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A MILP model for design of flotation circuits with bank/column and regrind/no regrind selection

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Abstract

A method for the design of flotation circuits is presented. The design problem is represented by several superstructures. The first superstructure represents separation tasks (STS), which include: feed processing superstructure (FPS), concentrate processing superstructure (CPS), and tail processing superstructure (TPS). The FPS commonly uses a single stage, i.e., rougher. The CPS represents the circuit needed to carry out the cleaner task, and the TPS represents the circuit needed to carry out the scavenger task. These superstructures are flow networks between several separation stages. In each separation stage two kinds of cells are allowed, bank and column. In several streams in the CPS and TPS, the incorporation of regrind mills is also included.

The optimal selection of the circuit is made with an appropriate objective function, upon which the values of the operational and structural variables may be determined. The problem is formulated using disjunctive programming, which is converted to a Mixed Integer Linear Programming (MILP) problem. The model includes mass balance, equipment models, operational conditions, and logic relationship. The approach is illustrated for a copper concentration plant.

Keywords: process design; flotation circuits; mathematical programming

1. Introduction

Froth flotation is one of the most widely accepted industrial practice for separation of valuable components from associated gangue materials in metallic minerals. The aim of the flotation process is to achieve maximum recovery and highest grade. Due to many reasons the separation is rarely complete, and in order to improve the process efficiency it is normal practice to make use of a number of interconnected flotation cell banks, flotation column and regrind mills. Optimization of flotation circuits has been considered because a typical flotation circuit may process thousands of ore ton/year, and a marginal improvement can have a considerable economic impact.

The early attempts at developing numerical methods to determine the optimal design of flotation cells started with the work of Jowett and Ghosh (1965), however their work only considered flotation cells in series. Mehrotra and Kapur (1974), were the first to consider the optimization of the circuit configuration and operation conditions. At least two reviews have been published on optimization of flotation circuits (Yingling, 1993;

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Mehrotra, 1988), nevertheless, a number of new studies have been published since that time (Schena et al., 1996, 1997; Abu-Ali and Abdel Sabour, 2003; Cisternas et al., 2004; Guria et al., 2005a,b). The review upgrade is outside the objective of this paper.

Cisternas et al. (2004) presented a procedure for the design or improvement of mineral flotation circuits based on a mathematical programming model with disjunctive equations. The procedure is characterized by the use of two level hierarchized superstructures: in the first level, a separation tasks superstructure (STS) is used to represent the flow network between three-second level superstructures. In the second level, three superstructures are used to represent the flow network between flotation bank cells. However, the work of Cisternas et al. (2004) did not allow the use of flotation column and regrind mills. The objective of this work is to incorporate these options in the procedure developed.

The outline of this paper is as follows: In Section 2 we present the strategy and the mathematical model developed. A hypothetical example is utilized in Section 3 to illustrate the procedure. Concluding remarks are offered in Section 4.

2. Model development

2.1. Strategy

The design strategy includes the use of flow network superstructures, which embeds many alternative configurations for a flotation circuits. In each superstructure, discrete decision variables are employed to represent configuration alternatives for the circuit, e.g., existence or non-existence of a flotation column, stream interconnection

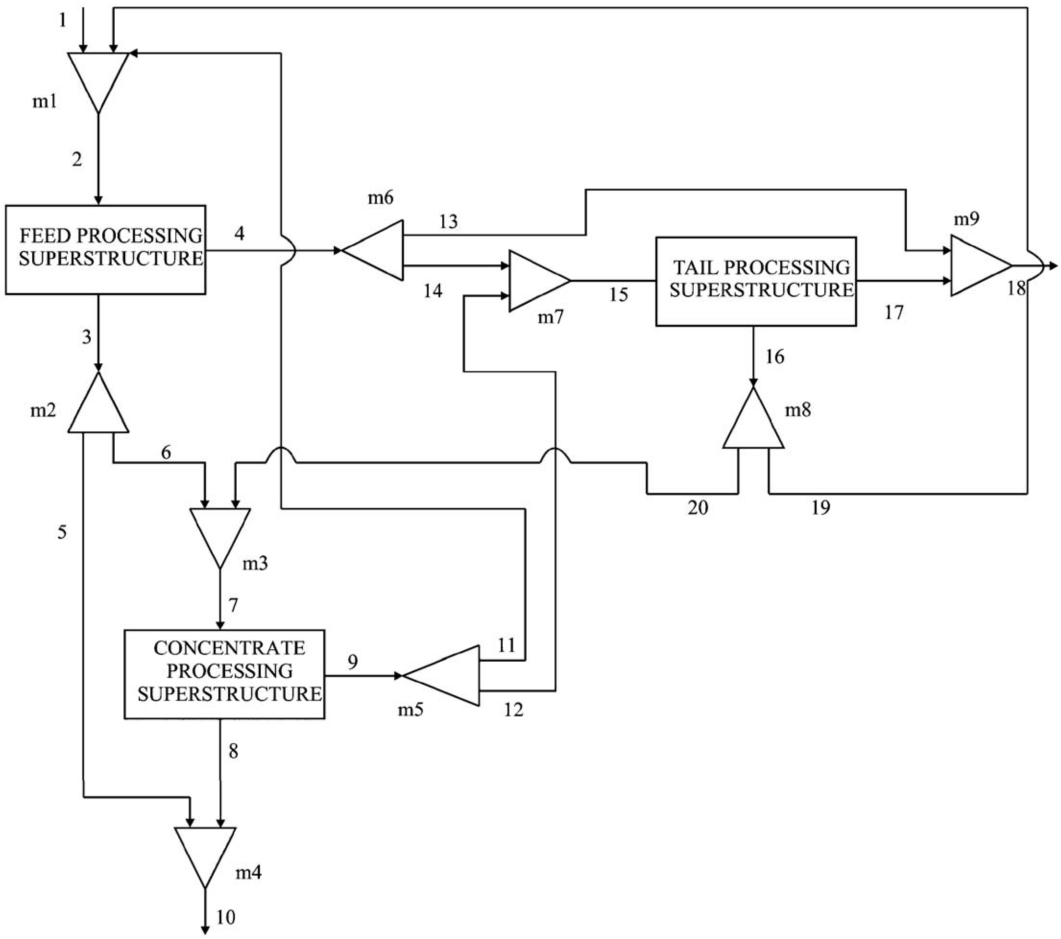

Fig. 1. STS, separation task superstructure.

arrangements, stream splits, etc.; and continuous decision variables are used to represent alternative steady state design specifications. The formulation may be supplemented with performance requirements that the flotation circuit must meet, e.g., constraints specifying minimum production or maximum stream flow rates.

The upper level includes a separation task superstructure (STS), in which a flow network is included between three processing superstructure. Fig. 1 shows the superstructure utilized, where the triangles represent mixers or stream division which permit the presentation of a group of alternatives for mineral processing upon which the search can be made for the best alternative. The superstructure has 20 streams, 5 mixers and 4 splitters. Not all stream connections are allowed with the objective to avoid symmetrical structures.

At the second level, three superstructures are considered. Feed processing superstructure (FPS), concentrate processing superstructure (CPS), and tail processing superstructure (TPS). The FPS commonly uses a single stage, i.e., rougher. The CPS represents the circuit needed to carry out the cleaner task, and the TPS represents the circuit needed to carry out the scavenger task. These superstructures are flow networks between several separation stages. The CPS and TPS have the same appearance with the STS, but each processing superstructure in the STS is changed by a flotation stage in the CPS and TPS. This same appearance makes easy the mathematical representation. Also the superstructures representation avoids the presence of symmetrical structures avoiding double counting and reducing the number of flow sheet configurations. In addition some streams (e.g., streams between cleaner and scavenger subsystems) can be eliminated as needed. Fig. 2 shows the first and second level superstructures.

In each stage two kinds of cells are allowed, bank and column flotation cells. Also the regrind mill is incorporated for some streams. This superstructure is called equipment selection superstructure, ESS. Fig. 3A shows this

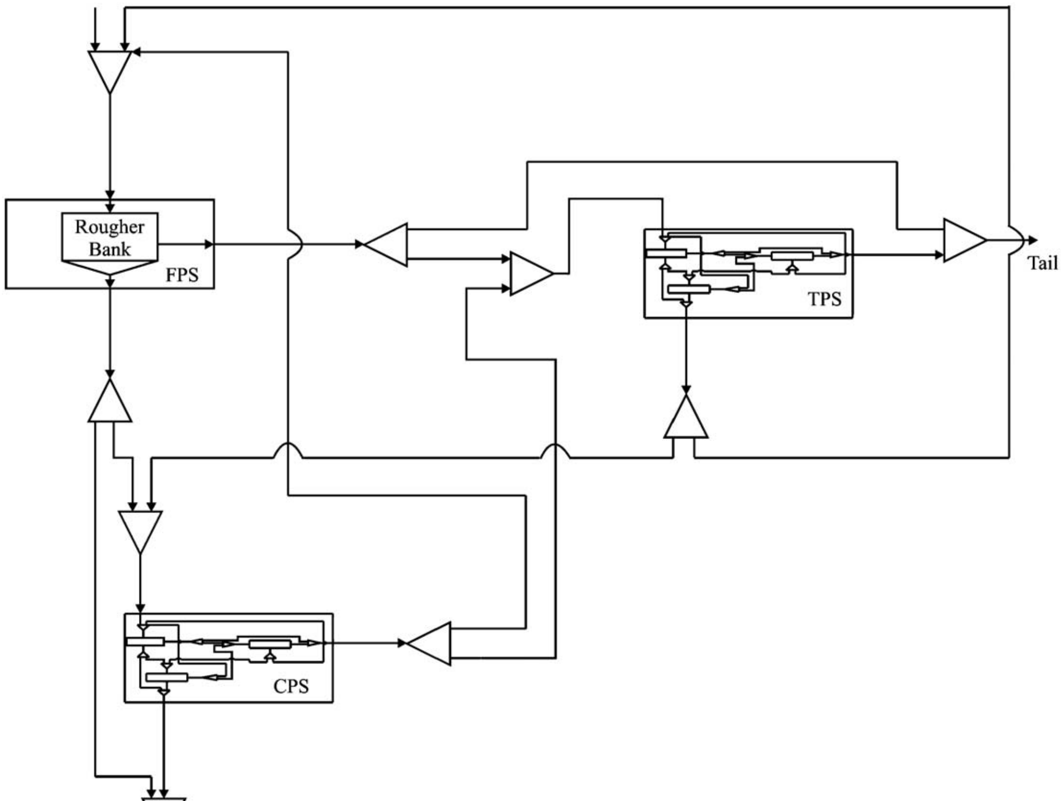

Fig. 2. STS including processing superstructure, FPS, CPS and TPS. The FPS only includes the rougher banks, however the CPS and TPS are analogous to the STS.

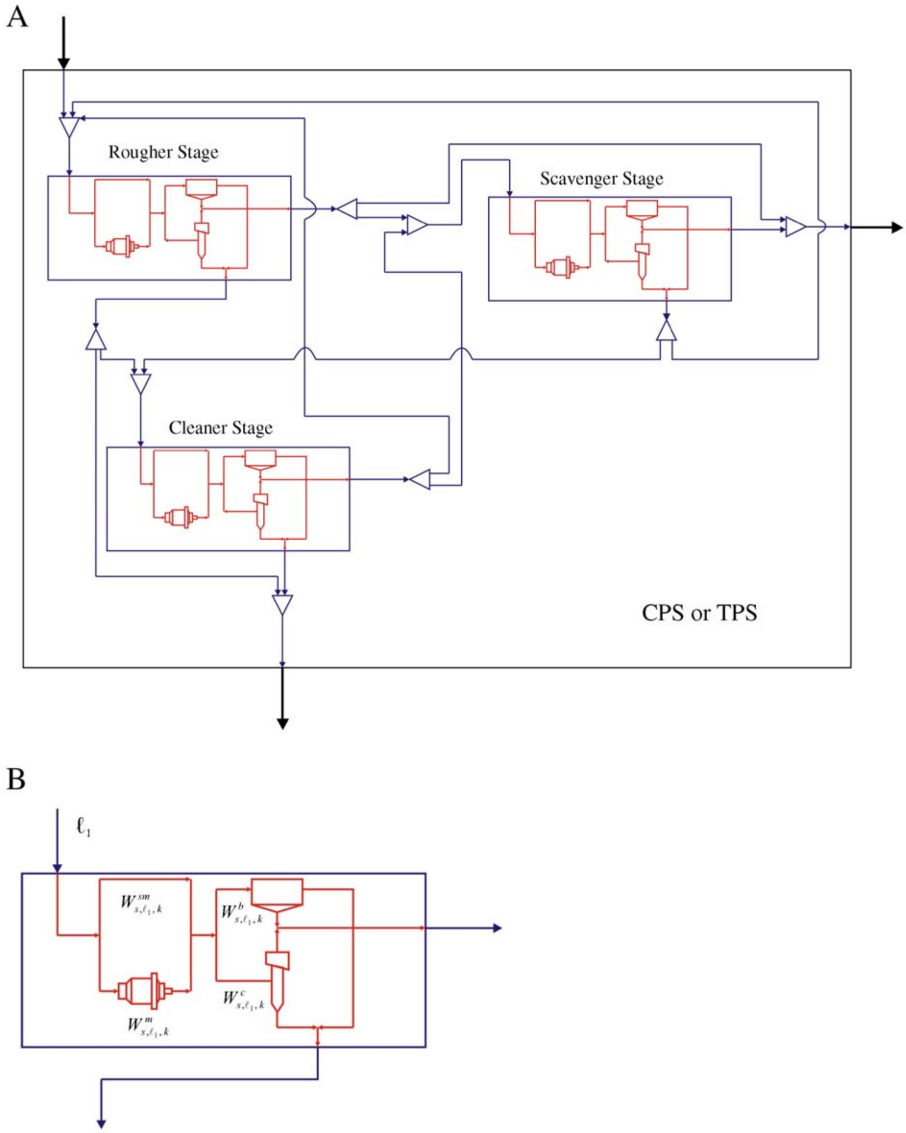

Fig. 3. A: CPS or TPS including the equipment selection superstructure, ESS. B: details of ESS.

superstructure, in this case the superstructure is not a flow network, instead the superstructure represents disjunction decisions: regrind mill or not regrind, column flotation cell or bank flotation cell.

The problem can then be defined as follows: given its technical characteristics, costs, prices, and the feed mass flow find the topology or structure of the process circuits, by searching through a group of hierarchized superstructures, as well as the operational conditions that maximize the profits of the plant. The strategy to be utilized includes establishing the previously mentioned superstructures, and then generating a mathematical model for the superstructure. The mathematical solution to the problem should deliver mass flows (zero mass flows represent the non-existence of streams), equipment selection and size, and the configuration of the plant.

2.2. Mathematical model

The mathematical models for the STS, CPS, TPS are similar to those developed by Cisternas et al. (2004), and therefore these equations are not given again in this paper. However, some basic definitions (sets, parameters, and variables) are given again to facilitate the reading. The mathematical model for the equipment selection superstructure is developed in this section.

The principal sets are: $S=\{s/s$ is a processing superstructure$\}$, $L=\{\ell/\ell$ is a stream$\}$ and $K=\{k/k$ is species$\}$. LA, LC and LT are a subset of L, which include the feeds, concentrates and tailings, respectively, of each system or bank of cells.

Also, the following sets are necessary:

$$\mathrm{Lcc}=\{(\ell_a,\ell_c)/\ell_a\in\mathrm{LA},\ell_c\in\mathrm{LC},\ell_c \text{ is the concentrate produced from } \ell_a\}$$

$$\mathrm{Ltt}=\{(\ell_a,\ell_t)/\ell_a\in\mathrm{LA},\ell_t\in\mathrm{LT},\ell_t \text{ is the tailing produced from } \ell_a\}$$

Since the superstructures are analogous, the sets L, M, LA, LC, LT, Lcc and Ltt are the same for CPS and TPS.

Each stream ℓ of the general superstructure is associated with the variable that represents the mass flow of species k, $W_{\ell,k}$. Similarly, each stream ℓ in each processing superstructure s is associated with the variable that represents the mass flow of the species k, $\mathrm{WI}_{s,\ell,k}$.

The mathematical model in the STS, CPS, and TPS includes material balances in the mixers and splitters, disjunction expressions for the selection of the level of division in splitters, the assignment of the feeds flows for each species k to the mass flow of circuit feed, streams connection between superstructures of different levels, upper and lower bounds for mass flows of each stream, and logic expressions

The mass balance equations for one stage (rougher) FPS are

$$W_{\ell_c,k}=T_{\ell_a,k}W_{\ell_a,k}\quad(\ell_a,\ell_c)\in\mathrm{Lcc},k\in K \tag{1}$$

$$W_{\ell_t,k}=(1-T_{\ell_a,k})W_{\ell_a,k}\quad(\ell_a,\ell_t)\in\mathrm{Ltt},k\in K \tag{2}$$

where $T_{\ell_a,k}$ is the ratio of flow of concentrate ℓ_c and feed ℓ_a, of species k, in the rougher stage. The ratio $T_{\ell_a,k}$, may be obtained from plant data, values from pilot plants, or theoretical or empirical models. For example:

$$T_{\ell_a,k}=1-\frac{1}{(1+k_{\ell_a,k}\tau_{\ell_a})^{N_{\ell_a}}} \tag{3}$$

where $T_{\ell_a,k}$ is the flotation rate for species k, N_{ℓ_a} the number of cells and τ_{ℓ_a} the retention time in one cell in the rougher bank fed by ℓ_a. Multiple values of $T_{\ell_a,k}$ may be implemented as was shown by Cisternas et al. (2004).

2.2.1. Equipment Selection Superstructure, ESS

In the CPS and TPS each stage can include bank cells or column cells with or without grinding mill (Fig. 3A). As there are six stages in the CPS and TPS, there are six ESS. Fig. 3B shows one of these superstructures for equipment selection. The mathematical model for these superstructures is:

$$\left[\begin{array}{c} y^m_{s,\ell_1} \\ W^m_{s,\ell_1,k}=\sum_{k'}\mathrm{WI}_{s,\ell_1,k}\alpha_{k,k'} \\ \left[\begin{array}{c} y^b_{s,\ell_1} \\ C^f_{s,\ell_1}=C^f_m+C^f_b \\ C^V_{s,\ell_1}=C^V_m\sum_k W^m_{s,\ell_1,k}+C^V_b\sum_k W^b_{s,\ell_1,k} \\ W^b_{s,\ell_1,k}=W^m_{s,\ell_1,k} \\ \mathrm{WI}_{s,\ell_2,k}=T^b_{s,\ell_1,k}W^b_{s,\ell_1,k} \\ \mathrm{WI}_{s,\ell_3,k}=(1-T^b_{s,\ell_1,k})W^b_{s,\ell_1,k} \end{array}\right] \vee \left[\begin{array}{c} y^c_{s,\ell_1} \\ C^f_{s,\ell_1}=C^f_m+C^f_c \\ C^V_{s,\ell_1}=C^V_m\sum_k W^m_{s,\ell_1,k}+C^V_c\sum_k W^c_{s,\ell_1,k} \\ W^c_{s,\ell_1,k}=W^m_{s,\ell_1,k} \\ \mathrm{WI}_{s,\ell_2,k}=T^c_{s\ell_1,k}W^c_{s,\ell_1,k} \\ \mathrm{WI}_{s,\ell_3,k}=(1-T^c_{s,\ell_1,k})W^c_{s,\ell_1,k} \end{array}\right] \end{array}\right]$$
$$\vee\left[\begin{array}{c} \neg y^m_{s,\ell_1} \\ W^{sm}_{s,\ell_1,k}=\mathrm{WI}_{s,\ell_1,k} \\ \left[\begin{array}{c} y^b_{s,\ell_1} \\ C^f_{s,\ell_1}=C^f_b \\ C^V_{s,\ell_1}=C^V_b\sum_k W^b_{s,\ell_1,k} \\ W^b_{s,\ell_1,k}=W^{sm}_{s,\ell_1,k} \\ \mathrm{WI}_{s,\ell_2,k}=T^b_{s,\ell_1,k}W^b_{s,\ell_1,k} \\ \mathrm{WI}_{s,\ell_3,k}=(1-T^b_{s,\ell_1,k})W^b_{s,\ell_1,k} \end{array}\right] \vee \left[\begin{array}{c} y^c_{s,\ell_1} \\ C^f_{s,\ell_1}=C^f_c \\ C^V_{s,\ell_1}=C^V_c\sum_k W^c_{s,\ell_1,k} \\ W^c_{s,\ell_1,k}=W^{sm}_{s,\ell_1,k} \\ \mathrm{WI}_{s,\ell_2,k}=T^c_{s,\ell_1,k}W^c_{s,\ell_1,k} \\ \mathrm{WI}_{s,\ell_3,k}=(1-T^c_{s,\ell_1,k})W^c_{s,\ell_1,k} \end{array}\right] \end{array}\right] \tag{4}$$

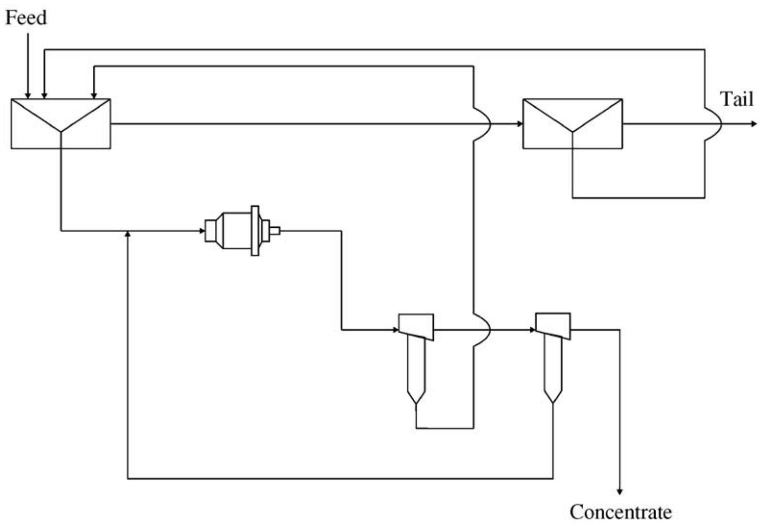

Fig. 4. Copper concentration plant obtained for the base case.

Where $\ell_1 \in \mathrm{LA}, \ell_2 \in \mathrm{LC}, \ell_3 \in \mathrm{LT}$, and $(\ell_2, \ell_2) \in \mathrm{Lcc}, (\ell_1, \ell_3) \in \mathrm{Ltt}$.

In Eq. (4) $T^b_{s,\ell_1,k}$ is the ratio of mass flow of concentrate to feed ℓ_1, of species k in processing superstructure s if bank cells are used, $T^c_{s,\ell_1,k}$ is the ratio of mass flow of concentrate to feed ℓ_1, of species k in processing superstructure s if column are used. $\alpha_{k,k'}$ is the conversion fraction or liberation fraction in the grinding mill of specie k from specie k'(Wei and Gay, 1999). C^f_m, C^f_b, C^f_c are the fixed costs for grinding, bank cell and column respectively. Correspondingly, C^V_m, C^V_b, C^V_c are the variable cost coefficients for grinding, bank cell and column. The following variables are utilized for CPS and TPS,: mass flow rate of specie k, in the stage that is fed with stream ℓ_1, with grinding $W^m_{s,\ell_1,k}$ and without grinding $W^{sm}_{s,\ell_1,k}$(see Fig. 4); mass flow rate of specie k, in the stage that is fed with stream ℓ_1, if column is utilized, $W^c_{s,\ell_1,k}$ and if bank cell is utilized $W^b_{s,\ell_1,k}$; fixed and variable costs in the stage that it is fed with stream ℓ_1,C^f_{s,ℓ_1}, C^V_{s,ℓ_1}; and binary variables related with the existence of the grinding mill, flotation column and bank cell in the stage that it is fed with stream ℓ_1, $y^m_{s,\ell_1}, y^c_{s,\ell_1}, y^b_{s,\ell_1}$.

The disjunctive Eq. (4) may be expressed as the following set of mixed-integer linear equations:

2.2.2. For grinding selection

$$
\begin{aligned}
&W^m_{s,\ell_1,k} \le \textstyle\sum_{k'} \mathrm{WI}_{s,\ell_1,k}\alpha_{k,k'}\\
&W^{sm}_{s,\ell_1,k} \le \mathrm{WI}_{s,\ell_1,k}\\
&\textstyle\sum_k \mathrm{WI}_{s,\ell_1,k} = \sum_k W^m_{s,\ell_1,k} + \sum_k W^{sm}_{s,\ell_1,k}\\
&\textstyle\sum_k W^m_{s,\ell_1,k} - U y^m_{s,\ell_1} \le 0\\
&\textstyle\sum_k W^{sm}_{s,\ell_1,k} - U(1 - y^m_{s,\ell_1}) \le 0
\end{aligned}
\tag{5}
$$

2.2.3. For flotation equipment selection

$$
\begin{aligned}
&W^b_{s,\ell_1,k} \le W^m_{s,\ell_1,k} + W^{sm}_{s,\ell_1,k}\\
&W^c_{s,\ell_1,k} \le W^m_{s,\ell_1,k} + W^{sm}_{s,\ell_1,k}\\
&W^m_{s,\ell_1,k} + W^{sm}_{s,\ell_1,k} = W^b_{s,\ell_1,k} + W^c_{s,\ell_1,k}\\
&\textstyle\sum_k W^b_{s,\ell_1,k} - U y^b_{s,\ell_1,k} \le 0\\
&\textstyle\sum_k W^c_{s,\ell_1,k} - U y^c_{s,\ell_1,k} \le 0\\
&y^b_{s,\ell_1} + y^c_{s,\ell_1} \le 1\\
&\mathrm{WI}_{s,\ell_2,k} = T^c_{s,\ell_1,k} W^c_{s,\ell_1,k} + T^b_{s,\ell_1,k} W^b_{s,\ell_1,k}\\
&\mathrm{WI}_{s,\ell_3,k} = (1 - T^c_{s,\ell_1,k}) W^c_{s,\ell_1,k} + (1 - T^b_{s,\ell_1,k}) W^b_{s,\ell_1,k}
\end{aligned}
\tag{6}
$$

In these equations $T_{s,\ell,k}$ is the ratio of flow of concentrate to feed of species k, and may be obtained from plant data, values from pilot plants, or theoretical or empirical models. For cell banks an example was given in Eq. (3). In the case of flotation column the following equation can be used (Levenspiel, 1998; Finch et al., 1990):

$$T_k^c = \left[1 - \frac{4a\exp\left(\frac{1}{2D/uL}\right)}{(1+a)^2\exp\left(\frac{a}{2D/uL}\right) - (1-a)^2\exp\left(\frac{-a}{2D/uL}\right)}\right] T_{k_\infty} \tag{7}$$

$$a = \sqrt{1 + 4k_k\tau\frac{D}{uL}} \tag{8}$$

$$T^c_{s,l_a,k} = \frac{T_k^c T_f}{T_k^c T_f + 1 - T_k^c} \tag{9}$$

Several logic expressions are also included, for example the designer may want to include only one grinding mill. The procedure of Raman and Grossmann (1991) is utilized for this purpose.

2.2.4. Objective function

The optimal selection of the circuit requires that an appropriate objective function be defined upon which the values of the operational and structural variables may be determined. Since the income depends on the structure and operational conditions, a useful function is the difference between income and costs. Different relations may be applied for calculation of the income depending on the type of product and its market. For base metals the formula net-smelter-return may be utilized, (Cisternas et al., 2004).

$$I = \left(\sum_k g_k W_{10,k}\right) \cdot p \cdot (q - \mathrm{Rfc})H - \sum_k W_{10,k}[pu(q - \mathrm{Rfc}) + Trc]H \tag{10}$$

where g_k is the mineral grade of each species k present in the concentrate, u is the grade deduction, Trc is the treatment charge, and Rfc is the refinery charge. H is the number of hours per year of plant operation, when the flows are in tons per hour. The grade deduction and the fraction of metal paid depend on the recovery efficiency of the smelter. The formula for the calculation of income incorporates the metallurgical efficiency of the plant, that is, the recovery and mineral content are opposite functions. Eq. (10) is a linear function of the mass flows of the species k in the concentrate. It should be noted that as the mass flows of the species with a high-grade increase, so does the profit. However, this increase in flows brings with it an increase in mass flows of low-grade value, which decrease the profits.

Table 1
Values of concentrate to feed ratio

	Concentrate to feed ratio for species k					
	Bank cells			Column cells		
	1	2	3	1	2	3
FPS						
Rougher	0.8830	0.3701	0.2573			
CPS						
Rougher	0.6580	0.2748	0.0391	0.7565	0.1388	0.0281
Cleaner	0.6580	0.2748	0.0198	0.7565	0.1371	0.0147
TPS						
Rougher	0.6580	0.0748	0.1168	0.5960	0.3667	0.0609

Table 2
Mass flow rates (ton/h) for flow sheet of Fig. 4

	$k=1$	$k=2$	$k=3$
Feed	5.0	4.0	291.0
Concentrate	4.941	0.050	0.049
Tail	0.307	3.454	291.199
Concentrate FPS	6.782	2.193	114.224
Tail FPS	0.899	3.733	329.709

The annualized costs of the plant may be considered as the sum of fixed and variable costs:

$$C^f_{s,\ell_1} = C^f_m \cdot y^m_{s,\ell_1} + C^f_b y^b_{s\ell_1} + C^f_c y^c_{s\ell_1} + C^f_{bb}(y^b_{s\ell_1} + y^c_{s,\ell_1})$$

$$C^v_{s,\ell_1} = C^v_m \sum_k W^m_{s,\ell_1,k} + C^v_b \sum_k W^b_{s,\ell_1,k} + C^v_c \sum_k W^c_{s,\ell_1,k} + C^v_{bb} \sum_k \mathrm{WI}_{s,\ell_2,k} \tag{11}$$

Where C^f_{bb} y C^v_{bb} are the fixed and variable coefficients of concentrate pumping.

The objective functions and the subject to restrictions represent a problem of mixed integer linear programming (MILP).

3 . Example

The procedure was applied to the design of a copper concentration plant, whose species are: $k=1$ (100% chalcopyrite), $k=2$ (50% silica, 50% chalcopyrite) and $k=3$ (100% silica). Most of the data were taken from Cisternas et al. (2004). The problem does not include scavenger stages in the CPS and TPS. Also the cleaner stage was not included in the TPS. Flow division was not considered for all the flow splitters in each of the superstructures. The feed flow for species 1, 2 and 3 were 5, 4 and 291 ton/h respectively. Table 1 gives the principal data utilized. Logic relationships where included, so that if column cells are selected, then grinding must be included, also regrinding was not allowed. These conditions can be easy eliminated or changed for other requirements.

The optimal integer solution gives an annual profits of 3.5 millions US$, while income from sales was 16.2 million US$. The problem, including a total of 594 equations and 408 variables, was solved using the GAMS and OSL2 solver, in a Pentium IV processor in 0.33 s. Fig. 4 shows the circuit obtained. The primary cell bank receives fresh feed and the tailing from scavenger and first cleaner stages. Bank cells were selected for the scavenger stage where the final tail is produced. On the other hand, the cleaner circuit includes grinding the rougher concentrate and the tail from the second cleaner

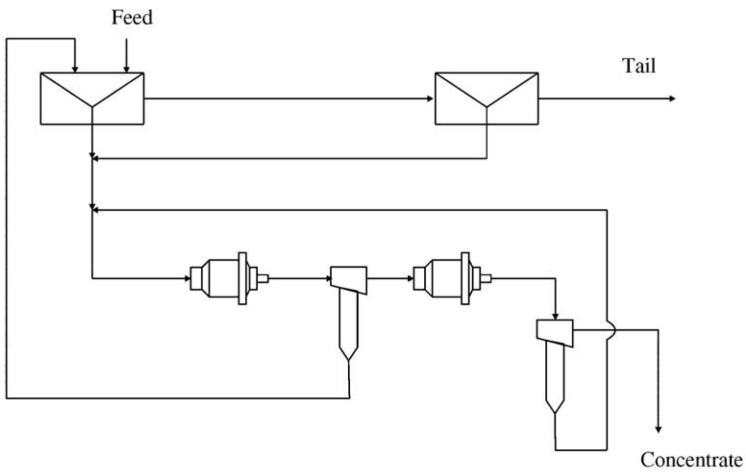

Fig. 5. New configuration when the metal price is higher that 1.06 US$/copper pound. Scavenger concentrate is fed to the mill in the cleaner stages.

Table 3
Feed mass flow rate for the studied cases

Case	Feed mass flow rate, ton/hr			Copper grade, %
	$k=1$	$k=2$	$k=3$	
Base	5	4	291	0.81
1	6	2	292	0.81
2	4	6	290	0.81
3	4	4.3	291.7	0.71
4	5	2.3	292.7	0.71

stage and two column banks. Table 2 gives the mass balance for the flow sheet of Fig. 4. As can be observed the liberation in the mill is small, however this is enough to justify its existence. It is well known that this kind of circuits may produce large amount of fine particles, a situation not considered in this work.

3.1. Sensitivity studies

The sensitivity of the solution to levels of stream division was considered. The streams were split between two and five levels. The solution obtained for all cases were the same as of Fig. 4. These results agree with the work by Cisternas et al. (2004) and with practice, because streams division is rarely used in flotation circuits. Of course, when the number of stream division levels increases the number of equations and variables in the model increases too.

To study the potential effect metal price on the structure of the flotation circuit, the metal price was changed until to find a different structure. In the base case a copper price of 0.85 US$/pound was considered. The copper price was decreased and no changes were observed in the circuit structure until 0.72 US$/pound, where the circuit becomes an alternative nonprofitable. Then the copper price was increased and a change in the structure was observed for prices over 1.06 US$/lb. Fig. 5 shows the circuit obtained. However, it can be observed that the only change is in the scavenger concentrate, which it is not sent to the rougher stage but to the cleaner stages. A potential reason for this behavior is that at higher prices it becomes profitable to liberate the copper available in specie k=2. This confirms that the metal price can affect the design, and as it is known the most of the metal prices are very volatile.

Other considered variables were the grade and species distribution in the feed stream to the circuit. Table 3 shows the cases studied. Cases 1 and 2 consider the same copper grade considered in the base case, and the same total feed mass flow rate, but the species relative composition was changed in the feed. The Case 1 considers a smaller proportion of specie $k=2$ (and higher for specie $k=1$) while the Case 2 considers a higher proportion of specie $k=2$ (and smaller for $k=1$). Fig. 6 shows the result of the Case 1, while Fig. 7 shows the results for Case 2. However it is logical that Case 2 had less profitability than the Case 1 because of the small presence the species $k=1$ and because the mill does not have the capacity to liberate the available valuable material in the specie $k=2$ and the specie $k=2$ has less recovery. The circuit for the Case 1 differ from

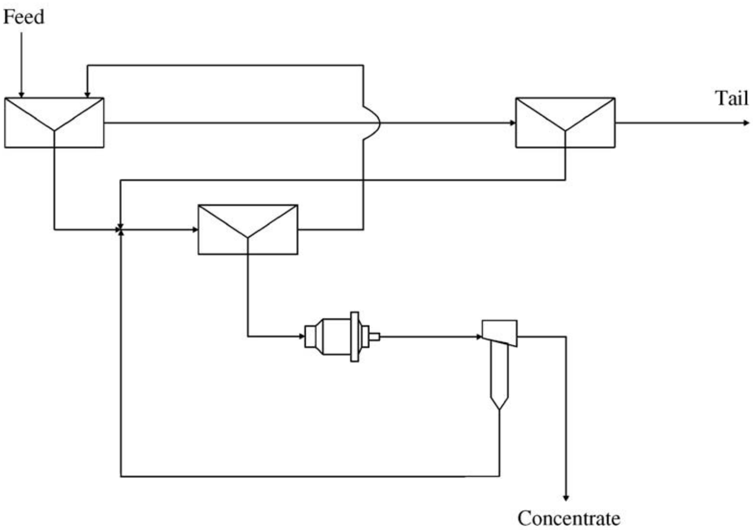

Fig. 6. New configuration when there is a low concentration of specie $k=2$. Bank cells are used in the first cleaner stage. Milling is considered in the second cleaner stage.

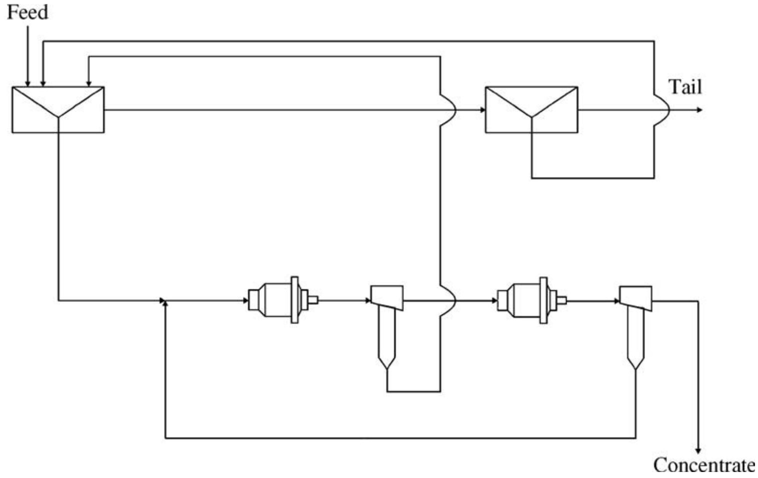

Fig. 7. New configuration when there is a high concentration of specie $k=2$. Two mills are used in the cleaner stages.

Case 2 circuit in the following: a) the circuit of Case 1 considered the incorporation of bank cells in the cleaning stage instead of column as in the Case 2, since the cell banks have higher recovery for the species $k=2$. b) the concentrate of scavenger stage is sent to the rougher stage in the Case 2, while it is sent to cleaning in the Case 1. c) The mill is only justified, in the Case 1, because the stream is more concentrated and because there is less material to liberate.

The optimal circuit in Case 3 is similar to the Case 1, while the optimal circuit in Case 4 is similar to Case 2. Cases 3 and 4 have a copper grade (0.71%) lower than Cases 1 and 2 (0.81%), but the specie distribution is similar among circuits 1 and 3, and among 2 and 4. This means that the species distribution has a bigger effect on the circuit structure than the change in the ore grade.

Finally, to show that the procedure is able to select the number of stages, all stages were allowed for the CPS and TPS. The circuit selected included one additional stage: scavenger–scavenger using bank cells. The new optimal integer solution gives an annual profits of 3.8 million US$ compared with the 3.5 million US$ of the original solution, while income from sales was increased from 16.2 to 16.7 million US$.

4. Conclusions

A procedure for the design and improvement of mineral concentration plants including equipment selection is presented. The mass balances and equipment selection in flotation stages and grinding were represented by disjunctive equations, and changed to MILP expressions. The results showed that the division of flows had no effect on determination of the most efficient circuits. This result agrees with practice since it is unusual to divide a stream in mineral concentration circuits. Also it has been demonstrated that a particular circuit configuration will be optimal only for interval value of the metal price. This is important because there is a great uncertainty on the future value of several metal prices. Based on the examples studied, it can be concluded that the valuable mineral mass distribution can be more important than the metal grade in the selection of a flotation circuit.

We have examined a hypothetical copper flotation plant, and the situations studied prove that the model can be useful in the analysis and design of circuits for mineral concentration. Several situations can be evaluated, such as: removal of crossover streams, changes in logic relationships, adjustment in the number of banks, selection of number of cells in each bank.

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