Eutrophication control in lakes and reservoirs using simultaneous dynamic optimization approaches

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Outline

- Motivation
- Objective
- Biological and biochemical determinations
- Global sensitivity analysis
- Dynamic parameter estimation problem
- Optimal control problem
- Simultaneous approach for dynamic optimization
- Discussion of results
- Conclusions

Motivation

- Eutrophication as natural process of aging of water body
- Water bodies increasingly eutrophic due to anthropogenic inputs of nutrients
- Application of restoration strategies requires systematic study, modeling and optimization of eutrophication processes

Cultural Eutrophication

- Main anthropogenic source of nutrients: Agricultural activities (fertilization)
- Main point source: discharge of agricultural, industrial and urban wastewater
- Over enrichment of nutrients (mainly P and N)
- Increase in the production levels and biomass
- Very strong development of phytoplankton community
- Decrease in water depth caused by sediment accumulation

Objective

- ✓ Development of ecological water quality (eutrophication) model
- ✓ Analysis of the trophic state of a water body through its composition and abundance of plankton
- ✓ Global sensitivity analysis and determination of sensitivity indices
- ✓ Parameter estimation based on available data:
- ✓ Model validation
- ✓ Study of the effect of nutrients concentration and environmental parameters on plankton population dynamics
- ✓ Determination of optimal bio-restoration policies

Trophic classification of water bodies

Oligotrophic

Mesotrophic

Eutrophic

Hipereutrophic

< [nutrients]

< Productivity

> [nutrients]

> Productivity

Trophic classification of water bodies

	OLIGOTROPHIC	MESOTROPHIC	EUTROPHIC	HYPEREUTROPHIC
Inorganic phosporus ($\mu g l^{-1}$)	1-10	10-20	20-100	>100
Inorganic nitrogen ($\mu g l^{-1}$)	<150	150-300	>300	
Phytoplankton ($cell ml^{-1}$)	2000	2000-5000	>5000	
Superficial chlorophyll a ($\mu g l^{-1}$)	1-2	2-5	5-50	>50
Depth of Secchi disk (m)	5-10	3-5	1-3	<1

Paso de las Piedras Reservoir

Paso de las Piedras Reservoir

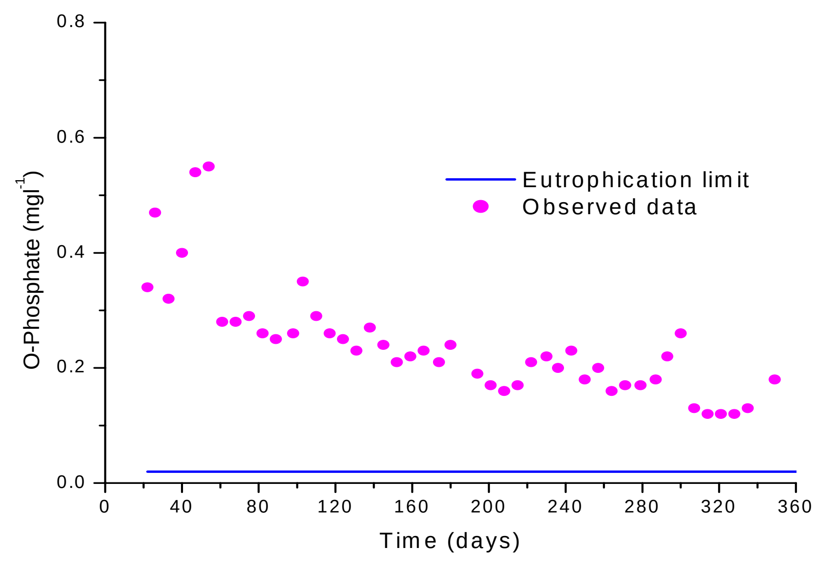
Provides drinking water to more than 450.000 inhabitants from Bahía Blanca, Punta Alta and to a petrochemical complex

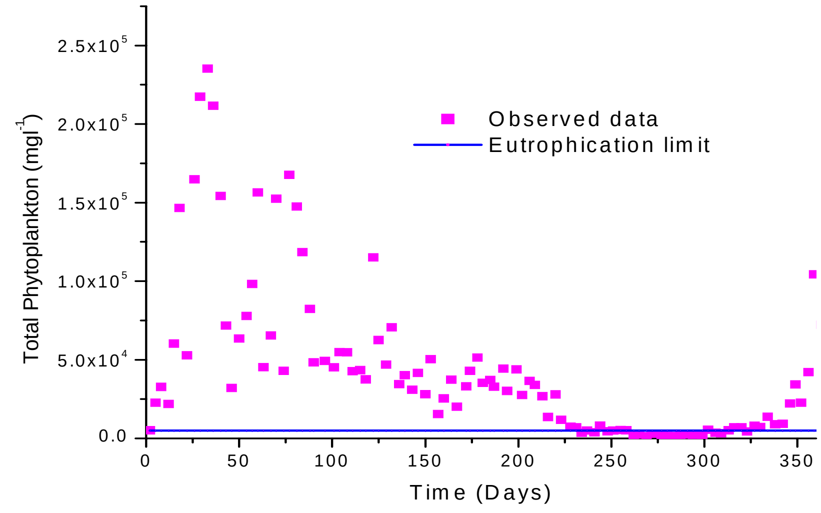
Lake Characteristics

Area of drainage basin	1620 km^2	Maximum depth	28 m
Perimeter of coastline	60 km	Maximum volume	328 Hm^3
Surface	36 km^2	Retention time	4 years
Mean depth	8.2 m		

Paso de las Piedras Reservoir

- ✓ Eutrophic
- ✓ Main source of nutrients: agricultural activities
- ✓ High phytoplankton concentration during spring and summer: surface water blooms





Surface water blooms

- ✓ Natural phenomena caused by phytoplankton.
- ✓ Phytoplankton: microscopic floating algae (first link of the trophic chain).
- ✓ In favorable environmental conditions they are multiplied and concentrated in the surface,
  ➡ fast increase in algal biomass.

Photosynthesis

$$106\ CO_2 + 16\ NO_3^- + HPO_4^{2-} + 122\ H_2O + 18\ H^+$$

Solar radiation

$$\longrightarrow C_{106}H_{263}N_{16}P + 138\ O_2$$

algae

Problems caused by water blooms

For man

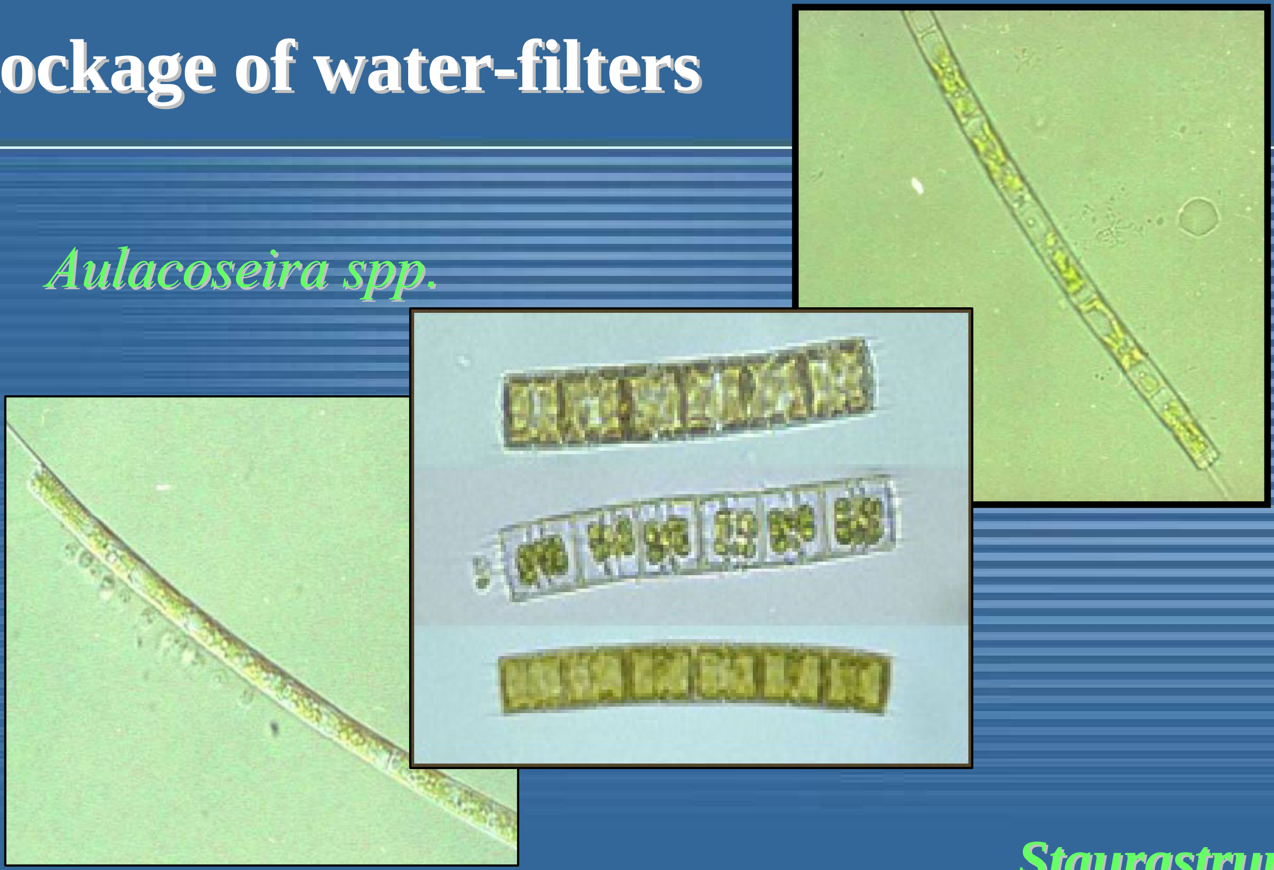
- Blockage of water-filters
- Unpleasant odor and taste
- Aesthetics
- Presence of potentially toxic algae

For ecosystem

- Reduction of biodiversity
- Anoxic conditions
- Shade
- Blockage of fish gills

Blockage of water-filters

Aulacoseira spp.



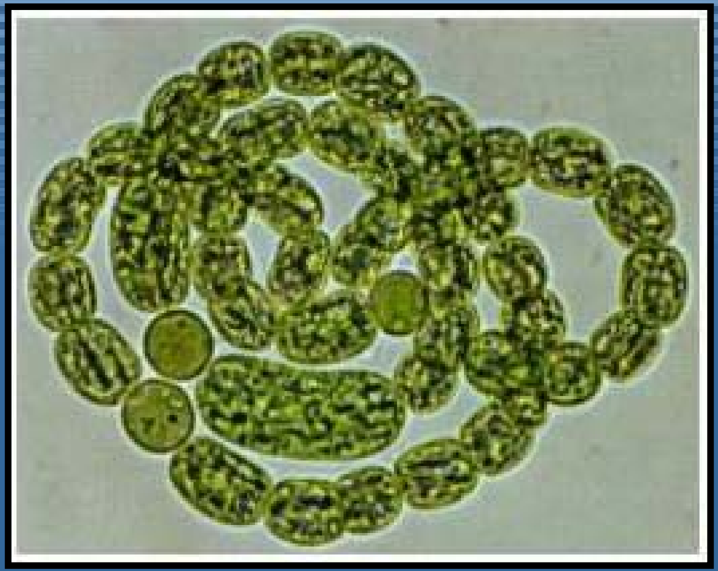
Staurastrum spp.

Closterium spp.

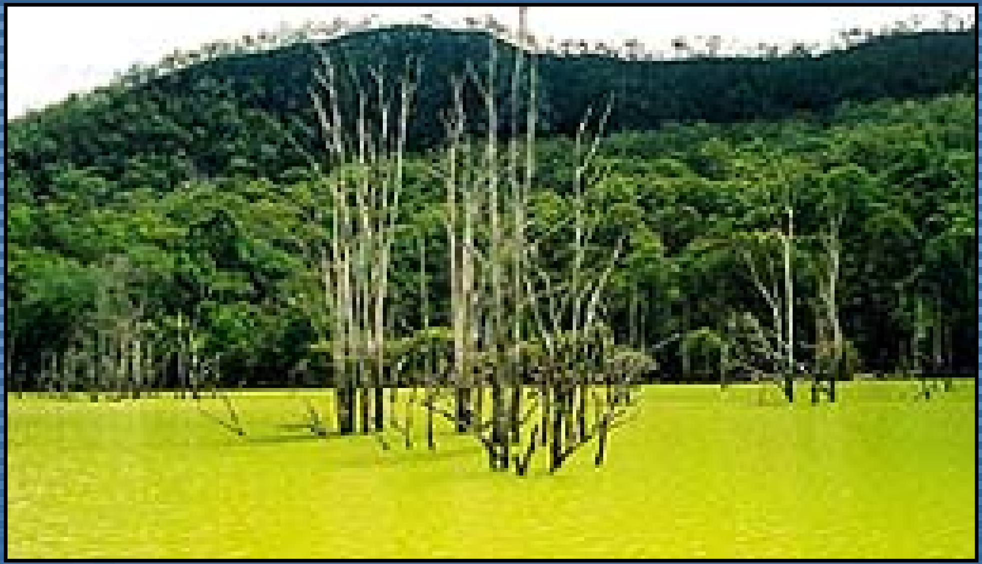
Unpleasant odor and taste

Ceratium hirundinella

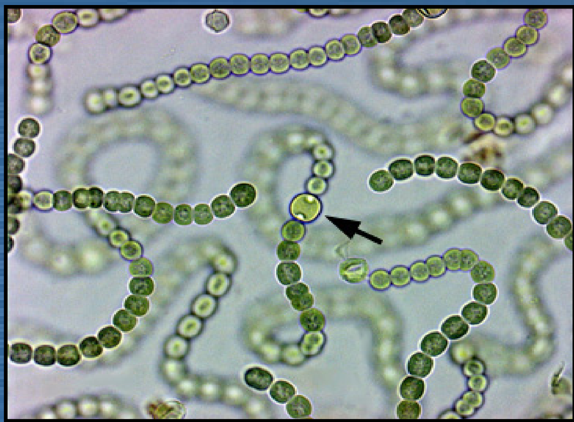
Anabaena circinalis

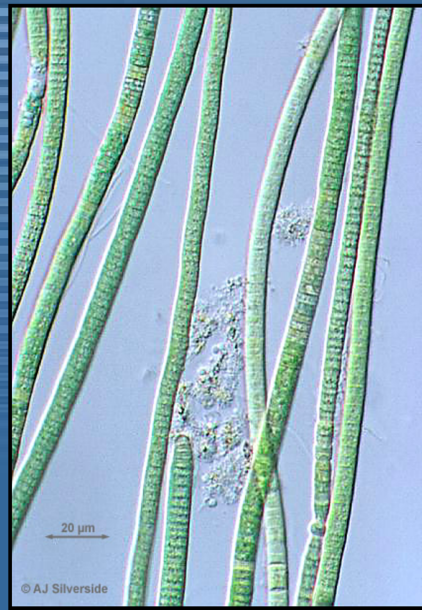


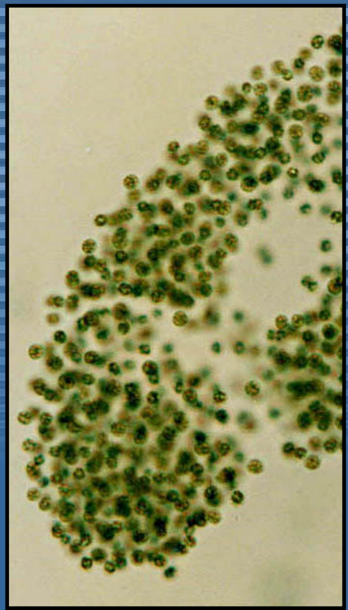
Aesthetics

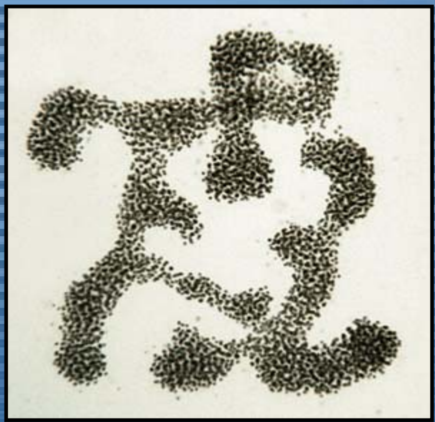


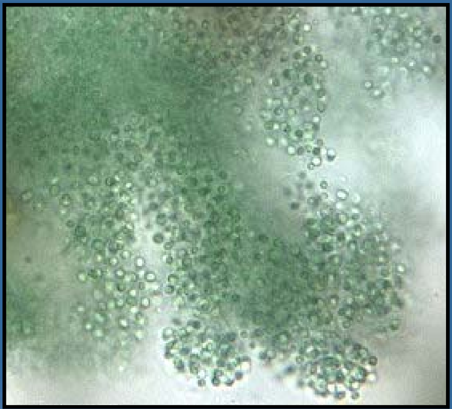
Toxic Cyanobacteria

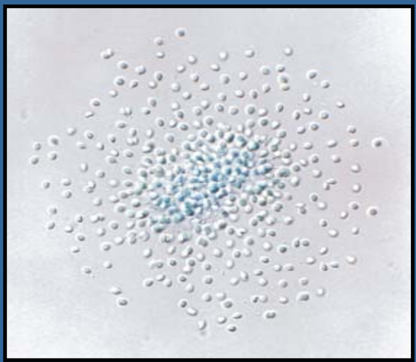


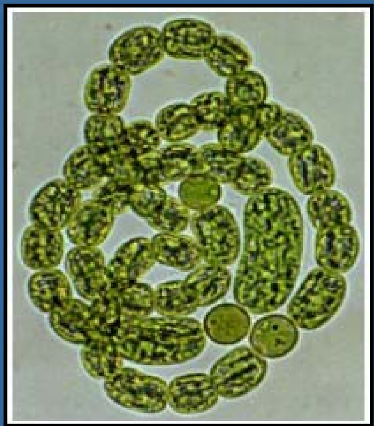












Biological determinations

Qualitative analysis

- ✓ Plankton net (30 µm)
- ✓ Observation to the optical microscope of the alive and fixed samples (formol 4%)
- ✓ Determination based on keys

Quantitative analysis

- ✓ Rutner water sampler
- ✓ *In situ* fixation with Lugol`s solution
- ✓ Phytoplankton enumeration in inverted microscope by Utermöhl method (1958)
- ✓ Phytoplankton biovolume
- ✓ Calculation of mgC.

- ✓ Cyanobacteria
- ✓ Diatoms
- ✓ Chlorophytes

Physico-chemical determinations

- Nitrates
- Nitrites
- Ammonium
- Organic Nitrogen
- Phosphates
- Organic Phosporus
- Silice

- Water temperature
- Solar radiation
- pH
- Dissolved Oxygen
- Biochemical Demand of Oxygen
- Depth of Secchi disk

Ecological Water Quality Model

Dynamic mass balances for nutrients and phytoplankton

↓

Concentration gradients along water column height

↓

Partial Differential Equations System

↓

Spatially Discretization: Horizontal layers

↓

Differential Algebraic System (DAE)

Assumptions

- ✓ Horizontally averaged concentration
- ✓ Phosphorus limiting nutrient (for algae growth)
- ✓ Constant density
- ✓ Constant transversal area in lake

Mass balance for horizontal layers

$$\left[\frac{\text{mg}}{\text{day}}\right]$$

$$\frac{d(C_{ij}V_i)}{dt} = \sum_{k=1}^{NIN} QIN_{i,k}CIN_{ijk} - \sum_{m=1}^{NOUT} QOUT_{im}CIN_{ij} + r_{ij}V_i - \frac{kdA}{\Delta h_i}(C_{ij} - C_{i+1j}) - \frac{kdA}{\Delta h_{i-1}}(C_{ij} - C_{i-1,j}$$

$$\frac{dC_{ij}}{dt} = \sum_{k=1}^{NIN} \frac{QIN_{i,k}}{V_i}CIN_{ijk} - \sum_{m=1}^{NOUT} \frac{QOUT_{i,m}}{V_i}CIN_{ij} + r_{ij} - \frac{kdA}{\Delta h_i h_i}(C_{ij} - C_{i+1,j}) - \frac{kdA}{\Delta h_i h_{i-1}}(C_{ij} - C_{i-1,j}) - \frac{C_{ij}}{h_i}\frac{dh_i}{dt}$$

i= horizontal layer

k = Tributary inflows: El Divisorio, Sauce Grande

m = Withdrawals: drinking water+complex, Sauce Grande

j = Cyanobacteria, Diatoms, Chlorophyta, NO_3,NH_4, ON, PO_4,OP, BDO, DO

Mass balance for horizontal layers

Upper Layer

$$\frac{dC_{Uj}}{dt} = \sum_{k=1}^{NIN} \frac{QIN_{U,k}}{V_U} CIN_{Ujk} - \sum_{m=1}^{NOUT} \frac{QOUT_U}{V_U} C_{Uj} + r_{Uj} - \frac{kdA}{\Delta h_U h_U}(C_{Uj} - C_{Lj}) - \frac{C_{Uj}}{h_U}\frac{dh_U}{dt}$$

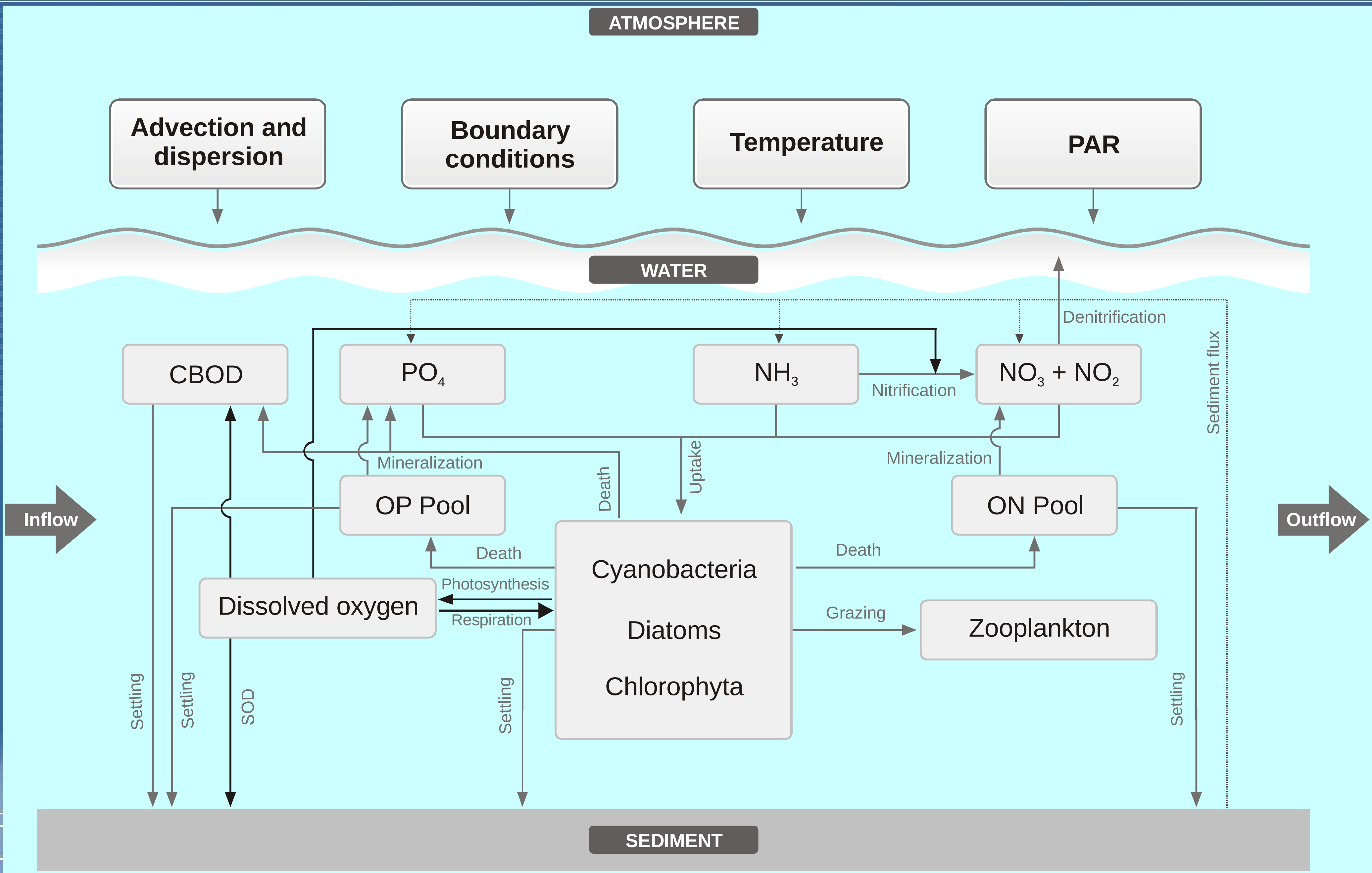
$i = U$

Lower Layer

$$\frac{dC_{Lj}}{dt} = \sum_{m=1}^{NOUT} \frac{QOUT_L}{V_L} C_{Lj} + r_{Lj} + \frac{kdA}{\Delta h_L h_L}(C_{Lj} - C_{U,j}) - \frac{C_{Lj}}{h_L}\frac{dh_L}{dt}$$

$i = L$

State variables and biogeochemical processes



Rate equations (r_{ij})

$$r_{ij} = R_{ij,growth} - R_{ij,resp} - R_{ij,death} - R_{ij,settling} - R_{ij,graz}$$

Growth

$$R_{ij,growth} = k_{j,growth} * f(T) * f(I) * f(N) * C_{ij}$$

$$f(T)_{ij} = -\frac{(T_j - T_{opt_i})^2}{T^2{}_{opt_i}} \qquad f(I) = \frac{I}{Ij} exp\left(1 - \frac{Ii}{Ij}\right) \qquad f(N) = \frac{C_{iPO4}}{C_{iPO4} + kpj}$$

Respiration

$$R_{ij,resp} = k_{i,resp}\, \theta_r^{(T-20)} C_{ij}$$

Death

$$R_{ij,death} = k_{i,death}\, \theta_m^{(T-20)} C_{ij}$$

Settling

$$R_{ij,sedim} = k_{j,sedim} * \frac{1}{h_i} * C_{ij}$$

Grazing

$$R_{ij,graz} = k_{j,graz} \frac{C_{ij}}{C_{ij} + K_{graz}} C_{Zoo\, j}$$

i = upper and lower layers

j = Cyanobacteria, Diatoms, Chlorophytes

Rate equations (r_{ij})

$$r_{ij} = R_{ij,death} - R_{ij,miner} - R_{ij,sedim}$$

Death

$$R_{ij,death} = \sum_{m=1}^{3} (a_{jc} * k_{m,death} * f_j * C_{im})$$

Mineralization

$$R_{ij,miner} = k_{miner} * \theta_{miner} * exp(Temp - 20) * \frac{\sum_{m=1}^{3} Cim * C_{ij}}{kmjc + \sum_{m=1}^{3} C_{im}}$$

Settling

$$R_{ij,sedim} = \frac{k_{j,sedim} * (1 - fDj)}{Di} * C_{ij}$$

i = upper and lower layers

j = **ON, OP**

Rate equations (r_{ij})

$$r_{ij} = R_{ij,death} + R_{ij,miner} - R_{ij,uptake}$$

Death

$$R_{ij,death} = \sum_{m=1}^{3} (apc * k_{m,death} * (1 - f_{po}) * C_{im})$$

Mineralization

$$R_{ij,miner} = k_{miner} * \theta_{miner} * exp(Temp - 20) * \frac{\sum_{m=1}^{3} C_{im} * C_{iOP}}{km_{pc} + \sum_{j=1}^{3} C_{im}}$$

Uptake

$$R_{ij,uptake} = \sum_{m=1}^{3} (R_{im,growth} * a_{pc} * C_{im})$$

i = upper and lower layers

***j* = PO_4**

Rate equations (r_{ij})

$$r_{ij} = R_{ij,nitri} - R_{ij,uptake} - R_{ij,denitr}$$

Nitrification

$$R_{ij,nitri} = k_{nitri} * \theta_{nitri} * exp(Temp - 20) * \frac{C_{iNH4} * C_{iDO}}{k_{nio} + C_{iDO}}$$

Uptake

$$R_{ij,uptake} = \sum_{m=1}^{3} (a_{nc} * R_{im,growth} * (1 - PNH4) * C_{im}))$$

Denitrification

$$R_{ij,denitr} = k_{denitr} * \theta_{denitr} * exp(Temp - 20) \frac{C_{iNO3} * k_{no3}}{k_{no3} + C_{iDO}}$$

i = upper and lower layers

j = NO_3

Rate equations (r_{ij})

$$r_{ij} = R_{ij,death} + R_{ij,miner} - R_{ij,uptake}$$

Death

$$R_{ij,death} = \sum_{m=1}^{3} (anc * k_{m,death} * (1 - f_{ON}) * C_{im})$$

Mineralization

$$R_{ij,miner} = k_{miner} * \theta_{miner} * exp(Temp - 20) * \frac{\sum_{m=1}^{3} C_{im} * C_{iON}}{kmpc + \sum_{m=1}^{3} C_{im}}$$

Uptake

$$R_{ij,uptake} = \sum_{m=1}^{3} (a_{nc} * R_{im,growth} * PNH4 * C_{im})$$

i = upper and lower layers

$j = NH_4$

Global sensitivity analysis

- ✓ Local vs. Global Sensitivity Analysis

- ✓ Quantitative variance-based Global Sensitivity Analysis
  - ❖ Model independent
  - ❖ Incorporate the effect of range of input variation and its pdf
  - ❖ Allow multidimensional averaging
  - ❖ Allow parameter grouping

Global sensitivity analysis

Given model output $Y=f(\mathbf{x})$, $\mathbf{x}$ vector of input factors

Output variance $$V(Y) = E\left(V\left(Y|x_i\right)\right) + V\left(E\left(Y|x_i\right)\right)$$

$E\left(V\left(Y|x_i\right)\right)$ mean output variance that remains if x_i fixed (known)

$V\left(E\left(Y|x_i\right)\right)$ expected reduction in output variance if x_i fixed

Sensitivity index

input x_i

$$S_i = \frac{V\left(E\left(Y|x_i\right)\right)}{V(Y)}$$

Global sensitivity analysis

Decompose model output *Y=f(**x**)*, as the sum of terms of increasing dimensionality

$$f(x_1,\ldots,x_k) = f_0 + \sum_{i=1}^{k} f_i(x_i) + \sum_{1\le i<j\le k} f_{ij}(x_i,x_j) + \ldots + f_{1,2,\ldots,k}(x_1,\ldots,x_k)$$

If input parameters are mutually independent ($\int_0^1 f_{i_1,\ldots,i_s}\,dx_k = 0$) unique decomposition of *f* such that the summands are orthogonal.

$$V = \sum_{i=1}^{k} V_i + \sum_{1\le i<j\le k} V_{ij} + \ldots + V_{1,2,\ldots,k}$$

V_i, V_{ij}, $V_{1,2,\ldots,k}$: Variance of *fi, fij, f1,2,…,k*

Global sensitivity analysis

$$1 = \sum_{i=1}^{k} \frac{V_i}{V} + \sum_{1 \le i < j \le k} \frac{V_{ij}}{V} + \ldots + \frac{V_{1,2,\ldots,k}}{V}$$

$$1 = \sum_{i=1}^{k} S_i + \sum_{1 \le i < j \le k} S_{ij} + \ldots + S_{1,2,\ldots,k}$$ **Sobol' sensitivity indices**

Higher orders indices calculation: computationally expensive

Total sensitivity index $$S_i^T = \frac{E\left(V\left(Y \middle| x_{-i}\right)\right)}{V(Y)}$$

Global sensitivity analysis

Calculation of Sobol' sensitivity indices (Monte Carlo)

- ✓ Generate two independent random sets ξ and ξ',
  let $\xi = (\eta, \zeta)$; $\xi' = (\eta', \zeta')$

- ✓ Evaluate

  - $f(\eta, \zeta)$
  - $f(\eta, \zeta')$
  - $f(\eta', \zeta)$

Global sensitivity analysis

- ✓ Calculate

$$\frac{1}{N}\sum_{i=1}^{N} f(\xi_i) \xrightarrow{p} E(Y)$$

$$\frac{1}{N}\sum_{i=1}^{N} f^2(\xi_i) \xrightarrow{p} V(Y) + E^2(Y)$$

$$\frac{1}{N}\sum_{i=1}^{N} f(\xi_i) f(\eta_i, \zeta'_i) \xrightarrow{p} V_y + E^2(Y)$$

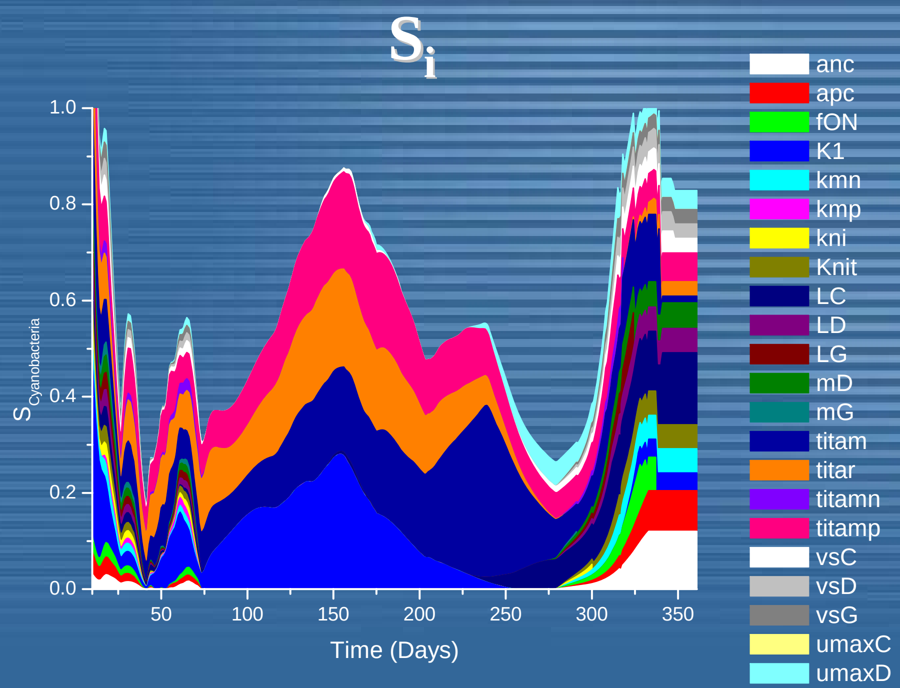
$$\frac{1}{N}\sum_{i=1}^{N} f(\xi_i) f(\eta'_i, \zeta_i) \xrightarrow{p} V_y^T + E^2(Y)$$

Sobol' indices

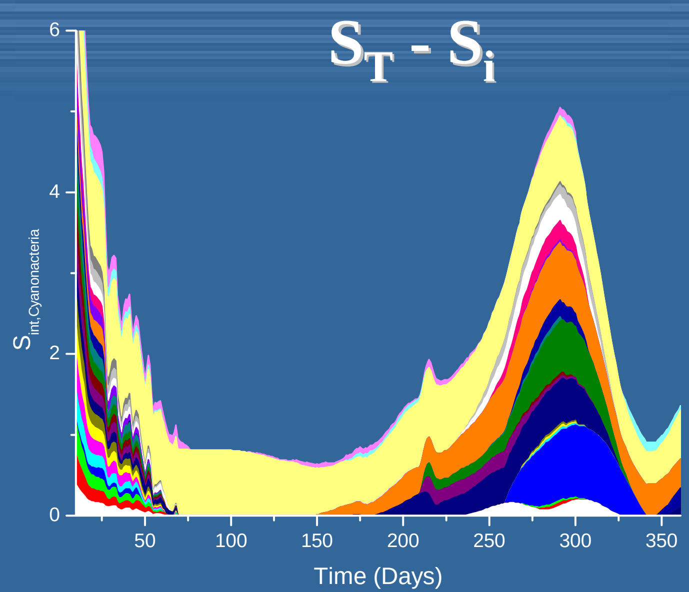
$$S_i = \frac{V_y}{V(Y)}$$

$$S_i^T = 1 - \frac{V_y^T}{V(Y)}$$

Sensitivity Indices: S_i

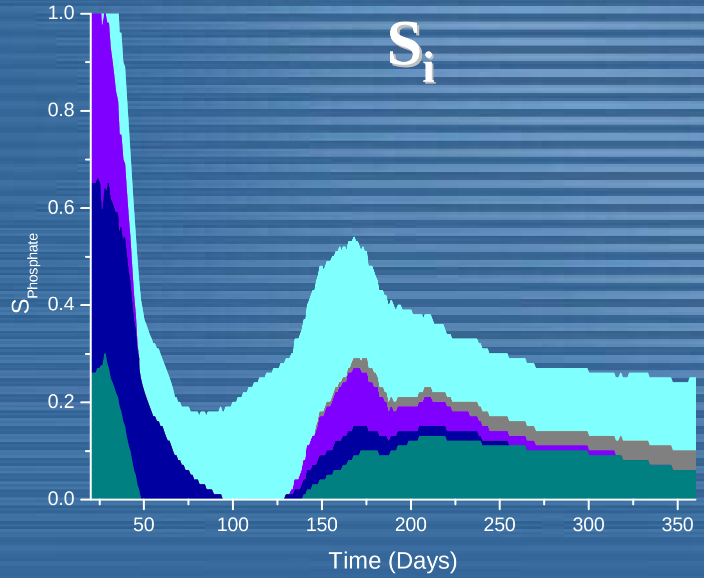


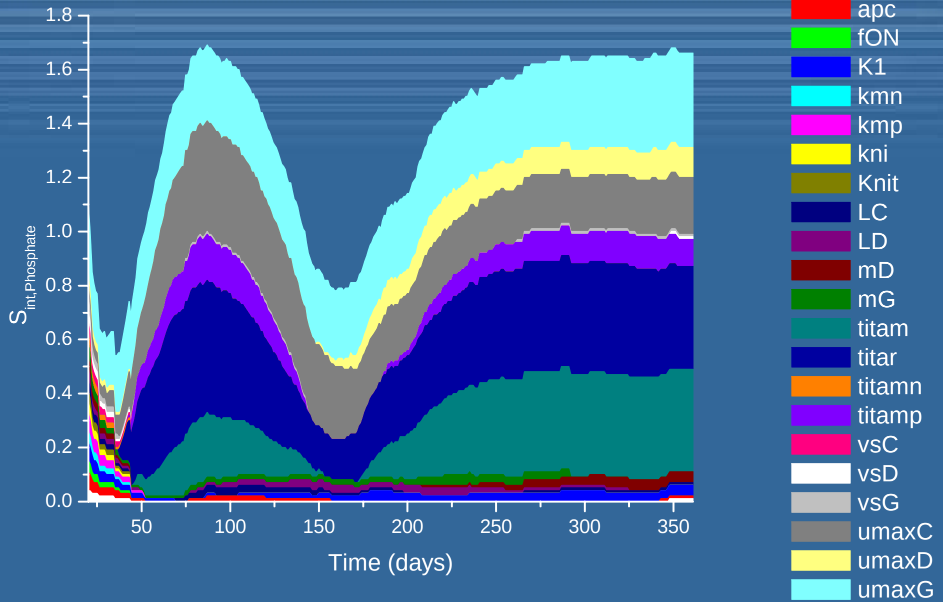
Cyanobacteria



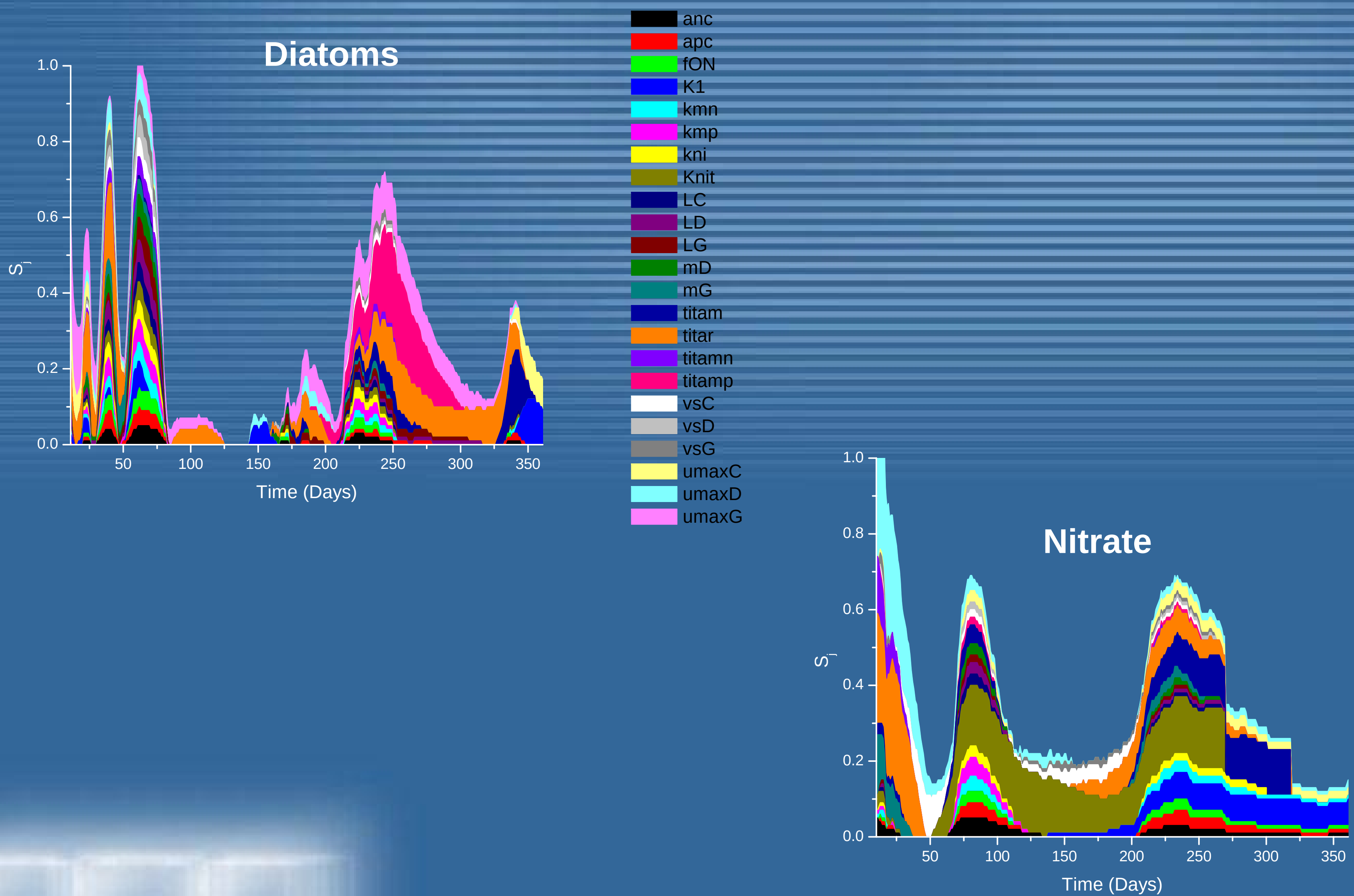
Sensitivity Indices: S_i

Phosphate





Sensitivity Indices: S_i



Dynamic Parameter Estimation Model

Parameter Estimation Problem

Process Model

$$f\left(\dot{x}, x, y, u, p, t\right) = \mathbf{0}$$

$$g\left(x, y, u, p, t\right) = \mathbf{0}$$

Variable Bounds

$$x_L \le x \le x_U,\ \ y_L \le y \le y_U$$

$$u_L \le u \le u_U,\ \ p_L \le p \le p_U$$

Initial Conditions

$$x(t_0) = x_0$$

Objective Function

$$min\,\phi = \frac{1}{2}\sum_{i=1}^{NI}\sum_{j=1}^{NV}\sum_{k=1}^{NL}\left(x_{ij}^{M} - x_{ij}\right)^{T} V^{-1}\left(x_{ij}^{M} - x_{ij}\right),$$

$$V = diag\left\{\sigma^2\right\}$$

Simultaneous Approach for Dynamic Optimization

Dynamic Parameter Estimation Problem

Process Model

$$f\left(\dot{x}, x, y, u, p, t\right) = \mathbf{0}$$

$$g\left(x, y, u, p, t\right) = \mathbf{0}$$

Variable Bounds

$x_L \le x \le x_U,\ y_L \le y \le y_U$

$u_L \le u \le u_U,\ p_L \le p \le p_U$

Initial Conditions

$$x(t_0) = x_0$$

Objective Function

$$\min \phi = \frac{1}{2} \sum_{i=1}^{NI} \sum_{j=1}^{NV} \sum_{k=1}^{NL} \left(x_{ij}^M - x_{ij}\right)^T V^{-1} \left(x_{ij}^M - x_{ij}\right),$$

$$V = diag\left\{\sigma^2\right\}$$

Nonlinear DAE optimization problem

↓

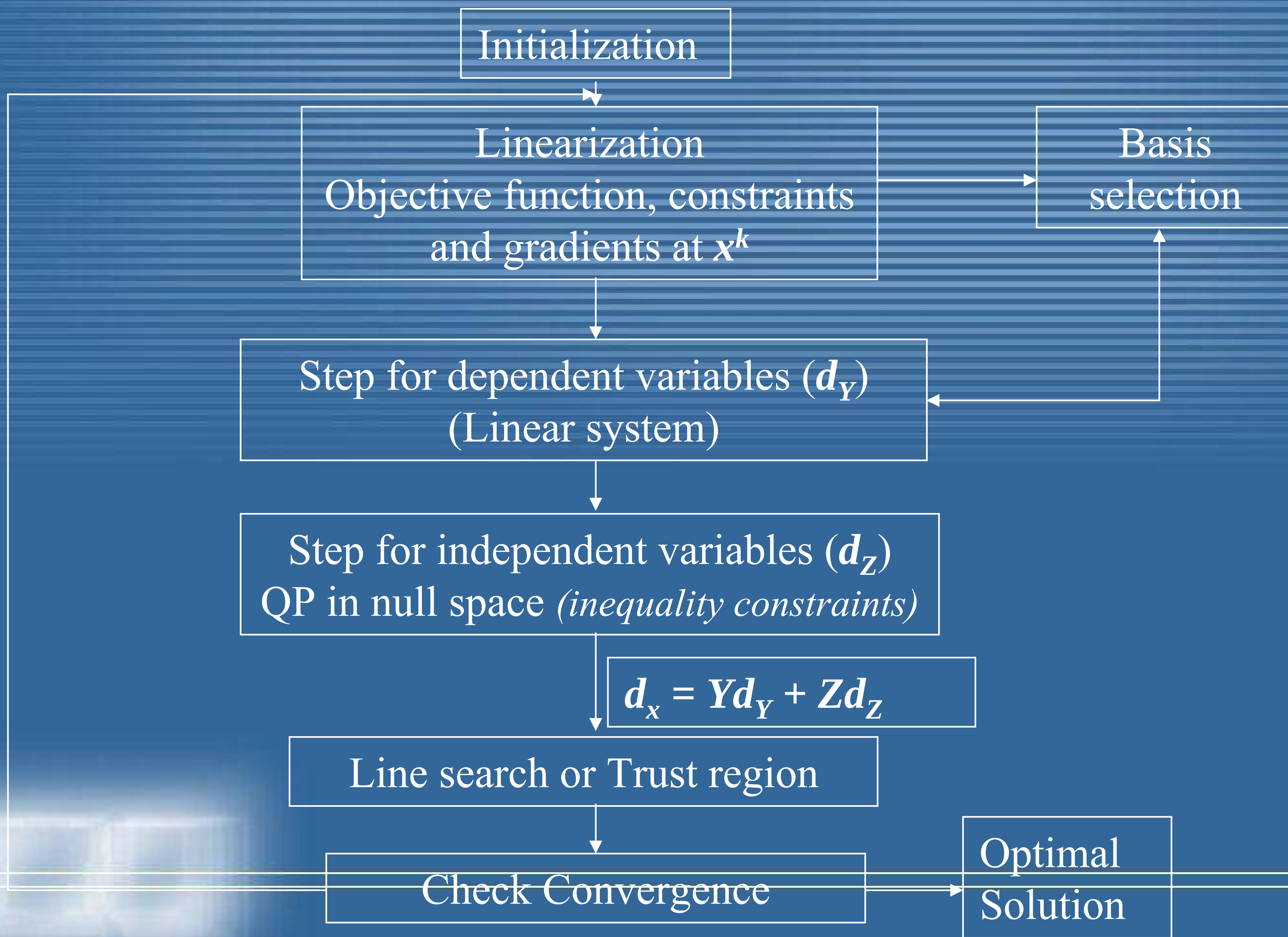
Discretization of Control and State variables
Collocation on finite elements

↓

Large-Scale Nonlinear Programming Problem
Interior Point Method (Biegler et al., 2002)

rSQP Algorithm

(Biegler et al., 2002)



Nonlinear Programming Problem

Application of Barrier method

$$\min f(x)$$

$$\text{s.t} \quad c(x) = 0$$

$$x \geq 0$$

$\Rightarrow$

$$\min f(x) - \mu \sum_{j=1}^{n} \ln x_j$$

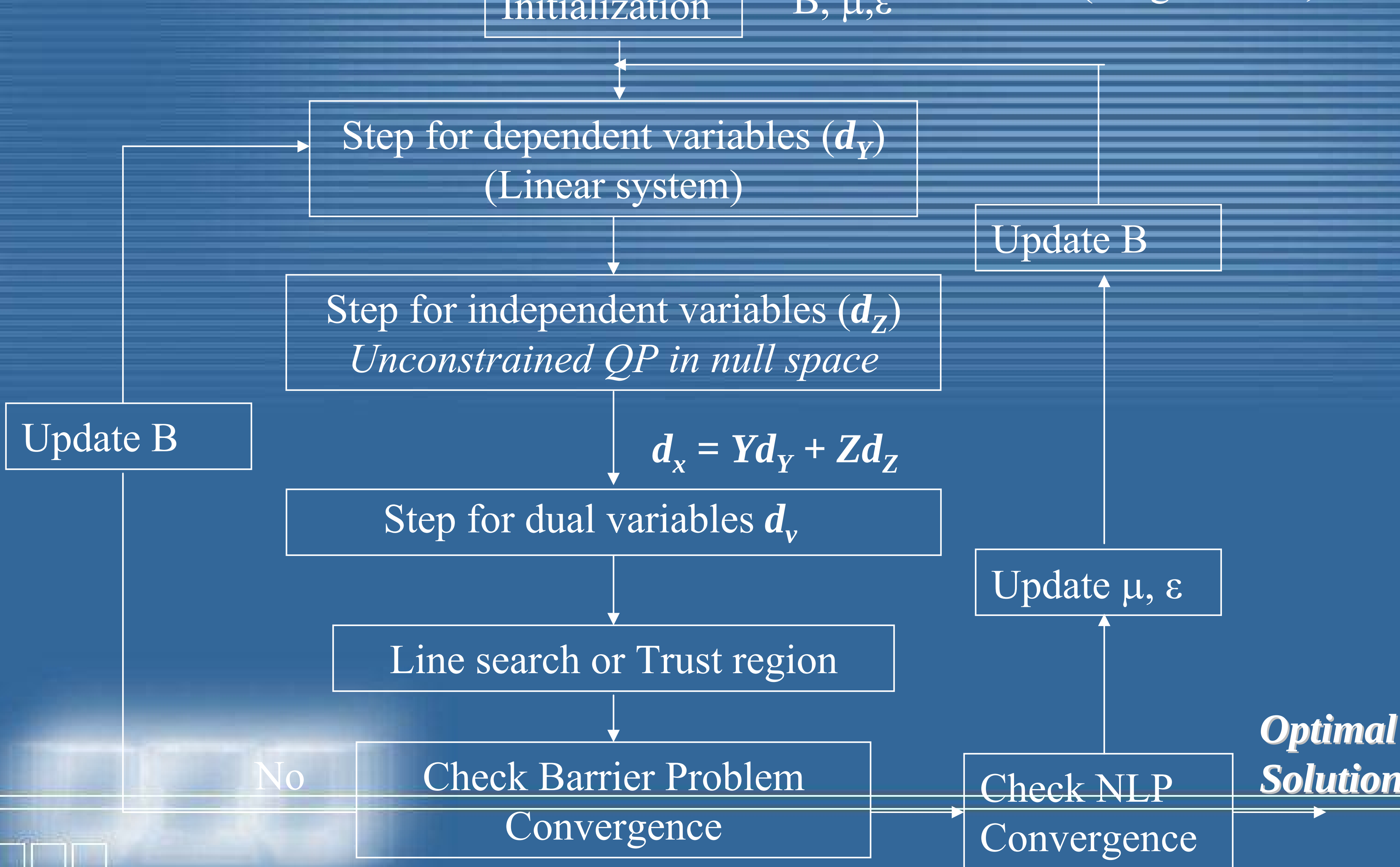
$$\text{s.t} \quad c(x) = 0$$

As $\mu \rightarrow 0$, $x^*(\mu) \rightarrow x^*$

Sequence of barrier problems for decreasing μ values

Barrier Method Algorithm: Primal-Dual Approach

(Biegler et al., 2002)



Parameter Estimation Problem

Input data

- ✓ Descriptive data for lake
- ✓ Temperature, solar radiation, lake depth time profiles
- ✓ Inflows and outflows time profiles
- ✓ Nutrients concentration profiles in inflows
- ✓ Initial conditions
- ✓ State variables profiles for upper and lower layer (measured)

Numerical Results

(Estrada et al., 2008a,b)

Symbol	Parameter	Estimated
$k_{cyanob,growth}$ (d^{-1})	Max growth of cyanobacteria	0.210
Ic (ly/d)	Optimal growth radiation cyano	109.9
$k_{diatom,growth}$ (d-1)	Max growth of diatoms	0.405
Id (ly/d)	Optimal growth radiation of diatoms	24.52
$k_{cyanoph,growth}$ (d^{-1})	Max growth of chlorophytes	0.654
Ig (ly/d)	Optimal growth radiation chlorophytes	89.74
kON,miner (d-1)	Rate coeff. mineralization ON	0.092
kOP,miner (d-1)	Rate coeff. mineralization OP	0.015
Knio (mg/l)	Half-sat. conc. for oxygen lim. of nitrification	0.343

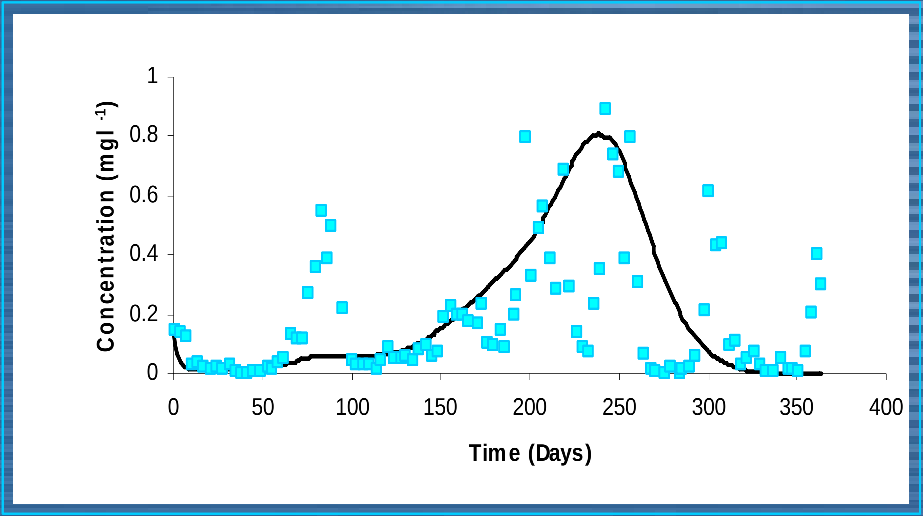
Time horizon: 365 days – Data frequency: twice a week

DAE: 20 differential equations, 60 algebraic equations, NE =40 NC=3

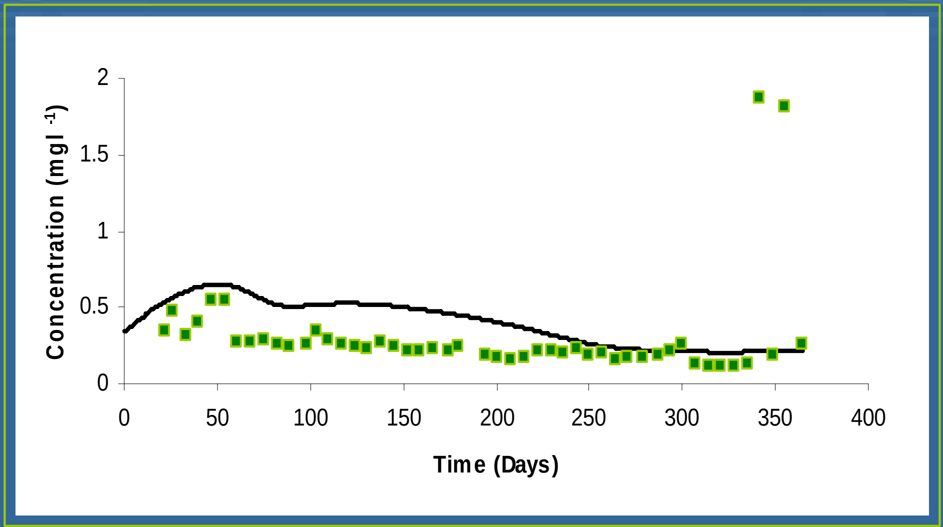
NLP: 10432 nonlinear equations, 52 Iterations, 4 barrier problems

Numerical Results

Diatoms

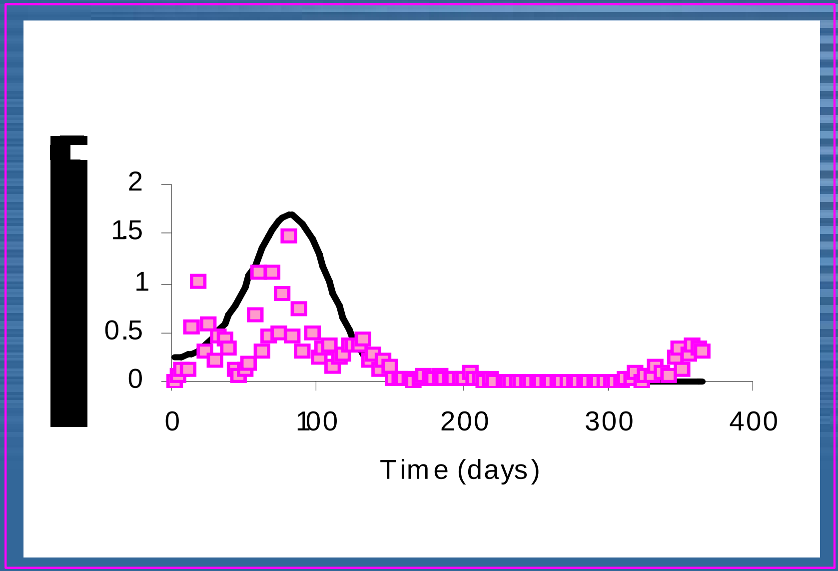


Phosphate

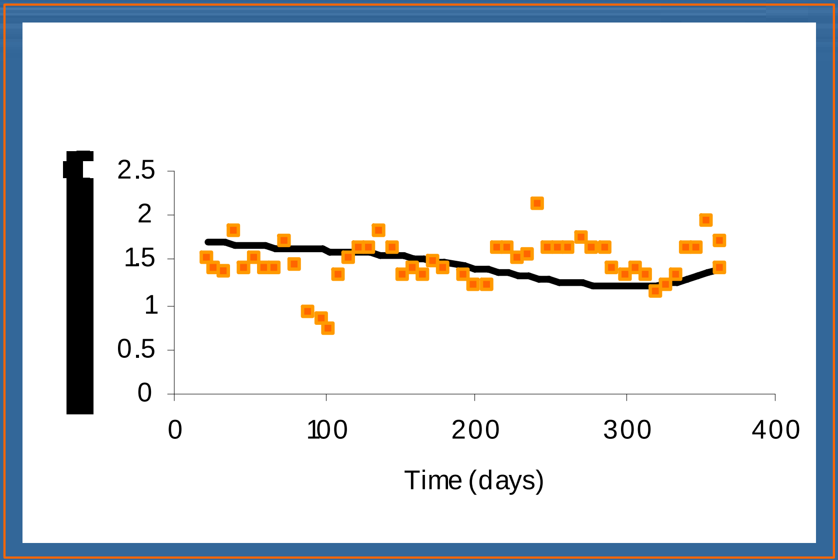


Numerical Results

Cyanobacteria



Nitrate



Bio-restoration policies

- ✓ Excessive nutrients that promote algal growth were identified as the most important problem in 44% of all U.S. lakes surveyed in 1998 (U.S. EPA 2000)
- ✓ Nutrient management:
  - How much do nutrients have to be reduced to eliminate algal blooms?
  - How long will it take for lake water quality to improve once controls are in place?
  - How successful will restoration be, based on water quality management goals?
  - Are proposed lake management goals realistic and cost effective?

Bio-restoration policies

Artificial wetland

- ✓ Built for nitrogen and phosphorus removal (Lopez et al., 2007)
- ✓ Next to El Divisorio Stream
- ✓ Global retention: 64% (phosphate)
- ✓ Derivation of tributary inflows through wetland

Optimal control problem: Tributary inflows derivation

- Case 1: Control variable: Tributary inflows profiles derivation to wetland for bio-remediation

$$min \int_0^{tf} \left(\sum_{j=phyto} C_j(t)_, - 0.25 \right)^2 dt$$

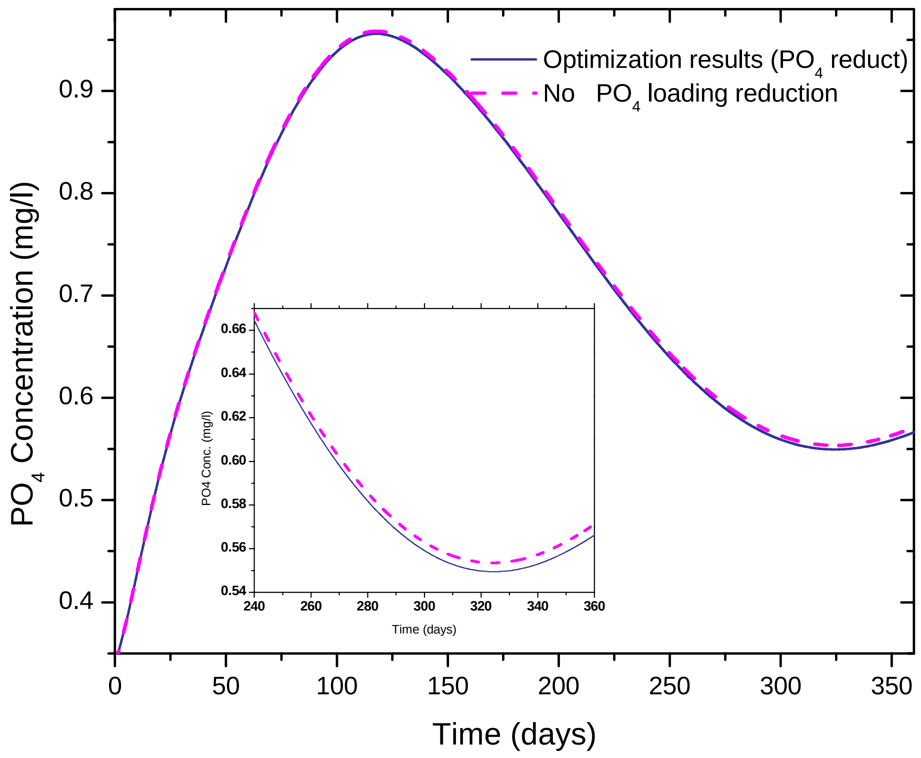
st

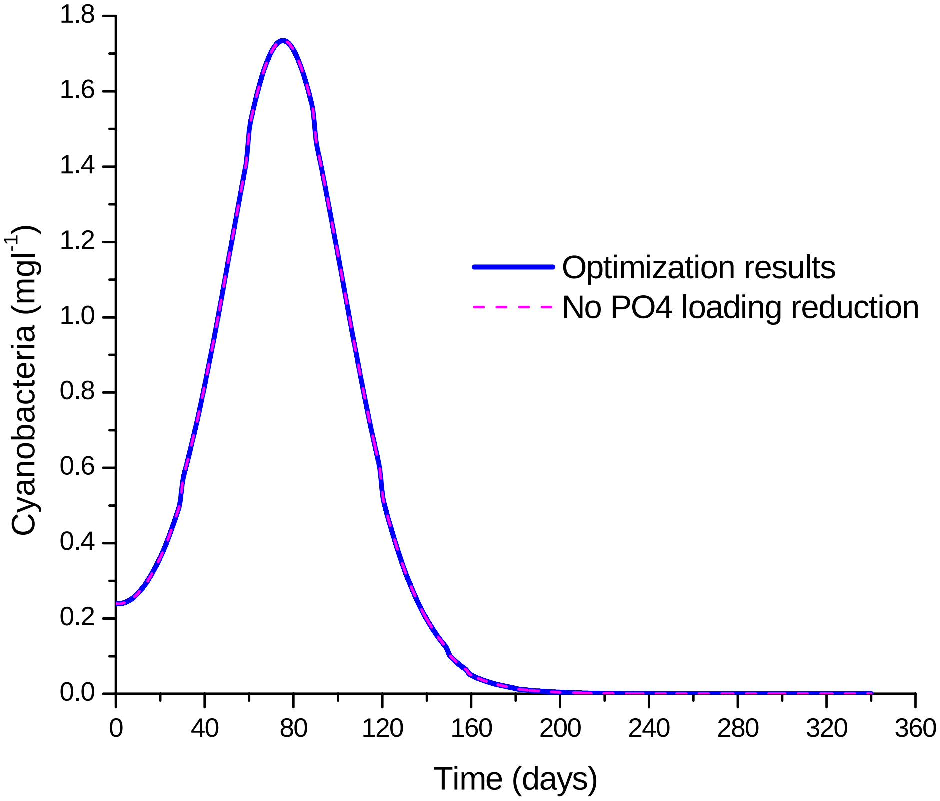
DAE Eutrophication model

$$0 \le F_{WETLAND} \le 0.5 F_{DIVISORIO} (1/d)$$

Case 1: Numerical results

(Estrada et al., 2008d)





NE = 40, NC = 3

NLP: 10581 nonlinear equations

Optimal control problem: Inlake bio-restoration

- Case 2: Control variables: Tributary inflows profiles derivation to wetland for bio-remediation and Zooplankton concentration profiles (Removal of zoo-planktivorous fish)

$$min \int_0^{tf} \left(\sum_{j=phyto} C_j(t)_, - 0.25 \right)^2 dt$$

$$st$$

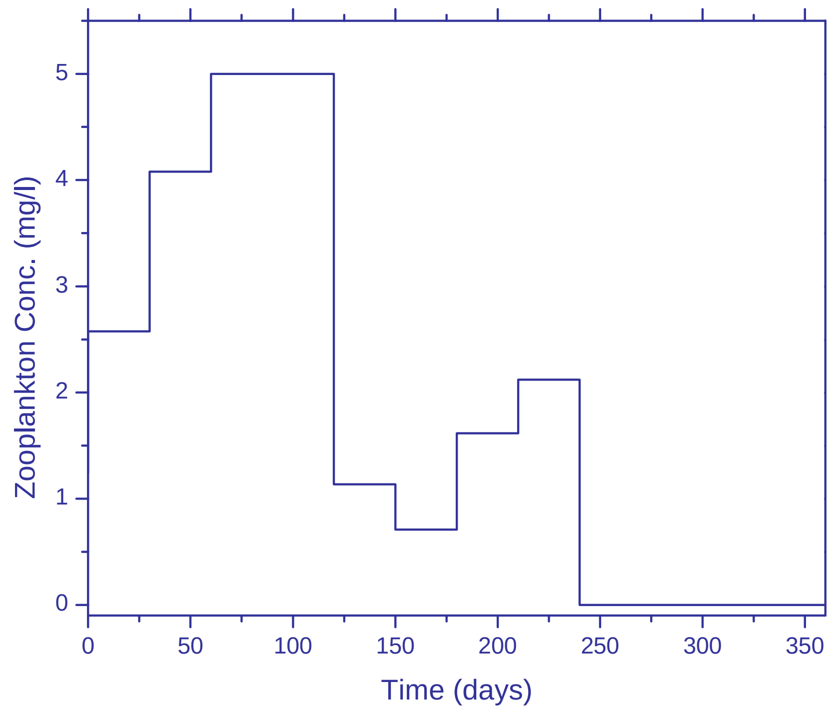
$$DAE\ Eutrophication\ model$$

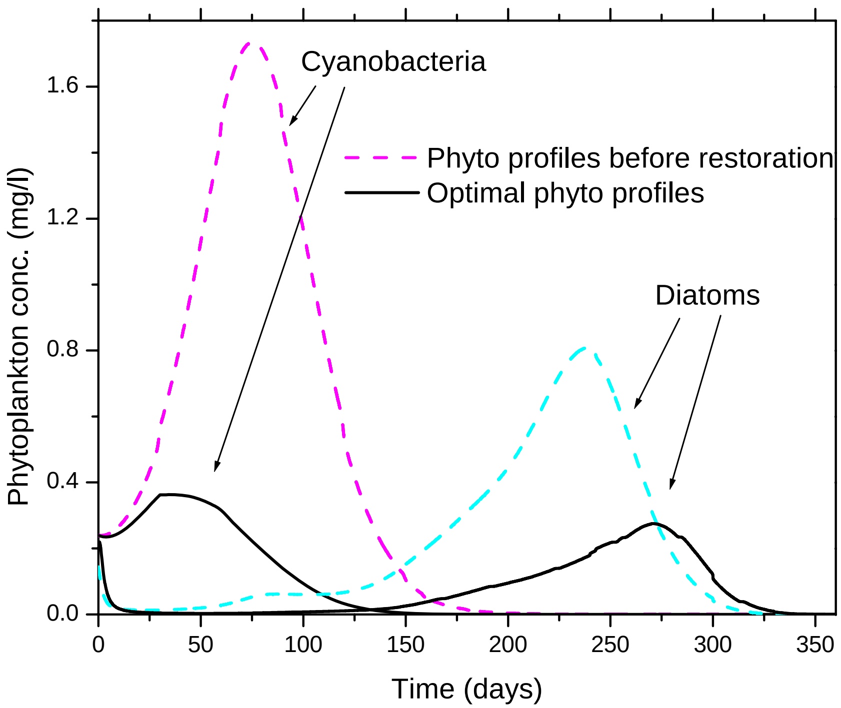
$$0 \le F_{WETLAND} \le 0.5 F_{DIVISORIO}^{(l/d)}$$

$$0.01 \le C_{zoo} \le 5 (mg/l)$$

Case 2: Control and state variables profiles

(Estrada et al., 2008d)





NE = 40, NC = 3

NLP: 10741 nonlinear equations

Conclusions

- ✓ Biological and physico chemical determinations at two depth level in Paso de las Piedras Reservoir. Current data collection at eight levels.
- ✓ Development of rigorous eutrophication model
- ✓ Global sensitivity analysis: ranking of input parameters
- ✓ Formulation of parameter estimation problem subject to DAE system
- ✓ Parameter estimation problem solved with advanced dynamic optimization techniques: simultaneous approach
- ✓ Resolution of optimal control problem: bio-restoration policies

References

- Arhonditsis, G. B. and Brett, M. T., Eutrophication model for Lake Washington (USA) Part. I. Model description and sensitivity análisis. *Ecol. Model.* 187, 140-178, 2005
- Biegler, L.T., A. Cervantes, A.Waechter, Advances in simultaneous strategies for dynamic process optimization. *Chem. Eng. Sci.* 57: 575-593, 2002
- Estrada V., E. Parodi, M.S. Diaz,"Dynamic Parameter Estimation Problem For A Water Quality Model", *Chem. Eng. Transactions*, **11**, 247-252, 2007
- Estrada V., E. Parodi, M.S. Diaz, Developing a Lake Eutrophication Model And Determining Biogeochemical Parameters: A Large Scale Parameter Estimation Problem, *Comp. Aided Chem. Eng.*, 23, 1113-1118, 2008
- Estrada V., E. Parodi, M.S. Diaz, Development of eutrophication biogeochemical models: global sensitivity analysis and dynamic parameter estimation, submitted to *J. Appl. Ecology*, 2008
- Estrada V., E. Parodi, M.S. Diaz, A simultaneous dynamic optimization approach for addressing the control problem of algae growth in water reservoirs through biogeochemical models, *FOCAPO*, June 2008, Boston, USA
- Jeppesen, E., Søndergaard, M., Jensen, Havens, Anneville, Hampton, Hilt, Kangur, Köhler, Lammens, Lauridsen, Portielje, Schelske, Straile, Tatrai, Willén, Winder. Lake responses to reduced nutrient loading: an analysis of contemporary long-term data from 35 case studies.Freshwater Biology, 50, 1747–1771, 2005.

References

- López, N., Alioto, Schefer, Belleggia, Siniscalchi, E.Parodi. Diseño de un humedal artificial para remoción de nutrientes de un afluente al Embalse Paso de las Piedras (Argentina). *AIDIS*, Uruguay, 10-15, 2007.
- Parodi, E.R., V. Estrada, N. Trobbiani, G. Argañaraz Bonini, Análisis del estado trófico del Embalse Paso de las Piedras (Buenos Aires, Argentina). *Ecología en tiempos de Cambio*. 178, 2004.
- Raghunathan, A., M.S. Diaz, L.T. Biegler, An MPEC formulation for dynamic optimization of distillation operations, *Comp. Chem. Eng*., 28, 2037, 2004.
- Rodriguez, M., M.S. Diaz, Dynamic modelling and optimisation of cryogenic systems, *Applied Thermal Engineering*, 27, 1182-1190, 2007.
- Sobol´, I. M., Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates. *Math. Comput. Simulation* 55, 271-280, 2001.
- Søndergaard, M., Jeppesen, E. Anthropogenic impacts on lake and stream ecosystems, and approaches to restoration. J. Applied Ecology, 44, 1089–1094, 2007.