Process Design for Mineral Operations

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Outline

Motivation
General Strategy
Crystallization Design Problem
Flotation Circuit Design Problem
Final Remark
Motivation

- High price cycle unprecedented
- Lowering the cost of production
- Achieving the balance of acceptable economic, environmental and social effects.
- Improve energy efficiency

Copper Technology Roadmap Review 2006, AMIRA
The engineer’s lover
Carlo Carrá
Italian 1881-1966
Problems:
Process design has multiple dimensions

Roberto Matta
Chilean 1911-2002
Solution:
Look as Picasso

Pablo Picasso
Spanish 1881-1973
The Onion Model
Crystallization Design Problem

Crystallization design problem overview
Fractional Crystallization
Fractional Crystallization with Heat Integration
& Cake Washing
Crystallization design problem overview

Crystallization is extensively used in different industrial applications, including the production of a wide range of materials such as fertilizers, detergents, foods, and pharmaceutical products, as well as in the treatment of waste effluents.
Problems

The crystallization stages are usually accompanied by other separation techniques. Leaching. Various types of crystallization exist: cooling, evaporation, reactions, and drowning-out. The characteristics of the product affects a series of other associated operations. filtration & washing. The separation is limited by multiple saturation points. Temperature changes & external chemical agents. Kinetic factors and metastability may affect the design.
Phase Diagram

The greatest advantages obtained in the use of the phase diagram are the possibilities for the visualization of the behavior of phase equilibria, describing the processes, and obtaining mass balances with the help of the lever arms rule. The phase diagrams, however, also have a series of limitations as a design tool.
Goals

Determine optimal stream configuration.
Determine operational conditions & flowrates.
Selection of equipment type.
Determine solid-liquid separation. Washing & Filtration.
Determine heat integration.
Fractional Crystallization

Basic Crystallization Separation
Relative Composition Diagram
Feasible Pathway Diagram
State Superstructure
Connectivity Matrix
Mathematical model
Examples
Basic Crystallization Separation

Isothermal Cut
KCl+NaCl+H2O
KNO3+NaNO3+H2O
L serine acid + L aspartic acid + water
Basic Crystallization Separation

Isothermal Cut
KCl+NaCl+H2O
KNO3+NaNO3+H2O
L serine acid + L aspartic acid + water
Basic Crystallization Separation

(a) Basic Cycle

- **Heat and Evaporate**
- **Cool and Dilute**
Relative Composition Diagram

\[ R = \frac{\text{Weight Composition of B}}{\text{Weight Composition of A}} \]

Diagram showing the relative compositions and their order: \( R^B > R^C > R^H > R^A \).
Feasible Pathway Diagram

(a) RB > RC > RH > RA

RB → ∞

RA = 0
State Superstructure

\[ R^B > R^C > R^H > R^A \]
Connectivity Matrix

\[ R^B > R^C > R^H > R^A \]

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>H</th>
<th>S_2</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S_1</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>8</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Mathematical model

General model

\[
\begin{align*}
\text{Min} & \sum_{i} w_i \\
& w_1 x_{1,i} + w_3 x_{3,i} + w_6 x_{6,i} = w_5 x_{5,i} + w_7 x_{7,i} + w_8 x_{8,i} \\
& w_2 x_{2,i} + w_4 x_{4,i} + w_5 x_{5,i} = w_6 x_{6,i} + w_9 x_{9,i} + w_{10} x_{10,i} \\
& w_1 x_{1,i} + w_2 x_{2,i} = C_{1,i}^F \\
& w_i \geq 0; \quad i = A, B, S
\end{align*}
\]
### Equilibrium Data

<table>
<thead>
<tr>
<th>key</th>
<th>Temperature [°C]</th>
<th>Weight Composition</th>
<th>Solid Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>KCl</td>
<td>NaCl</td>
</tr>
<tr>
<td>C1</td>
<td>30</td>
<td>11.7</td>
<td>20.25</td>
</tr>
<tr>
<td>H1</td>
<td>100</td>
<td>22.2</td>
<td>15.9</td>
</tr>
</tbody>
</table>

$$
\begin{array}{cccc}
R_{KCl} & R_{H1} & R_{C1} & R_{NaCl} \\
\infty & 1.40 & 0.58 & 0.0 \\
\end{array}
$$
Examples - Sylvinite

Relative Composition Diagram

$R^\text{KCl} > R^\text{H1} > R^\text{C1} > R^\text{NaCl}$

Feasible Pathway Diagram

$R^\text{KCl} > R^\text{H1} > R^\text{C1} > R^\text{NaCl}$

State Diagram

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Examples-Sylvinitite

State Diagram

Flow Sheet

Sylvinite

NaCl

KCl

LEACHING AT 100 °C

LEACHING AT 30 °C

NaCl

KCl
### Equilibrium data for $\text{MgSO}_4 + \text{Na}_2\text{SO}_4 + \text{H}_2\text{O}$ system.

<table>
<thead>
<tr>
<th>T °C</th>
<th>keys</th>
<th>Saturated solution, % w</th>
<th>Solid phase</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\text{MgSO}_4$</td>
<td>$\text{Na}_2\text{SO}_4$</td>
<td></td>
</tr>
<tr>
<td>18.7</td>
<td>C</td>
<td>20.57</td>
<td>11.8</td>
<td>Mg$<em>7$ + Na$</em>{10}$</td>
</tr>
<tr>
<td>25</td>
<td>D1</td>
<td>21.15</td>
<td>13</td>
<td>Mg$_7$ + SD1</td>
</tr>
<tr>
<td>25</td>
<td>D2</td>
<td>16.6</td>
<td>17.8</td>
<td>SD1 + Na$_{10}$</td>
</tr>
<tr>
<td>50</td>
<td>E1</td>
<td>31.32</td>
<td>4.74</td>
<td>Mg$_6$ + SD1</td>
</tr>
<tr>
<td>50</td>
<td>E2</td>
<td>11.98</td>
<td>23.25</td>
<td>SD1 + Na</td>
</tr>
<tr>
<td>97</td>
<td>F1</td>
<td>32.2</td>
<td>5.55</td>
<td>Mg$_1$ + SD2</td>
</tr>
<tr>
<td>97</td>
<td>F2</td>
<td>14.4</td>
<td>19.15</td>
<td>SD2 + SD3</td>
</tr>
<tr>
<td>97</td>
<td>F3</td>
<td>5.88</td>
<td>26.9</td>
<td>SD3 + Na</td>
</tr>
<tr>
<td></td>
<td>SD1</td>
<td>35.99</td>
<td>42.48</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SD2</td>
<td>45.86</td>
<td>54.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SD3</td>
<td>22.02</td>
<td>77.98</td>
<td></td>
</tr>
</tbody>
</table>

Mg7 = MgSO4.7H2O; Mg1 = MgSO4.1H2O; Mg6 = MgSO4.6H2O; Na10 = Na2SO4.10H2O; Na = Na2SO4; SD1 = Na2SO4.MgSO4.4H2O; SD2 = Na2SO4.MgSO4; SD3 = MgSO4.3Na2SO4

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Examples-Astrakanite

Relative Composition Diagram

Feasible Pathway Diagram

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Examples - Astrakanite

Astrakanite Water

REACTIVE CRYSTALLIZATION 18.7 °C

Na₂SO₄·10H₂O

COOLING CRYSTALLIZATION 25 °C

MgSO₄·6H₂O

EVAPORATIVE CRYSTALLIZATION 50 °C

Water

astrakanite.gms

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Fractional Crystallization with Heat Integration & Cake Washing

State Superstructure
Task superstructure.
Heat integration.
Cake Washing

Task Superstructure
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Heat Integration

Cake Washing

Parallel Options
Mathematical Model

Mass balance for each component in multiple saturation nodes \((SM)\):

\[
\sum_{l \in Lq \cap S^{in}(s)} h x_{l,i} + \sum_{l \in S^{in}(s)} w_{l} \cdot x_{l,i} - \sum_{l \in S^{out}(s)} w_{l} \cdot x_{l,i} - \sum_{l \in (Lw \cup Lq) \cap S^{out}(s)} w_{l} h x_{l',i} = 0 \quad s \in S_{M}, i \in I
\]

Mass balance for each component in intermediate product nodes \((SI)\):

\[
\sum_{l \in Lq \cap S^{in}(s)} w_{l} \cdot x_{l,i} - \sum_{l \in Lq \cap S^{out}(s)} w_{l} \cdot x_{l,i} = 0 \quad s \in S_{I}, i \in I
\]

\[
\sum_{l \in Lq \cap S^{out}(s)} h x_{l,i} - \sum_{l \in Lq \cap S^{in}(s)} w_{l} h x_{l',i} = 0 \quad s \in S_{I}, i \in I
\]

\[
\sum_{i \in I} (w_{l} \cdot x_{l,i} + h x_{l,i}) - U_{l} \quad y m_{l} \leq 0 \quad l \in Lq \cap S^{out}(s), s \in S_{I}
\]

\[
\sum_{l \in Lq \cap S^{out}(s)} y m_{l} - 1 = 0 \quad s \in S_{I}
\]

Specification for feeds flow rates in feed nodes \((SF)\):

\[
\sum_{l \in S^{out}(s)} w_{l} \cdot x_{l,i} = C_{s,i}^{F} \quad s \in S_{F}, i \in I_{F}(s)
\]
**Task Superstructure**

Mass balance between the thermodynamic state network and task network

\[ w_l + \sum_{i \in I} h x_{l,i} = \sum_{t \in T(s)} G_{l,t}^{in}, \quad l \in S_{in}^{in}(s), s \in S_M \]

\[ w_l + w_l h_l = \sum_{t \in T(s)} G_{l,t}^{out}, \quad l \in S_{out}^{out}(s), s \in S_M \]

Mass balance in the task network:

\[ \sum_{l \in S_{in}(s)} G_{l,t}^{in} = \sum_{l \in S_{out}(s)} G_{t,l}^{out}, \quad t \in T, s \in S_M \]

Task selection and energy balance:

IF carnallite is fed to node 3 (stream 14)

THEN the task is reactive crystallization

\[ y_{n3,rc}^t - y_{14}^w \geq 0 \quad t \in T(s), s \in S_M(s) \]

\[ Q_{t,s}^C = HQ_{t,s}^C \]

\[ g(y_{t,s}) = \text{True} \]
Mass balance for each component in wash/reslurry stage:

Efficiency constraint for wash/reslurry stage:

Degree of impurity:

Wash or reslurry/filter selection:

\[
\begin{align*}
\begin{bmatrix}
y_{w_{i,e}} \\
\neg y_{r_{i,e}} \\
y_{l,e,d} = y_{m_\text{w}_{l,e,d}} \\
y_{p_{w_{l,e,d}}} = y_{l,e-1,d} \\
y_{m_{r_{l,e,d}}} = 0 \\
y_{p_{r_{l,e,d}}} = 0 \\
z_{w_{l,e,d}} = z_{l,e,d} \\
z_{r_{l,e,d}} = 0 \\
Q_{w_{l,e}} = n w_{l,e} h_i w_l^0 \\
Q_{r_{l,e}} = 0 \\
C_{f_{l,e}} = C_{f_w} \\
C_{v_{l,e}} = C_{v_w} Q_{w_{l,e}} + C_s Q_{w_{l,e}}
\end{bmatrix} & \Rightarrow & \begin{bmatrix}
\neg y_{w_{l,e}} \\
y_{r_{l,e}} \\
y_{l,e,d} = y_{m_{r_{l,e,d}}} \\
y_{p_{r_{l,e,d}}} = y_{l,e-1,d} \\
y_{m_{w_{l,e,d}}} = 0 \\
y_{p_{w_{l,e,d}}} = 0 \\
z_{w_{l,e,d}} = 0 \\
z_{r_{l,e,d}} = z_{l,e,d} \\
Q_{w_{l,e}} = 0 \\
Q_{r_{l,e}} = n r_{l,e} h_i w_l^0 \\
C_{f_{l,e}} = C_{f_r} + C_{f_f} \\
C_{v_{l,e}} = C_{v_r} (Q_{r_{l,e}} + w_l^0) + C_{v_f} Q_{r_{l,e}} + C_s Q_{r_{l,e}}
\end{bmatrix} & \Rightarrow & \begin{bmatrix}
\neg y_{w_{l,e}} \\
\neg y_{r_{l,e}} \\
y_{l,e,d} = y_{l,e,d} \\
y_{m_{r_{l,e,d}}} = 0 \\
y_{p_{r_{l,e,d}}} = 0 \\
y_{m_{w_{l,e,d}}} = 0 \\
y_{p_{w_{l,e,d}}} = 0 \\
z_{w_{l,e,d}} = 0 \\
z_{r_{l,e,d}} = z_{l,e,d} \\
Q_{w_{l,e}} = 0 \\
Q_{r_{l,e}} = 0 \\
C_{f_{l,e}} = 0 \\
C_{v_{l,e}} = 0
\end{bmatrix}
\end{align*}
\]
Heat Integration

\[ R_k - R_{k-1} - \sum_{m \in V_k} Q_m^V + \sum_{n \in U_k} Q_n^U = \sum_{l \in H_k} w_l (C_p \Delta T)^H_{lk} - \sum_{l \in C_k} w_l (C_p \Delta T)^C_{lk} \quad k \in K \]

Objective Function

The objective function minimizes the total venture cost:

\[
\text{min } \sum_{s \in S_m} \sum_{t \in T(s)} (FC_{t,s} + VC_{t,s} + c_{t,s}^C Q_{t,s}^C + c_{t,s}^S Q_{t,s}^S) + \sum_{m \in V} c_m Q_m^V + \sum_{n \in U} c_n Q_n^U + \sum_{l \in L_w} \sum_{e} (Cf_{l,e} + CV_{l,e})
\]
Example: Sylvinite

The MILP formulation contains 299 equations, 218 continuous variables, and 27 binary variables.

Sylvinite.gms
Sylvinite_heat.gms
Sylvinite_wash.gms
Sylvinite_wash_heat.gms
The MILP formulation contains 1209 equations, 1201 continuous variables, and 145 binary variables. Solution time was 84 s for OSLv2 (GAMS) with a 1.7 GHz Pentium 4 processor.
Flotation Circuit Design Problem

• Flotation Circuit problem overview
• Superstructures for task, state and equipment selection
• Mathematical model
• Examples
Flotation design problem overview

Mineral flotation processes consist of several units that are grouped into banks and interconnected in a predefined manner in order to divide the feed into concentrate and tailing. The behavior of these processes depends on the configuration of the circuit and the physical and chemical nature of the slurry treated.
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Figures From
http://cache.eb.com/eb/image?id=1534&rendTypeId=4
http://cape.uwaterloo.ca/images/pal1.gif

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The design of these circuits is carried out based on the experience of the designer, with the help of laboratory tests and simulations. Some attempts have been described in the literature on automated methods for the design of these types of circuits. However, methods for the design of flotation circuits have not yet progressed to the stage where an optimum circuit configuration can be completely derived automatically.
Superstructures for task, state & equipment

Superstructures are developed in a hierarchical form:
• First level: separation task superstructure
• Second level: processing systems are presented which must carry out rougher, cleaner, and scavenger operations and define states.
• Third level: equipment selection (column versus mechanical bank; grinding-classification circuit)
Task Superstructure
State Superstructures

Mixing with/without grinding
Mixing without grinding
Mechanical cell bank or column
R = 1 - \frac{1}{(1 + \omega \tau)^N}

R_{k} = 1 - \frac{4a \exp\left(\frac{1}{2D/u h}\right)}{(1 + a)^2 \exp\left(\frac{a}{(2D/u h)}\right) - (1 - a)^2 \exp\left(-a \frac{2D/u h}{2D/u h}\right)}

Mechanical cell bank or column

Grinding-classification circuits

Equipment Superstructures

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Floatability Index

\[ \Gamma = \frac{\sum_{k} \sum_{j} \gamma_{k,j} W_{P,k,j}}{\sum_{k} \sum_{j} W_{P,k,j}} \]
Mathematical Model

Material Balances

\[ \sum_{l \in M^{\text{in}}(m)} W_{l,k} - \sum_{l \in M^{\text{out}}(m)} W_{1,k} = 0 \quad k \in K, \quad m \in M \quad (1) \]

Feedstock flows

\[ W_{1,k} = F_k \quad (4) \]

Material Balances Flotation Steps

\[ W_{I_{s,l_c,k}} = T_{s,l_a,k} W_{I_{s,l_a,k}} \quad (l_a, l_c) \in \text{Lcc}, \quad k \in K, \quad s \in S \]

\[ W_{I_{s,l_h,k}} = (1 - T_{s,l_a,k}) W_{I_{s,l_a,k}} \]

\[ T_{l_a,k} = 1 - \frac{1}{(1 + k_{l_a} \tau_{l_a})^{N_{l_a}}} \]
Equipment Selection

\[
\begin{align*}
\begin{bmatrix}
    y_{s,\ell_1}^b \\
    C_{s,\ell_1}^f = C_b^f \\
    C_{s,\ell_1}^V = C_b^V \sum_k W_{s,\ell_1,k}^b \\
    W_{s,\ell_1,k}^b = WI_{s,\ell_1,k}^b \\
    WI_{s,\ell_2,k}^b = T_{s,\ell_1,k}^b W_{s,\ell_1,k}^b \\
    WI_{s,\ell_3,k}^b = (1 - T_{s,\ell_1,k}^b) W_{s,\ell_1,k}^b
\end{bmatrix} & \quad \begin{bmatrix}
    y_{s,\ell_1}^c \\
    C_{s,\ell_1}^f = C_c^f \\
    C_{s,\ell_1}^V = C_c^V \sum_k W_{s,\ell_1,k}^c \\
    W_{s,\ell_1,k}^c = WI_{s,\ell_1,k}^c \\
    WI_{s,\ell_2,k}^c = T_{s,\ell_1,k}^c W_{s,\ell_1,k}^c \\
    WI_{s,\ell_3,k}^c = (1 - T_{s,\ell_1,k}^c) W_{s,\ell_1,k}^c
\end{bmatrix}
\end{align*}
\]
Objective Function

\[ \text{Profit} = \text{Income} - \text{Costs} = I - C \]

\[ I = \left( \sum_k g_k W_{10,k} \right) p (q - R_{fc}) H - \sum_k W_{10,k} [pu(q - R_{fc}) + Tr_{c}] H \]

Table 1
Typical values for a copper concentration plant (minimum grade of concentrate 28%)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>0.975</td>
<td>( R_{fc} )</td>
<td>$200/ton metal</td>
</tr>
<tr>
<td>( u )</td>
<td>0.015</td>
<td>( Tr_{c} )</td>
<td>$85/ton concentrate</td>
</tr>
<tr>
<td>( q )</td>
<td>$1764/ton metal</td>
<td>( H )</td>
<td>7200 h/year</td>
</tr>
</tbody>
</table>
Example

The procedure was applied to the design of a copper concentration plant, whose species are: $k=1$ (100% chalcopyrite), $k=2$ (50% silica, 50% chalcopyrite) and $k=3$ (100% silica).
Economic results in millions of US$ for a metal price of 1.00 US$/lb where the k=3 recovery is comparable to the k=2 recovery.

(NG)= no grinding, (G)= grinding w/o classification, (G-C)= grinding – classification, (C-G)= classification – grinding.

<table>
<thead>
<tr>
<th>Case</th>
<th>Options</th>
<th>Selection</th>
<th>Profit</th>
<th>Revenues</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 9</td>
<td>NG</td>
<td>NG</td>
<td>11.964</td>
<td>37.627</td>
<td>26.421</td>
</tr>
<tr>
<td>Case 10</td>
<td>G/G-C</td>
<td>G-C</td>
<td>12.748</td>
<td>28.639</td>
<td>26.726</td>
</tr>
<tr>
<td>Case 11</td>
<td>G/C-G</td>
<td>C-G</td>
<td>12.996</td>
<td>38.771</td>
<td>26.571</td>
</tr>
<tr>
<td>Case 12</td>
<td>G-C/C-G</td>
<td>C-G</td>
<td>12.996</td>
<td>38.771</td>
<td>26.571</td>
</tr>
</tbody>
</table>
Final Remark

Complete list of references on design of separation based on crystallization, design of flotation circuits, design of leaching process, and design of solvent extraction circuits

Can be found on
http://www.uantof.cl/d2p