Process Design for Mineral Operations

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Outline

Motivation

General Strategy

Crystallization Design Problem

Flotation Circuit Design Problem

Final Remark

Motivation

- High price cycle unprecedented
- Lowering the cost of production
- Achieving the balance of acceptable economic, environmental and social effects.
- Improve energy efficiency

Copper Technology Roadmap Review 2006, AMIRA

Price of copper

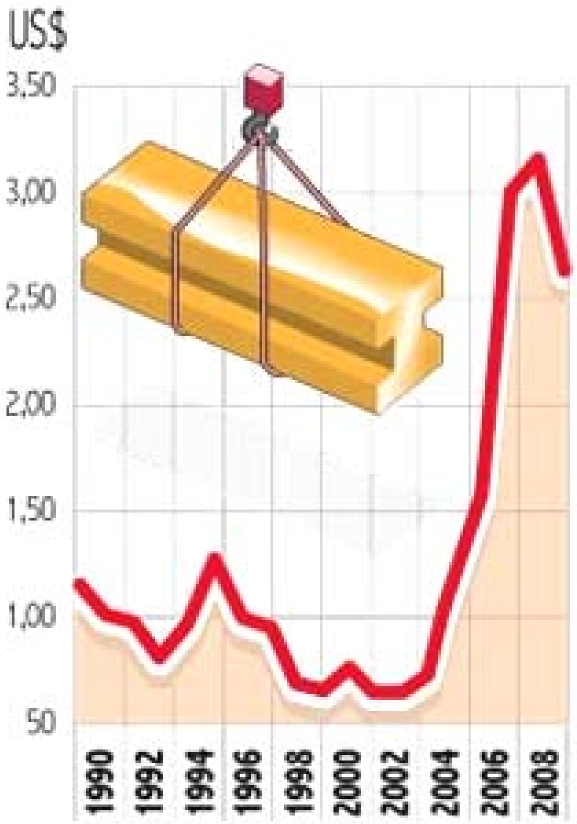

Figure From:
http://www.lanacion.cl/prontus_noticias/site/artic/20070802/pags/20070802215453.html

CICITEM

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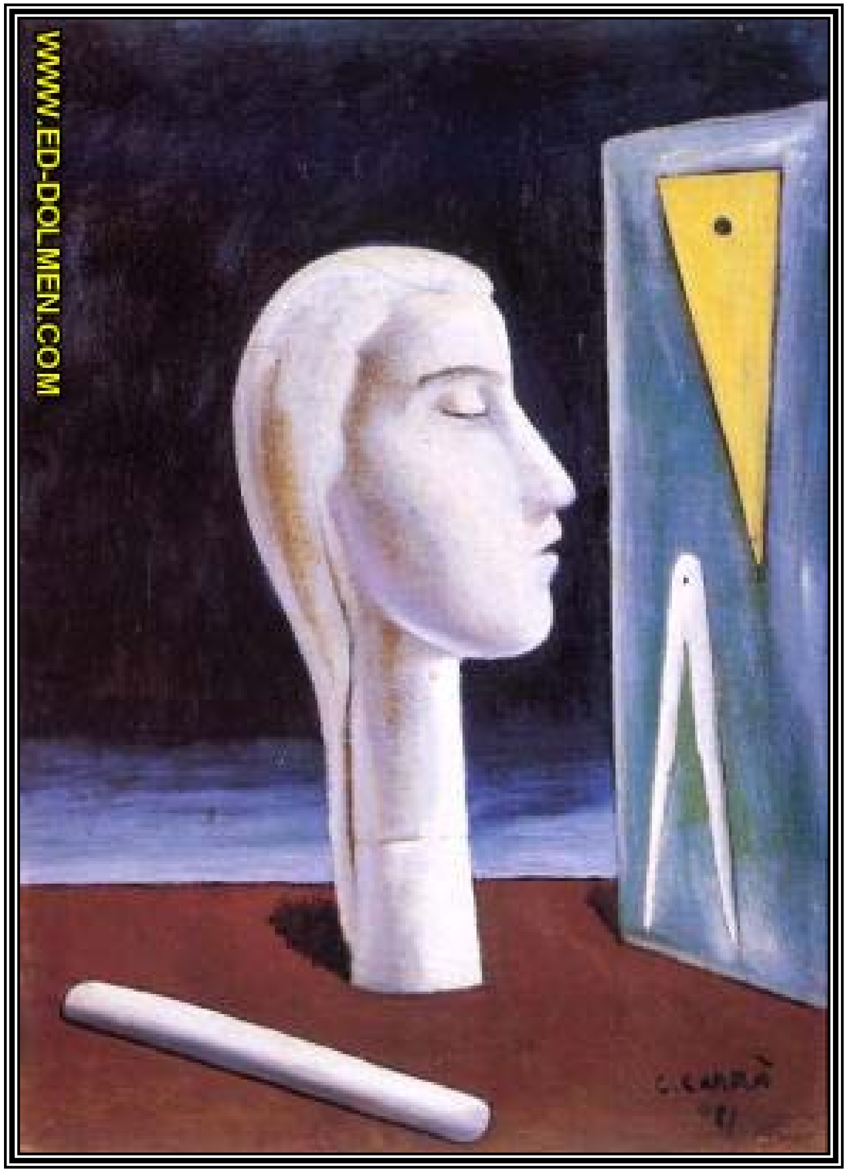
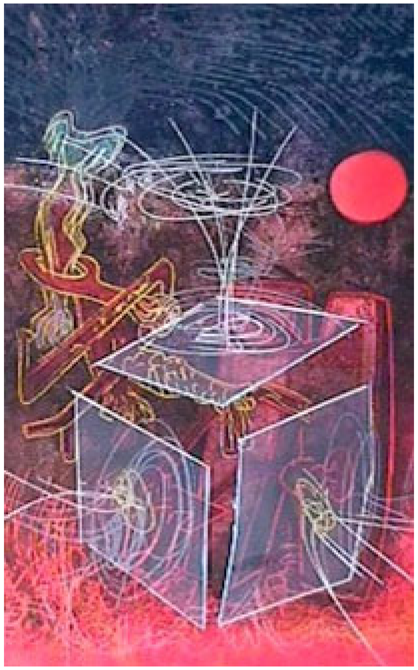


The engineer`s lover
Carlo Carrá
Italian 1881-1966

Problems:

Process design has multiple dimensions

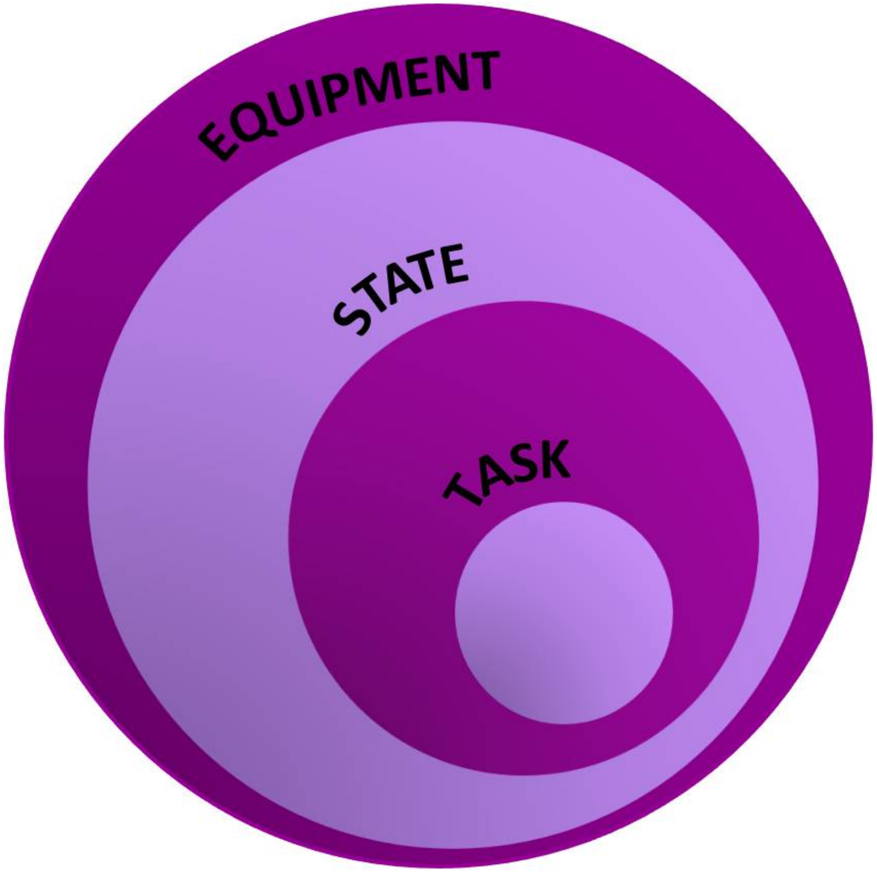


Roberto Matta
Chilean 1911-2002

Solution:
Look as Picasso

Pablo Picasso
Spanish 1881-1973

The Onion Model



Crystallization Design Problem

Crystallization design problem overview

Fractional Crystallization

Fractional Crystallization with Heat Integration & Cake Washing

Crystallization design problem overview

Crystallization is extensively used in different industrial applications, including the production of a wide range of materials such as fertilizers, detergents, foods, and pharmaceutical products, as well as in the treatment of waste effluents

Problems

The crystallization stages are usually accompanied by other separation techniques. Leaching.

Various types of crystallization exist: cooling, evaporation, reactions, and drowning-out

The characteristics of the product affects a series of other associated operations. filtration & washing.

The separation is limited by multiple saturation points. Temperature changes & external chemical agents.

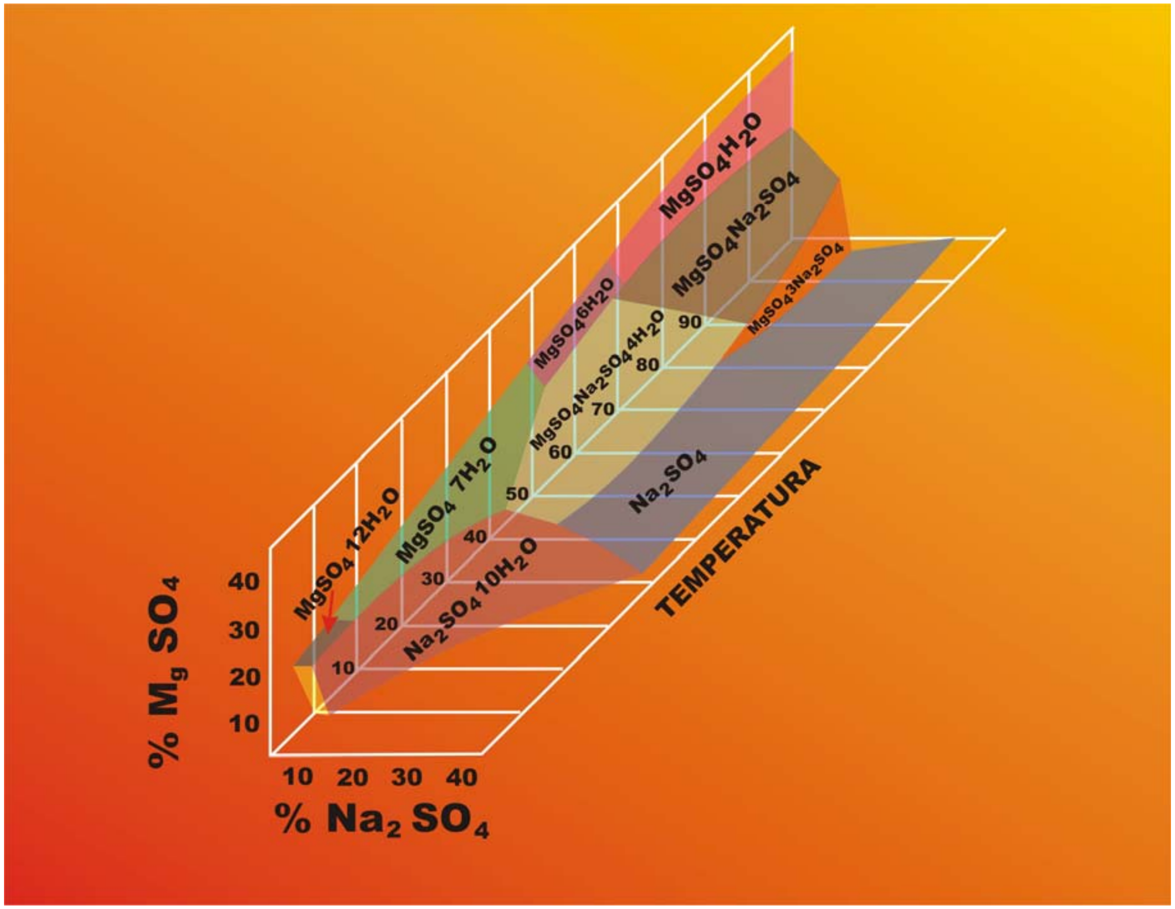
Kinetic factors and metastability may affect the design.

Phase Diagram

The greatest advantages obtained in the use of the phase diagram are the possibilities for the visualization of the behavior of phase equilibria, describing the processes, and obtaining mass balances with the help of the lever arms rule.

The phase diagrams, however, also have a series of limitations as a design tool

Phase Diagram



Goals

Determine optimal stream configuration.

Determine operational conditions & flowrates.

Selection of equipment type.

Determine solid-liquid separation. Washing & Filtration.

Determine heat integration.

Fractional Crystallization

Basic Crystallization Separation

Relative Composition Diagram

Feasible Pathway Diagram

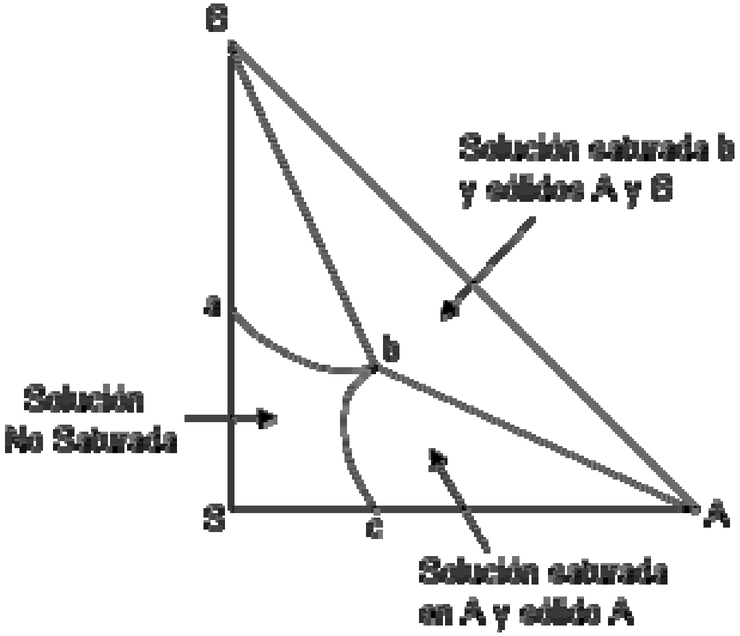
State Superstructure

Connectivity Matrix

Mathematical model

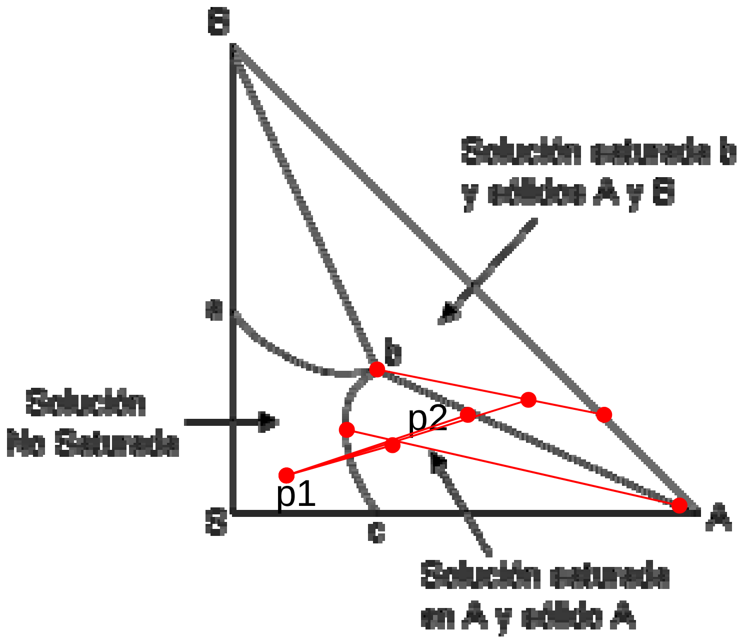
Examples

Basic Crystallization Separation



Isothermal Cut
$KCl+NaCl+H_2O$
$KNO_3+NaNO_3+H_2O$
L serine acid + L aspartic acid + water

Basic Crystallization Separation



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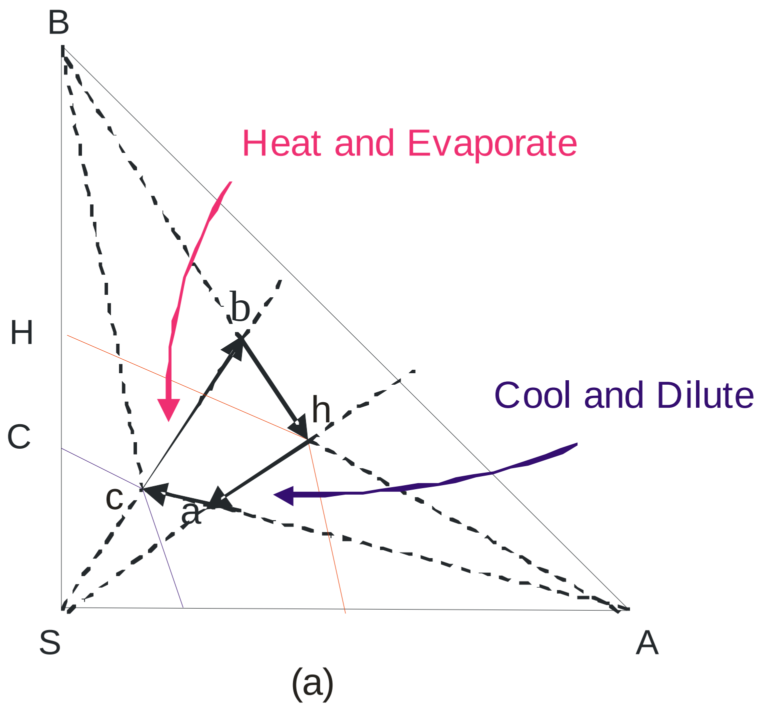
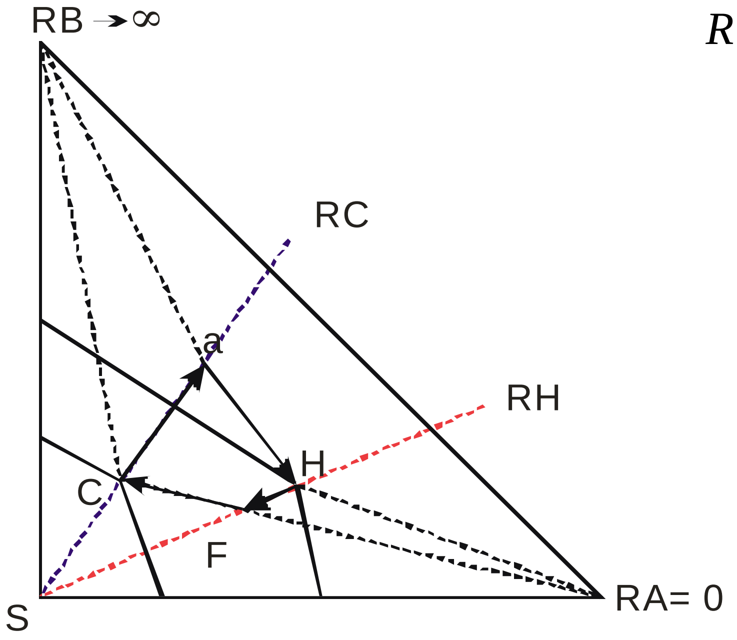
Basic Crystallization Separation



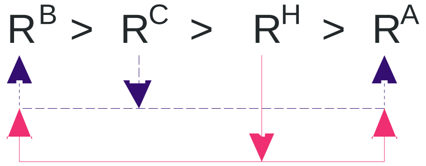
Basic Cycle

Relative Composition Diagram



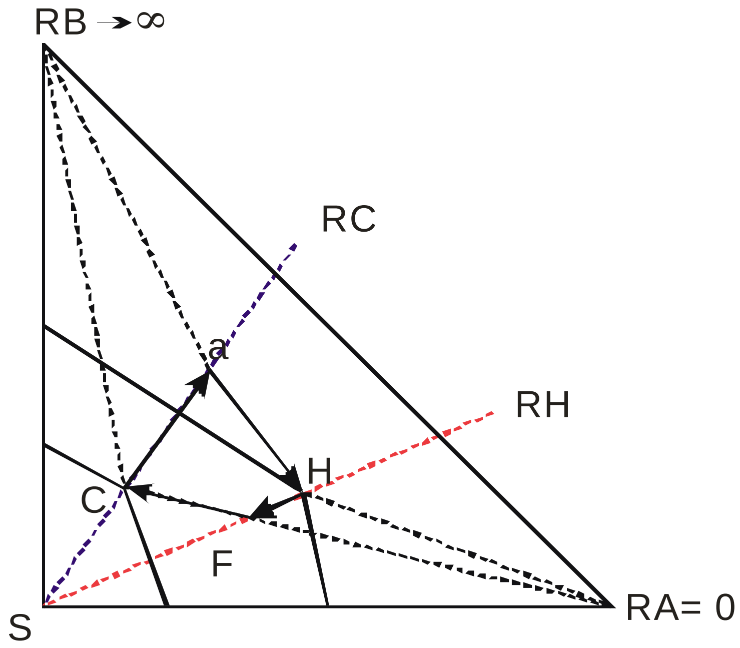
(a)

$$R = \frac{\text{Weight Composition of B}}{\text{Weight Composition of A}}$$

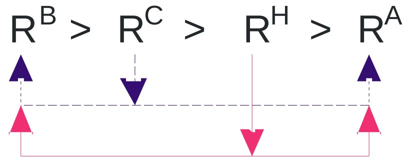


$R^B > R^C > R^H > R^A$

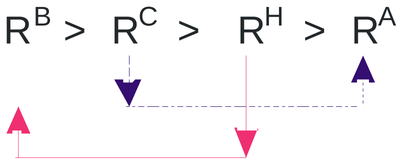
Feasible Pathway Diagram



(a)

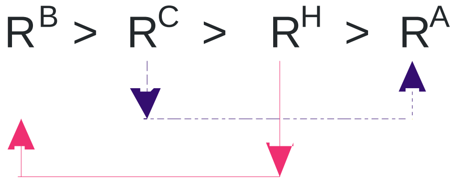


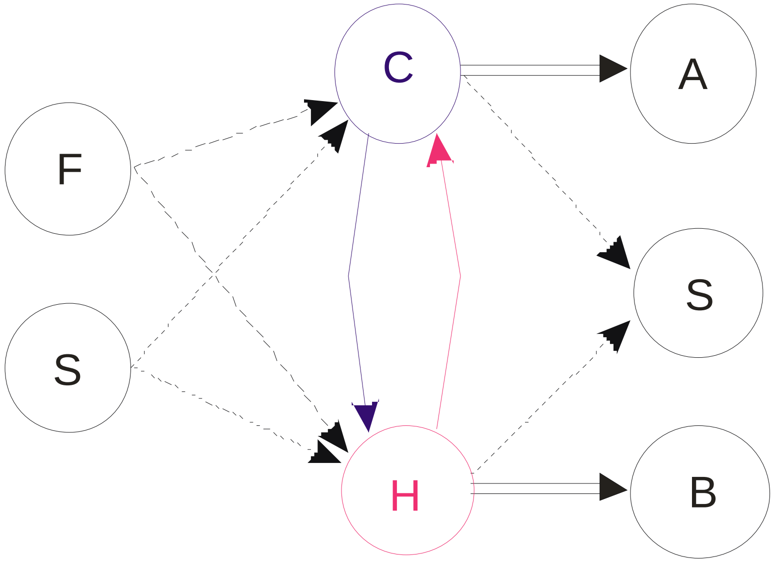
$R^B > R^C > R^H > R^A$



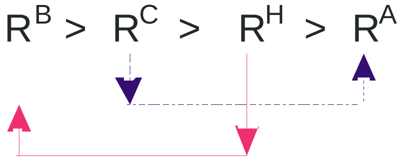
$R^B > R^C > R^H > R^A$

State Superstructure

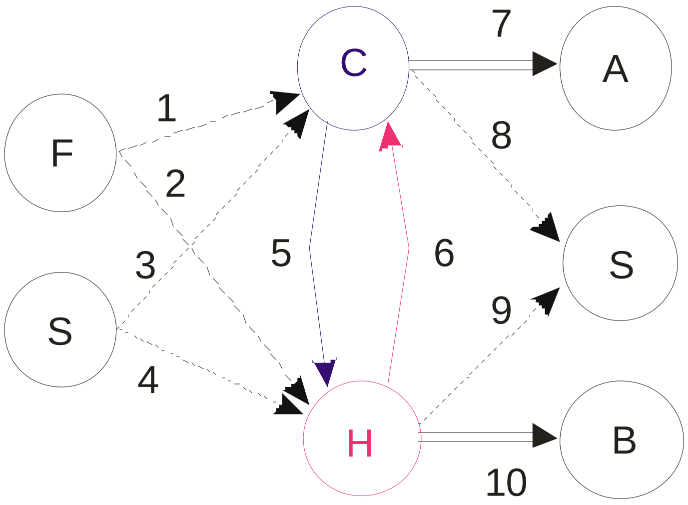

$R^B > R^C > R^H > R^A$



Connectivity Matrix

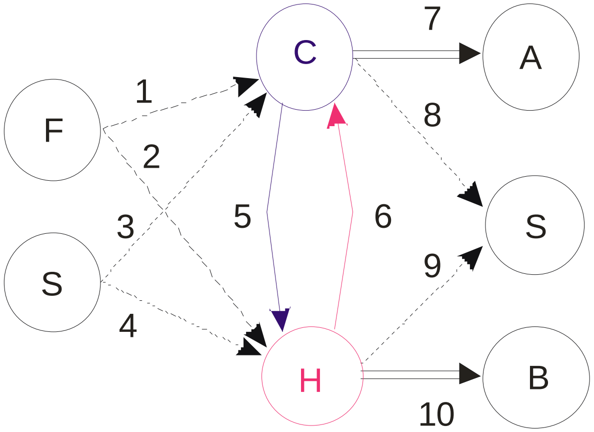


$R^B > R^C > R^H > R^A$



	C	H	S_2	A	B
F	1	2			
S_1	3	4			
C		5	8	7	
H	6		9		10

Mathematical model



General model
Cisternas, L.A. (1999), Optimal design of crystallization-based separation schemes, *AIChE J.*, 45, 1477-1487.

$$\underset{w}{Min} \sum_{l} w_l$$

$$w_1 \ x_{1,i} + w_3 \ x_{3,i} + w_6 \ x_{6,i} = w_5 \ x_{5,i} + w_7 \ x_{7,i} + w_8 \ x_{8,i}$$

$$w_2 \ x_{2,i} + w_4 \ x_{4,i} + w_5 \ x_{5,i} = w_6 \ x_{6,i} + w_9 \ x_{9,i} + w_{10} \ x_{10,i}$$

$$w_1 \ x_{1,i} + w_2 \ x_{2,i} = C_{1,i}^F$$

$$w_l \geq 0; \quad i = A, B, S$$

Examples-Sylvinite

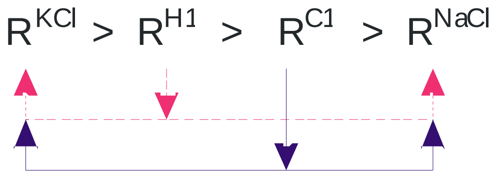
Equilibrium Data

key	Temperature [°C]	Weight Composition		Solid Phase
		KCl	NaCl	
C1	30	11.7	20.25	KCl+NaCl
H1	100	22.2	15.9	KCl+NaCl

R_{KCl}	R_{H1}	R_{C1}	R_{NaCl}
∞	1.40	0.58	0.0

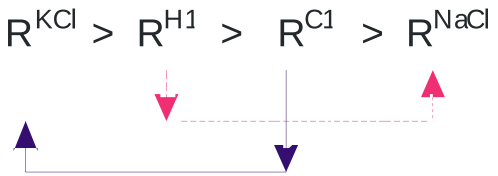
Examples-Sylvinite

Relative Composition Diagram



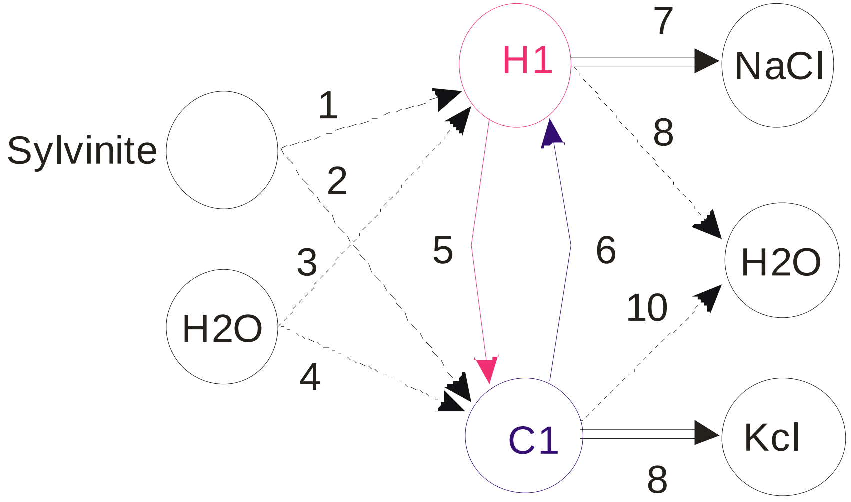
$R^{KCl} > R^{H1} > R^{C1} > R^{NaCl}$

Feasible Pathway Diagram



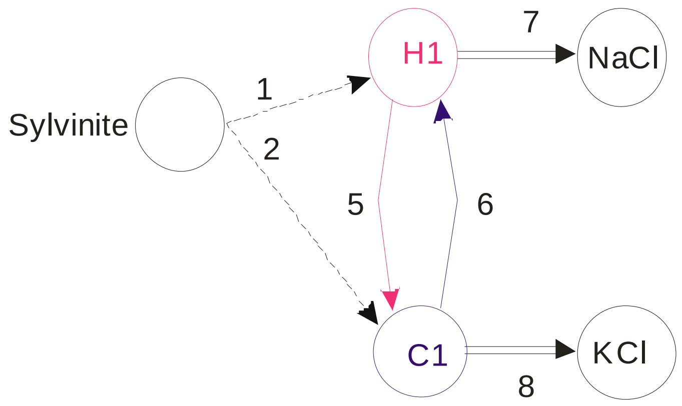
$R^{KCl} > R^{H1} > R^{C1} > R^{NaCl}$

State Diagram

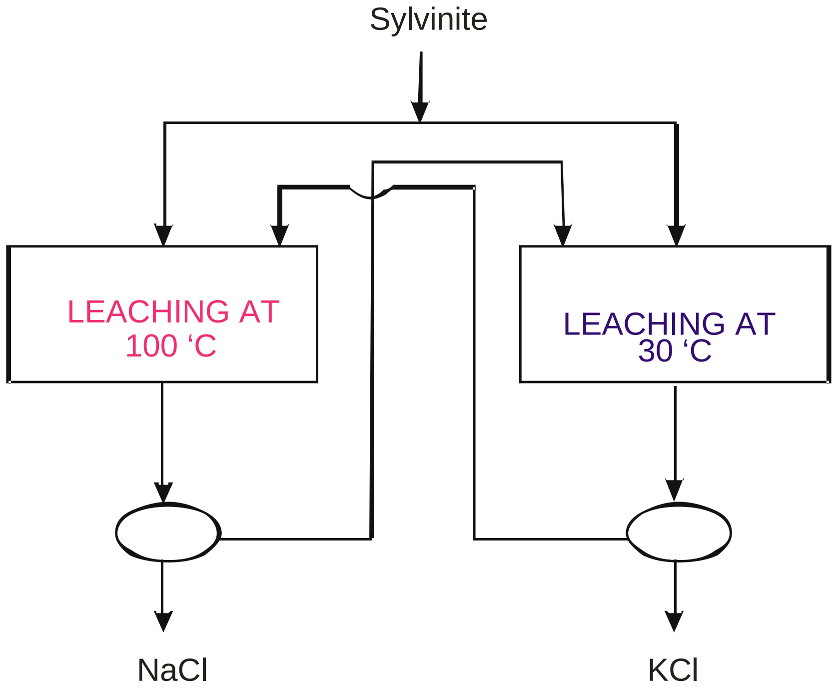


Examples-Sylvinite

State Diagram



Flow Sheet



Examples-Astrakanite

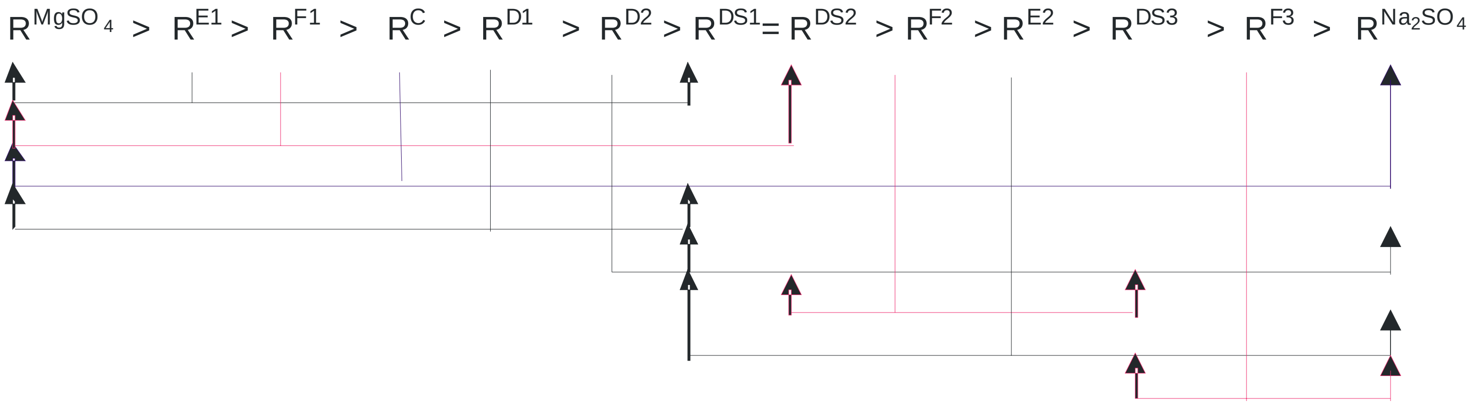
Equilibrium data for $MgSO_4+Na_2SO_4+H_2O$ system.

T ºC	keys	Saturated solution, % w		Solid phase	R
		$MgSO_4$	Na_2SO_4		
18.7	C	20.57	11.8	Mg_7 + Na_{10}	1.7
25	D1	21.15	13	Mg_7 + SD1	1.6
25	D2	16.6	17.8	SD1 + Na_{10}	0.9
50	E1	31.32	4.74	Mg_6 + SD1	6.6
50	E2	11.98	23.25	SD1 + Na	0.5
97	F1	32.2	5.55	Mg_1 + SD2	5.8
97	F2	14.4	19.15	SD2 + SD3	0.8
97	F3	5.88	26.9	SD3 + Na	0.2
	SD1	35.99	42.48		0.8
	SD2	45.86	54.14		0.8
	SD3	22.02	77.98		0.3

Mg7=MgSO4.7H2O; Mg1=MgSO4.1H2O; Mg6=MgSO4.6H2O; Na10=Na2SO4.10H2O; Na=Na2SO4; SD1= Na2SO4.MgSO4.4H2O; SD2= Na2SO4.MgSO4; SD3= MgSO4.3Na2SO4

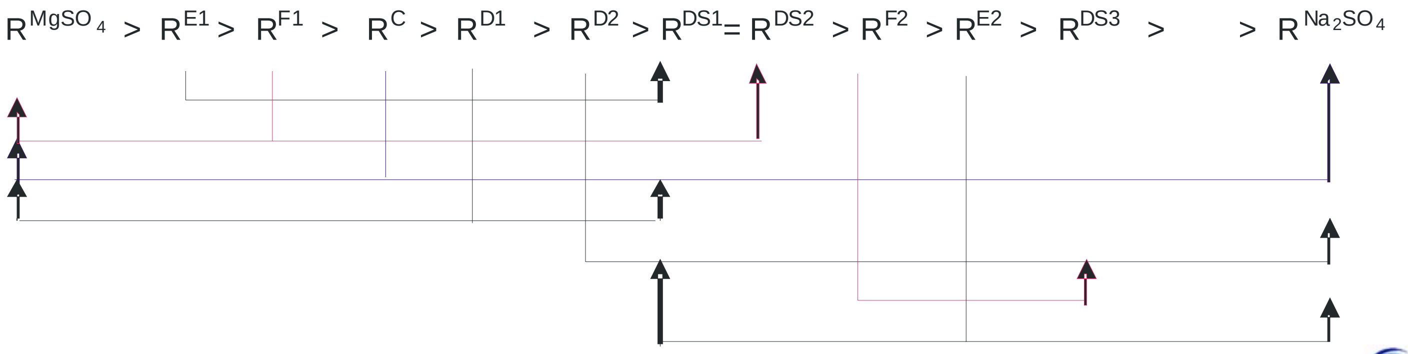
Examples-Astrakanite

Relative Composition Diagram



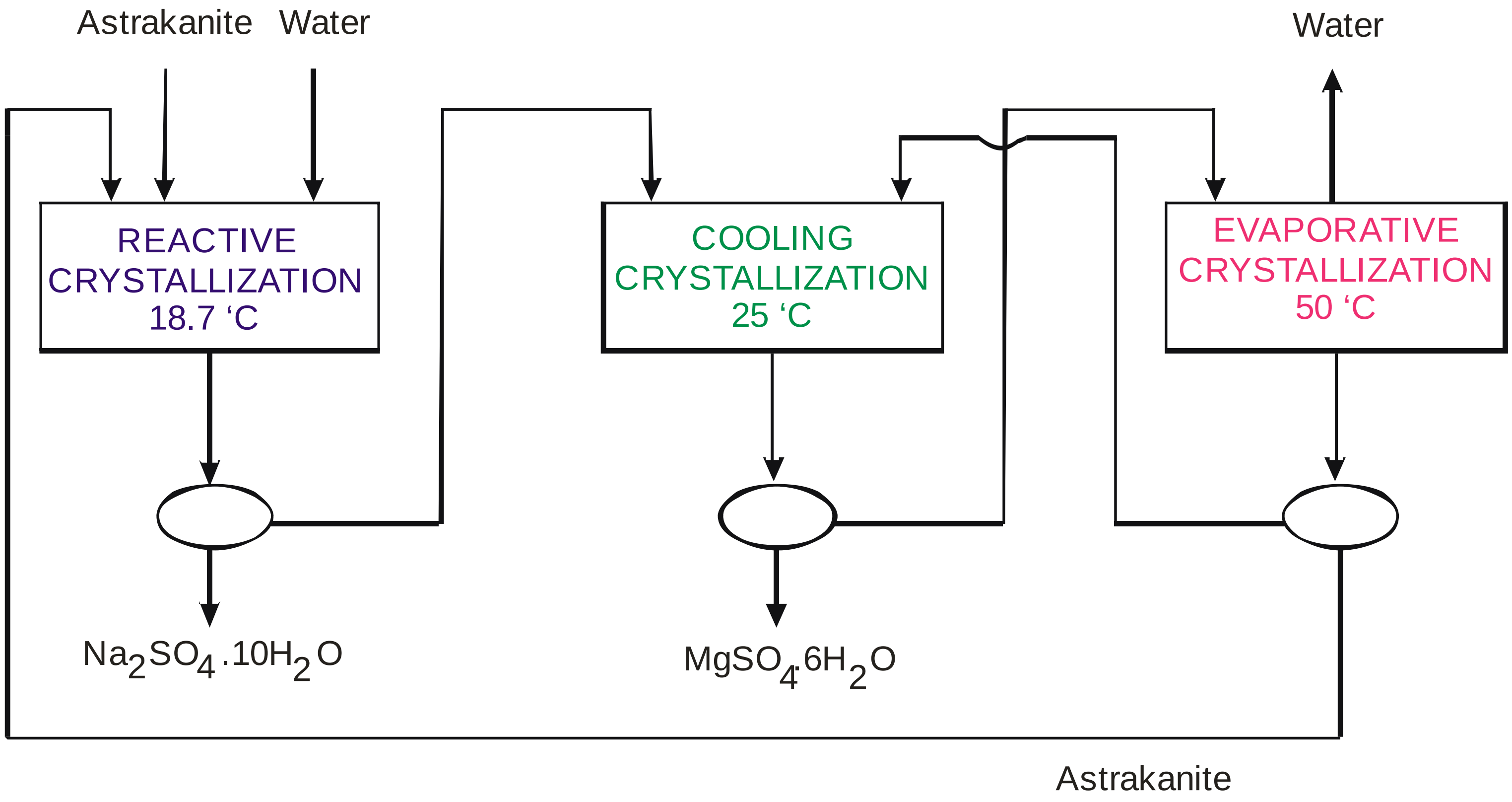
$R^{MgSO_4} > R^{E1} > R^{F1} > R^{C} > R^{D1} > R^{D2} > R^{DS1} = R^{DS2} > R^{F2} > R^{E2} > R^{DS3} > R^{F3} > R^{Na_2SO_4}$

Feasible Pathway Diagram



$R^{MgSO_4} > R^{E1} > R^{F1} > R^{C} > R^{D1} > R^{D2} > R^{DS1} = R^{DS2} > R^{F2} > R^{E2} > R^{DS3} > \quad > R^{Na_2SO_4}$

Examples-Astrakanite



astrakanite.gms

Fractional Crystallization with Heat Integration & Cake Washing

State Superstructure

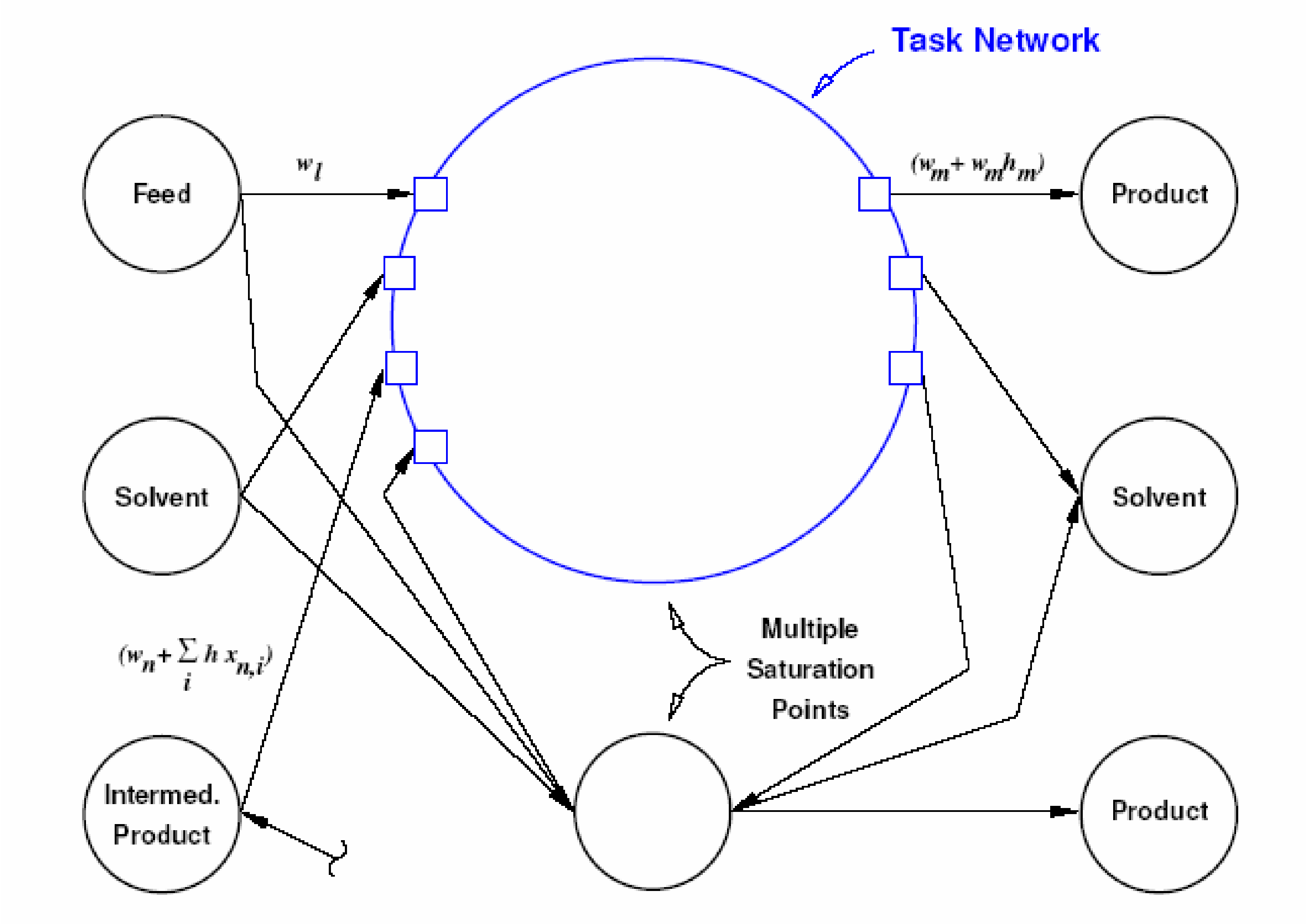
Task superstructure.

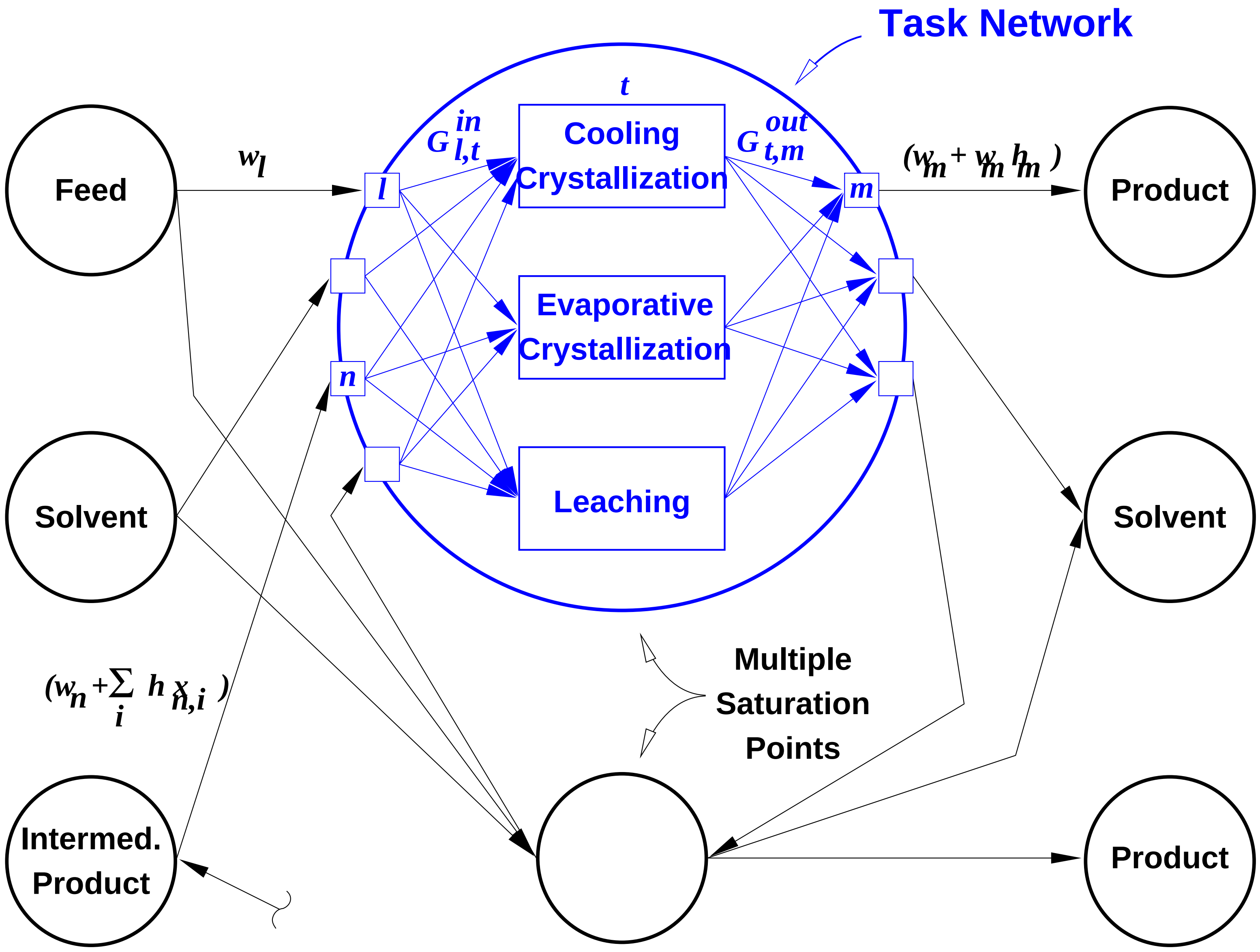
Heat integration.

Cake Washing

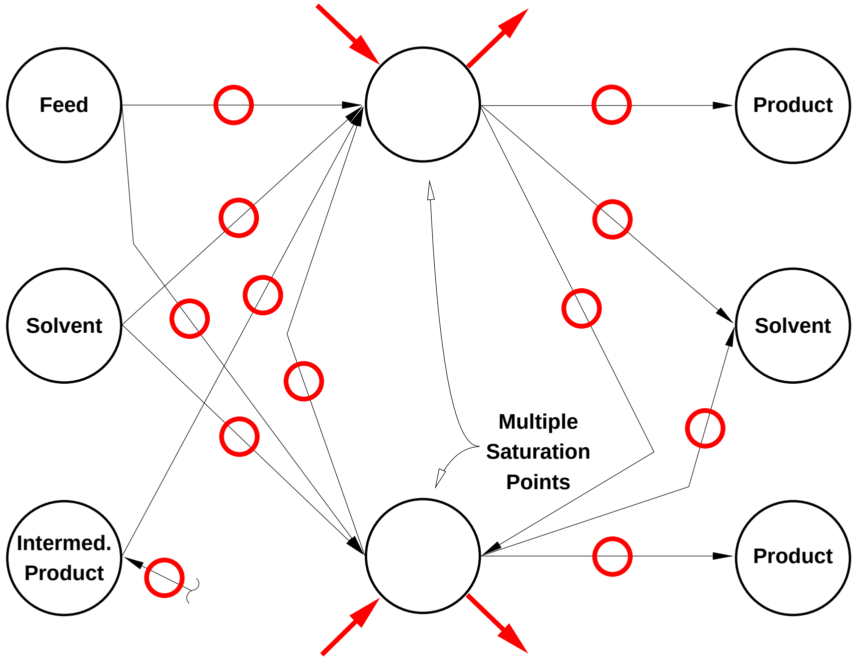
- Cisternas L.A., J.Y. Cueto and R.E. Swaney, “Flowsheet Synthesis of Fractional Crystallization Process with Cake Washing”, *Computer and Chemical Engineering*, 28, 613-623 (2004)
- Cisternas L.A., C. Guerrero and R. Swaney,, “Separation System Synthesis of Fractional Crystallization Processes with Heat Integration”, *Computer and Chemical Engineering*, 25, 595-602 2001

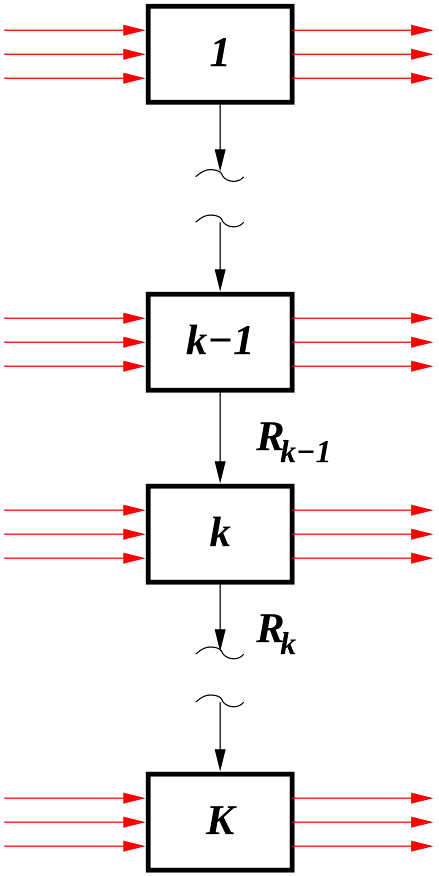
Task Superstructure




Task Network
t
Feed
w_l
l
$G^{in}_{l,t}$
Cooling Crystallization
$G^{out}_{t,m}$
m
$(w_m + w_m h_m)$
Product
Evaporative Crystallization
n
Solvent
Leaching
Solvent
$(w_n + \sum_i h\, x_{n,i})$
Multiple Saturation Points
Intermed. Product
Product

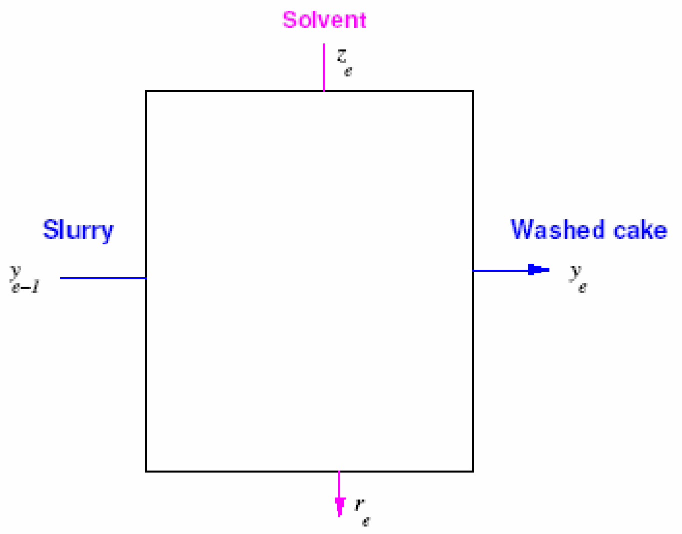
Heat Integration

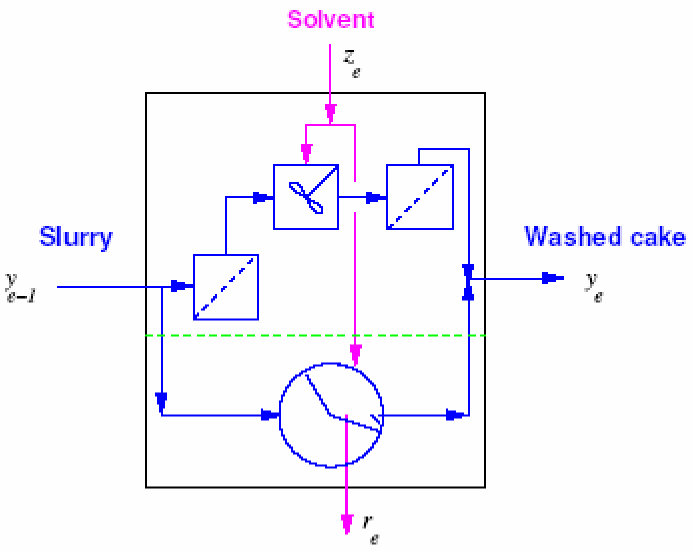




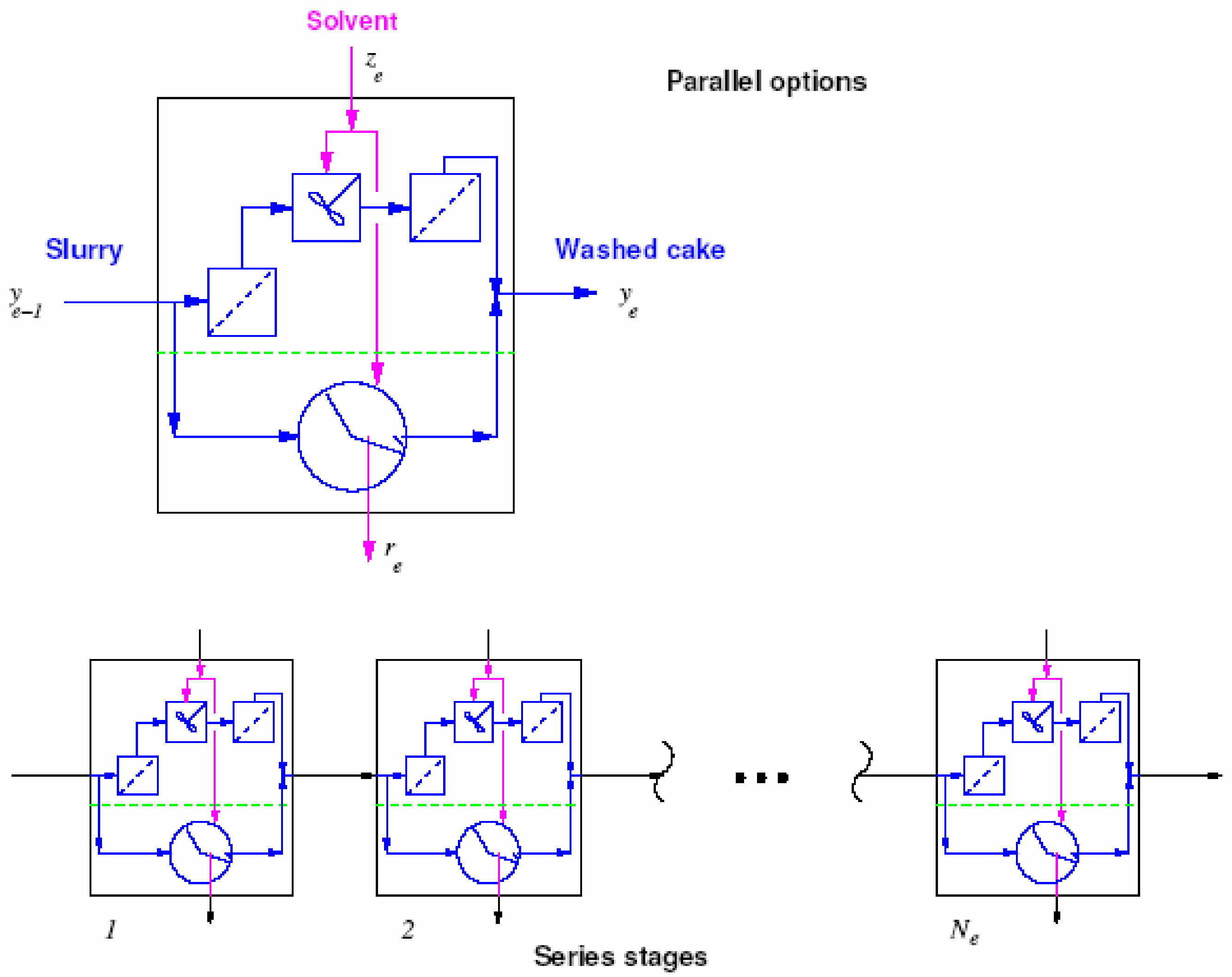
Papoulias S.A., & I.E. Grossmann (1983), A structural optimization approach to process synthesis-II. Heat recovery networks. Comp. and Chem. Engng., 7, 707

Cake Washing





Parallel Options


Solvent
z_e
Parallel options
Slurry
Washed cake
y_e−1
y_e
r_e
1
2
N_e
Series stages

Mathematical Model

State Superstructure

Mass balance for each component in multiple saturation nodes (*SM*):

$$\sum_{l \in Lq \cap S^{in}(s)} hx_{l,i} + \sum_{l \in S^{in}(s)} w_l \cdot x_{l,i} - \sum_{l \in S^{out}(s)} w_l \cdot x_{l,i} - \sum_{l \in (Lw \cup Lq) \cap S^{out}(s)} w_l h_l x_{l',i} = 0 \qquad s \in S_M, i \in I$$

Mass balance for each component in intermediate product nodes (S_I):

$$\sum_{l \in Lq \cap S^{in}(s)} w_l \cdot x_{l,i} - \sum_{l \in Lq \cap S^{out}(s)} w_l \cdot x_{l,i} = 0 \qquad s \in S_I, i \in I$$

$$\sum_{l \in Lq \cap S^{out}(s)} hx_{l,i} - \sum_{l \in Lq \cap S^{in}(s)} w_l \; h_l x_{l',i} = 0 \qquad s \in S_I, i \in I$$

$$\sum_{i \in I} (w_l \cdot x_{l,i} + hx_{l,i}) - U_l \; ym_l \le 0 \qquad l \in Lq \cap S^{out}(s), s \in S_I$$

$$\sum_{l \in Lq \cap S^{out}(s)} ym_l - 1 = 0 \qquad s \in S_I$$

Specification for feeds flow rates in feed nodes (S_F):

$$\sum_{l \in S^{out}(s)} w_l \cdot x_{l,i} = C^F_{s,i} \qquad s \in S_F, i \in I_F(s)$$

Task Superstructure

Mass balance between the thermodynamic state network and task network

$$w_l + \sum_{i\in I} hx_{l,i} = \sum_{t\in T(s)} G^{in}_{l,t}, \qquad l \in S^{in}(s), s \in S_M$$

$$w_l + w_l h_l = \sum_{t\in T(s)} G^{out}_{t,l}, \qquad l \in S^{out}(s), s \in S_M$$

Mass balance in the task network:

$$\sum_{l\in S^{in}(s)} G^{in}_{l,t} = \sum_{l\in S^{out}(s)} G^{out}_{t,l}, \qquad t \in T, s \in S_M$$

Task selection and energy balance:

IF carnallite is fed to node 3 (stream 14)

THEN the task is reactive crystallization

$$y^t_{n3,rc} - y^w_{14} \geq 0 \qquad t \in T(s), s \in S_M(s)$$

$$Q^C_{t,s} = HQ^C_{t,s}($$

$$g(y_{t,s}) = \mathrm{Tru}$$

Mass balance for each component in wash/reslurry stage:

Efficiency constraint for wash/reslurry stage:

Degree of impurity:

Wash or reslurry/filter selection:

Cake Washing

$$\begin{bmatrix} yw_{l,e} \\ \neg yr_{l,e} \\ y_{l,e,i} = ymw_{l,e,i} \\ ypw_{l,e,i} = y_{l,e-1,i} \\ ymr_{l,e,i} = 0 \\ ypr_{l,e,i} = 0 \\ zw_{l,e,i} = z_{l,e,i} \\ zr_{l,e,i} = 0 \\ Qw_{l,e} = nw_{l,e} h_l \; w_l^0 \\ Qr_{l,e} = 0 \\ Cf_{l,e} = Cfw \\ Cv_{l,e} = Cvw \; Qw_{l,e} + \\ Cs \; Qw_{l,e} \end{bmatrix} \vee \begin{bmatrix} \neg yw_{l,e} \\ yr_{l,e} \\ y_{l,e,i} = ymr_{l,e,i} \\ ypr_{l,e,i} = y_{l,e-1,i} \\ ymw_{l,e,i} = 0 \\ ypw_{l,e,i} = 0 \\ zw_{l,e,i} = 0 \\ zr_{l,e,i} = z_{l,e,i} \\ Qw_{l,e} = 0 \\ Qr_{l,e} = nr_{l,e} h_l \; w_l^0 \\ Cf_{l,e} = Cfr + Cff \\ Cv_{l,e} = Cvr \; (Qr_{l,e} + w_l^0) \\ Cvf \; Qr_{l,e} + Cs \; Qr_{l,e} \end{bmatrix} \vee \begin{bmatrix} \neg yw_{l,e} \\ \neg yr_{l,e} \\ y_{l,e,i} = y_{l,e,i} \\ ymr_{l,e,i} = 0 \\ ypr_{l,e,i} = 0 \\ ymw_{l,e,i} = 0 \\ ypw_{l,e,i} = 0 \\ zw_{l,e,i} = 0 \\ zr_{l,e,i} = z_{l,e,i} \\ Qw_{l,e} = 0 \\ Qr_{l,e} = 0 \\ Cf_{l,e} = 0 \\ Cv_{l,e} = 0 \end{bmatrix} \quad \begin{array}{c} e \in E(Lw), \\ l \in Lw, i \in I \end{array}$$

Heat Integration

$$R_k - R_{k-1} - \sum_{m\in V_k} Q_m^V + \sum_{n\in U_k} Q_n^U = \sum_{l\in H_k} w_l\,(C_p\Delta T)_{lk}^H - \sum_{l\in C_k} w_l\,(C_p\Delta T)_{lk}^C \qquad k\in K$$

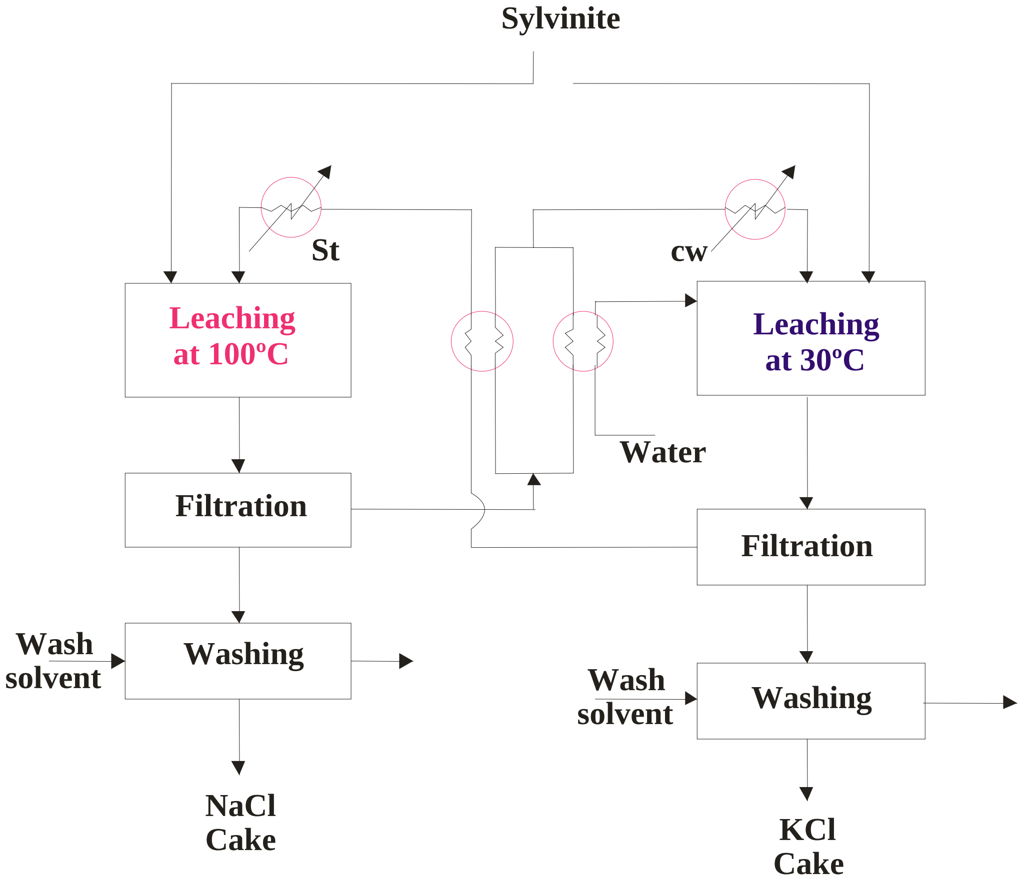
Objective Function

objective function minimizes the total venture cost:

$$\min \sum_{s\in S_M}\sum_{t\in T(s)}(FC_{t,s} + VC_{t,s} + c_{t,s}^C Q_{t,s}^C + c_{t,s}^S Q_{t,s}^S) + \sum_{m\in V} c_m Q_m^V + \sum_{n\in U} c_n Q_n^U + \sum_{l\in Lw}\sum_{e}(Cf_{l,e} + Cv_{l,e})$$

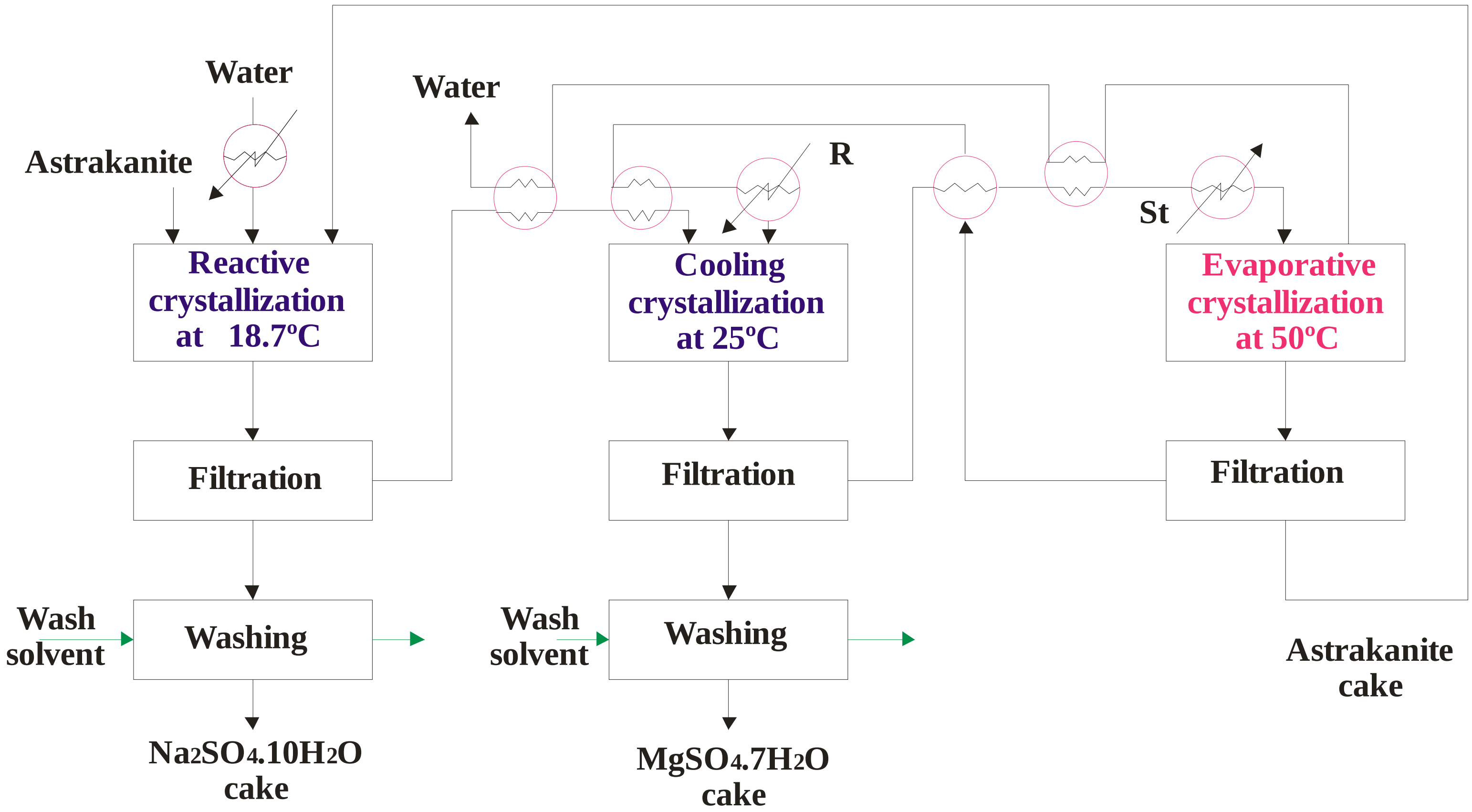
Example: Sylvinite

The MILP formulation contains 299 equations, 218 continuous variables, and 27 binary variables.



Sylvinite.gms
Sylvinite_heat.gms
Sylvinite_wash.gms
Sylvinite_wash_heat.gms

The MILP formulation contains 1209 equations, 1201 continuous variables, and 145 binary variables. Solution time was 84 s for OSLv2 (GAMS) with a 1.7 GHz Pentium 4 processor.



Example: Astrakanite

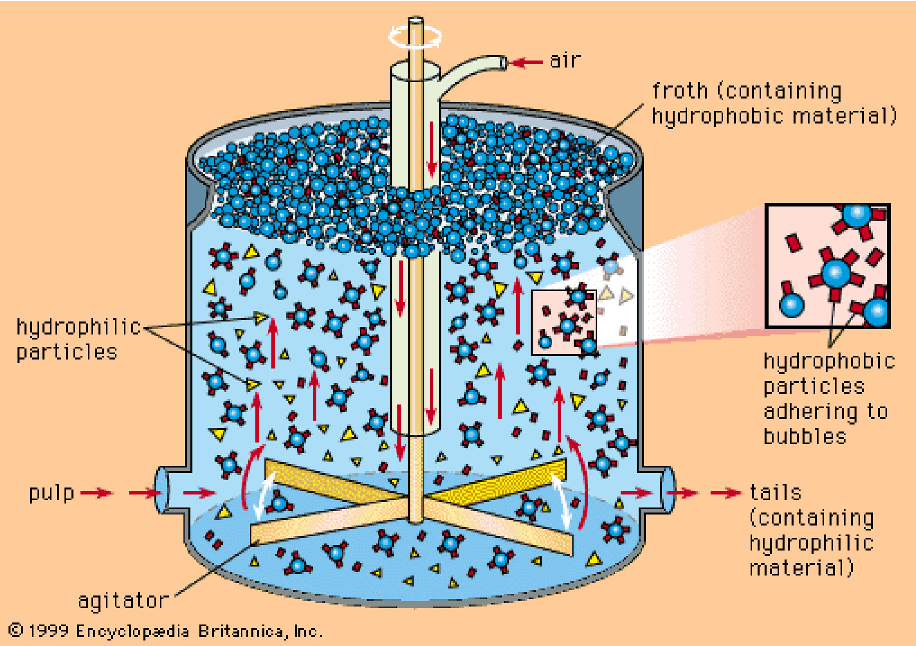
$MgSO_4 \cdot Na_2SO_4 \cdot 4H_2O$

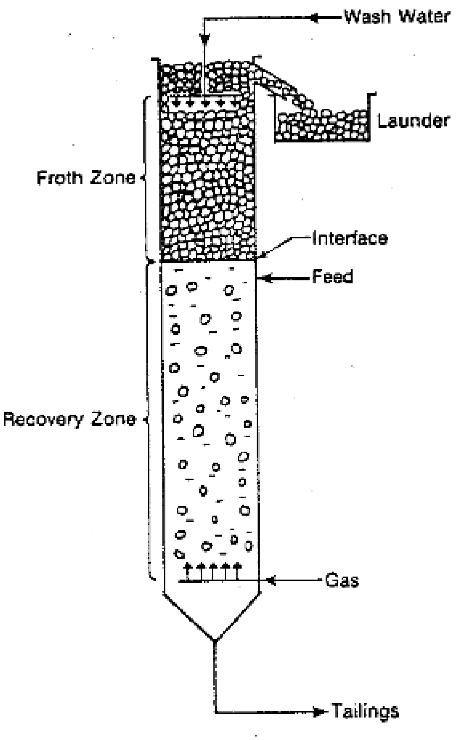
Flotation Circuit Design Problem

- •Flotation Circuit problem overview
- •Superstructures for task, state and equipment selection
- •Mathematical model
- •Examples

Flotation design problem overview

Mineral flotation processes consist of several units that are grouped into banks and interconnected in a predefined manner in order to divide the feed into concentrate and tailing. The behavior of these processes depends on the configuration of the circuit and the physical and chemical nature of the slurry treated





Figures From
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http://cape.uwaterloo.ca/images/pal1.gif

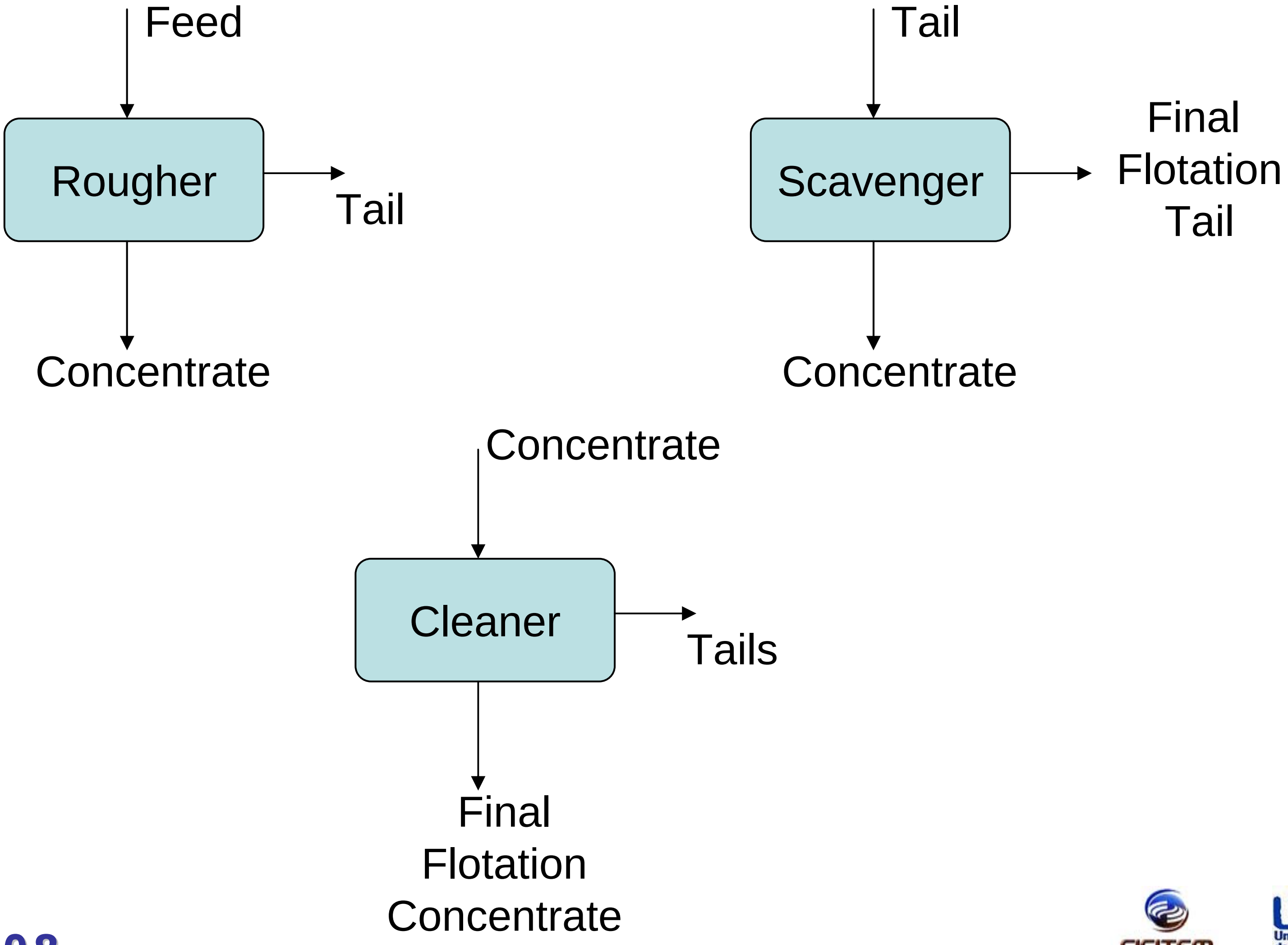
The design of these circuits is carried out based on the experience of the designer, with the help of laboratory tests and simulations. Some attempts have been described in the literature on automated methods for the design of these types of circuits. However, methods for the design of flotation circuits have not yet progressed to the stage where an optimum circuit configuration can be completely derived automatically.

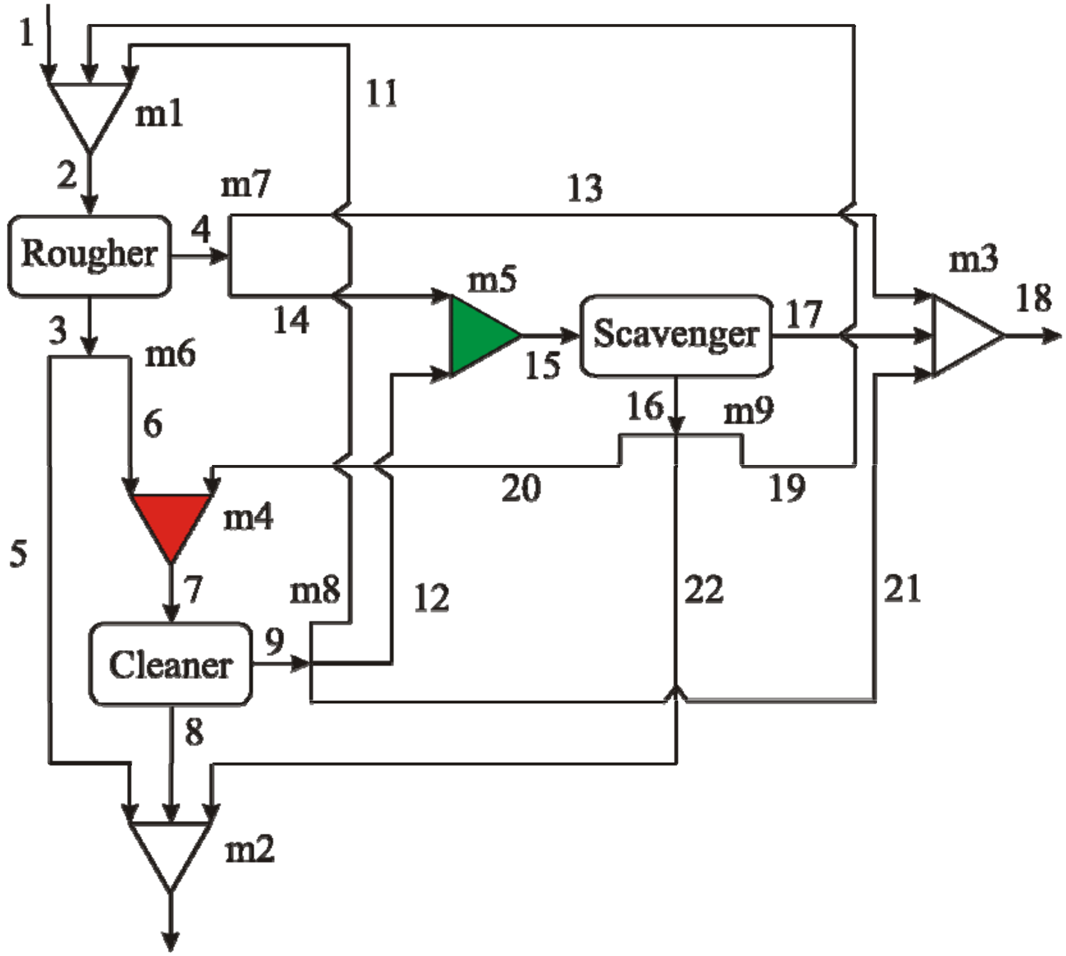
Superstructures for task, state & equipment

Superstructures are developed in a hierarchical form:

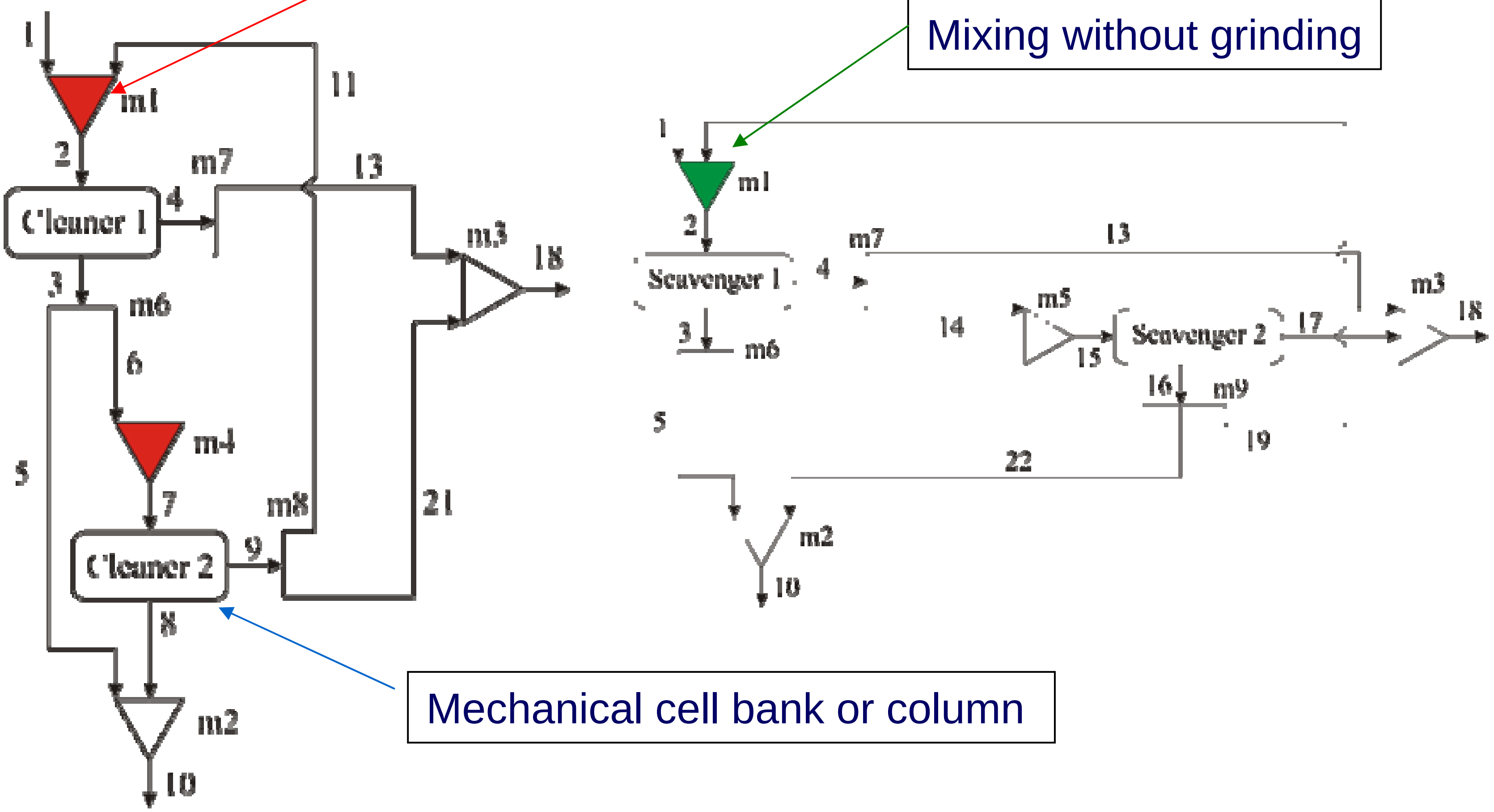
- First level: separation task superstructure
- Second level: processing systems are presented which must carry out rougher, cleaner, and scavenger operations and define states.
- Third level: equipment selection (column versus mechanical bank; grinding-classification circuit)

Tasks


Feed
Rougher
Tail
Concentrate
Tail
Scavenger
Final Flotation Tail
Concentrate
Concentrate
Cleaner
Tails
Final Flotation Concentrate



Task Superstructure



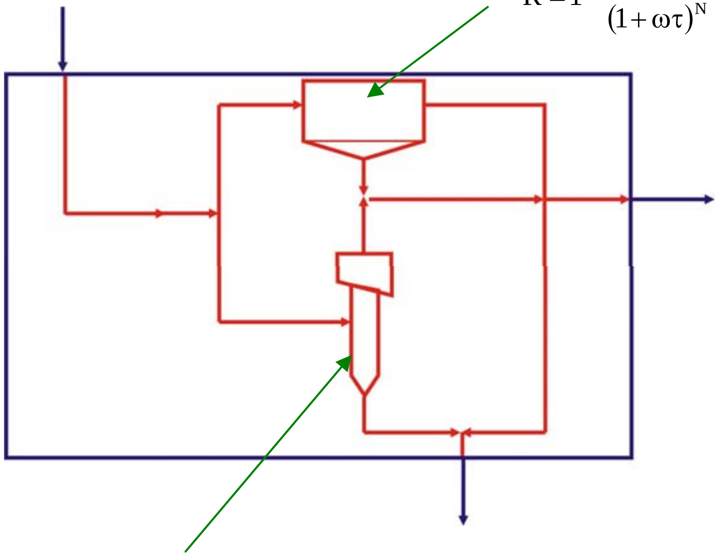
Mixing with/without grinding

Mixing without grinding

Mechanical cell bank or column

State Superstructures

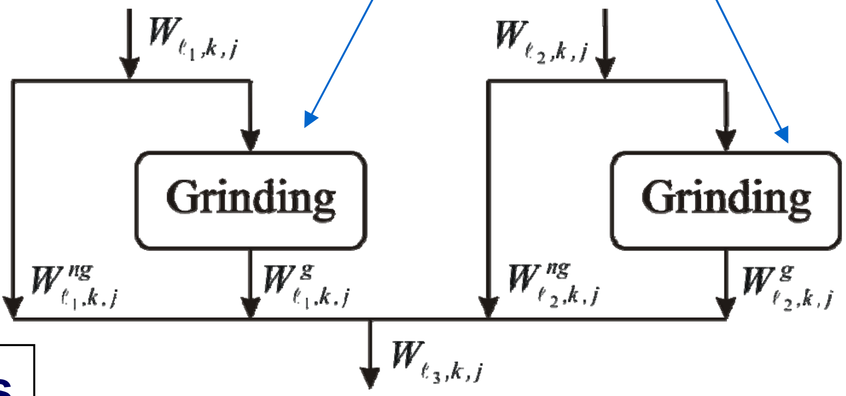
$$R = 1 - \frac{1}{(1+\omega\tau)^N}$$



$$R_C = 1 - \frac{4a\,exp\left(\frac{1}{2D/u\,h}\right)}{(1+a)^2\,exp\left(a/(2D/u\,h)\right) - (1-a)^2\,exp\left(-a/(2D/u\,h)\right)}$$

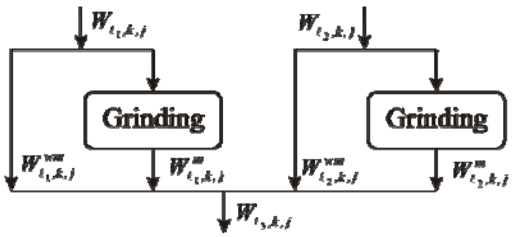
Mechanical cell bank or column

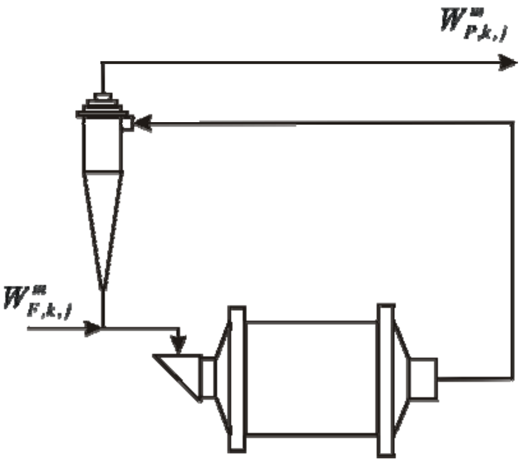
Grinding-classification circuits

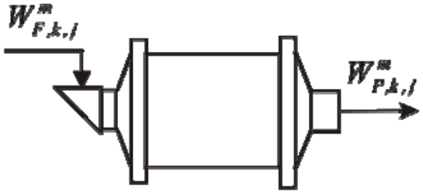


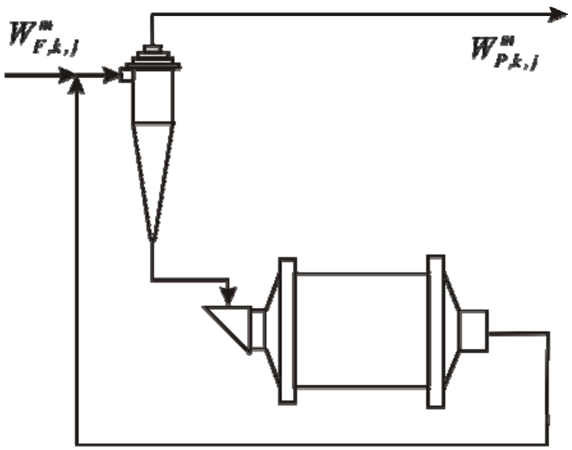
Equipment Superstructures

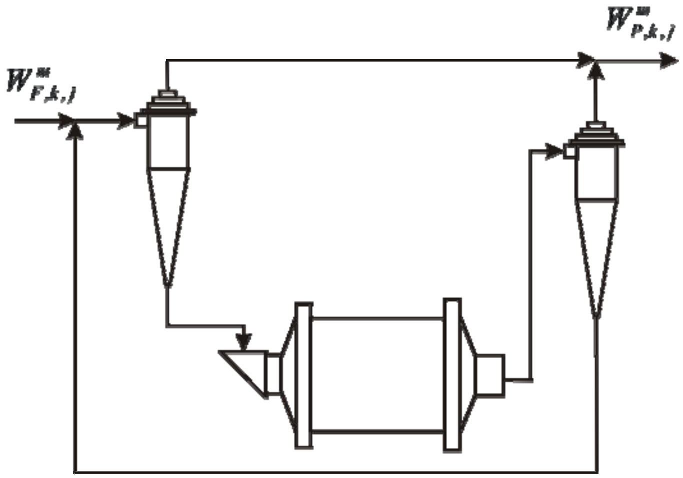
Grinding circuits



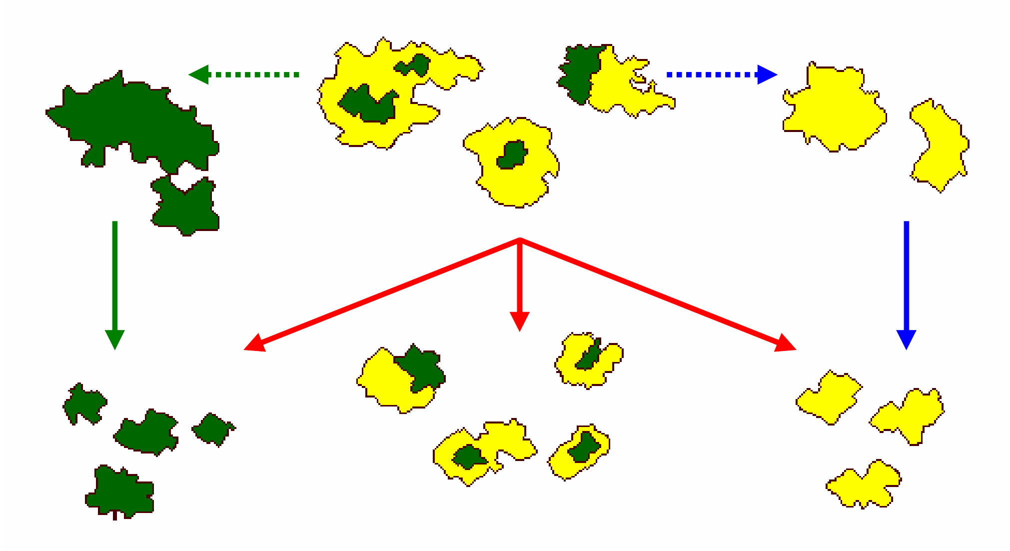




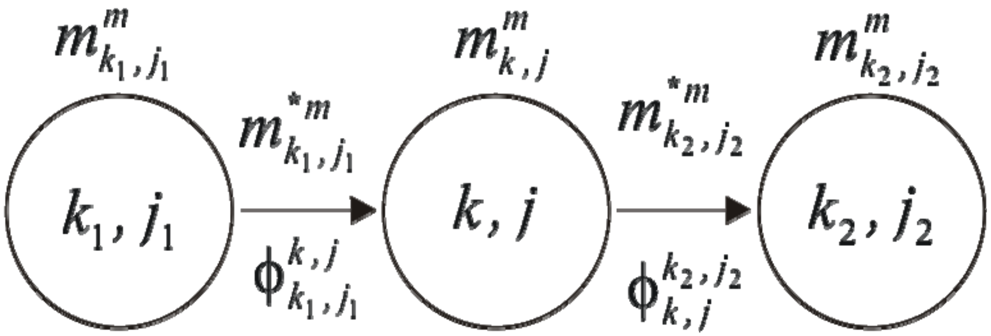


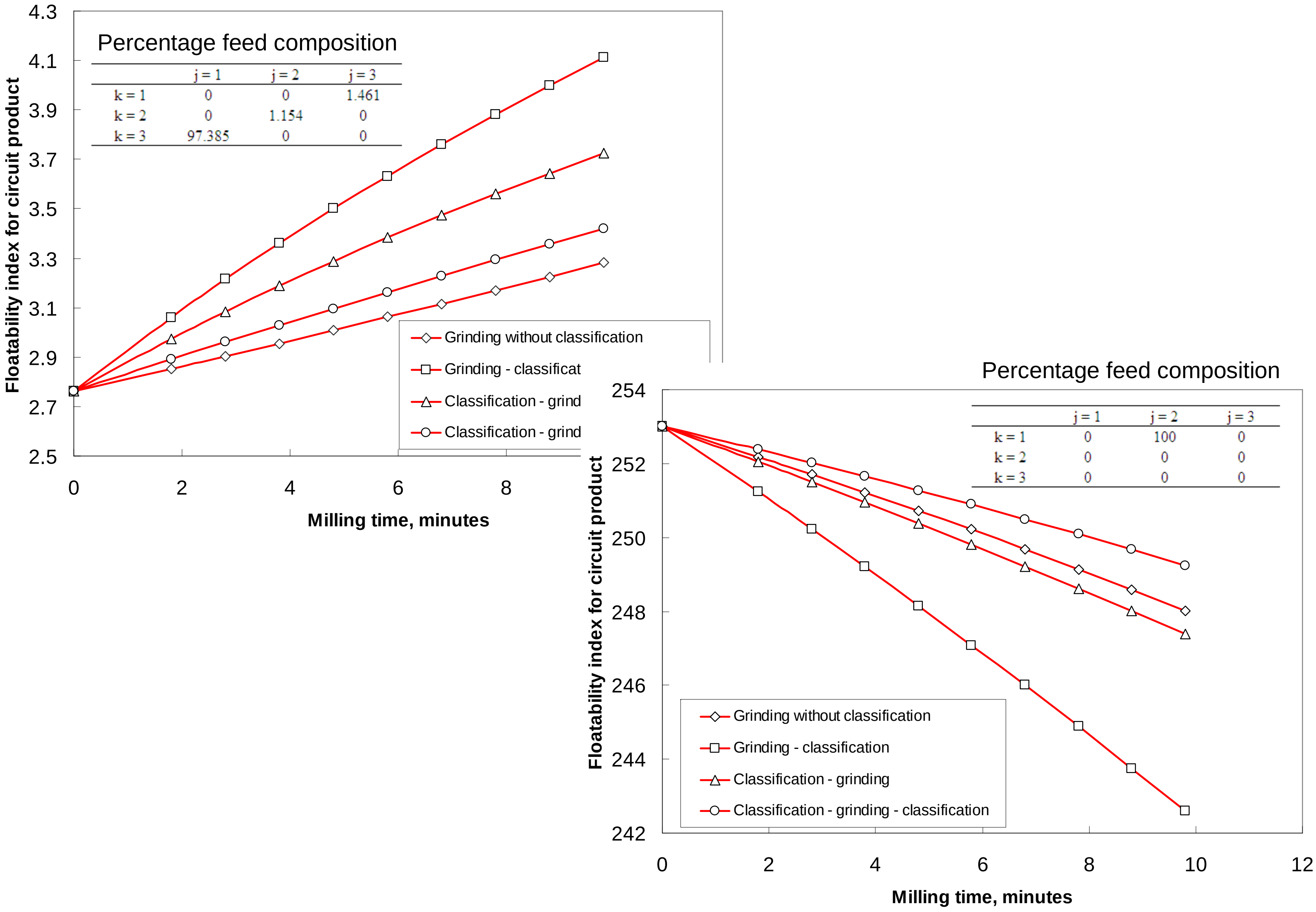


Floatability Index



$$\Gamma = \frac{\sum_{K}\sum_{J}\gamma_{k,j}W_{P,k,j}}{\sum_{K}\sum_{J}W_{P,k,j}}$$





Percentage feed composition

	j = 1	j = 2	j = 3
k = 1	0	0	1.461
k = 2	0	1.154	0
k = 3	97.385	0	0

Percentage feed composition

	j = 1	j = 2	j = 3
k = 1	0	100	0
k = 2	0	0	0
k = 3	0	0	0

Mathematical Model

Material Balances

$$\sum_{1\in M^{\mathrm{in}}(m)} W_{l,k} - \sum_{l\in M^{\mathrm{out}}(m)} W_{1,k} = 0 \quad k\in K,\ m\in M \qquad (1)$$

Feedstock flows

$$W_{1,k} = F_k \qquad (4)$$

Material Balances Flotation Steps

$$\mathrm{WI}_{s,l_c,k} = T_{s,l_a,k}\mathrm{WI}_{s,l_a,k} \qquad (l_a, l_c)\in\mathrm{Lcc},\ k\in K,\ s\in S$$

$$\mathrm{WI}_{s,l_t,k} = \left(1 - T_{s,l_a,k}\right)\mathrm{WI}_{s,l_a,k}$$

$$T_{\ell_a,k} = 1 - \frac{1}{\left(1 + k_{\ell_a,k}\tau_{\ell_a}\right)^{N_{\ell_a}}}$$

Equipment Selection

$$
\begin{bmatrix}
y^{b}_{s,\ell_1} \\
C^{f}_{s,\ell_1} = C^{f}_{b} \\
C^{V}_{s,\ell_1} = C^{V}_{b} \sum_{k} W^{b}_{s,\ell_1,k} \\
W^{b}_{s,\ell_1,k} = WI_{s,\ell_1,k} \\
WI_{s,\ell_2,k} = T^{b}_{s,\ell_1,k} \; W^{b}_{s,\ell_1,k} \\
WI_{s,\ell_3,k} = \left(1 - T^{b}_{s,\ell_1,k}\right) W^{b}_{s,\ell_1,k}
\end{bmatrix}
\vee
\begin{bmatrix}
y^{c}_{s,\ell_1} \\
C^{f}_{s,\ell_1} = C^{f}_{c} \\
C^{V}_{s,\ell_1} = C^{V}_{c} \sum_{k} W^{c}_{s,\ell_1,k} \\
W^{c}_{s,\ell_1,k} = WI_{s,\ell_1,k} \\
WI_{s,\ell_2,k} = T^{c}_{s,\ell_1,k} \; W^{c}_{s,\ell_1,k} \\
WI_{s,\ell_3,k} = \left(1 - T^{c}_{s,\ell_1,k}\right) W^{c}_{s,\ell_1,k}
\end{bmatrix}
$$

Objective Function

$$\text{Profit} = \text{Income} - \text{Costs} = I - C$$

$$I = \left(\sum_k g_k W_{10,k}\right) p\cdot(q - \text{Rfc})H - \sum_k W_{10,k}[pu(q - \text{Rfc}) + Trc]H$$

Table 1

Typical values for a copper concentration plant (minimum grade of concentrate 28%)

Parameter	Value	Parameter	Value
p	0.975	Rfc	US$200/ton metal
u	0.015	Trc	US$85/ton concentrate
q	US$1764/ton metal	H	7200 h/year

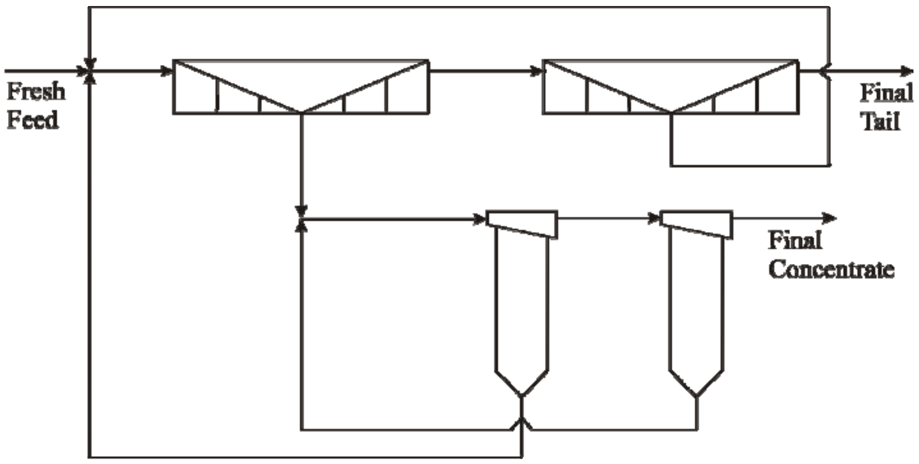
Example

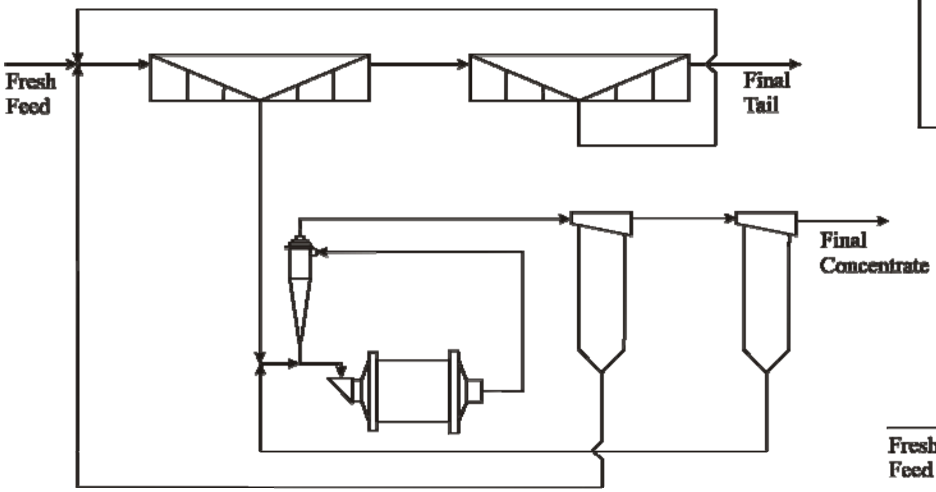
The procedure was applied to the design of a copper concentration plant, whose species are: k=1 (100% chalcopyrite), k=2 (50% silica, 50% chalcopyrite) and k=3 (100% silica).

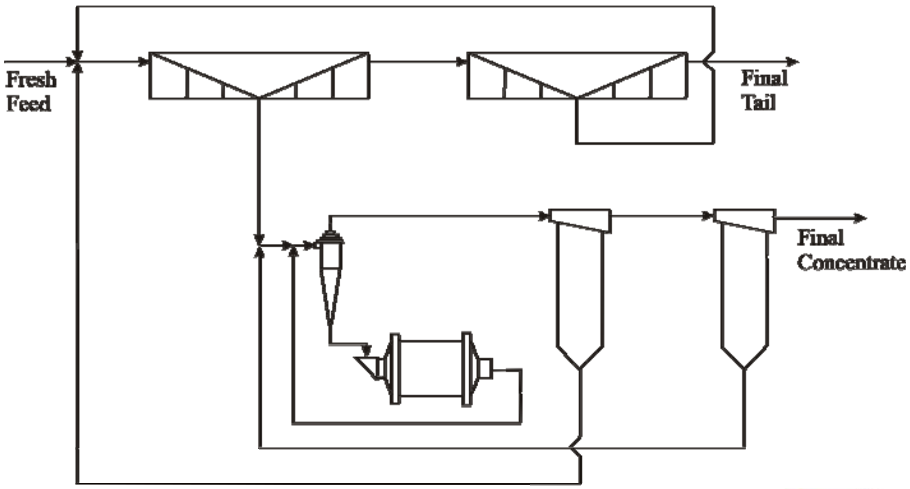
Economic results in millions of US$ for a metal price of 1.00 US$/lb where the k=3 recovery is comparable to the k=2 recovery.

(NG)= no grinding, (G)= grinding w/o classification, (G-C)= grinding – classification, (C-G)= classification – grinding.

Case	Options	Selection	Profit	Revenues	Costs
Case 9	NG	NG	11.964	37.627	26.421
Case 10	G/G-C	G-C	12.748	28.639	26.726
Case 11	G/C-G	C-G	12.996	38.771	26.571
Case 12	G-C/C-G	C-G	12.996	38.771	26.571


Fresh
Feed
Final
Tail
Final
Concentrate

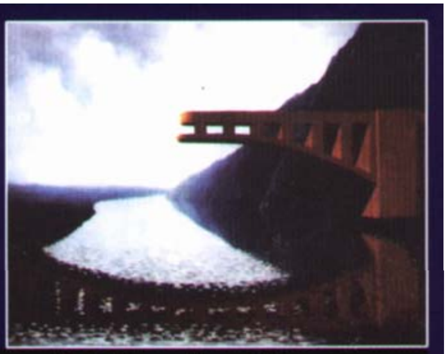

Fresh
Feed
Final
Tail
Final
Concentrate


Fresh
Feed
Final
Tail
Final
Concentrate

Final Remark


Combustion, Yerka

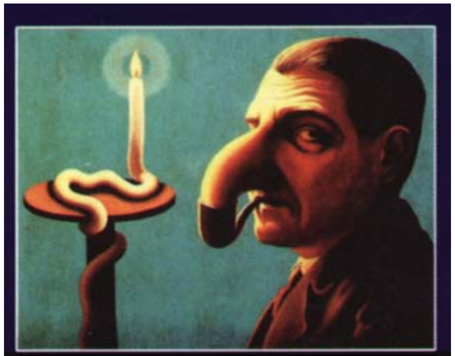

Diffusion, Wick


Sublimation, Magritte

Complete list of references on design of separation based on crystallization, design of flotation circuits, design of leaching process, and design of solvent extraction circuits

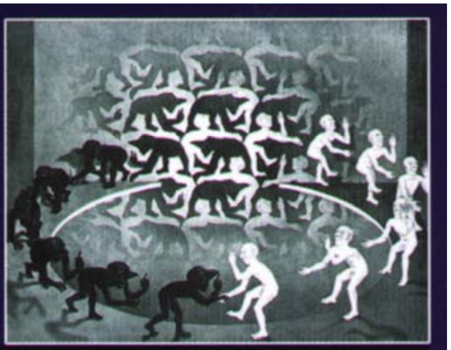
Can be found on

http:/www.uantof.cl/d2p


Cycle Process, Magritte


Radiation, Van Gogh


Compression, Baldaccini


fluids, Escher