeled. Furthermore, inventory management is not considered to have been studied in previous works. Taking into account the distribution centers, real operation storage capacities, as well as operational restrictions, are modeled. For the latter, the existence of quality control tasks are introduced with associated settling periods that constrain the product availability for clients.

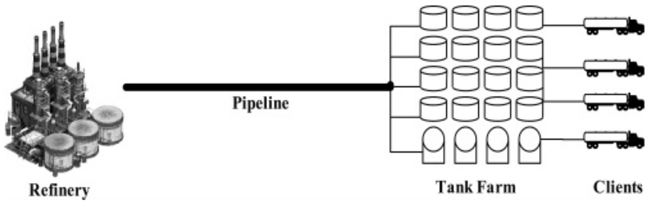

Figure 1. Schematic depiction of the problem operating system.

The pipeline schedule defines the product sequence to transport and the lot volumes and associated timing issues, beginning and ending times of each lot pumping, and discharging. The schedule enables the maintenance of a feasible inventory level during the entire time horizon, considering settling periods, maximum and minimum tank capacity, and satisfaction of client demands.

The general model developed is applied to a real-case study of a Portuguese distribution company.

The paper is structured as follows. The next section describes the general problem that supports the present work. Section 3 is divided in two main sections: the first section is dedicated to the modeling considerations concerning the mathematical representation of the problem under study, and the second section is reserved for the detailed mathematical model description. (This last section includes an extension of the mathematical model that accounts for the modeling of the settling period existence for new lots.) In section 4, a real-world case study is examined, where different scenarios are derived to test the base model and its extension. All the results are presented, compared, and discussed. Finally, section 5 includes the conclusions and some guidelines for future work.

2. Problem Description

Figure 1 gives a schematic overview of the considered problem. The system comprises a pipeline that pumps oil derivatives to a single distribution center located in a strategic local market. The distribution center contains a tank farm, where each tank has a fixed product service; i.e., each product is always stored in the same set of tanks. The clients are directed to the distribution center, where they are supplied with the respective products.

The process involves unloading oil derivatives from the pipeline to the respective distribution center's tanks and then making them available to the local market. Because there is only one pipeline, only one lot of any product is arriving at each moment. Each tank can assume three different states (at any given moment): loading from pipeline, full and performing settling and approving tasks, unloading for clients. Therefore, the problem not only relies on the scheduling but also on the tanks' inventory management. On the other hand, clients provide a monthly plan of their demands that are to be satisfied on a daily basis.

Based on this characterization, the distribution center's schedulers develop a monthly pipeline schedule and inventory management that allows the tracking all the operations.

In summary, the problem addressed in this paper can be stated as follows:

Given (a) the number of products to be transported, (b) the matrix of possible sequences between pairs of products in pipeline transit, (c) the maximum storage capacity for each product, (d) the pipeline capacity, (e) the time horizon extent and the total number of days to be considered, (f) the maximum number of allowable lots to be pumped to the pipeline during the time horizon, (g) the pumping rate, (h) the initial inventory of each product, (i) the daily clients' demands, and (j) the minimum settling period, one must *determine* (1) the optimal pipeline schedule, with the sequence of products and pumping and discharging conditions; and (2) the inventory management, including daily volume balances for each product and monitoring of arrivals, settling, and approving tasks, as well as satisfaction of the client demands, to *satisfy* a predefined objective function that can be either economical or operational.

The development of a rigorous mathematical model to represent the aforementioned problem requires a detailed analysis of the process, where some assumptions arise, such as the following:

(A1) To maintain a fixed flow rate during the entire time horizon, this assumption will be lifted in future work;

(A2) For scheduling purposes, do not consider the volume of the interfaces, which are much smaller when compared to lot volumes. In addition, the current sequence constraints already account for interface interference minimization;

(A3) The tanks in the distribution center always assume one of the following set of states: filled to the total capacity, settling, or distributing the assigned product to clients.

3. Continuous-Time Mixed-Integer Linear Programming (MILP) Model

3.1. Model Building Considerations. When modeling the problem stated in the previous section, three key aspects determine the approach to develop the mathematical model. These are (i) the time and volume scales adopted, (ii) the products sequence assumptions, and (iii) the client information. In the following sections, some considerations will be addressed for these topics.

3.1.1. Continuous Time and Volume Scales. The modeling of time issues is largely influenced by the characteristics of the problem under study. Two main types of time representations are often used: time discretization or time as a continuous variable. In discrete-time formulations, good accuracy is associated with an excessive number of time periods and, thus, a large amount of information is generated. On the other hand, a continuous-time representation overcomes this problem by using fewer time intervals but results in large integrality gaps problems, requiring complex and/or less-rigorous formulations to eliminate possible nonlinearities.

Looking at the problem described in section 2, different aspects influence the choice of the time and volume representations. The problem involves a continuous pipeline that transports oil derivatives. Also, the market requirements are different for the products that are transported and, taking into account the tanks operability (see section 2, assumption A3), the volume of the lots that are transported should be a multiple of the available tank capacity of the products, which results in very different values, regarding the product. Therefore, the usage of a discretization approach would require a short interval length to comport such a variety of information, spread over two different scale types: time and volume, leading to large models. For these reasons, the time and volume scales adopted in this work are represented through continuous approaches (see Figure 2).

The continuous time scale is controlled by the time when each lot i finishes to be pumped to the pipeline (event point-driven). At each time interval, there can be two distinct situations: either the pipeline is working over the complete time interval (Figure 2, for event 5, situation B) or there is a stopping time, which is allocated at the beginning of the interval (Figure 2, for event 5, situation A).

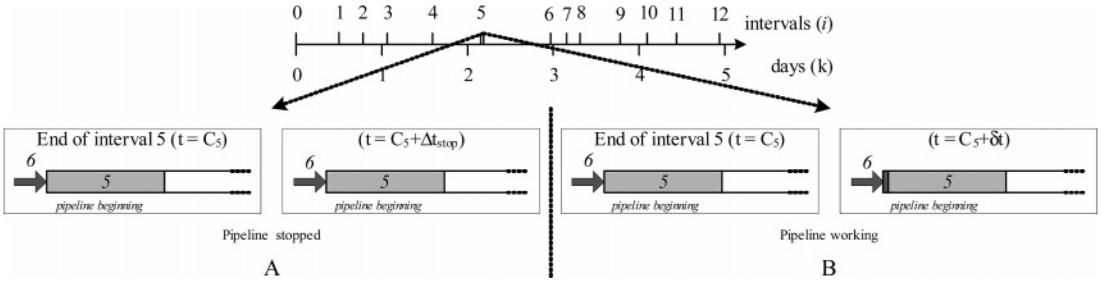

Figure 2. Model representations: continuous-time and pipeline volume and lot sequencing.

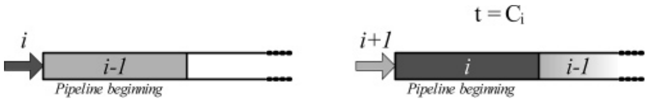

Figure 3. Physical allocation of lots inside the pipeline.

On the other hand, the continuous pipeline volume scale is controlled by the upper volumetric coordinate of each lot i' inside the pipeline when a later lot i has finished being pumped to the pipeline. Chronologically, pumping lot i is situated farther from the pipeline origin than lot $i + 1$ (Figure 3). This representation approach follows the work developed by Cafaro and Cerdá,[22] where C_i represents the completion time of pumping lot i to the pipeline.

3.1.2. Product Sequence. One of the main aspects of this problem is the practiced product sequence inside the pipeline. Because of the products' final specifications, there are some transportation incompatibilities between products inside the pipeline. In this system, no plugs are used to separate consecutive lots. In this way, the common sequence will have a tendency to lie on a cycle of products that is a result of systematic practical approaches developed by schedulers to avoid undesired product contamination. Furthermore, because of time and volume scales built within a sequential approach, the product sequence is the key aspect for the solution reported by this model.

The following approaches concerning the product sequence are explored in this paper (these guarantee the product sequence restrictions):

(S1) free sequence, where the product sequence is obtained by optimization but obeys the elimination of forbidden consecutive products;

(S2) fixed sequence, where the sequence is predefined based on real used procedures and is given to the model; and

(S3) mixed sequence, where an incomplete predefined sequence is given to the model, which means that the choice of open positions can be filled by different allowed products.

3.1.3. Daily Client Information: From Discrete to Continuous Representation. A very important point of the entire process is the pipeline discharge at the distribution center. Although in the first part of the process the operation is continuous, exception made if there is any pipeline stop, and the main objective is to provide a schedule for the current time horizon; in the second part, the operation relies on the batch field and the objective is to develop valid resource management.

Because the approach of this formulation is to use a general continuous-time scale for the entire process description, then the discrete information must be handled so that it can be introduced as continuous information. For this reason, the daily client demands are translated into continuous information, using two decision variables: one represents a time position, $\mathrm{dme}_{i,k}$, and the other is an event variable that matches the discrete-time scale with the continuous-time scale, $\mathrm{dm}_{i,k}^{\mathrm{Final}}$.

Taking, as an example, the time scales presented in Figure 2, where the continuous-time scale is represented by events i and the discrete-time scale is represented by days k, the event variable states that the zero hour of a certain day k relies on the time interval $[C_{i-1},C_i]$. For example, the zero hour of day 2 is located in time interval $[C_4,C_5]$. The event variable $\mathrm{dm}_{5,2}^{\mathrm{Final}}$ takes a value of 1 for this grid matching point. To model this approach, another decision variable is included. This variable takes a value of 1 whenever the zero hour of day k relies on time interval $[0,C_i]$. The discontinuity on the values of this variable for each day gives the event variable. For the example of matching day 2, $\mathrm{dme}_{i,2} = 1$ for $i = 1, ..., 5$.

3.2. The Mathematical Model. The basis for this model is the mathematical formulation of Cafaro and Cerdá[22] (referenced hereafter as the CC model, and, for this reason, in the model formulation, the same nomenclature will be used in similar parts, namely time and volume scales for pipeline representation). The main differences between this work and the work of the CC model rely both on the system studied and the modeling of market behavior and distribution center internal dynamics:

(a) Our approach relies on a system with a single distribution center, whereas their approach assumes a multicenter system. However, the generalization of the model of this work is direct.

(b) On modeling issues, the CC model represents clients at the end of the time horizon, whereas we generalized this approach and considered that clients' supply can be performed in several intermediate discrete time points, e.g., daily.

(c) Finally, and for the first time, this paper considers the dynamics of a common distribution center, where usual operational issues that have impact on scheduling and inventory management issues are modeled explicitly, e.g., the settling period.

The model is composed of the pipeline as a transportation resource and the inventory management of a distribution center. An initial model (the base model) was initially developed that evolutes in the way of more-rigorous descriptions of a real-world scenario and comprises some extensions. In section 3.2.1, the base model will be described. In this section, the innovative constraints of this model will be differentiated, in comparison to the CC model. In the subsequent section, an extension to the base model will be presented where the settling period for each new lot discharged from the pipeline, which is required for quality and approving tasks, are modeled. Finally, section 3.2.3 includes some notes on the objective function.

The different indices/sets, variables, and constraints are defined in Table 1.

3.2.1. The Base Model. Different model constraints that describe the aforementioned multiproduct pipeline transportation and inventory management are presented subsequently.

3.2.1.1. Lot Sequencing. The beginning of the pumping of lot i should not start before the end of the pumping of lot $i - 1$; thus,

$$C_i - L_i \geq C_{i-1} \qquad \forall\, i \in I^{\mathrm{new}} \tag{1}$$

Table 1. Indices/Sets, Parameters, and Variables Used in the Model Formulation[a]

symbol	description
	Indices/Sets
$i \in I$	lots
$p \in P$	products
$lt \in LT$	volumetric lots
$k \in K$	days
$I \supset I^{\mathrm{new}}$	lots whose pumping will be performed during the time horizon
	Parameters
$h_{\max}$	time horizon (h)
vb	flow rate (v.u./h)
$D_{\max}$	upper limit on the volume of any lot that can be transferred from the pipeline to the distribution center (v.u.)
$D_{\min}$	lower limit on the volume of any lot that can be transferred from the pipeline to the distribution center (v.u.)
σ	volumetric coordinate of the distribution center (v.u.)
M_{vol}	large number, related to pipeline volume
$\mathrm{SS}_{\min}$	minimum safety stock fraction
T_{rep}	minimum settling period (h)
$\mathrm{ID}_{\max,\mathrm{p}}$	maximum storage capacity for product p (v.u.)
Wo_i	volume of lots i that originally are inside of the pipeline (v.u.)
TDem_k	daily based (k) time scale for information on clients demands (h)
$\mathrm{lots}_{\mathrm{p,lt}}$	matrix of possible lots' volumes lt for each product p (v.u.)
$\mathrm{Dem}_{\mathrm{p},k}$	clients' demands of each product p for each day k ((v.u.)/day)
$\mathrm{sequence}_{p,p'}$	matrix of possible sequences between subsequent products inside the pipeline, p and p'
M	large value related to time horizon extent
	Continuous Variables
C_i	completion time of pumping lot i (h)
L_i	length of pumping lot i (h)
Q_i	volume of lot i (v.u.)
$F_i^{i'}$	upper volumetric coordinate of lot i while injecting a later lot i' (v.u.)
$W_i^{i'}$	volumetric fraction of lot i still inside of the pipeline while injecting a later lot i' (v.u.)
$D_i^{i'}$	volumetric fraction of lot i discharged to the distribution center while injecting a later lot i' (v.u.)
$D_{\mathrm{clients},i}^{i'}$	volume of lot i made available for clients after accomplishing the settling period while injecting a later lot i' (v.u.)
$\mathrm{qd}_{\mathrm{p}}^{i}$	demand of product p while injecting lot i (v.u.)
$DV_{i,\mathrm{p}}^{i'}$	volumetric fraction of lot i of product p discharged to the distribution center while injecting a later lot i' (v.u.)
$DV_{\mathrm{clients},i,\mathrm{p}}^{i'}$	volume of lot i of product made available for clients after accomplishing the settling period while injecting a later lot i' (v.u.)
$\mathrm{ID}_{\mathrm{total,o}}^{\mathrm{i}}$	total inventory of product p available at the distribution center while injecting lot i (v.u.)
$\mathrm{ID}_{\mathrm{p}}^{i}$	inventory of product p available for clients while injecting lot i (v.u.)
$T_{\mathrm{set},i}^{i'}$	total time that lot i had already settled while injecting a later lot i' (h)
	Binary Variables
$y_{i,\mathrm{p}}$	indicates that product p is contained in lot i whenever $y_{i,\mathrm{p}} = 1$
$x_i^{i'}$	indicates that a fraction of lot i can be transferred to the distribution center while a later lot i' being pumped, whenever $x_i^{i'} = 1$
$\mathrm{ls}_{i,\mathrm{p,lt}}$	indicates that lot i has lot volume lt of product p whenever $ls_{i,p,\mathrm{lt}} = 1$
$\mathrm{dme}_{i,k}$	indicates that day k (0 h) is located in time interval $[0,C_i]$ whenever $\mathrm{dme}_{i,k} = 1$
$\mathrm{dm}_{i,k}^{\mathrm{Final}}$	indicates that 0 h of day k are located in time interval $[C_{i-1}, C_i]$ whenever $\mathrm{dm}_{i,k}^{\mathrm{Final}} = 1$
$x_{\mathrm{set},i}^{i'}$	indicates that lot i has already performed a settling period over T_{rep} whenever $x_{\mathrm{set},i}^{i'} = 1$
$x_{\mathrm{clients},i}^{i'}$	indicates that lot i made available for clients when a later lot i' is being pumped whenever $x_{\mathrm{clients},i}^{i'} = 1$

[a] Throughout this table, the abbreviation v.u. denotes volumetric unit.

where C_i is the completion time (end of pumping) of lot i and L_i is the pumping duration of lot i. Compared with the CC model formulation, this constraint decreased the changeover time between consecutive products. Also, the pumping duration of lot i is always inferior to the completion time of pumping lot i:

$$L_i \leq C_i \qquad \forall\ i \in I^{\mathrm{new}} \tag{2}$$

Furthermore, it is imposed that all the assigned lots finish their pumping during the time horizon:

$$C_i \leq h_{\max} \qquad \forall\ i \in I^{\mathrm{new}} \tag{3}$$

3.2.1.2. Relation between Volume and Pumping Duration. Using a constant flow rate vb, the relation between the volume of the lot (Q_i) and pumping duration is given by

$$\mathrm{vb} \times L_i = Q_i \qquad \forall\ i \in I^{\mathrm{new}} \tag{4}$$

In the original model (the CC model), the flow rate can vary. In the present model, it was assumed a constant. This describes a real plant procedure, where the goal of the pipeline is to maintain a constant flow rate.

3.2.1.3. Forbidden Sequences. Because of operability restrictions that concern the product quality, some product sequences are not allowed. These are defined in the sequence matrix ($\mathrm{sequence}_{p,p'}$). If product p can be followed by product p', then $\mathrm{sequence}_{p,p'} = 1$; otherwise, $\mathrm{sequence}_{p,p'} = 0$. Therefore, any feasible schedule must verify the following restriction:

$$y_{i-1,p} + y_{i,p'} \leq 1 + \mathrm{sequence}_{p,p'} \qquad \forall\ i \in I^{\mathrm{new}}, \forall\ p,p' \in P \tag{5}$$

where $y_{i,p}$ represents the allocation of product p to lot i.

3.2.1.4. Upper and Lower Volume Coordinates of Lot *i*. Any event that occurs in the process other than the end of pumping of each lot must be reported to that event. At any given instant (reported as i'), the upper volumetric coordinate of lot i is $F_i^{i'}$. If $W_i^{i'}$ represents the volumetric fraction of lot i still inside of the pipeline while injecting a later lot i', then the lower volumetric coordinate is the same as the upper volumetric coordinate of the following lot, $i + 1$. Consequently, we have

$$F_{i+1}^{i'} + W_i^{i'} = F_i^{i'} \qquad \forall\, i \in I, \forall\, i' \in I^{\text{new}}, i' > i \qquad (6)$$

3.2.1.5. Pipeline End Tasks. When a lot i arrives at the end of the pipeline, two distinct situations may happen at the pipeline beginning: (1) either lot i is still being pumped to the pipeline or (2) a later lot i' is now being pumped. If situation 1 is verified, the sum of the total volume of lot i inside the pipeline at the time interval i (W_i^i) and the volume of lot i transferred from the pipeline to the distribution center (D_i^i) is equal to the lot volume:

$$Q_i = W_i^i + D_i^i \qquad \forall\, i \in I^{\text{new}} \qquad (7)$$

In this particular situation, the upper volumetric coordinate of lot i is equivalent to the total volume of lot i inside the pipeline:

$$F_i^i = W_i^i \qquad \forall\, i \in I^{\text{new}} \qquad (8)$$

Note that the conditions described by eqs 7 and 8 are still valid for situation 2. The only difference is that $D_i^i = 0$.

Situation 2 will probably happen more often than situation 1. For this case, the volume of lot i discharged to the distribution center must be referred to a subsequent pumping lot, $D_i^{i'}$, and is obtained through

$$W_i^{i'} = W_i^{i'-1} - D_i^{i'} \qquad \forall\, i \in I, \forall\, i' \in I^{\text{new}}, i' > i \qquad (9)$$

It is then necessary to verify when it is possible to start unloading a lot to the distribution center. This is obtained through the restrictions described as eqs 10−13. If lot i can be unloaded to the distribution center by the time that lot i' has finished being pumped, the associated discharged volume is given by ($D_i^{i'}$):

$$D_i^{i'} \le D_{\max} x_i^{i'} \qquad \forall\, i \in I, \forall\, i' \in I^{\text{new}}, i' \ge i \qquad (10)$$

where $x_i^{i'}$ is the decision variable for the discharge operation and $D_{\max}$ is an upper limit to $D_i^{i'}$.

On the other hand, the upper volumetric coordinate of lot i ($F_i^{i'}$) is less than the volumetric coordinate that represents the distribution center, σ, unless it is being discharged:

$$F_i^{i'} \ge \sigma \times x_i^{i'} \qquad \forall\, i \in I, \forall\, i' \in I^{\text{new}}, i' \ge i \qquad (11)$$

In the CC model formulation, this restriction also accounts for interface volume. Equation 12 enforces the definition of discharge operation, which has proven to improve model performance, as stated by Cafaro and Cerdá:[22]

$$F_{i+1}^{i'-1} \le \sigma - D_i^{i'} + (1 - x_i^{i'}) M_{\text{vol}} \qquad \forall\, i \in I, \forall\, i' \in I^{\text{new}}, i' > i \qquad (12)$$

The third term in the right-hand side of this constraint becomes apparently redundant for a system with a single distribution center. However, it has performed in a positive manner to improve the solution finding process. Future work will keep this constraint into consideration.

Furthermore, to enforce the zero values of the decision variable $x_i^{i'}$ if no fraction i is inside the pipeline, we have

$$W_i^{i'-1} \ge D_{\min} x_i^{i'} \qquad \forall\, i \in I, \forall\, i' \in I^{\text{new}}, i' > i \qquad (13)$$

where $D_{\min}$ is a lower bound for variable $D_i^{i'}$.

This condition improves the original CC model, because of the fact that it was verified that after a lot i was completely discharged from the pipeline, occasionally $x_i^{i'}$ values were equal to 1. This state for this binary variable is undesired for the model extension presented in the subsequent section, but is conditioned to the type of formulation presented in this paper (due to the establishment of parameter $D_{\min}$).

Finally, any volume of lot i discharged to the distribution center, $D_i^{i'}$, is limited by the volume of lot i still inside of the pipeline in the previous time interval [$C_{i'-2}$,$C_{i'-1}$], $W_i^{i'-1}$:

$$D_i^{i'} \le W_i^{i'-1} \qquad \forall\, i \in I, \forall\, i' \in I^{\text{new}}, i' > i \qquad (14)$$

From the original CC model, this constraint decreased interface handling issues.

3.2.1.6. Product Allocation Constraints. Considering that, at most, each lot contains one product, we have

$$\sum_{p \in P} y_{i,p} \le 1 \qquad \forall\, i \in I^{\text{new}} \qquad (15)$$

If no product is allocated to a lot, it means that this lot is fictitious and, therefore, it should be placed at the end of the sequence, not influencing the final schedule:

$$\sum_{p \in P} y_{i,p} \le \sum_{p \in P} y_{i-1,p} \qquad \forall\, i \in I^{\text{new}} \qquad (16)$$

If one desires to work with a fixed number of lots and/or a mixed sequence of products, the restrictions described by eqs 15 and 16 can be replaced by

$$\sum_{p} y_{i,p} = 1 \qquad \forall\, i \in I^{\text{new}} \qquad (17)$$

Furthermore, the restriction described by eq 17 can be eliminated if a fixed sequence of products is used.

3.2.1.7. Choice of Lot Volumes. Considering that the tanks in the distribution center are either being filled up to maximum capacity; settling, with full capacity occupied; or distributing products, then a lot i will assume one of the possible lot volumes, which corresponds to tank capacity, instead of a free positive value as in the CC model:

$$\sum_{lt \in LT} ls_{i,p,lt} = y_{i,p} \qquad \forall\, i \in I^{\text{new}}, p \in P \qquad (18)$$

$ls_{i,p,lt}$ is a disaggregated decision variable from $y_{i,p}$ that chooses the lot volume, lt.

The volume of lot i (Q_i) is then obtained directly by

$$Q_i = \sum_{p \in P} \sum_{lt \in LT} (ls_{i,p,lt} \text{lots}_{p,lt}) \qquad i \in I^{\text{new}} \qquad (19)$$

where $\text{lots}_{p,lt}$ is the parameter for possible lot volume specification. This formulation will also enforce the calculation of pumping time of lot i by the constraint described in eq 4.

3.2.1.8. Overall Volume Balance to the Pipeline Ends while Injecting Lot i'. Because the event considered to generate the continuous-time scale is the pumping of each lot i, at any interval, the volume balance between the pipeline ends is accomplished by the total volume of the lot fractions i discharged in the current time interval and the volume of the lot $i' \ge i$ pumped in that interval:

$$\sum_{i \in I, i \le i'} D_i^{i'} = Q_{i'} \qquad \forall\, i' \in I^{\text{new}} \qquad (20)$$

3.2.1.9. Initial Conditions inside the Pipeline. Parameter Wo_i indicates the volumes of lots i already inside the pipeline at the beginning of the time horizon. These lots are also ordered as indicated in Figure 3. To initialize the model, the following restriction is included:

$$W_i^{i'-1} = \mathrm{Wo}_i \qquad \forall\ i \notin I^{\mathrm{new}}, i' = \mathrm{first}(I^{\mathrm{new}}) \qquad (21)$$

This condition is only valid for pumping lot i' such that it is the first to be pumped.

3.2.1.10. Inventory Control at the Distribution Center. Previous sections addressed the pipeline operation, where scheduling is the main issue. After a lot reaches the end of the pipeline, one should identify the product contained in the lot and direct it to the respective tank set. Because distribution center operations reside in the management field, the most important issue is to control inventory during the time horizon, with respect to all the end-of-pipe operations.

To identify the product contained in each discharged volume, the following conditions are written:

$$\mathrm{DV}_{i,p}^{i'} \leq D_{\max} y_{i,p} \qquad \forall\ i \in I, i' \in I^{\mathrm{new}}, p \in P, i' \geq i \quad (22)$$

$$\sum_{p \in P} \mathrm{DV}_{i,p}^{i'} = D_i^{i'} \qquad \forall\ i \in I, i' \in I^{\mathrm{new}}, i' \geq i \qquad (23)$$

The continuous variable $\mathrm{DV}_{i,p}^{i'}$ indicates the volumetric fraction of lot i of product p discharged to the distribution center while injecting a later lot i' and is obtained through the restrictions given as eqs 22 and 23.

The restriction described by eq 22 is confined to only, at most, one nonzero value in p for variable $\mathrm{DV}_{i,p}^{i'}$ and further indicates an upper limit for this variable ($D_{\max}$). On another hand, the restriction described by eq 23 obliges that the sum of the nonzero variable $\mathrm{DV}_{i,p}^{i'}$ assumes the same value as the aggregate corresponding variable $D_i^{i'}$.

Furthermore, inventory control is done through a volume balance to each tank, at each time interval.

$$\mathrm{ID}_{\mathrm{total},p}^{i'} = \mathrm{ID}_{\mathrm{total},p}^{i'-1} + \sum_{i \in I, i \leq i'} \mathrm{DV}_{i,p}^{i'} - qd_p^{i'} \qquad p \in P, i' \in I^{\mathrm{new}} \qquad (24)$$

Under these conditions, the update of inventory of product p accounts for the inputs, discharges from the pipeline ($\mathrm{DV}_{i,p}^{i'}$) on that interval, and the outputs for clients (qd_p^{i}). $\mathrm{ID}_{\mathrm{total},p}^{i'}$ represents the inventory of product p at interval i'. Initial inventories ($\mathrm{ID}_{\mathrm{total},p}^{i'-1}$, where $i' = \mathrm{first}(I^{\mathrm{new}})$) should be provided to the model. The condition described by eq 25 imposes the maximum capacity for each product, through parameter $\mathrm{ID}_{\max,p}$.

$$\mathrm{ID}_{\mathrm{total},p}^{i'} \leq \mathrm{ID}_{\max,p} \qquad p \in P, i' \in I^{\mathrm{new}} \qquad (25)$$

In addition, some operational constraints can also be included when a minimum stock percentage ($\mathrm{SS}_{\min}$) must be guaranteed. This condition is optional and represents a hard constraint on inventory. No considerations on safety stocks were made in the original CC model.

$$\mathrm{ID}_{\mathrm{total},p}^{i'} \geq \mathrm{SS}_{\min} \mathrm{ID}_{\max,p} \qquad p \in P, i' \in I^{\mathrm{new}} \qquad (26)$$

3.2.1.11. Client Demands. Daily client information is an important issue. In previous work, demands were considered at the end of the time horizon or at few intermediate points. This is a rough model formulation, because a tank farm may not be able to accommodate the arriving lot at a given moment. Furthermore, previous works focused on the belief that the pipeline schedule should account for client demands (in shorter time horizons, initial inventories are sufficient to satisfy clients, so the pipeline schedule does not reflect the needs of the client), but they ignored the fact the pipeline schedule should account for tank availability at any moment in the time horizon.

In section 3.1.3, a simple approach to transform the discrete information concerning client demands on continuous information was presented. The decision variables are then the position variable ($\mathrm{dme}_{i,k}$) and the event variable ($\mathrm{dm}_{i,k}^{\mathrm{Final}}$). The position variable indicates whether the midnight of day k is before C_i and is calculated through the following restrictions:

$$C_i \geq \mathrm{TDem}_k \mathrm{dme}_{i,k} \qquad \forall\ i \in I^{\mathrm{new}}, k \in K \qquad (27)$$

$$C_i \leq \mathrm{TDem}_k + h_{\max} \mathrm{dme}_{i,k} \qquad \forall\ i \in I^{\mathrm{new}}, k \in K \quad (28)$$

where TDem_k represents the discrete-time scale with an interval length of 1 day. The discontinuity on the position variable $\mathrm{dme}_{i,k}$ allows the calculation of the event variable:

$$\mathrm{dm}_{i,k}^{\mathrm{Final}} = \mathrm{dme}_{i,k} - \mathrm{dme}_{i-1,k} \qquad \forall\ i \in I^{\mathrm{new}}, k \in K \quad (29)$$

The continuous information of client demands ($\mathrm{qd}_{i,p}$) is then obtained using the matching variable between discrete and continuous-time scales, as stated in the condition described by eq 30.

$$\mathrm{qd}_{p,i} = \sum_{k \in K} \mathrm{Dem}_{p,k} \mathrm{dm}_{i,k}^{\mathrm{Final}} \qquad \forall\ i \in I^{\mathrm{new}}, p \in P \qquad (30)$$

The model receives, as a parameter, the discrete daily information on client demands through parameter $\mathrm{Dem}_{p,k}$.

3.2.1.12. Auxiliary Conditions. To improve the model performance, some auxiliary constraints were defined. These are essentially sequencing constraints that are applied to pipeline operation, which explore the spatial representation of pipeline volume.

The condition described by eq 31 enforces the upper volumetric coordinate variable $F_i^{i'}$ to be sequential for the same lot.

$$F_i^{i'} \geq F_i^{i'-1} \qquad \forall\ i \in I, \forall\ i' \in I^{\mathrm{new}}, i' > i \qquad (31)$$

The same assumption can be made for the lower volumetric coordinate, $F_i^{i'} - W_i^{i'}$, which is included in the model by the restriction described by eq 32.

$$F_i^{i'} - W_i^{i'} \geq F_i^{i'-1} - W_i^{i'-1} \qquad \forall\ i \in I, \forall\ i' \in I^{\mathrm{new}}, i' > i \qquad (32)$$

Another condition can be included that uses, as an upper limit for the current volume of lot i inside the pipeline $W_i^{i'}$, the upper volumetric coordinate of lot i:

$$F_i^{i'} \geq W_i^{i'} \qquad \forall\ i \in I, \forall\ i' \in I^{\mathrm{new}}, i' \geq i \qquad (33)$$

The inclusion of the choice of lot volume in the model formulation allows the possibility to write the volume balance in a different way for the pipeline beginning. Using the right term of eq 19, an equivalent volume balance can be included:

$$\sum_{i \in I, i \leq i'} D_i^{i'} = \sum_{p \in P} \sum_{lt \in LT} (ls_{i,'p,lt} \mathrm{lots}_{p,lt}) \qquad \forall\ i' \in I^{\mathrm{new}} \qquad (34)$$

Using both forms—eq 20 from the CC model and the new volume balance for the present model (described by eq 34)—the formulation makes it more efficient.

Finally, we examine some considerations that help the establishment of the value of the event $\mathrm{dm}_{i,k}^{\mathrm{Final}}$:

$$\sum_{i \in I^{new}} dm_{i,k}^{Final} = 1 \qquad k \in K \tag{35}$$

This condition states that day is allocated only once in the continuous-time scale.

$$\sum_{i' = first(I^{new})}^{i' \leq i} dm_{i',k}^{Final} \leq \sum_{i' = first(I^{new})}^{i' \leq i} dm_{i',k-1}^{Final} \qquad \forall\, i \in I^{new}, k \in K, k > 1 \tag{36}$$

In turn, the condition described by eq 36 enforces an ordered sequence to allocate days in the continuous-time scale.

3.2.2. Model Extension: The Settling Period. In the previous section, a general formulation was presented that accounts for the pipeline scheduling, where different oil derivatives are transported from a refinery to a distribution center, and for the maintenance of a feasible inventory throughout the entire time horizon. Client demands are considered on a daily basis. This extension is entirely new, if compared to the CC model, and was developed to build up a more-rigorous model that describes real-world internal operations in a distribution center. These are related to the need of guaranteeing settling periods that account for quality control and lot-approving tasks. In this way, at any given moment of the time horizon i, there will be a certain inventory (ID_p^i) for each product p that is available for clients; however, there is also a total inventory of product p ($ID_{total,p}^i$) such that $ID_{total,p}^i \geq ID_p^i$, which accounts further for the inventory on settling time.

The key idea for this extension is to start to count the settling period at the moment each lot has completely left the pipeline. For this purpose, one can use the previously introduced decision variable $x_i^{i'}$. Unless both of the next situations have already happened, the settling time is set to zero:

(a) lot i has not been pumped to the pipeline or is still inside the pipeline;

(b) lot i is being discharged from the pipeline.

Considering that variable $x_i^{i'}$ is defined such that $x_i^{i'} = 0$, for $i' < i$, because of the problem formulation, then it is necessary to add two restrictions to translate situations a and b completely. The former is granted by the condition described by eq 37, and the latter is granted by the condition described by eq 38.

$$T_{set,i}^{i'} \leq h_{max} \sum_{i'' \geq i}^{i'} x_i^{i''} \qquad \forall\, i \in I, i' \in I^{new}, i' \geq i \tag{37}$$

$$T_{set,i}^{i'} \leq (1 - x_i^{i'}) h_{max} \qquad \forall\, i \in I, i' \in I^{new}, i' > i \tag{38}$$

$T_{set,i}^{i'}$ is the settling time of lot i while injecting a later lot i'.

The settling period, when nonzero, is updated using the length of each time interval. This is achieved using the following restrictions:

$$T_{set,i}^{i'} \leq T_{set,i}^{i'-1} + C_{i'} - C_{i'-1} \qquad \forall\, i \in I, i' \in I^{new}, i' > i \tag{39}$$

$$T_{set,i}^{i'} \geq T_{set,i}^{i'-1} + C_{i'} - C_{i'-1} - x_i^{i'} h_{max} - M\left(\frac{W_i^{i'}}{\sigma}\right) \qquad \forall\, i \in I, i' \in I^{new}, i' > i \tag{40}$$

where M represents a large value that is related to the time horizon extent. The last term on the condition described by eq 40 behaves as an activation/deactivation term; the value of M should be large enough to use the ratio between $W_i^{i'}$ and σ as a decision variable. For this specific case, a value of M that is equivalent to $|h_{max}| \times h_{max}$ is advisable.

After controlling the values of the settling period of each lot, it is necessary to include a decision variable, $x_{set,i}^{i'}$, that takes a value of 1 whenever lot i has already completed a settling time of at least T_{rep} by the time lot i' is being pumped. Its values are obtained through

$$T_{set,i}^{i'-1} \geq T_{rep} x_{set,i}^{i'} \qquad \forall\, i \in I, i' \in I^{new}, i' > i \tag{41}$$

$$T_{set,i}^{i'-1} \leq T_{rep} + h_{max} x_{set,i}^{i'} \qquad \forall\, i \in I, i' \in I^{new}, i' > i \tag{42}$$

In this way, it is possible to determine when the product is available for clients through the event variable $x_{clients,i}^{i'}$. Therefore,

$$x_{set,i}^{i'} - x_{set,i}^{i'-1} = x_{clients,i}^{i'} \qquad \forall\, i \in I, i' \in I^{new}, i' > i \tag{43}$$

The associated volume made available for clients at time interval i' is given by

$$\sum_{i \in I^{new}, i' > i}^{I} D_{clients,i}^{i'} \leq Q_i \qquad \forall\, i \in I \tag{44}$$

$$D_{clients,i}^{i'} \leq x_{clients,i}^{i'} D_{max} \qquad \forall\, i \in I, i' \in I^{new}, i' > i \tag{45}$$

which gives the inventory available to clients (ID_p^i), calculated using the expressions

$$ID_p^{i'} = ID_p^{i'-1} + \sum_{i \in I, i \leq i'} DV_{clients,i,p}^{i'} - qd_p^{i'} \qquad p \in P, i' \in I^{new} \tag{46}$$

$$ID_p^{i'} \leq ID_{max,p} \qquad p \in P, i' \in I^{new} \tag{47}$$

The initial inventory available for clients must be input into the model.

Equation 46 uses the volume that is made available for clients from lot i and product p, at time interval i' , $DV_{clients,i,p}^{i'}$. This variable is obtained from the corresponding aggregated continuous variable $D_{clients,i}^{i'}$ through eq 48. Equations 49 and 50 relate that continuous variable to decision variables $y_{i,p}$ and $x_{clients,i}^{i'}$.

$$\sum_p DV_{clients,i,p}^{i'} = D_{clients,i}^{i'} \qquad \forall\, i' \in I^{new}, i' \geq i \tag{48}$$

$$DV_{clients,i,p}^{i'} \leq D_{max} y_{i,p} \qquad \forall\, i' \in I^{new}, p \in P, i' \geq i \tag{49}$$

$$DV_{clients,i,p}^{i'} \leq D_{max} x_{clients,i}^{i'} \qquad \forall\, i' \in I^{new}, p \in P, i' \geq i \tag{50}$$

The model will track the total inventory ($ID_{total,p}^i$) as well as the inventory available for clients (ID_p^i). At any moment in the time horizon, both of these inventories must be feasible. The first determines if it is possible to accommodate the incoming lots that are being discharged from the pipeline, whereas the second states that, even with some amounts of product participating in the required settling period, it is possible to fulfill the client demands.

3.2.3. The Objective Function. The objective function to be used may be operationally or economically oriented. The orientation observed most often in previously published works is the economical field. However, in real-world scenarios, operational objectives are frequently used. This will be assumed in the present work, where the objective function is essentially the maximization of the amount of products transported plus the total inventory at the end of the time horizon.

$$\max \frac{\sum_{i \in I^{new}} L_i}{h_{max}} + \frac{\sum_{i \in I^{new}} Q_i}{h_{max} vb} + \frac{\sum_{p \in P} (ID_{I,p}/ID_{max,p})}{P} - \frac{I - \sum_{i \in I} \sum_{i' \in I^{new}, i' > i} x^{i'}_{clients,i}}{I} \quad (51)$$

The first term maximizes the total working time of the pipeline and, the second term enforces the maximization of the amount of products transported. The third term maximizes the inventory at the end of the time horizon, whereas the last term will be only used when the minimum settling period is considered. In this case, the latter term penalizes solutions where the number of lots that participate in the settling period is not the maximum possible. All the terms in the objective are normalized.

4. Scenario Analysis and Computational Results

This section is reserved to present a real-world scenario, which is the case study of a Portuguese company. This particular situation enables some testing on the previous model, namely, with different parameters and market demands, among other scenarios.

The first part of this section presents a brief overview of the company. In the second part of this section, some scenarios are described and the respective results are presented and analyzed.

4.1. Case Study. The present work is applied to the real world scenario of Companhia Logística de Combustíveis, S. A. (abbreviated hereafter as CLC), which is a Portuguese oil derivatives distribution company operating in the central area of Portugal. This company owns a distribution center in Portugal and receives six different oil derivatives from the Portuguese refinery, located in southern Portugal (Figure 4), by means of a pipeline. This distribution center is one of the major clients of the feeding refinery. The refinery takes into account the demands of CLC's distribution center to fulfill the monthly production plan. In this way, CLC's complete plan information is exchanged with refinery production planners to meet the needs of both parties.

Four of the products are white products and the remaining two are liquefied gases. The tank farm is composed by storage tanks for liquid products and spheres for gases (Figure 5). The products are identified as P1−P6.

The pipeline has a length of 147 km and the total volume is 18 000 v.u. (where v.u. denotes volumetric units) The operating flow rates can be in the range of 400−720 v.u./h. Because of product specifications and/or interface volumes, some sequences cannot be allowed inside the pipeline. Table 2 represents the possibilities allowed (which are noted by a checkmark, √) or not allowed (which are noted by a cross, ×) to combine two consecutive products.

Because the formulation of the model accounts for the maximum storage capacity of each product, Table 3 shows the limit values for CLC's case. This table also shows typical lot volumes for each problem that fill up exactly one tank or multiple tanks. In addition, this table introduces the color code for each product, which is depicted in the first column of the table.

The possible sequence, inventory management, and market demands that characterize CLC's operation reinforce the pipeline schedule to be based on a typical cycle of products (see Figure 6). This cycle is used by CLC's schedulers to build up their monthly plan. Because the cycle is used as input, it consists of a fixed sequence.

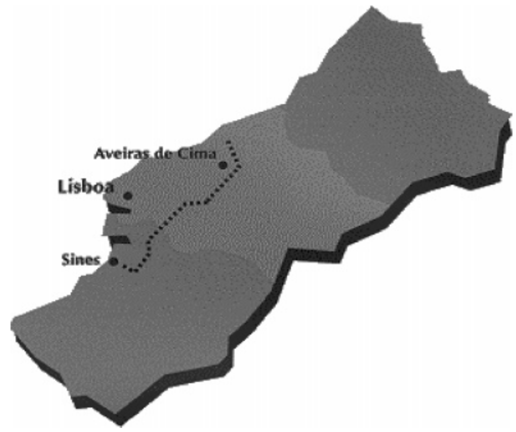

Figure 4. Map of the country of Portugal, showing the location of Companhia Logística de Combustíveis (CLC)'s facilities and pipeline path.

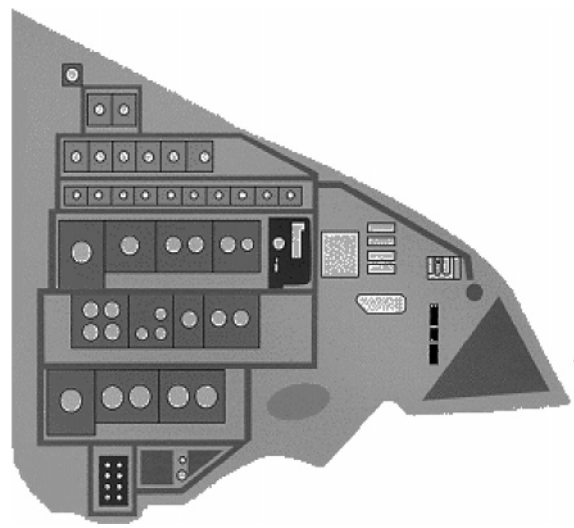

Figure 5. Layout of CLC's tank farm in Portugal.

Table 2. Matrix of Possibilities of Sequences on Two Consecutive Products Inside the Pipeline (sequence$_{p,p'}$): CLC's Case Study

	P1	P2	P3	P4	P5	P6
P1	-	√	√	√	×	×
P2	√	-	×	×	×	×
P3	√	×	-	√	√	×
P4	√	×	√	-	√	×
P5	×	×	√	√	-	√
P6	×	×	×	×	√	-

However, market evolution has led to a new situation where the product sequences should be more flexible. At CLC, the flexibility often relies on the choice of two single products that fit in the same position (Figure 7), instead of the inclusion of both products in the cycle. Because some of the information of this sequence is given as input, this resembles a mixed sequence.

In a heuristic-based plan, both of these types of product sequences can be very helpful to obtain the pipeline schedule. They leave behind a series of different options, and they are a result of several years of planning practice. However, from a modeling point of view, the optimization should define the optimal product sequence. This is modeled through a free sequence, which only has the forbidden sequences described in Table 2 as constraints.

CLC's operation is based on a monthly pipeline schedule, including the product sequence, lot volumes and timings, and inventory management, including the control of product stock, the achievement of a settling period, and client satisfaction.

Table 3. Maximum Storage Capacity and Practiced Lot Volumes: CLC's Case Study

Product (p)	$ID_{max,p}$ (v.u.)	$lots_{p,1}$ (v.u.)	$lots_{p,2}$ (v.u.)	$lots_{p,3}$ (v.u.)
P1	81500	21 800	18 000	17 300
P2	32 000	16 000	8 000	-
P3	24 000	16 000	8 000	-
P4	27 800	16 000	8 000	3 800
P5	10 320	3 440	1 720	860
P6	13 120	6 560	4 920	8 200

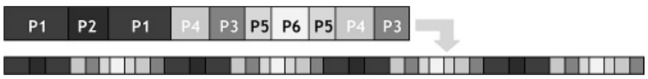

Figure 6. Base unit of the product sequence.

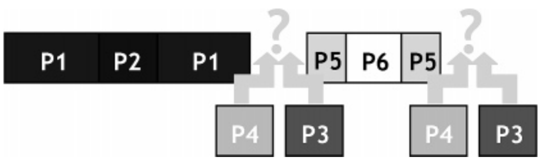

Figure 7. Evolution of the base unit.

At the beginning of each month, the pipeline is totally filled with product P1 and it starts to be pumped after the initial inventory control of the tanks, usually at 10 AM of day 1.

The flow rate can vary during the time horizon, but CLC's schedulers usually practice a constant flow rate. Typical values are 500, 550, or 600 v.u./h.

Finally, because of product certification, the minimum settling period that should be performed essentially consists of 24 or 48 h.

The first provisory client demands are sent to CLC usually two weeks before the beginning of each month. By this time, the first pipeline schedule and inventory management are produced. These documents will be used to develop the real schedule and inventory management that is a response to the tradeoff between refinery capabilities, the restrictions of the tank farm, or the clients' information fluctuations.

4.2. Results. In this section, some scenarios are presented and the respective model performance and results are shown and discussed. The model was developed using GAMS 21.5,[26] coupled with CPLEX 9.0. All the reported results were obtained using a personal computer (PC) with a Pentium IV 2.8 GHz processor and 256 MB RAM that was running Microsoft Windows XP Professional software.

As scenario data, the initial information corresponding to a typical month at CLC is used. The practiced flow rate will be set at the medium flow rate verified at the initial plan for this month, which is 519.4 v.u./h. The total client demands (aggregated information on the daily client demands) and initial inventories are shown in the Appendix. During the month in study, 35 lots were pumped in 31 days, and, therefore, this number will be used as a reference.

All scenarios were conducted using, as stopping criteria, either a maximum resource time of 7200 CPU seconds (which is a reasonable amount of time, considering CLC operation and the work of schedulers) or a final solution within a tolerance of 5%.

4.2.1. The Base Model. The main objective of the first set of scenarios is to test the base model and the model performance

Table 4. Model Performance for the Base Model

item	Value	
	fixed sequence	mixed sequence
number of equations	17067	17067
number of continuous variables	9831	9831
number of binary variables	3497	3505
model status	integer solution	integer solution
CPU time	62.785 s	105.631 s
number of iterations	3623	6733
number of nodes explored	140	110
best possible objective	2.761008	2.780304
objective	2.665586	2.672470
relative gap	3.58%	4.04%

Table 5. Model Performance for the Base Model, Free Sequence

item	Value	
	free sequence, no decomposition	free sequence, with decomposition
number of equations	17067	4489 + 6179
number of continuous variables	9831	2664 + 3638
number of binary variables	3531	981 + 1263
model status	no solution found	integer solution
CPU time	95000 s[a]	(3600 + 3600) s
number of iterations	1969665	4281699 + 3249632
number of nodes explored	500	6704 + 3848
best possible objective	(2.8599)	2.823282/2.884984
objective		2.661010/2.638498
relative gap		6.10%/9.34%

[a] CPU time limit was increased to find a solution with a gap of <5%.

when using the different proposed product sequence. Three scenarios will be analyzed for the base model, corresponding to the fixed, mixed, and free sequences of products (see Tables 4 and 5). When considering a solution that uses only one instance of the model, no solution was observed for the free sequence case, within the imposed limit of 2 h, corresponding to an increase of 27 binary variables when compared to the mixed sequence (see Table 5). The model was conducted, for this scenario, using a decomposition technique that divided the time horizon in two parts. The final conditions of the first part were given as initial conditions for the second part.

Table 6 compares the final inventories and the pipeline usage for the solutions obtained.

For the two first test scenarios, the model developed was quite efficient and the final solution for a 31-day scheduling period was reached within 1−2 CPU minutes, as indicated in Table 4. It can be observed that the mixed sequence was capable of achieving a higher objective value, which is reflected in higher pipeline usage (see Table 6). When leaving open the possibility for the model to decide over the products' sequence, as referred previously, a single model run was unable to obtain a solution in an extended period. In this way, a simple temporal decomposition strategy was applied, where the time horizon was split in two similar parts (see Table 5). The resulting models are composed of fewer variables and equations and are now able to build a valid solution for this scenario. For a CPU time limit of 2 h, they obtained solutions for both parts. The final gap is higher than that obtained for the fixed and mixed scenarios, but, ultimately, the pipeline usage was similar to the previous scenarios (see Table 6).

Figure 8 shows the inventory profiles for every scenario considered.

Figure 9 presents the pipeline schedule for the free sequence scenario. The first line presents the initial conditions. The

Table 6. Final Inventory and Pipeline Usage for the Base Model

	Final Inventory (v.u.)							
	P1	P2	P3	P4	P5	P6	total	pipeline usage (%)
fixed sequence	42254	24765	19927	19444	8553	10034	124977	96.5
mixed sequence	46754	24065	19927	19444	7693	10034	127917	97.2
free sequence[a]	57154	8765	19927	26644	8553	6754	127797	97.1

[a] Obtained through the decomposition approach.

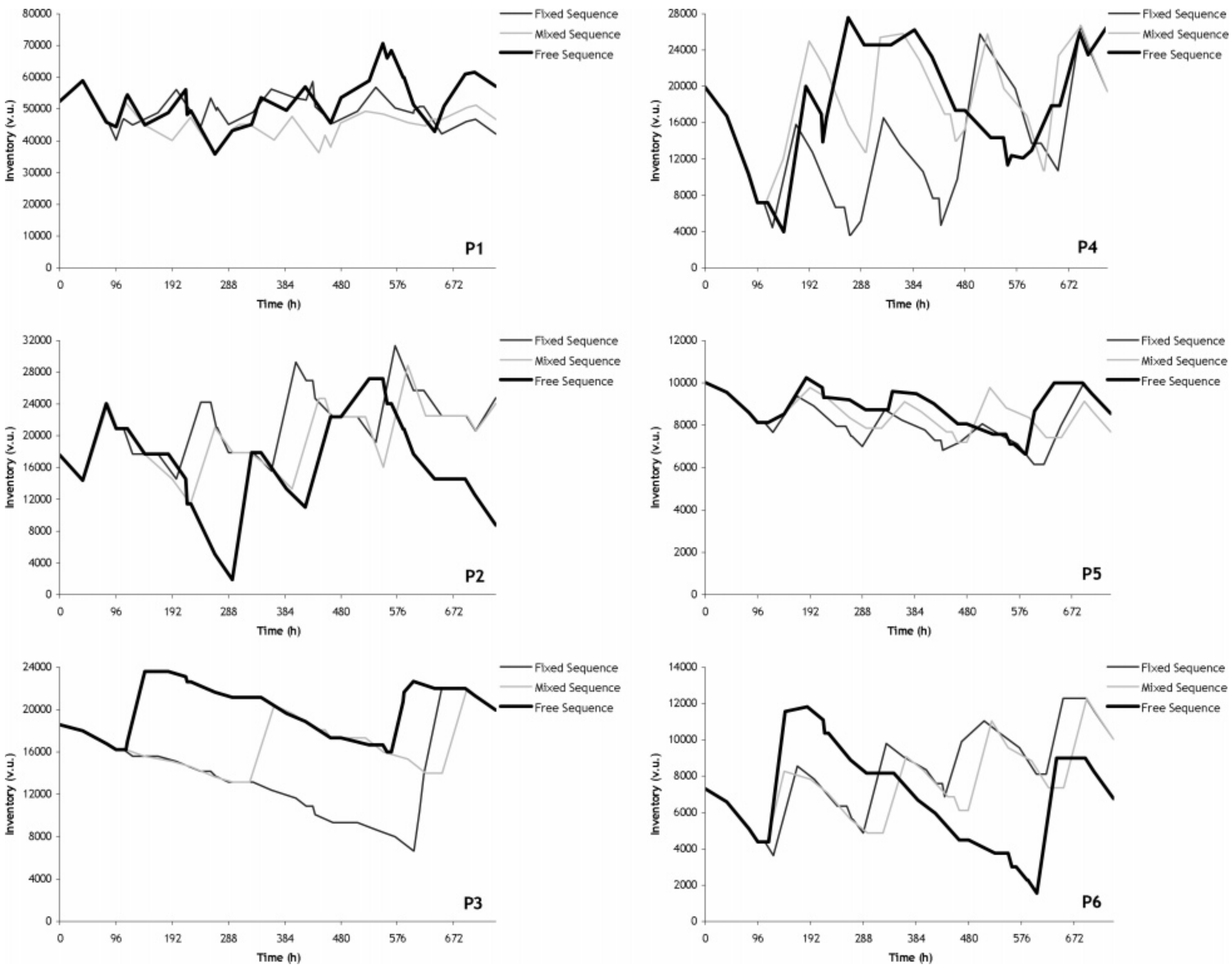

Figure 8. Inventory profiles for each product, considering the base model.

remaining lines represent each time interval returned by the model solution. On the left side, there is an axis with a continuous-time scale. The input arrows represent the inlet of the pipeline during the corresponding time interval. On the other hand, a single lot input corresponds to a discharge of one or more lots (or fractions). These are represented on the right side of Figure 9. The center of the figure presents the final state of the pipeline on the corresponding time interval. As stated previously, any pipeline stop is located at the beginning of the time interval where it occurs. Some stops were predicted by the model and are indicated on the far right side. The initial stop at 10 h is also indicated at the top.

Finally, Figure 10 compares the sequences of different scenarios. The fixed sequence corresponds to CLC's monthly plan. Changes between the fixed sequences and the mixed and free sequences are shown with a shaded/gray background. The mixed sequence allowed some changes in regard to open positions and, in the matter of results, obtained a slightly higher pipeline schedule. The free sequence presents a large amount of changes and, therefore, no maintenance of the usual product cycle was obtained. However, in regard to the results, the objective value was reasonable and pipeline usage and final inventories comparable to the previous scenarios.

There are three main conclusions from this first set of scenarios. First of all, the sequence of products is a very important result in any solution. The computational effort increases when the number of pre-fixed product−lot pairs decreases. This fact adds more complexity to the solution finding process, which is highly combinatorial. Only with an established sequence it is possible to obtain the remaining information. On the other hand, with the actual field of operation and market behavior, the mixed-sequence approach is capable of achieving better usage of the pipeline, which results in a higher inventory at the end of the time horizon, which is one desired objective. Finally, the decomposition of the initial problem results in smaller problems, making it possible to obtain a solution for the free sequence scenario. In this case, the product sequence verified in the pipeline schedule is quite different when compared to the usual cycle used by CLC's schedulers.

One note should be added regarding minimum stocks. Figure 8 shows that the P2 and P6 inventories, in free sequence,

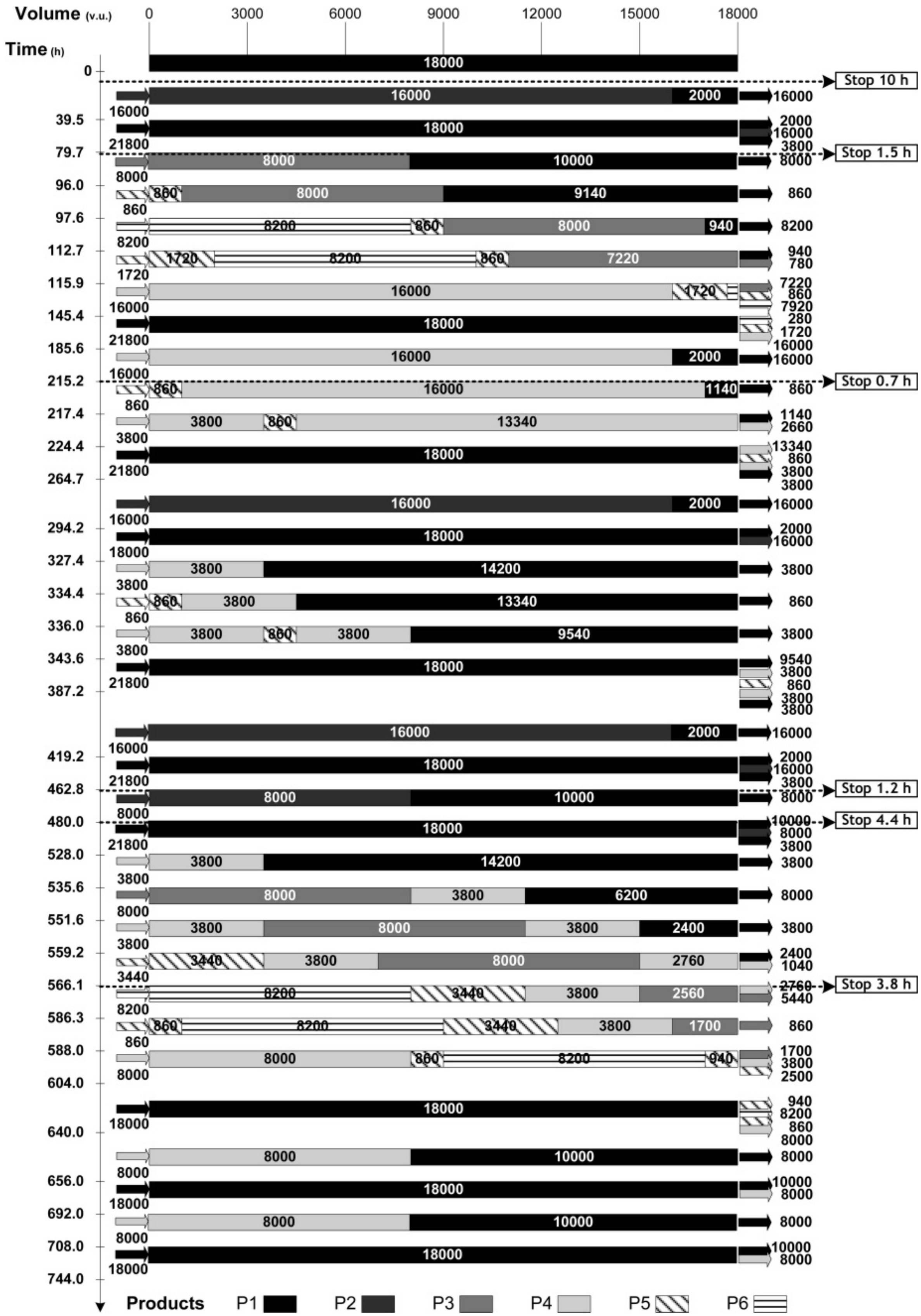

Figure 9. Pipeline schedule for the free sequence scenario, using the base model.

reach low values for certain time intervals. They correspond to 5.9% and 11.4%, respectively, of the total capacity for each product. In a more-refined study, the constraint that is defined by eq 26 should be used to guarantee a specific minimum level. In CLC's specific situation, the client information is quite accurate and no large fluctuations are verified on their demand. Therefore, no need for minimum stock levels is imposed by the company.

4.2.2. Extension To Account for the Settling Period. Because the settling period introduces a high level of complexity

Lot	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Fixed Sequence	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2	P1	P4
Mixed Sequence	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2	P1	P4	P5	P6	P5	P3	P1	P2	P1	P4
Free Sequence	P1	P2	P1	P3	P5	P6	P5	P4	P1	P4	P5	P4	P1	P2	P1	P4	P5	P4	P1	P2

Lot	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Fixed Sequence	P5	P6	P5	P4	P1	P2	P1	P3	P5	P6	P5	P4	P1	P2	P1
Mixed Sequence	P5	P6	P5	P4	P1	P2	P1	P4	P5	P6	P5	P3	P1	P2	P1
Free Sequence	P1	P2	P1	P4	P3	P4	P5	P6	P5	P4	P1	P4	P1	P4	P1

Figure 10. Comparison of product sequence between the three scenarios.

Table 7. Final Inventory and Pipeline Usage for Both Scenarios, Compared with the Initial Plan Developed at CLC

	Final Inventory (v.u.)							
	P1	P2	P3	P4	P5	P6	total	pipeline usage (%)
planned at CLC	41554	24765	11927	15244	5965	6054	105509	91.4
	Δ Final Inventory (v.u.)							
	P1	P2	P3	P4	P5	P6	Δtotal (v.u.)	pipeline usage (%)
fixed sequence	+1400	0	+8000	+3800	+2588	+3980	+19768	96.6
mixed sequence	+700	0	+8000	+4200	+2588	+3980	+19468	96.5

in the model (and in the associated solution performance), and based on the fact that it is company procedure, in the definition of a fixed sequence, only two scenarios are explored to study the influence of the settling period on the pipeline scheduling. These scenarios are described, respectively, by a fixed sequence and a mixed sequence, where a minimum settling period of 24 h is considered.

Table 7 compares the final inventories and the pipeline usage, for both scenarios, with the initial plan developed by CLC's schedulers. Table 8 presents model statistics and performance for both scenarios.

From Table 8, it can be easily stated that the initial plan developed by CLC's schedulers leaves ~9% of the total time horizon unused, whereas both scenarios run in GAMS/CPLEX cover a larger portion. This reinforces the belief that CLC's schedulers use heuristics that are not the most adequate to obtain the monthly plan. Consequently, as the pipeline transports more volume during the entire month, the final inventories calculated through the model are higher than those in CLC's plan. The comparison between both scenarios results in a high level of similarities. However, this is not true when model performance is compared. The mixed sequence has only more eight binary variables (corresponding to sequence "holes"), but the computational effort increases significantly (>1000%). The number of iterations and explored nodes is also larger in the mixed sequence. Finally, looking at the objective function and gap values attained, it is not possible to recognize if the mixed sequence would bring more benefits to the planning, when compared to the fixed sequence. Although in a real-world situation, the fixed sequence is associated with a situation of reduced flexibility and, thus, the mixed-sequence results are the most adequate for a more-flexible solution.

Table 8. Model Performance Using the Minimum Settling Time

	Value	
item	fixed sequence	mixed sequence
number of equations	30703	30703
number of continuous variables	16259	16259
number of binary variables	4687	4695
model status	integer solution	integer solution
CPU time	355.083 s	4120.795 s
number of iterations	59975	1183378
number of nodes explored	150	3200
best possible objective	2.556225	2.565428
objective	2.452544	2.451958
relative gap	4.23%	4.63%

Figure 11 represents the inventory profiles for each product during the entire time horizon. Product P2 is the only one where the initial CLC's plan performs better than the model. In the remaining profiles, it is easily seen that, at the end of the time horizon, the model plan achieves better results. Globally, the model results present a high level of inventory at the end of the time horizon.

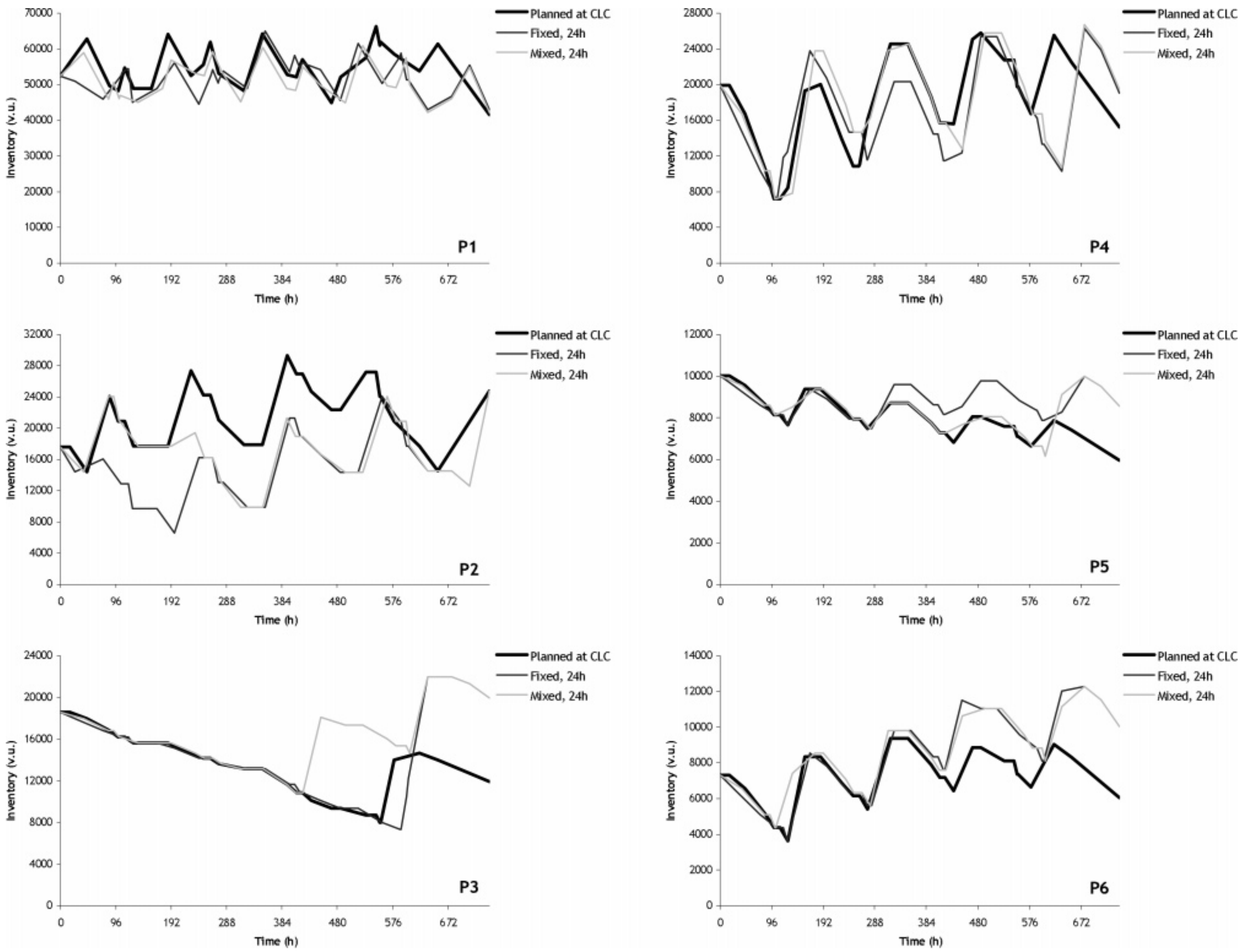

Figure 11. Inventory profiles for each product considering a minimum settling period.

When comparing these scenarios with those derived for the base model, the main difference verified is that model size has duplicated, in the equations, continuous and binary variables. However, the model extension allows an initial approximate comparison with the real operational scenario at CLC.

5. Discussion and Future Work

The main objective of this work was to provide a valid model of a real-world distribution center that combines pipeline operation with inventory management. The model represents a mixed-integer linear programming (MILP) approach, combined with continuous-time and volume representations. The model was used to simulate the scheduling and management procedures of a real-world distribution center, for a medium-term horizon. The most-common time horizons used in alternative works were small, when compared to real-world scenarios. In addition, this work closes a gap in the crude oil supply chain, focusing on the end of the chain. Usually distribution centers are only modeled as chain nodes, where no internal dynamics are considered. However, as seen through a real-world scenario, distribution centers need to address several strict conditions, mainly because they have to adjust the pipeline feeding stock with own conditions and market demands.

In regard to results, the base model proved to be a good approach; it returned feasible solutions in reduced amounts of time, compared to the time required by the schedulers at Companhia Logística de Combustíveis (CLC). However, the extension of the model to consider the existence of a settling period resulted in a complex model with a lower performance, when compared to the base model.

Furthermore, the product sequence established for the pipeline schedule is determinant in the solution procedure, both in terms of quality and computational effort of the solution. In future work, the decomposition approach used will be revised and efforts to build a more robust procedure will be enforced, not only considering a temporal decomposition, but also looking into the spatial dimension. In addition, other model characteristics will be studied. For instance, in the previous scenarios, no analysis was presented on the effect of a nonfixed number of lots, which would complicate the solution procedure even further. Therefore, combinatorial complexity versus model level of rigorousness will be the focus of future studies.

Finally, when compared with previous published works, the innovative contribution of the presented model relies on the daily client information and on the modeling of a minimum settling period.

Nevertheless, and to step closer to a real distribution center operation, effort should be exerted to account for individual tank management. Moreover, the model should be adequate to be used in rescheduling situations. As explained in section 4.1, there is a frequent trade of information between the refinery and CLC. When unpredicted situations happen, it results in a revision of the current schedule. Initially, CLC's schedulers plan a constant flow rate, as modeled in this work; however, variations in the flow rate often are observed in subsequent revisions. Future model revisions will account for the variable flow rate.

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Nomenclature

Acronyms

v.u. = volumetric units

CC model = model published by Cafaro and Cerdá[22]

Appendix: Total Client Demands and Initial Inventories

Table A1 presents the total client demands for a typical month (31 days) at Companhia Logística de Combustíveis (CLC) for each product. Table A2 indicates the initial inventory of each product.

Table A1. Total Client Demands for Each Product

product	P1	P2	P3	P4	P5	P6
demand (v.u./month)	198043	64800	14642	68244	10934	16955

Table A2. Initial Inventory of Each Product

product	P1	P2	P3	P4	P5	P6
initial inventory (v.u.)	52397	17565	18569	19888	10027	7309

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