SHORT-TERM SCHEDULING

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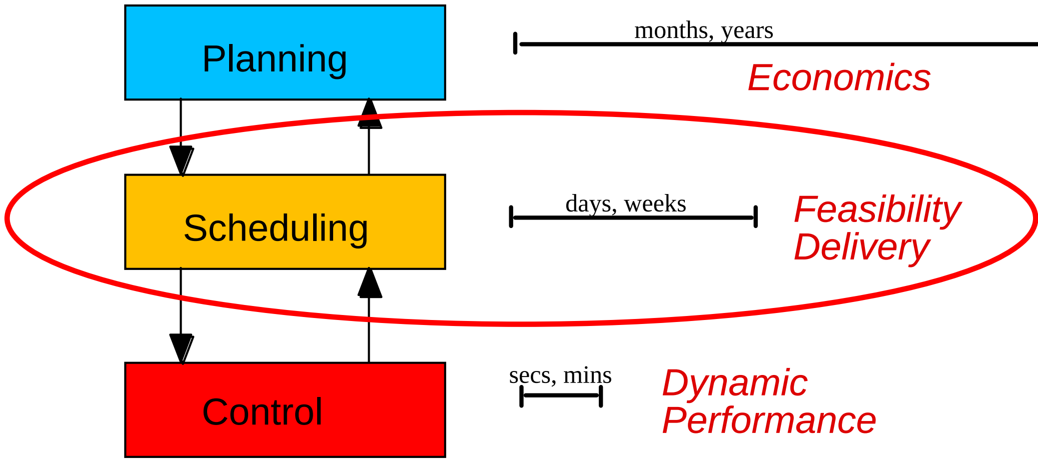
OUTLINE

- PROBLEM STATEMENT
- MAJOR FEATURES AND CHALLENGES
- SOLUTION METHODS
- MILP-BASED MODELS
- EXAMPLES AND COMPUTATIONAL ISSUES
- INDUSTRIAL-SCALE PROBLEMS

LITERATURE. REVIEW PAPERS

- Pekny, J.F., & Reklaitis, G.V. **(1998).** Towards the convergence of theory and practice: A technology guide for scheduling/planning methodology. In *Proceedings of the third international conference on foundations of computer-aided process operations* (pp. 91–111).
- Shah, N. **(1998).** Single and multisite planning and scheduling: Current status and future challenges. In *Proceedings of the third international conference on foundations of computer-aided process operations* (pp. 75–90).
- Pinto, J.M., & Grossmann, I.E. **(1998).** Assignments and sequencing models of the scheduling of process systems. *Annals of Operations Research*, *81*, 433–466.
- Kallrath, J. **(2002).** Planning and scheduling in the process industry. *OR Spectrum*, *24*, 219–250.
- Floudas, C A., & Lin, X. **(2004).** Continuous-time versus discrete-time approaches for scheduling of chemical processes: A review. *Computers and Chemical Engineering*, *28*, 2109–2129.
- Méndez, C.A., Cerdá, J., Harjunkoski, I., Grossmann, I.E. & Fahl, M. **(2006).** State-of-the-art review of optimization methods for short-term scheduling of batch processes. *Computers and Chemical Engineering*, 30, 6, 913 – 946,

- **<u>Plant Level:</u> Multilevel/Hierarchical Decisions**

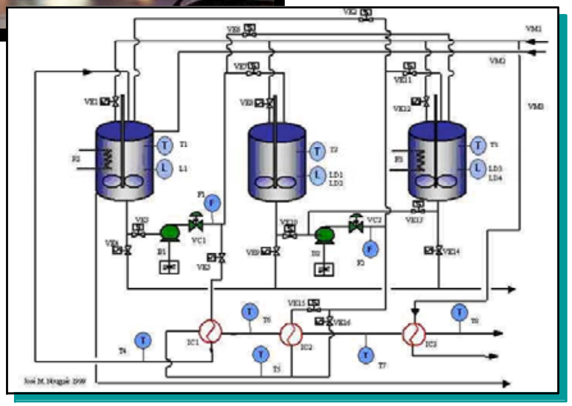


Allocation of limited resources over time to perform a collection of tasks

Information systems

Optimization-based computer tools

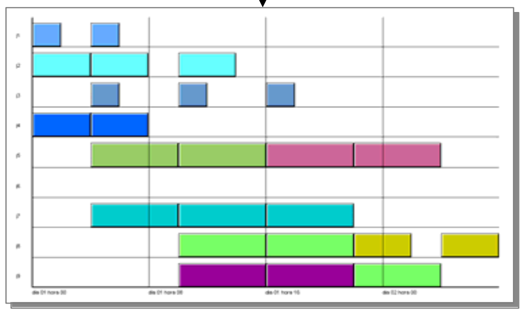
"Decision-making process with the goal of optimizing one or more objectives"



Plant configuration
Recipe data
Demands

Scheduler

Production Scheduling
Detailed plant production scheduling



Schedule

DECISION-MAKING PROCESS

Batches or campaigns to be processed

unit allocation

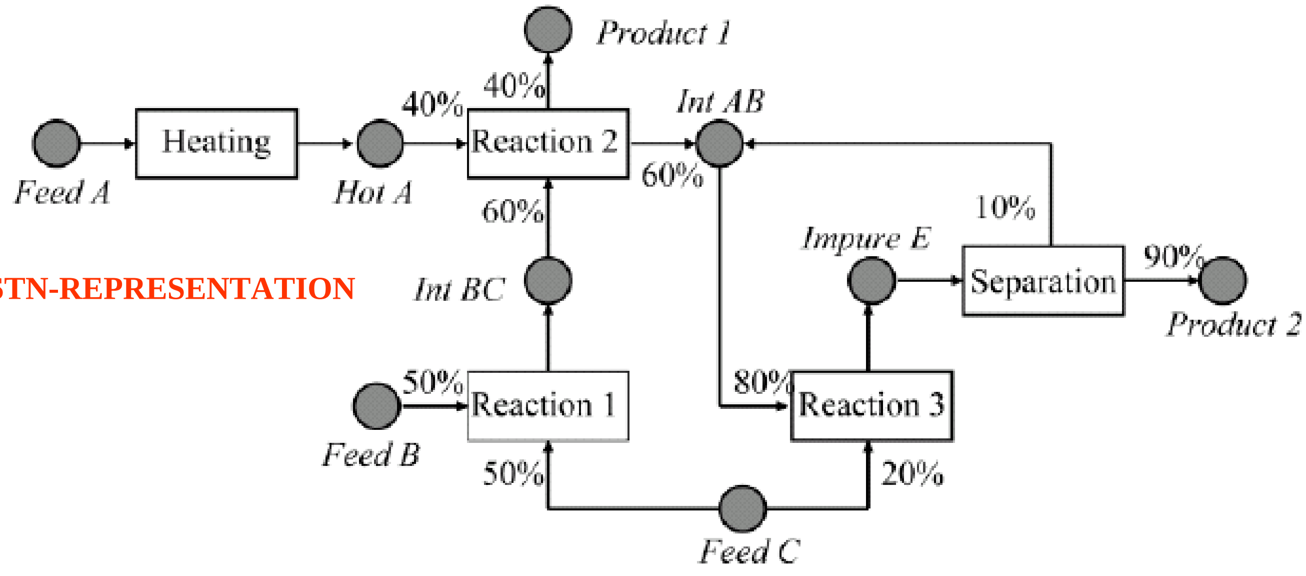
resource allocation: steam, electricity, raw materials, manpower

Timing of manufacturing operations

MAIN CHALLENGES

- ***High combinatorial complexity***
- ***Many problem features to be simultaneously considered***
- ***Time restrictions***

ILLUSTRATIVE EXAMPLE



STN-REPRESENTATION

EQUIPMENT
- HEATER
- 2 REACTORS
- STILL

BATCH TASKS
- HEATING
- 3 REACTIONS
- SEPARATION

GOAL
MAXIMIZE PROFIT

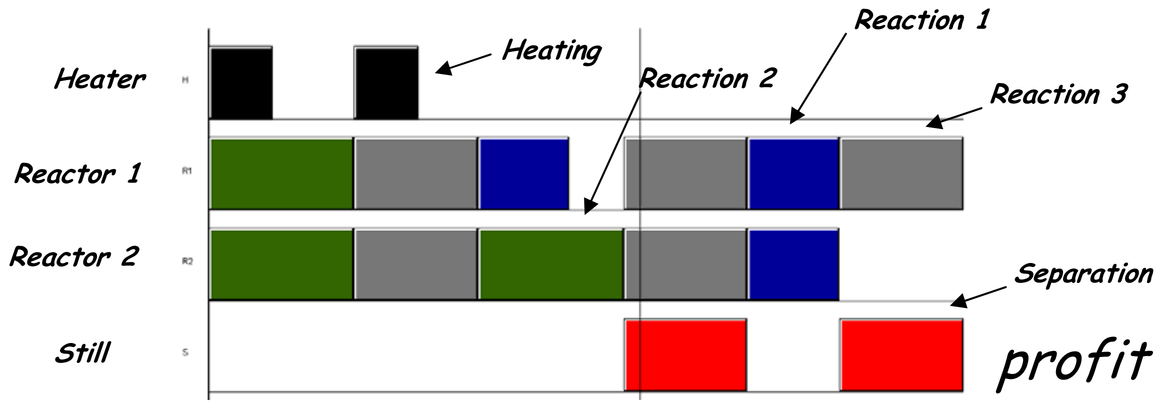
DECISIONS

Lot-sizing

Allocation

Sequencing

Timing



profit = 2805

PROBLEM STATEMENT - I

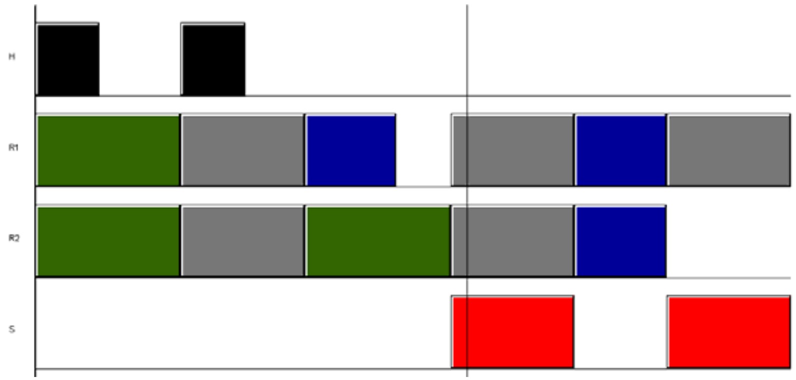
- **Given**:
  - plant configuration
    - plant equipment (processing units, storage tanks, transfer units, connecting networks)
    - resources (electricity, manpower, heating/cooling utilities, raw materials)
  - product recipes
  - product precedence relations
  - demands

PROBLEM STATEMENT - II

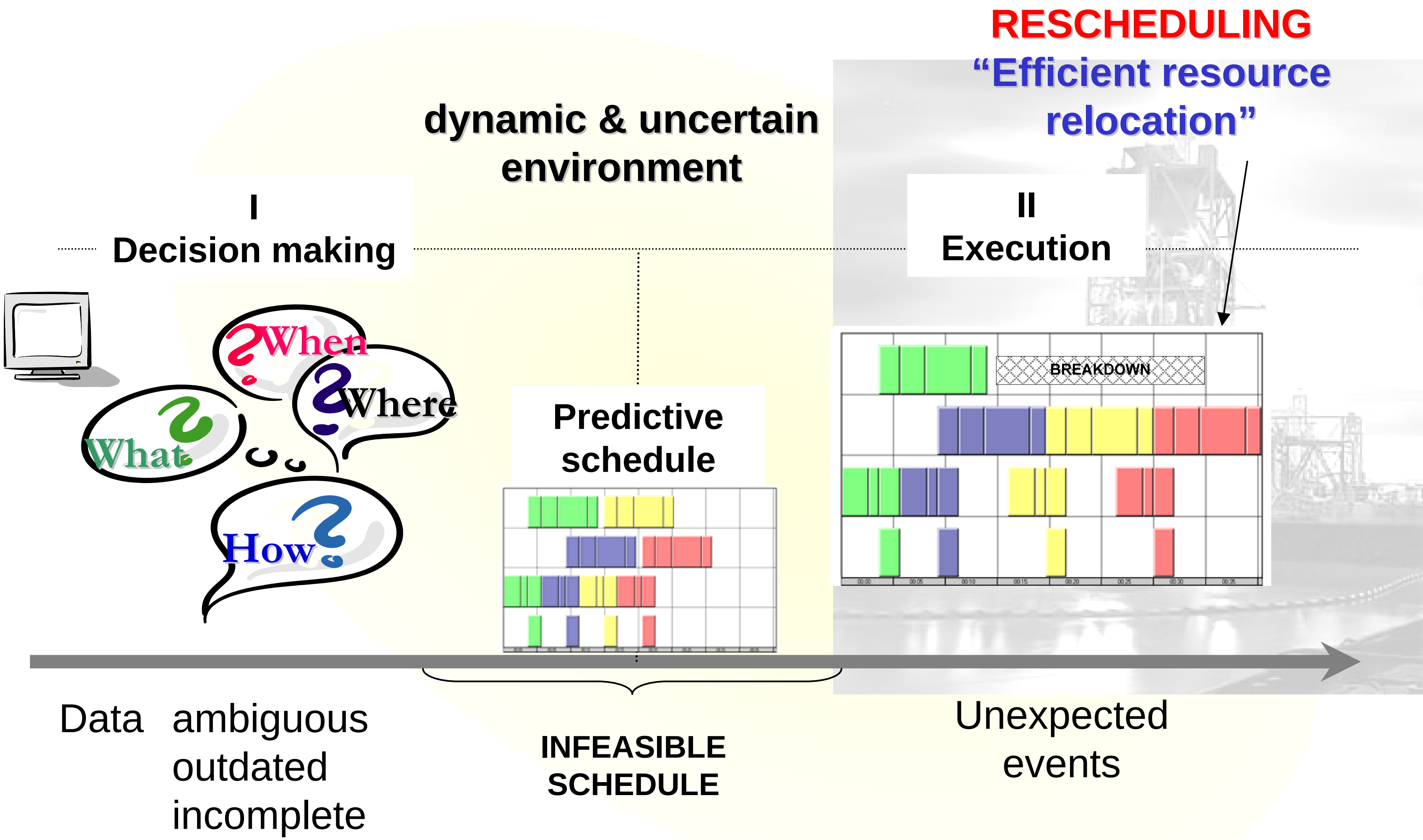
- **Determine**:
  - assignment of equipment and resources to tasks
  - production sequence
  - detailed schedule
    - start and end times
    - inventory levels
    - resources utilization profiles

PROBLEM STATEMENT - III

- **To** optimize one or more objectives:
  - time required to complete all tasks (makespan)
  - number of tasks completed after their due dates
  - plant throughput
  - customer satisfaction
  - profit
  - costs



SCHEDULING & RE-SCHEDULING



BATCH SCHEDULING FEATURES

(1) Process topology

- Sequential
  - Single stage
    - Single unit
    - Parallel units
  - Multiple stages
    - Multiproduct (Flow-shop)
    - Multipurpose (Job-shop)
- Network (arbitrary)

(2) Equipment assignment

- Fixed
- Variable

(3) Equipment connectivity

- Partial (restricted)
- Full

(4) Inventory storage policies

- Unlimited Intermediate Storage (UIS)
- Non-Intermediate Storage (NIS)
- Finite Intermediate Storage (FIS)
  - Dedicated storage units
  - Shared storage units
- Zero Wait (ZW)

(5) Material transfer

- Instantaneous (neglected)
- Time-consuming
  - No-resources
  - Pipes
  - Vessels (Pipeless)

BATCH SCHEDULING FEATURES

Large diversity of factors !
Developing general methods is quite difficult …

(6) Batch size
- Fixed
- Variable (Mixing and Splitting)

(7) Batch processing time
- Fixed
  - Unit independent
  - Unit dependent
- Variable (unit/batch-size dependent)

(8) Demand patterns
- Due dates
  - Single product demand
  - multiple product demands
- Scheduling horizon
  - Fixed requirements
  - Minimum / maximum requirements

(9) Changeovers
- None
- Unit dependent
- Sequence dependent
  - Product dependent
  - Product and unit dependent

(10) Resource Constraints
- None (only equipment)
- Discrete
- Continuous

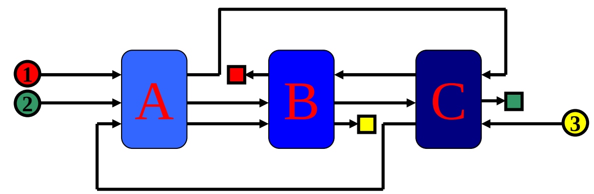
(11) Time Constraints
- None
- Non-working periods
- Maintenance
- Shifts

(12) Costs
- Equipment
- Utilities
- Inventory
- Changeover

(13) Degree of certainty
- Deterministic
- Stochastic

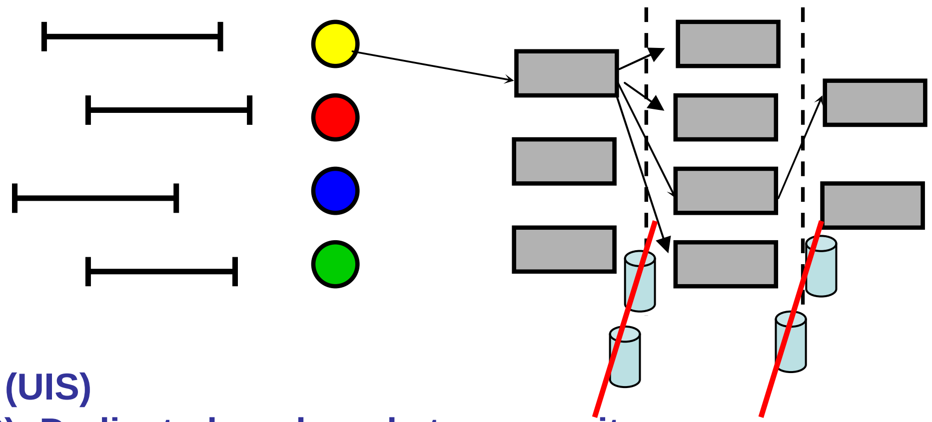
ROAD-MAP FOR BATCH SCHEDULING

(1) TASK TOPOLOGY:
- Single Stage (single unit or parallel units)
- Multiple Stage (multiproduct or multipurpose)
- Network



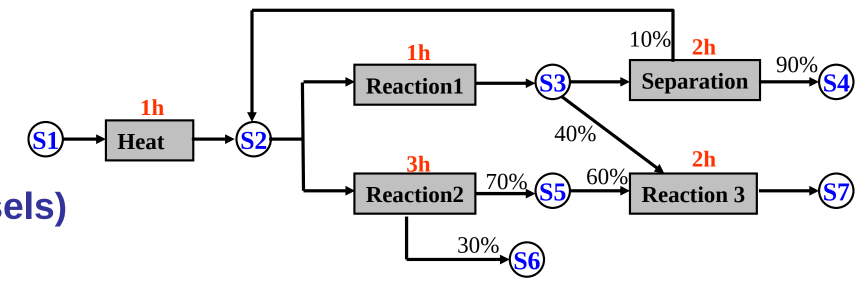
(2) EQUIPMENT ASSIGNMENT
- Fixed
- Variable

(3) EQUIPMENT CONNECTIVITY
- Partial
- Full



(4) INVENTORY STORAGE POLICIES
- Unlimited intermediate storage (UIS)
- Finite intermediate storage (FIS): Dedicated or shared storage units
- Non-intermediate storage (NIS)
- Zero wait (ZW)

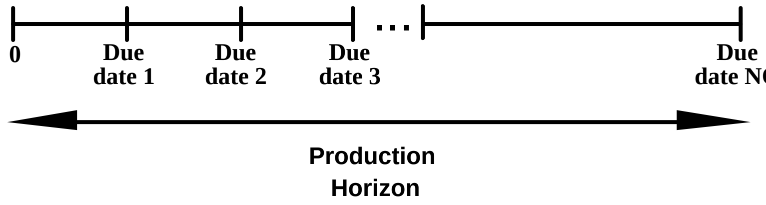
(5) MATERIAL TRANSFER
- Instantaneous (neglected)
-Time consuming (no-resource, pipes, vessels)



ROAD-MAP FOR BATCH SCHEDULING

(6) BATCH SIZE:
- Fixed
- Variable (mixing and splitting operations)

(7) BATCH PROCESSING TIME
- Fixed
- Variable (unit / batch size dependent)



(8) DEMAND PATTERNS
- Due dates (single or multiple product demands)
- Scheduling horizon (fixed, minimum/maximum requirements)

(9) CHANGEOVERS

- None
- Unit dependent
- Sequence dependent (product or product/unit dependent)

(10) RESOURCE CONSTRAINTS
- None (only equipment)
- Discrete (manpower)
- Continuous (utilities)
  (Fixed or time dependent)

ROAD-MAP FOR BATCH SCHEDULING

(11) TIME CONSTRAINTS
- None
- Non-working periods
- Maintenance
- Shifts

(12) COSTS
- Equipment
- Utilities (fixed or time dependent)
- Inventory
- Changeovers

(13) Degree of certainty
- Deterministic
- Stochastic

ROAD-MAP FOR SOLUTION METHODS

(1) Exact methods
MILP
MINLP

(2) Constraint programming (CP)
Constraint satisfaction methods

(3) Meta-heuristics
Simulated annealing (SA)
Tabu search (TS)
Genetic algorithms (GA)

(4) Heuristics
Dispatching rules

(5) Artificial Intelligence (AI)
Rule-based methods
Agent-based methods
Expert systems

(6) Hybrid-methods
Exact methods + CP
Exact methods + Heuristics
Meta-heuristics + Heuristics

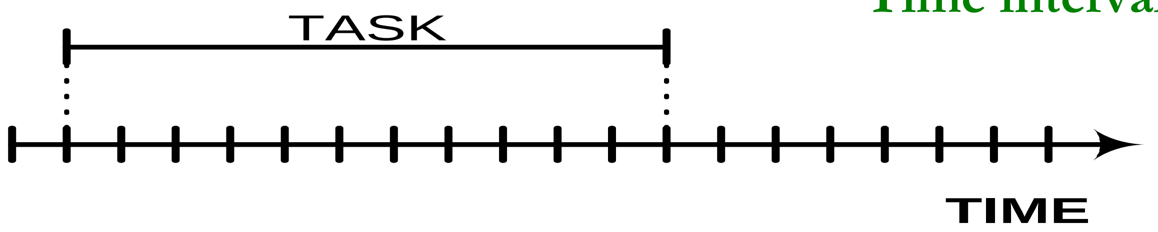
Rigorous mathematical representation
Non-linear constraints are avoided
Discrete and continuous variables
Mathematical-based solution methods
Systematic solution search
Feasibility and optimality

ROAD-MAP FOR OPTIMIZATION APPROACHES

TIME DOMAIN REPRESENTATION

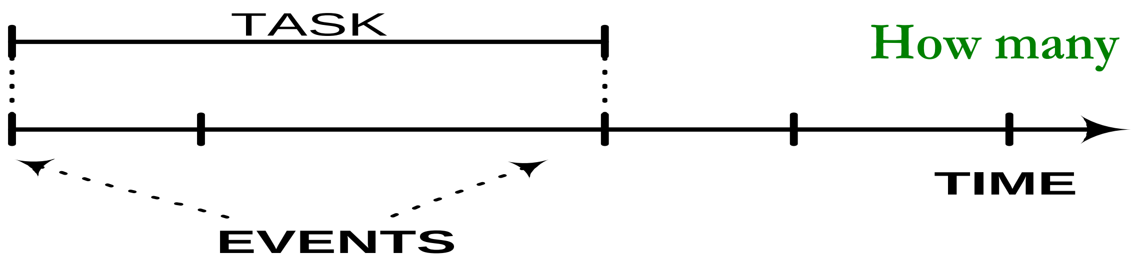
- Discrete time

Time interval duration ?

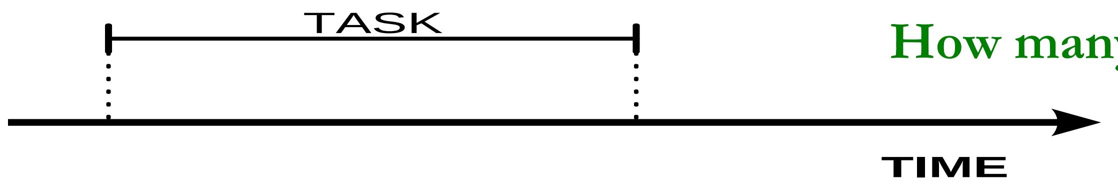


- Continuous time

How many events ?



How many tasks ?

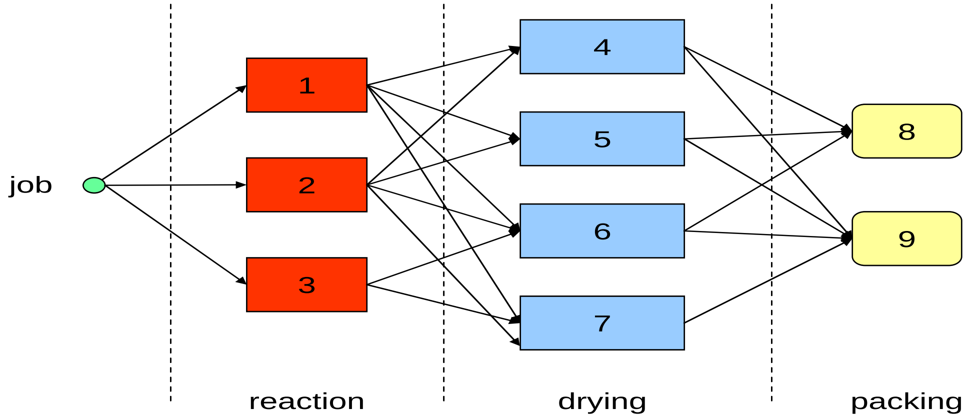


ROAD-MAP FOR OPTIMIZATION APPROACHES

INTEC UNL CONICET

MATERIAL BALANCES

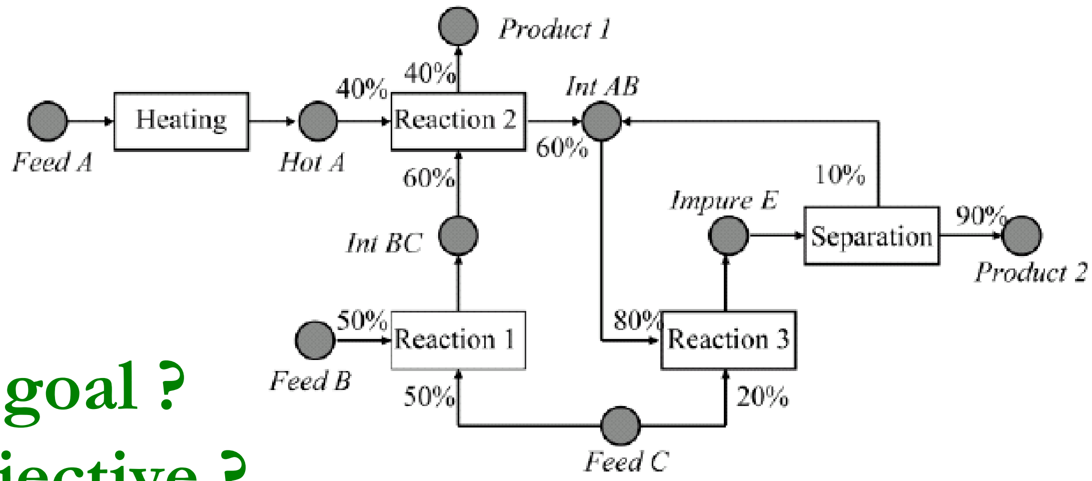
- Lots (Order or batch oriented)
- Network flow equations (STN or RTN problem representation)



Sequential process

Separate Batching from Scheduling ?
Batch mixing and splitting ?

Network process



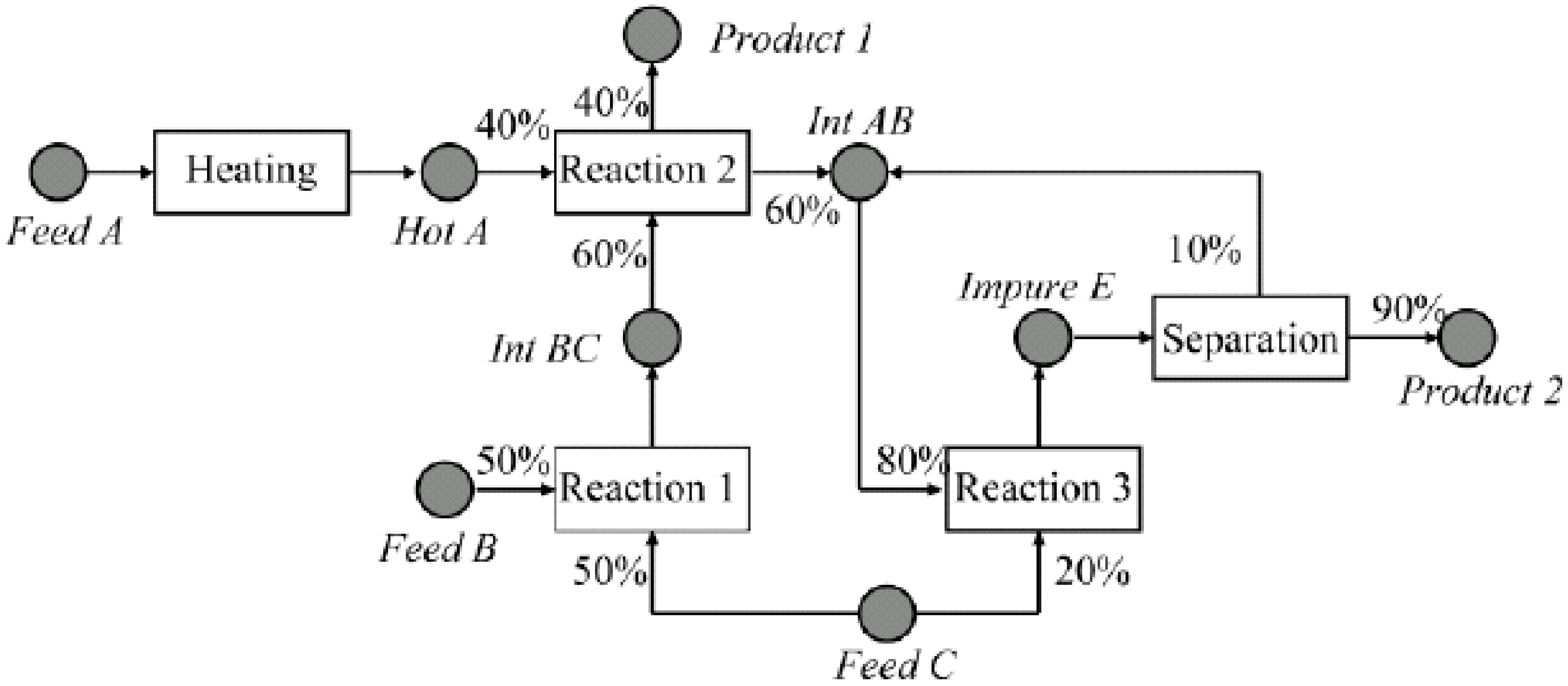
OBJECTIVE FUNCTION

- Makespan
- Earliness/ Tardiness
- Profit
- Inventory
- Cost

Which goal ?
Multi-objective ?

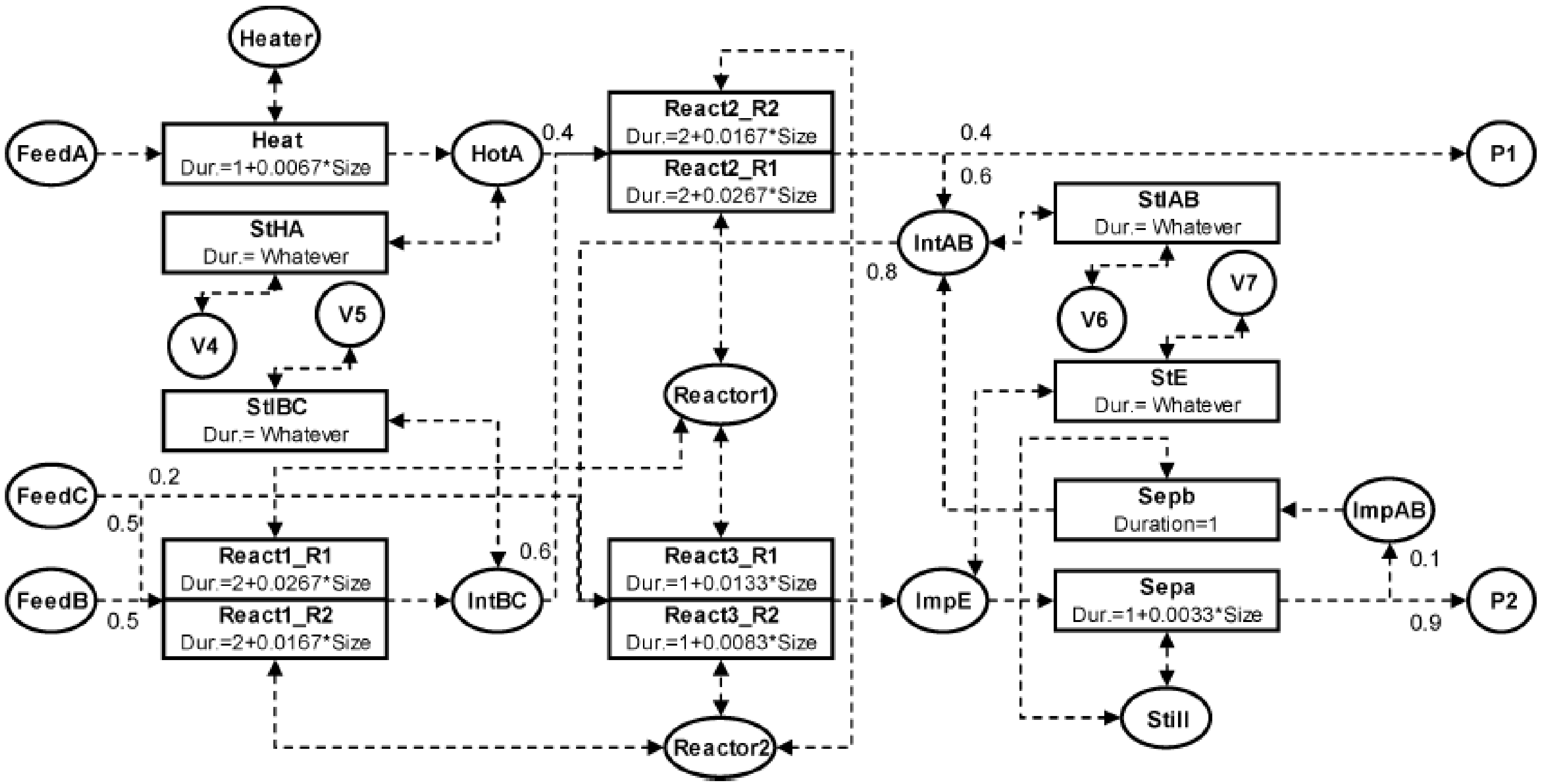
NETWORK PROCESS REPRESENTATION

State-Task Network (STN): assumes that processing tasks produce and consume states (materials). A special treatment is given to manufacturing resources aside from equipment.



NETWORK PROCESS REPRESENTATION

Resource-Task Network (RTN): employs a uniform treatment for all available resources through the idea that processing tasks consume and release resources at their beginning and ending times, respectively.



ROAD-MAP FOR OPTIMIZATION APPROACHES

Main events involve changes in:
- Processing tasks (start and end)
- Availability of any resource
- Resource requirement of a task

EVENT REPRESENTATION

NETWORK-ORIENTED PROCESSES
- DISCRETE TIME
  - Global time intervals (STN or RTN)
- CONTINUOUS TIME
  - Global time points (STN or RTN)
  - Unit- specific time event (STN)

Key point: reference points to check resources

BATCH-ORIENTED PROCESSES
- CONTINUOUS TIME
  - Time slots
  - Unit-specific direct precedence
  - Global direct precedence
  - Global general precedence

Key point: arrange resource utilization

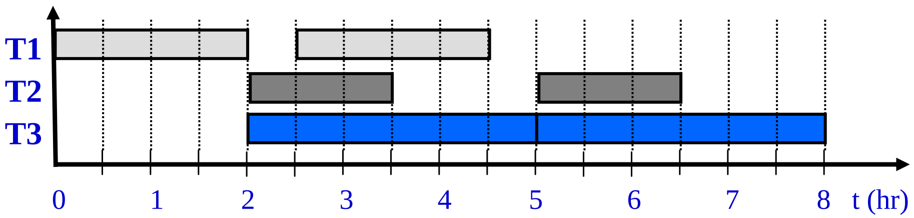
STN-BASED DISCRETE TIME FORMULATION

(Kondili et al., 1993; Shah et al., 1993; Rodrigues et al., 2000.)

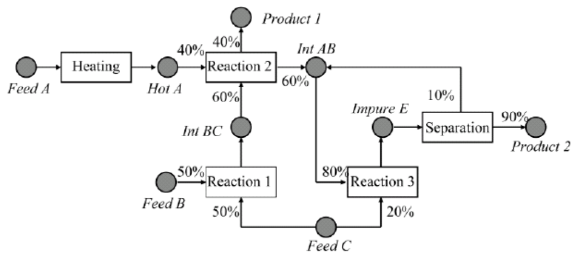
MAIN ASSUMPTIONS

- **The scheduling horizon is divided into a finite number of time intervals with known duration**
- **The same time grid is valid for all shared resources, i.e. global time intervals**
- **Tasks can only start or finish at the boundaries of these time intervals**

Discrete Time Representation (Global time intervals)



STATE-TASK NETWORK



ADVANTAGES

- **Resource constraints are only monitored at predefined and fixed time points**
- **Good computational performance**
- **Simple models and easy representation of a wide variety of scheduling features**

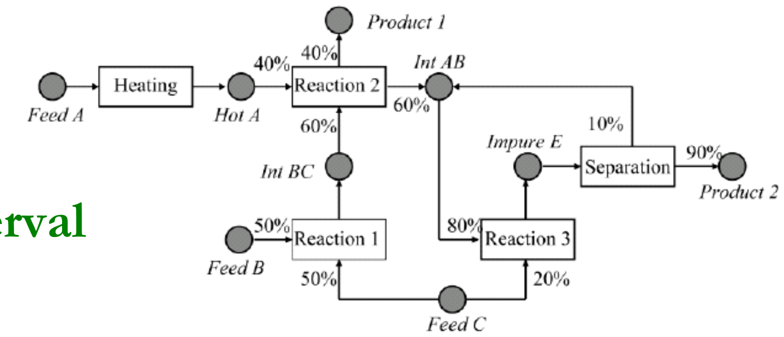
DISADVANTAGES

- **Model size and complexity depend on the number of time intervals**
- **Constant processing times are required**
- **Sub-optimal or infeasible solutions can be generated due to the reduction of the time domain**

STN-BASED DISCRETE TIME FORMULATION

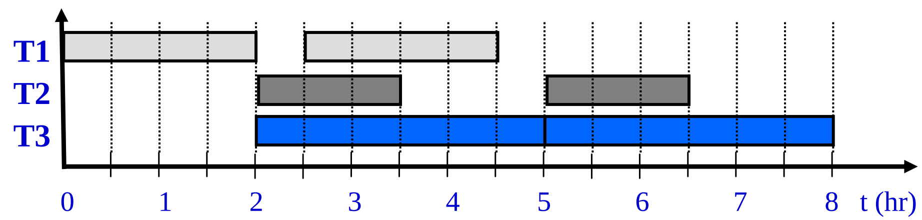
MAJOR MODEL VARIABLES

STATE-TASK NETWORK



BINARY VARIABLES: $W_{i,j,t}$ — task (i), unit (j), time interval (t)

$W_{i,j,t}$ = 1 only if the processing of a batch undergoing task **i** in unit **j** is started at time point **t**



CONTINUOUS VARIABLES:

$B_{i,j,t}$ = size of the batch (i,j,t)

$S_{s,t}$ = available inventory of state s at time point t

$R_{r,t}$ = availability of resource r at time point t

The number of time intervals is the critical point (data dependent)

STN-BASED DISCRETE TIME FORMULATION

(**Kondili et al., 1993; Shah et al., 1993**)

ALLOCATION AND SEQUENCING

$$\sum_{i \in I_j} \sum_{t'=t-pt_{ij}+1}^{t} W_{ijt'} \leq 1 \qquad \forall j, t$$

BATCH SIZE

$$V_{ij}^{\min} W_{ijt} \leq B_{ijt} \leq V_{ij}^{\max} W_{ijt} \qquad \forall i, j \in J_i, t$$

MATERIAL BALANCE

$$S_{st} = S_{s(t-1)} + \sum_{i' \in I_s^p} \rho_{is}^p \sum_{j \in J_i} B_{ij(t-pt_{is})} - \sum_{i' \in I_s^c} \rho_{is}^c \sum_{j \in J_i} B_{ijt} + \prod_{st} - D_{st} \qquad \forall s, t$$

$$C_s^{\min} \leq S_{st} \leq C_s^{\max} \qquad \forall s, t$$

RESOURCE BALANCE

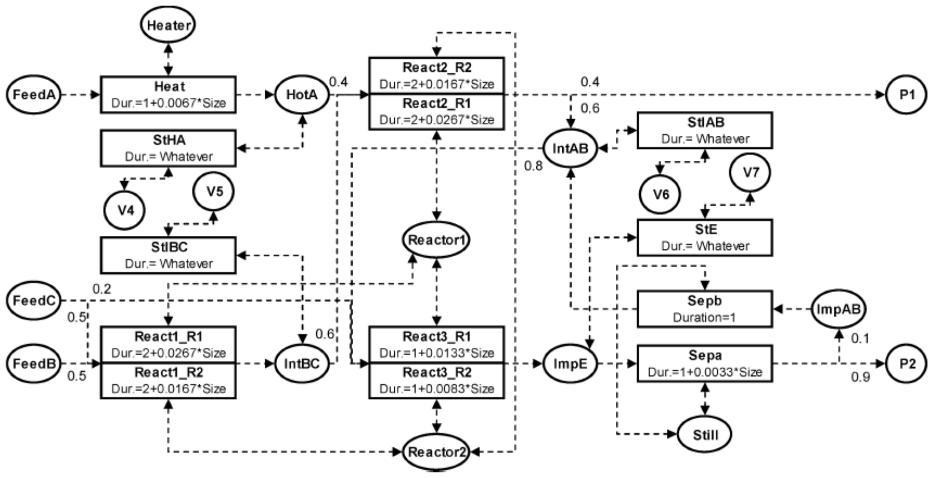
$$R_{rt} = \sum_{i} \sum_{j \in J_i} \sum_{t'=0}^{pt_{ij}-1} \left(\mu_{irt'} W_{ij(t-t')} + v_{irt'} B_{ij(t-t')} \right) \qquad \forall r, t$$

$$0 \leq R_{rt} \leq R_{rt}^{\max} \qquad \forall r, t$$

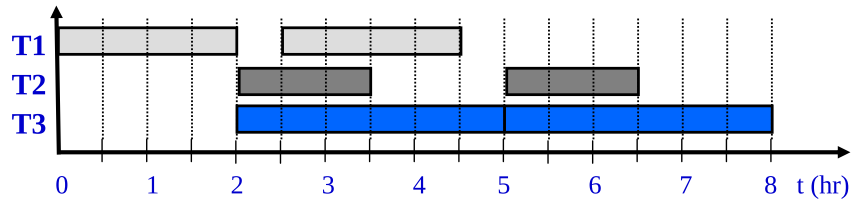
CHANGEOVER TIMES

$$\sum_{i \in I_j^f} W_{ijt} + \sum_{i \in I_j^{f'}} \sum_{t'=t-cl_{f'f}-pt_{ij}+1}^{t} W_{ijt'} \leq 1 \qquad \forall j, f, f', t$$

RTN-BASED DISCRETE TIME FORMULATION



RESOURCE-TASK NETWORK



(Pantelides, 1994).

ADVANTAGES

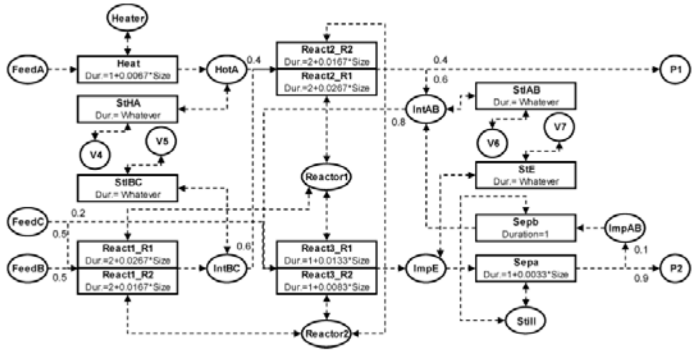
- **Resource constraints are only monitored at predefined and fixed time points**
- **All resources are treated in the same way**
- **Good computational performance**
- **Very Simple models and easy representation of a wide variety of scheduling features**

DISADVANTAGES

- **Model size and complexity depend on the number of time intervals**
- **Constant processing times are required**
- **Sub-optimal or infeasible solutions can be generated due to the reduction of the time domain**
- **Changeovers have to be considered as additional tasks**

RTN-BASED DISCRETE TIME FORMULATION

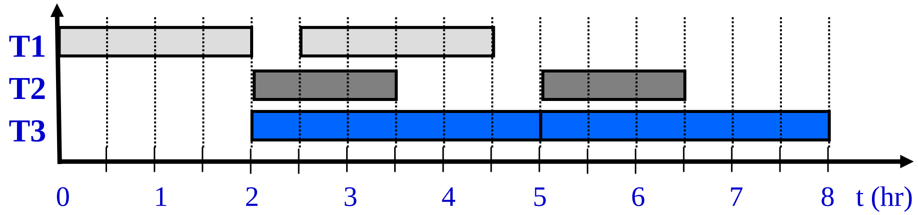
RESOURCE-TASK NETWORK



MAJOR MODEL VARIABLES

BINARY VARIABLES: $W_{i,t}$ → task, time interval

$W_{i,t}$ = 1 only if the processing of a batch task **i** is started at time point **t**



CONTINUOUS VARIABLES:

$B_{i,t}$ = size of the batch (**i,t**)

$R_{r,t}$ = availability of resource **r** at time point **t**

The number of time intervals is the critical point (data dependent)

RTN-BASED DISCRETE TIME FORMULATION

$$R_{rt} = R_{r(t-1)} + \sum_{i \in I_r} \sum_{t'=0}^{pt_i} \left(\mu_{irt'} W_{i(t-t')} + \nu_{irt'} B_{i(t-t')} \right) + \prod_{rt} \qquad \forall r, t$$

RESOURCE BALANCE

$$0 \leq R_{rt} \leq R_{rt}^{\max} \qquad \forall r, t$$

$$V_{ir}^{\min} W_{it} \leq B_{it} \leq V_{ir}^{\max} W_{it} \qquad \forall i, r \in R_i^J, t$$

BATCH SIZE

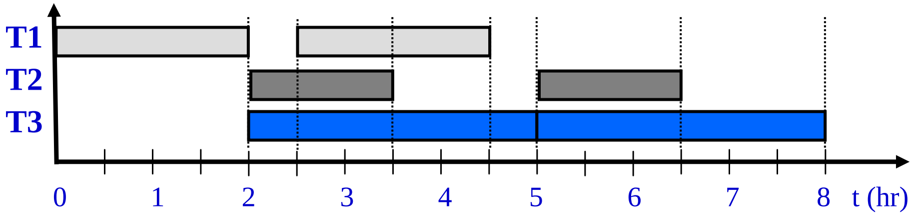
Changeovers must be defined as additional tasks

STN-BASED CONTINUOUS TIME FORMULATION
(GLOBAL TIME POINTS)

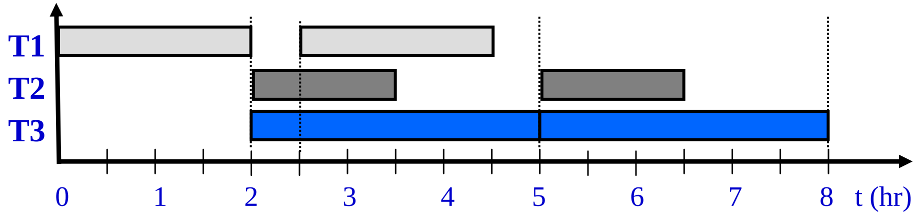
(Pantelides, 1996; Zhang and Sargent, 1996; Mockus and Reklaitis,1999; Mockus and Reklaitis, 1999; Lee et al., 2001, Giannelos and Georgiadis, 2002; Maravelias and Grossmann, 2003)

- Define a common time grid for all shared resources
- The maximum number of time points is predefined
- The time at which each time point takes place is a model decision (continuous domain)
- Tasks allocated to a certain time point *n* must start at the same time
- Only zero wait tasks must finish at a time point, others may finish before

Continuous Time Representation I



Continuous Time Representation II



ADVANTAGES

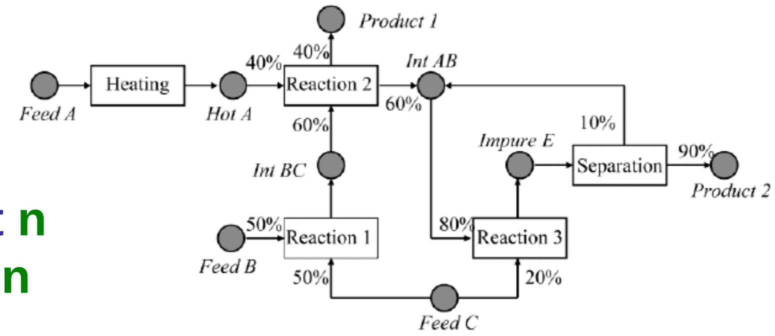
- Significant reduction in model size when the minimum number of time points is predefined
- Variable processing times
- A wide variety of scheduling aspects can be considered
- Resource constraints are only monitored at each time point

DISADVANTAGES

- Definition of the minimum number of time points
- Model size and complexity depend on the number of time points predefined
- Sub-optimal or infeasible solution can be generated if the number of time points is smaller than required

STN-BASED CONTINUOUS TIME FORMULATION

STATE-TASK NETWORK

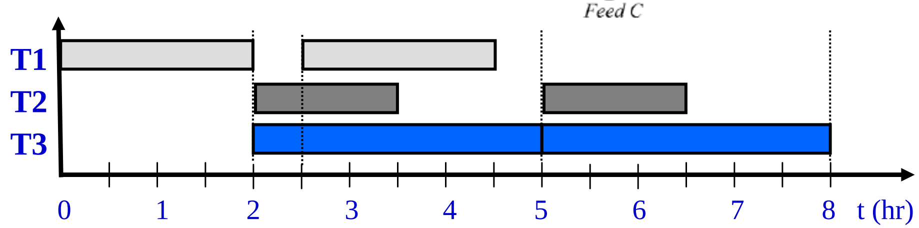


MAJOR MODEL VARIABLES

BINARY VARIABLES:

$Ws_{i,n}$ = 1 only if task **i** starts at time point **n**

$Wf_{i,n}$ = 1 only if task **i** ends at time point **n**



CONTINUOUS VARIABLES:

T_n = time for events allocated at time point **n**

$Ts_{i,n}$ = start time of task **i** assigned at time point **n**

$Tf_{i,n}$ = end time of task **i** assigned at time point **n**

$Bs_{i,n}$ = batch size of task **i** when it starts at time point **n**

$Bp_{i,n}$ = batch size of task **i** at an intermediate time point **n**

$Bf_{i,n}$ = batch size of task **i** when it ends at time point **n**

$S_{s,n}$ = inventory of state **s** at time point **n**

$R_{r,n}$ = availability of resource **r** at time point **n**

The number of time points n is the critical point

STN-BASED CONTINUOUS FORMULATION
(GLOBAL TIME POINTS)

(Maravelias and Grossmann, 2003)

ALLOCATION CONSTRAINTS

$$\sum_{i \in Ij} Ws_{in} \le 1 \qquad \forall j,n$$

$$\sum_{i \in Ij} Wf_{in} \le 1 \qquad \forall j,n$$

$$\sum_{i \in Ij} \sum_{n' \le n} (Ws_{in'} - Wf_{in'}) \le 1 \qquad \forall j,n$$

$$\sum_{n} Ws_{in} = \sum_{n} Wf_{in} \qquad \forall i$$

BATCH SIZE CONSTRAINTS

$$V_i^{\min} Ws_{in} \le Bs_{in} \le V_i^{\max} Ws_{in} \qquad \forall i,n$$

$$V_i^{\min} Wf_{in} \le Bf_{in} \le V_i^{\max} Wf_{in} \qquad \forall i,n$$

$$V_i^{\min}\left(\sum_{n'<n} Ws_{in'} - \sum_{n' \le n} Wf_{in'}\right) \le Bp_{in} \le V_i^{\max}\left(\sum_{n'<n} Ws_{in'} - \sum_{n' \le n} Wf_{in'}\right) \qquad \forall i,n$$

$$Bs_{in-1} + Bp_{i(n-1)} = Bp_{in} + Bf_{in} \qquad \forall i, n>1$$

MATERIAL AND RESOURCE BALANCES

$$S_{sn} = S_{s(n-1)} - \sum_{i \in I_s^c} \rho_{is}^c Bs_{in} + \sum_{i \in I_s^p} \rho_{is}^p Bf_{in} \qquad \forall s, n>1$$

$$S_{sn} \le C_s^{\max} \qquad \forall s,n$$

$$R_{rn} = R_{r(n-1)} - \sum_{i} \mu_{ir}^c Ws_{in} + \nu_{ir}^c Bs_{in} + \sum_{i} \mu_{ir}^p Wf_{in} + \nu_{ir}^p Bf_{in} \qquad \forall r,n$$

TIMING AND SEQUENCING CONSTRAINTS

$$T_{n+1} \ge T_n \qquad \forall n$$

$$Tf_{in} \le T_n + \alpha_i Ws_{in} + \beta_i Bs_{in} + H(1 - Ws_{in}) \qquad \forall i,n$$

$$Tf_{in} \ge T_n + \alpha_i Ws_{in} + \beta_i Bs_{in} - H(1 - Ws_{in}) \qquad \forall i,n$$

$$Tf_{i(n-1)} \le T_n + H(1 - Wf_{in}) \qquad \forall i, n>1$$

$$Tf_{i(n-1)} \ge T_n - H(1 - Wf_{in}) \qquad \forall i \in I^{ZW}, n>1$$

$$Ts_{i'n} \ge Tf_{i(n-1)} + cl_{ii'} \qquad \forall j, i \in I_j, i' \in I_j, n$$

SHARED STORAGE TASKS

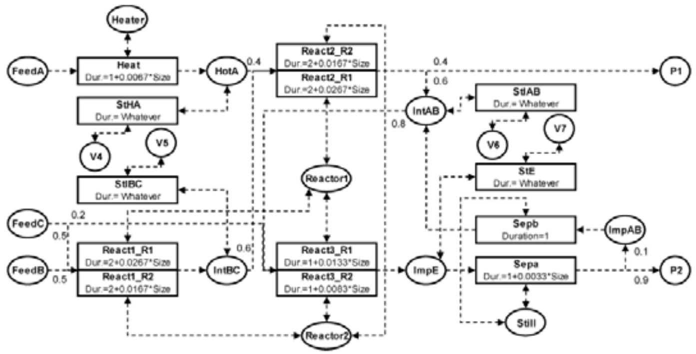
$$\sum_{s \in S_j} V_{jsn} \le 1 \qquad \forall j \in J^T, n$$

$$S_{sjn} \le C_j V_{jsn} \qquad \forall j \in J^T, s \in S_j, n$$

$$S_{sn} = \sum_{j \in J_s^T} S_{sjn} \qquad \forall s \in S^T, n$$

RTN-BASED CONTINUOUS TIME FORMULATION

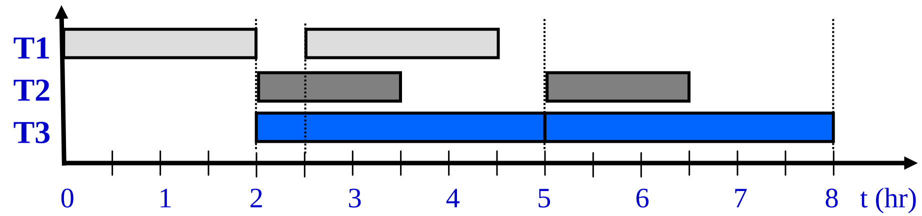
RESOURCE-TASK NETWORK



MAJOR MODEL VARIABLES

BINARY VARIABLES:

$W_{i,n,n'}$ = 1 only if task **i** starts at time point **n** and finishes at time point **n'**



CONTINUOUS VARIABLES:

T_n = time for events allocated at time point **n**

$B_{i,n,n'}$ = batch size of task **i** when it starts at time point **n** and finishes at time point **n'**

$R_{r,n}$ = availability of resource **r** at time point **n**

The number of time points n is the critical point

RTN-BASED CONTINUOUS FORMULATION
(GLOBAL TIME POINTS)

(Castro et al., 2004)

TIMING CONSTRAINTS

$$T_{n'} - T_n \geq \sum_{i \in I_r} \left(\alpha_i W_{inn'} + \beta_i B_{inn'} \right) \qquad \forall r \in R^J, n, n', (n < n')$$

$$T_{n'} - T_n \leq H \left(1 - \sum_{i \in I_r^{ZW}} W_{inn'} \right) + \sum_{i \in I_r^{ZW}} \left(\alpha_i W_{inn'} + \beta_i B_{inn'} \right) \qquad \forall r \in R^J, n, n', (n < n')$$

BATCH SIZE

$$V_i^{min} W_{inn'} \leq B_{inn'} \leq V_i^{max} W_{inn'} \qquad \forall i, n, n', (n < n')$$

RESOURCE BALANCE

$$R_{rn} = R_{r(n-1)} + \sum_{i \in I_r} \left[\sum_{n' < n} \left(\mu_{ir}^p W_{in'n} + \nu_{ir}^p B_{in'n} \right) - \sum_{n' > n} \left(\mu_{ir}^c W_{inn'} + \nu_{ir}^c B_{inn'} \right) \right] + \sum_{i \in I^S} \left(\mu_{ir}^p W_{i(n-1)n} - \mu_{ir}^c W_{in(n+1)} \right) \qquad \forall r, n > 1$$

$$R_r^{min} \leq R_{rn} \leq R_r^{max} \qquad \forall r, n$$

STORAGE CONSTRAINTS

$$V_i^{min} W_{in(n+1)} \leq \sum_{r \in R_i^S} R_{rn} \leq V_i^{max} W_{in(n+1)} \qquad \forall i \in I^s, n, (n \neq |N|)$$

$$V_i^{min} W_{i(n-1)n} \leq \sum_{r \in R_i^S} R_{rn} \leq V_i^{max} W_{i(n-1)n} \qquad \forall i \in I^s, n, (n \neq 1)$$

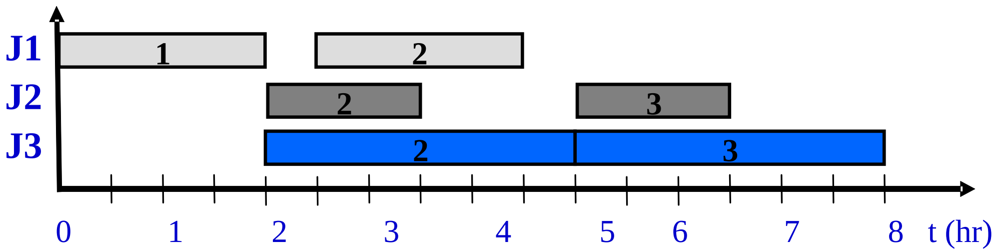
STN-BASED CONTINUOUS FORMULATION
(UNIT-SPECIFIC TIME EVENT)

(Ierapetritou and Floudas, 1998; Vin and Ierapetritou, 2000; Lin et al., 2002; Janak et al., 2004).

MAIN ASSUMPTIONS

- **The number of event points is predefined**
- **Event points can take place at different times in different units (global time is relaxed)**

Event-Based Representation



ADVANTAGES

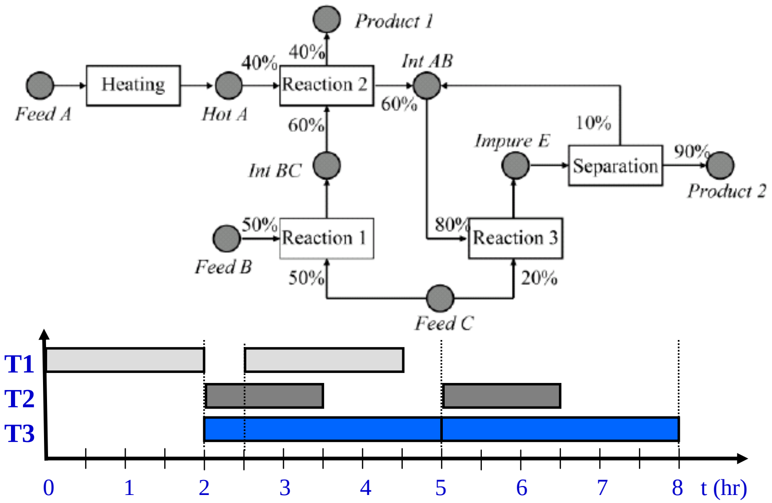
- **More flexible timing decisions**
- **Less number of event points**

DISADVANTAGES

- **Definition of event points**
- **More complicated models, no reference points to check resource availabilities**
- **Model size and complexity depend on the number of time points predefined**
- **Sub-optimal or infeasible solution can be generated if the number of time points is smaller than required**
- **Additional tasks for storage and utilities**

STN-BASED CONTINUOUS TIME FORMULATION
(UNIT-SPECIFIC TIME EVENT)

STATE-TASK NETWORK



MAJOR MODEL VARIABLES

BINARY VARIABLES:

- $W_{i,n}$ = 1 only if task i starts at time point n
- $Ws_{i,n}$ = 1 only if task i starts at time point n
- $Wf_{i,n}$ = 1 only if task i ends at time point n

CONTINUOUS VARIABLES:

- T_n = time for events allocated at time point n
- $Ts_{i,n}$ = start time of task i assigned at time point n
- $Tf_{i,n}$ = end time of task i assigned at time point n
- $Bs_{i,n}$ = batch size of task i when it starts at time point n
- $B_{i,n}$ = batch size of task i at an intermediate time point n
- $Bf_{i,n}$ = batch size of task i when it ends at time point n
- $S_{s,n}$ = inventory of state s at time point n
- $R_{i,r,n}$ = amount of resource r consumed by task i at time point n
- $R^A_{r,n}$ = availability of resource r at time point n

The number of time events n is the critical point

STN-BASED CONTINUOUS FORMULATION
(UNIT-SPECIFIC TIME EVENT)

INTEC UNL CONICET

(Janak et al., 2004)

ALLOCATION CONSTRAINTS

$$\sum_{i \in I_j} W_{in} \leq 1 \quad \forall j, n$$

$$\sum_{n' \leq n} Ws_{in'} - \sum_{n' < n} Wf_{in'} = W_{in} \quad \forall i, n$$

$$\sum_{n} Ws_{in} = \sum_{n} Wf_{in} \quad \forall i$$

$$\sum_{n} Ws_{in} \leq 1 - \sum_{n' < n} Ws_{in'} + \sum_{n' < n} Wf_{in'} \quad \forall i, n$$

$$Wf_{in} \leq \sum_{n' < n} Ws_{in'} - \sum_{n' < n} Wf_{in'} \quad \forall i, n$$

BATCH SIZE CONSTRAINTS

$$V_i^{min} W_{in} \leq B_{in} \leq V_i^{max} W_{in} \quad \forall i, n$$

$$B_{in} \leq B_{i(n-1)} - V_i^{\max}\left(1 - W_{i(n-1)} + Wf_{i(n-1)}\right) \quad \forall i, n > 1$$

$$B_{in} \geq B_{i(n-1)} - V_i^{\max}\left(1 - W_{i(n-1)} + Wf_{i(n-1)}\right) \quad \forall i, n > 1$$

$$Bs_{in} \leq B_{in} \quad \forall i, n$$

$$Bs_{in} \leq B_{in} + V_i^{\max} Ws_{in} \quad \forall i, n$$

$$Bs_{in} \geq B_{in} - V_i^{\max}\left(1 - Ws_{in}\right) \quad \forall i, n$$

$$Bf_{in} \leq B_{in} \quad \forall i, n$$

$$Bf_{in} \leq B_{in} + V_i^{\max} Wf_{in} \quad \forall i, n$$

$$Bf_{in} \geq B_{in} - V_i^{\max}\left(1 - Wf_{in}\right) \quad \forall i, n$$

MATERIAL BALANCE

$$S_{sn} = S_{s(n-1)} + \sum_{i \in I_s^p} \rho_{is}^p Bf_{i(n-1)} + \sum_{i^{st} \in I_s^{ST}} B_{i^{st}(n-1)} - \sum_{i \in I_s^c} \rho_{is}^c Bs_{in} - \sum_{i^{st} \in I_s^{st}} B_{i^{st}n} \quad \forall s, n$$

STORAGE CAPACITY

$$B_{i^{st}n} \leq C_s^{\max} \quad \forall s, i^{st} \in I_s^{st}, n$$

STN-BASED CONTINUOUS FORMULATION

(UNIT-SPECIFIC TIME EVENT)

$$Tf_{in} \geq Ts_{in} \qquad \forall i, n$$

$$Tf_{in} \leq Ts_{in} + H\, W_{in} \qquad \forall i, n$$

$$Ts_{in} \leq Tf_{i(n-1)} + H\left(1 - W_{i(n-1)} + Wf_{i(n-1)}\right) \qquad \forall i, n > 1$$

$$Tf_{in'} - Ts_{in} \geq \alpha_i Ws_{in} + \beta_i B_{in} + H\left(1 - Ws_{in}\right) + H\left(1 - Wf_{in'}\right) + H\left(\sum_{n \leq n'' \leq n'} Wf_{in''}\right) \qquad \forall i, n, n', (n \leq n')$$

$$Tf_{in'} - Ts_{in} \leq \alpha_i Ws_{in} + \beta_i B_{in} + H\left(1 - Ws_{in}\right) + H\left(1 - Wf_{in'}\right) + H\left(\sum_{n \leq n'' \leq n'} Wf_{in''}\right) \qquad \forall i \in I^{ZW}, n, n', (n \leq n')$$

$$Ts_{in} \geq Tf_{i(n-1)} \qquad \forall i, n > 1$$

$$Ts_{in} \geq Tf_{i'(n-1)} + cl_{i'i} + H\left(1 - Wf_{i'(n-1)} - Ws_{in}\right) \qquad \forall i, i', i \neq i', j \in J_{ii'}, n > 1$$

$$Ts_{in} \geq Tf_{i'(n-1)} + H\left(1 - Wf_{i'(n-1)}\right) \qquad \forall s, i \in I_s^c, i' \in I_s^p, j \in J_i, j' \in J_{i'}, j \neq j', n > 1$$

$$Ts_{in} \leq Tf_{i'(n-1)} + H\left(2 - Wf_{i'(n-1)} - Ws_{in}\right) \qquad \forall s \in S^{ZW}, i \in I_s^c, i' \in I_s^p, j \in J_i, j' \in J_{i'}, j \neq j', n > 1$$

TIMING AND SEQUENCING CONSTRAINTS (PROCESSING TASKS)

STN-BASED CONTINUOUS FORMULATION
(UNIT-SPECIFIC TIME EVENT)

TIMING AND SEQUENCING CONSTRAINTS (STORAGE TASKS)

$$Tf_{i^{st}n} \geq Ts_{i^{st}n} \qquad \forall i^{st}, n$$
$$Ts_{i^{st}n} \geq Tf_{i(n-1)} - H\left(1 - Wf_{i(n-1)}\right) \qquad \forall s, i \in I_s^p, i^{st} \in I_s^{ST}, n > 1$$
$$Ts_{i^{st}n} \leq Tf_{i(n-1)} + H\left(1 - Wf_{i(n-1)}\right) \qquad \forall s, i \in I_s^p, i^{st} \in I_s^{ST}, n > 1$$
$$Ts_{in} \geq Tf_{i^{st}(n-1)} \qquad \forall s, i \in I_s^c, i^{st} \in I_s^{ST}, n > 1$$
$$Ts_{in} \leq Tf_{i^{st}(n-1)} + H\left(1 - Ws_{in}\right) \qquad \forall s, i \in I_s^c, i^{st} \in I_s^{ST}, n > 1$$
$$Ts_{i^{st}n} = Tf_{i^{st}(n-1)} \qquad \forall i^{st}, n > 1$$

RESOURCE BALANCE

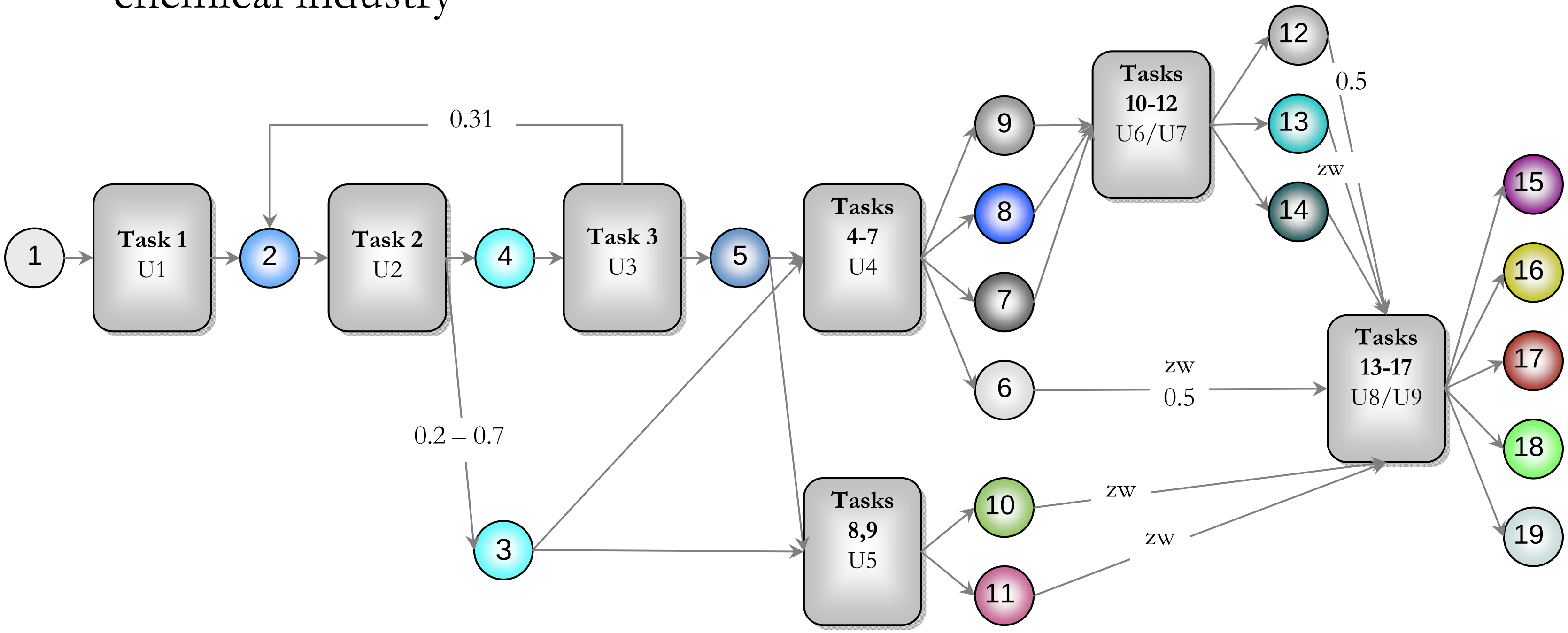
$$R_{irn} = \mu_{ir}^c W_{in} + \nu_{ir}^c B_{in} \qquad \forall r, i \in I_r, n$$
$$\sum_{i \in I_r} R_{irn} + R_{rn}^A = R_r^{\max} \qquad \forall r, n = 1$$
$$\sum_{i \in I_r} R_{irn} + R_{rn}^A = \sum_{i \in I_r} R_{ir(n-1)} + R_{r(n-1)}^A \qquad \forall r, n > 1$$

TIMING AND SEQUENCING OF RESOURCE USAGE

$$Tf_{rn} \geq Ts_{rn} \qquad \forall r, n$$
$$Tf_{i(n-1)} \geq Ts_{rn} - H\left(1 - W_{i(n-1)} + Wf_{i(n-1)}\right) \qquad \forall r, i \in I_r, n > 1$$
$$Tf_{i(n-1)} \leq Ts_{rn} - H\left(1 - W_{i(n-1)}\right) \qquad \forall r, i \in I_r, n > 1$$
$$Ts_{rn} \geq Ts_{in} - H\left(1 - W_{in}\right) \qquad \forall r, i \in I_r, n$$
$$Ts_{rn} \leq Ts_{in} + H\left(1 - W_{in}\right) \qquad \forall r, i \in I_r, n$$
$$Ts_{rn} = Tf_{r(n-1)} \qquad \forall r, n > 1$$

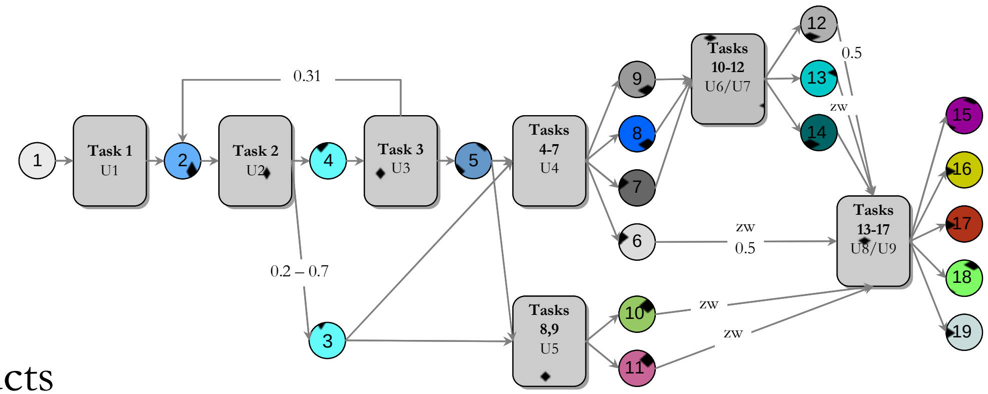
CASE STUDY: Westenberger & Kallrath (1995)

Benchmark problem for production scheduling in chemical industry

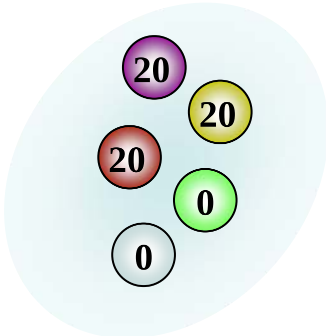


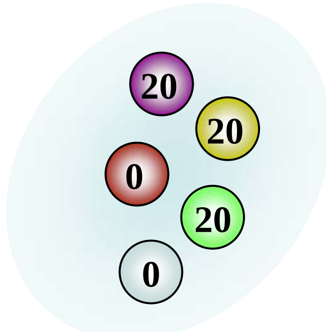
PROBLEM FEATURES

- 17 processing tasks, 19 states
- 9 production units
- 37 material flows
- Batch mixing / splitting
- Cyclical material flows
- Flexible output proportions
- Non-storable intermediate products
- No initial stock of final products
- Unlimited storage for raw material and final products
- Sequence-dependent changeover times



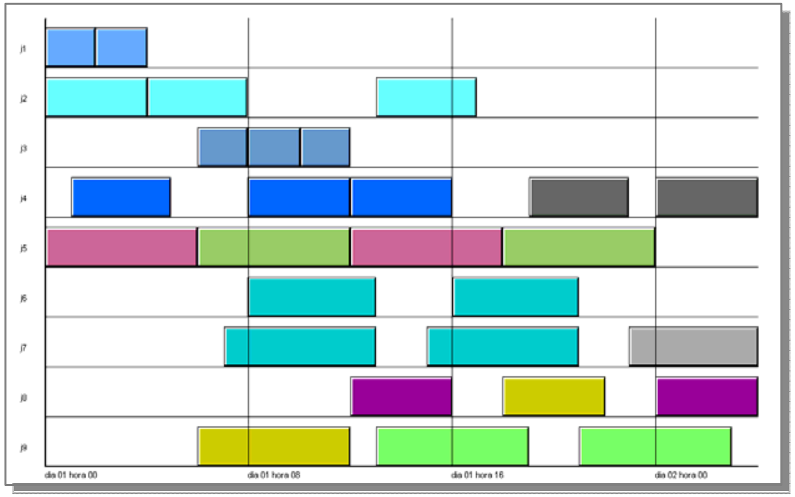
MAKESPAN MINIMIZATION





Instance	*A*			*B*		
Formulation	*Discrete*	*Continuous*		*Discrete*	*Continuous*	
time points	30	8	9	30	7	8
binary variables	720	384	432	720	336	384
continuous variables	3542	2258	2540	3542	1976	2258
constraints	6713	4962	5585	6713	4343	4964
LP relaxation	9.9	24.2	24.1	9.9	25.2	24.3
objective	**28**	**28**	**28**	**28**	**32**	**30**
iterations	728	78082	27148	2276	58979	2815823
nodes	10	1180	470	25	1690	63855
CPU time (s)	1.34	108.39	51.41	4.41	66.45	3600.21
relative gap	0.0	0.0	0.0	0.0	0.0	0.067

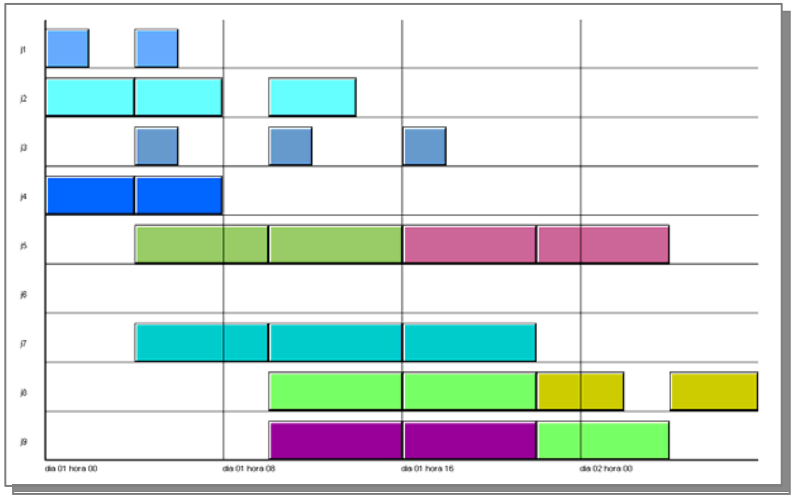
MAKESPAN MINIMIZATION



Discrete model

Time intervals: 30

Makespan: 28



Continuous model

Time points: 7

Makespan: 32

PROFIT MAXIMIZATION

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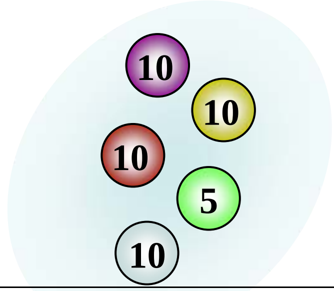
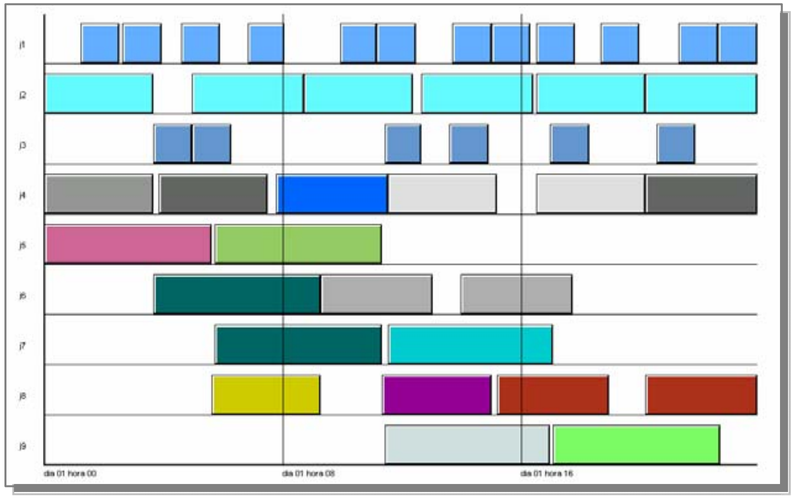
- H = 24 h



	Instance D			
	Discrete			*Continuous*
Formulation		*LB*	*UB*	
time points	240	24	24	14
binary variables	5760	576	576	672
continuous variables	28322	2834	2834	3950
constraints	47851	4794	4799	8476
LP relaxation	1769.9	1383.0	2070.9	1647.1
objective	**1425.8**	**1184.2**	**1721.8**	**1407.4**
iterations	449765	3133	99692	256271
nodes	5580	203	4384	1920
CPU time (s)	7202	6.41	58.32	258.54
relative gap	0.122	0.047	0.050	0.042

PROFIT MAXIMIZATION

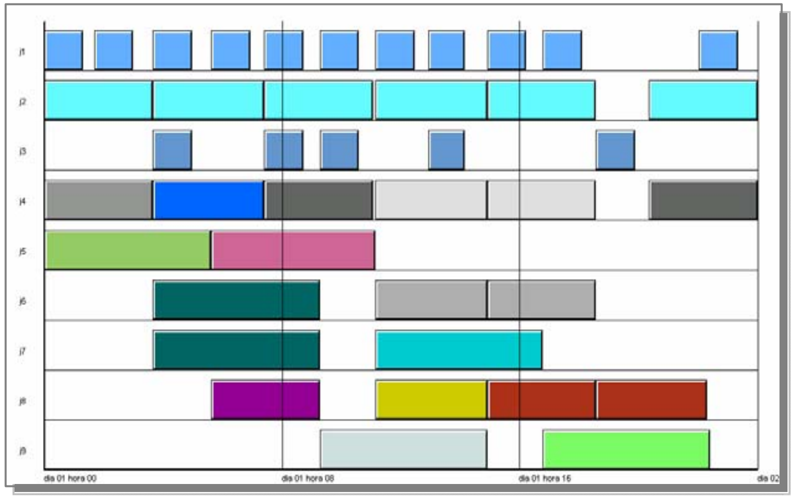
- H = 24 h



Discrete model

Time intervals: 240

Profit: **1425.8**



Continuous model

Time points: 14

Profit: **1407.4**

ROAD-MAP FOR OPTIMIZATION APPROACHES

INTEC
UNL CONICET

Main events involve changes in:
- Processing tasks (start and end)
- Availability of any resource
- Resource requirement of a task

EVENT REPRESENTATION

NETWORK-ORIENTED PROCESSES
- DISCRETE TIME
  - Global time intervals (STN or RTN)
- CONTINUOUS TIME
  - Global time points (STN or RTN)
  - Unit- specific time event (STN)

Key point: reference points to check resources

BATCH-ORIENTED PROCESSES
- CONTINUOUS TIME
  - Time slots
  - Unit-specific direct precedence
  - Global direct precedence
  - Global general precedence

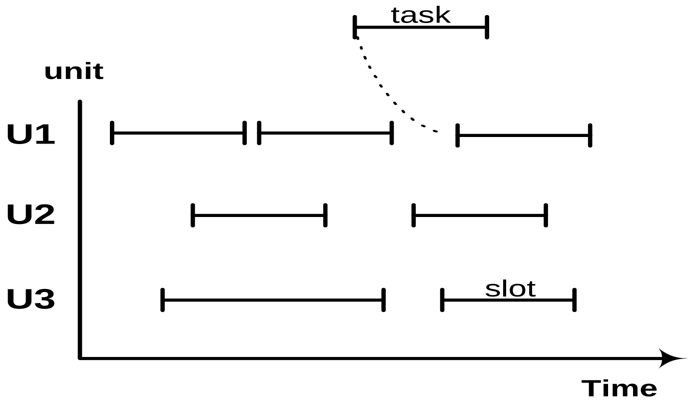
Key point: arrange resource utilization

SLOT-BASED CONTINUOUS TIME FORMULATIONS

(Pinto and Grossmann (1995, 1996); Chen et. al. ,2002; Lim and Karimi, 2003)

MAIN ASSUMPTIONS

- A number of time slots with unknown duration are postulated to be allocated to batches
- Batches to be scheduled are defined a priori
- No mixing and splitting operations
- Batches can start and finish at any time during the scheduling horizon



ADVANTAGES

- Significant reduction in model size when a minimum number of time slots is predefined
- Good computational performance
- Simple model and easy representation for sequencing and allocation scheduling problems

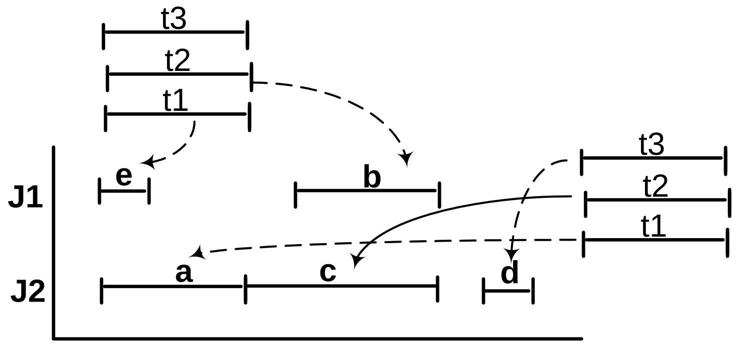
DISADVANTAGES

- Resource and inventory constraints are difficult to model
- Model size and complexity depend on the number of time slots predefined
- Sub-optimal or infeasible solution can be generated if the number of time slots is smaller than required

MAJOR MODEL VARIABLES

BINARY VARIABLES: $W_{i,j,k,l}$

batch (i), unit (j), slot (k), stage (l)



$W_{i,j,k,l}$ = 1 only if stage l of batch i is allocated to slot k of unit j

CONTINUOUS VARIABLES:

$Ts_{i,l}$ = start time of stage l of batch i
$Tf_{i,l}$ = end time of stage l of batch l

$Ts_{j,k}$ = start time of slot k in unit j
$Tf_{j,k}$ = end time of slot k in unit j

The number of time slots k is the critical point

SLOT-BASED CONTINUOUS TIME FORMULATIONS

(Pinto and Grossmann (1995)

$$\sum_{j}\sum_{k\in K_j} W_{ijkl} = 1 \quad \forall i, l \in L_i$$

BATCH ALLOCATION

$$\sum_{i}\sum_{l\in L_i} W_{ijkl} \le 1 \quad \forall j, k \in K_j$$

SLOT ALLOCATION

$$Tf_{jk} = Ts_{jk} + \sum_{i}\sum_{l\in L_i} W_{ijkl}\left(p_{ij} + su_{ij}\right) \quad \forall j, k \in K_j$$

SLOT TIMING

$$Tf_{il} = Ts_{il} + \sum_{j}\sum_{k\in K_j} W_{ijkl}\left(p_{ij} + su_{ij}\right) \quad \forall i, l \in L_i$$

BATCH TIMING

$$Tf_{jk} \le Ts_{j(k+1)} \quad \forall j, k \in K_j$$

SLOT SEQUENCING

$$Tf_{il} \le Ts_{i(l+1)} \quad \forall j, k \in K_j$$

STAGE SEQUENCING

$$-M\left(1 - W_{ijkl}\right) \le Ts_{il} - Ts_{jk} \quad \forall i, j, k \in K_j, l \in L_i$$

$$M\left(1 - W_{ijkl}\right) \ge Ts_{il} - Ts_{jk} \quad \forall i, j, k \in K_j, l \in L_i$$

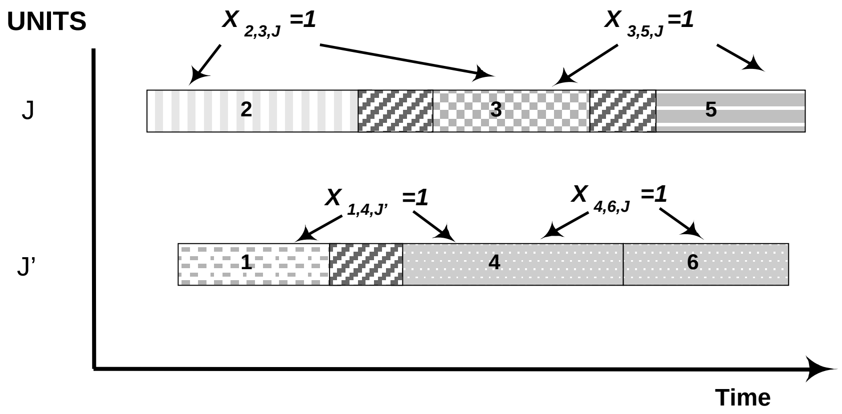
SLOT-BATCH MATCHING

UNIT-SPECIFIC DIRECT PRECEDENCE

(Cerdá et al., 1997).

MAIN ASSUMPTIONS

- Batches to be scheduled are defined a priori
- No mixing and splitting operations
- Batches can start and finish at any time during the scheduling horizon



6 BATCHES, 2 UNITS

6 x 5 x 2= 60 SEQUENCING VARIABLES

ADVANTAGES

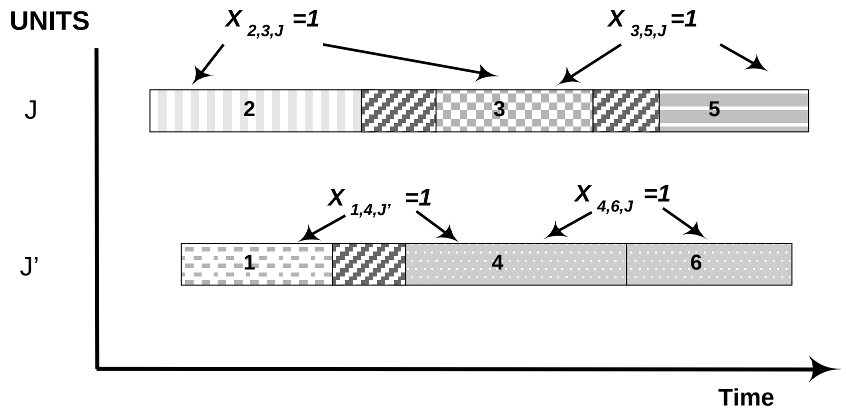
- Sequencing is explicitly considered in model variables
- Changeover times and costs are easy to implement

DISADVANTAGES

- Large number of sequencing variables
- Resource and material balances are difficult to model

UNIT-SPECIFIC DIRECT PRECEDENCE

MAJOR MODEL VARIABLES



BINARY VARIABLES: $X_{i,i',j}$ (i → batch, i' → batch, j → unit)

$X_{i,i',j}$ = 1 only if batch i' is processed immediately after that batch i in unit j

$Xf_{i,j}$ = 1 only if batch i is first processed in unit j

CONTINUOUS VARIABLES:

Ts_i = start time of batch i

Tf_i = end time of batch i

The number of predecessors and units is the critical point

UNIT-SPECIFIC DIRECT PRECEDENCE

(Cerdá et al., 1997).

$$\sum_{i \in I_j} XF_{ij} = 1 \quad \forall j$$

FIRST BATCH IN THE PROCESSING SEQUENCE

$$\sum_{j \in J_i} XF_{ij} + \sum_{j \in J_i} \sum_{i' \in I_j} X_{i'ij} = 1 \quad \forall i$$

FIRST OR WITH ONE PREDECESSOR

$$\sum_{i' \in I_j} X_{ii'j} \le 1 \quad \forall i$$

AT MOST ONE SUCCESSOR

$$XF_{ij} + \sum_{i' \in I_j} X_{i'ij} + \sum_{\substack{j' \in J_i \\ j \neq j'}} \sum_{i' \in I_j} X_{ii'j'} \le 1 \quad \forall i, j \in J_i$$

SUCCESSOR AND PREDECESSOR IN THE SAME UNIT

$$Tf_i = Ts_i + \sum_{j \in J_i} tp_i \left(XF_{ij} + \sum_{i' \in I_j} X_{i'ij} \right) \quad \forall i$$

PROCESSING TIME

$$Ts_i \ge Tf_{i'} + \sum_{j \in J_{ii'}} cl_{i'i} X_{i'ij} - M \left(1 - \sum_{j \in J_{ii'}} X_{i'ij} \right) \quad \forall i, i'$$

SEQUENCING

GLOBAL DIRECT PRECEDENCE

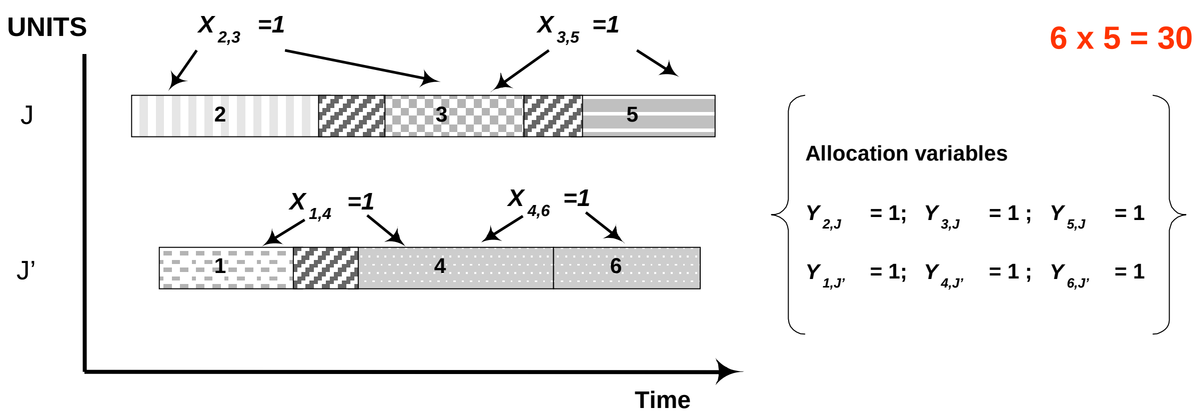
(Méndez et al., 2000; Gupta and Karimi, 2003)

MAIN ASSUMPTIONS

- Batches to be scheduled are defined a priori
- No mixing and splitting operations
- Batches can start and finish at any time during the scheduling horizon

6 BATCHES, 2 UNITS

6 x 5 = 30 SEQUENCING VARIABLES



ADVANTAGES

- Sequencing is explicitly considered in model variables
- Changeover times and costs are easy to implement

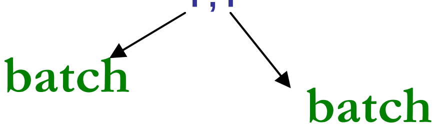
DISADVANTAGES

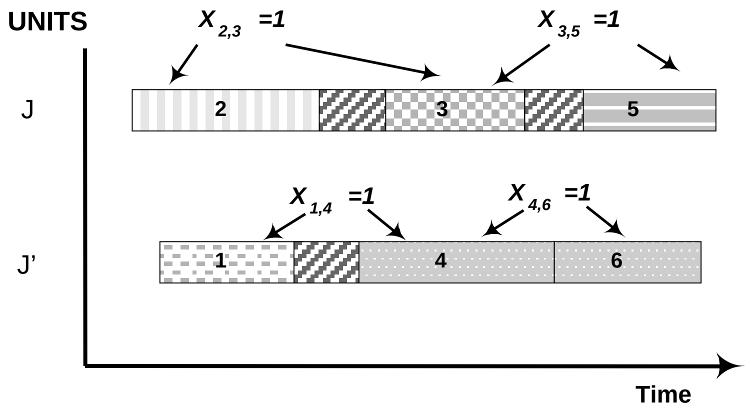
- Large number of sequencing variables
- Resource and material balances are difficult to model

GLOBAL DIRECT PRECEDENCE

MAJOR MODEL VARIABLES

BINARY VARIABLES: $X_{i,i'}$





$X_{i,i'}$ = 1 only if batch i' is processed immediately after that batch i in unit j
$W_{i,J}$ = 1 only if batch i' is processed in unit j
$Xf_{i,j}$ = 1 only if batch i is first processed in unit j

CONTINUOUS VARIABLES:

Ts_i = start time of batch i
Tf_i = end time of batch i

The number of predecessors is the critical point

GLOBAL DIRECT PRECEDENCE

(Méndez et al., 2000)

$$\sum_{i \in I_j} XF_{ij} \le 1 \quad \forall j$$

AT MOST ONE FIRST BATCH IN THE PROCESSING SEQUENCE

$$\sum_{j \in J_i} XF_{ij} + \sum_{j \in J_i} W_{ij} = 1 \quad \forall i$$

ALLOCATION CONSTRAINT

$$XF_{ij} + W_{ij} \le W_{i'j} - X_{ii'} + 1 \quad \forall i, i', j \in J_{ii'}$$

$$XF_{ij} + W_{ij} \le 1 - X_{ii'} \quad \forall i, i', j \in \left(J_i - J_{ii'}\right)$$

SEQUENCING-ALLOCATION MATCHING

$$\sum_{j \in J_i} XF_{ij} + \sum_{i'} X_{i'i} = 1 \quad \forall i$$

FIRST OR WITH ONE PREDECESSOR

$$\sum_{i'} X_{ii'} \le 1 \quad \forall i$$

AT MOST ONE SUCCESSOR

$$Tf_i = Ts_i + \sum_{j \in J_i} tp_i \left(XF_{ij} + W_{ij}\right) \quad \forall i$$

$$Ts_{i'} \ge Tf_i + \sum_{j \in J_i} \left(cl_{ii'} + su_{i'j}\right) W_{i'j} - M\left(1 - X_{ii'}\right) \quad \forall i$$

TIMING AND SEQUENCING

GLOBAL GENERAL PRECEDENCE

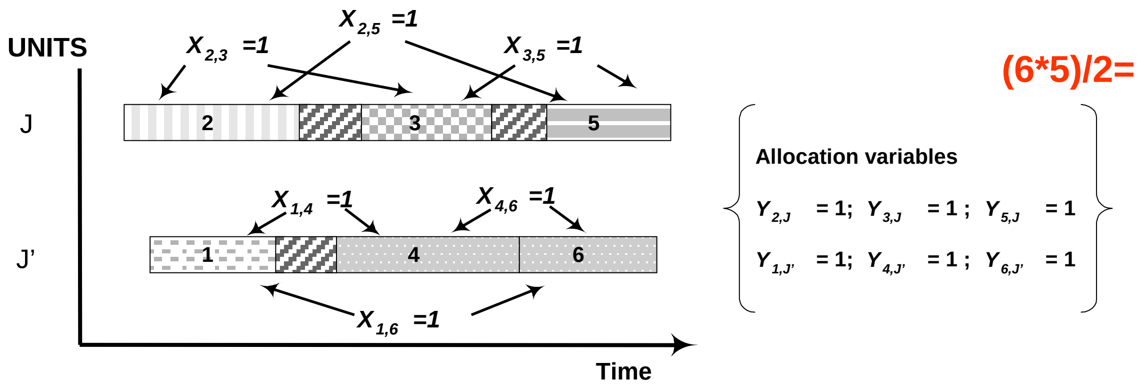
(Méndez et al., 2001; Méndez and Cerdá (2003,2004))

MAIN ASSUMPTIONS

- Batches to be scheduled are defined a priori
- No mixing and splitting operations
- Batches can start and finish at any time during the scheduling horizon

6 BATCHES, 2 UNITS

(6*5)/2= 15 SEQUENCING VARIABLES



ADVANTAGES

- General sequencing is explicitly considered in model variables
- Changeover times and costs are easy to implement
- Lower number of sequencing decisions
- Sequencing decisions can be extrapolated to other resources

DISADVANTAGES

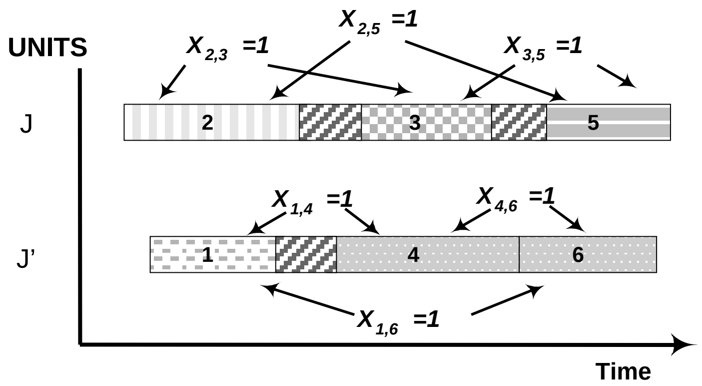
- Material balances are difficult to model, no reference points

GLOBAL GENERAL PRECEDENCE

MAJOR MODEL VARIABLES

BINARY VARIABLES: $X_{i,i'}$

batch (i) — batch (i')



$X_{i,i'}$ = 1 only if batch i' is processed after that batch i in unit j

$W_{i,J}$ = 1 only if batch i' is processed in unit j

CONTINUOUS VARIABLES:

Ts_i = start time of batch i

Tf_i = end time of batch i

CAN BE EASILY GENERALIZED TO MULTISTAGE PROCESSES AND TO SEVERAL RESORCES

The number of predecessors is the critical point

GLOBAL GENERAL PRECEDENCE

(Méndez and Cerdá, 2003)

$$\sum_{j \in J_{il}} W_{ilj} = 1 \quad \forall i, l \in L_i$$

ALLOCATION CONSTRAINT

$$Tf_{il} = Ts_{il} + \sum_{j \in J_{il}} tp_{ilj} W_{ilj} \quad \forall i, l \in L_i$$

PROCESSING TIME

$$Ts_{i'l'} \geq Tf_{il} + cl_{il,i'l'} + su_{i'l'} - M\left(1 - X_{il,i'l'}\right) - M\left(2 - W_{ilj} - W_{i'l'j}\right) \quad \forall i, i', l \in L_i, l' \in L_{i'}, j \in J_{il,i'l'}$$

SEQUENCING CONSTRAINTS

$$Ts_{il} \geq Tf_{i'l'} + cl_{i'l',il} + su_{il} - M\, X_{il,i'l'} - M\left(2 - W_{ilj} - W_{i'l'j}\right) \quad \forall i, i', l \in L_i, l' \in L_{i'}, j \in J_{il,i'l'}$$

$$Ts_{il} \geq Tf_{i(l-1)} \quad \forall i, l \in L_i, l > 1$$

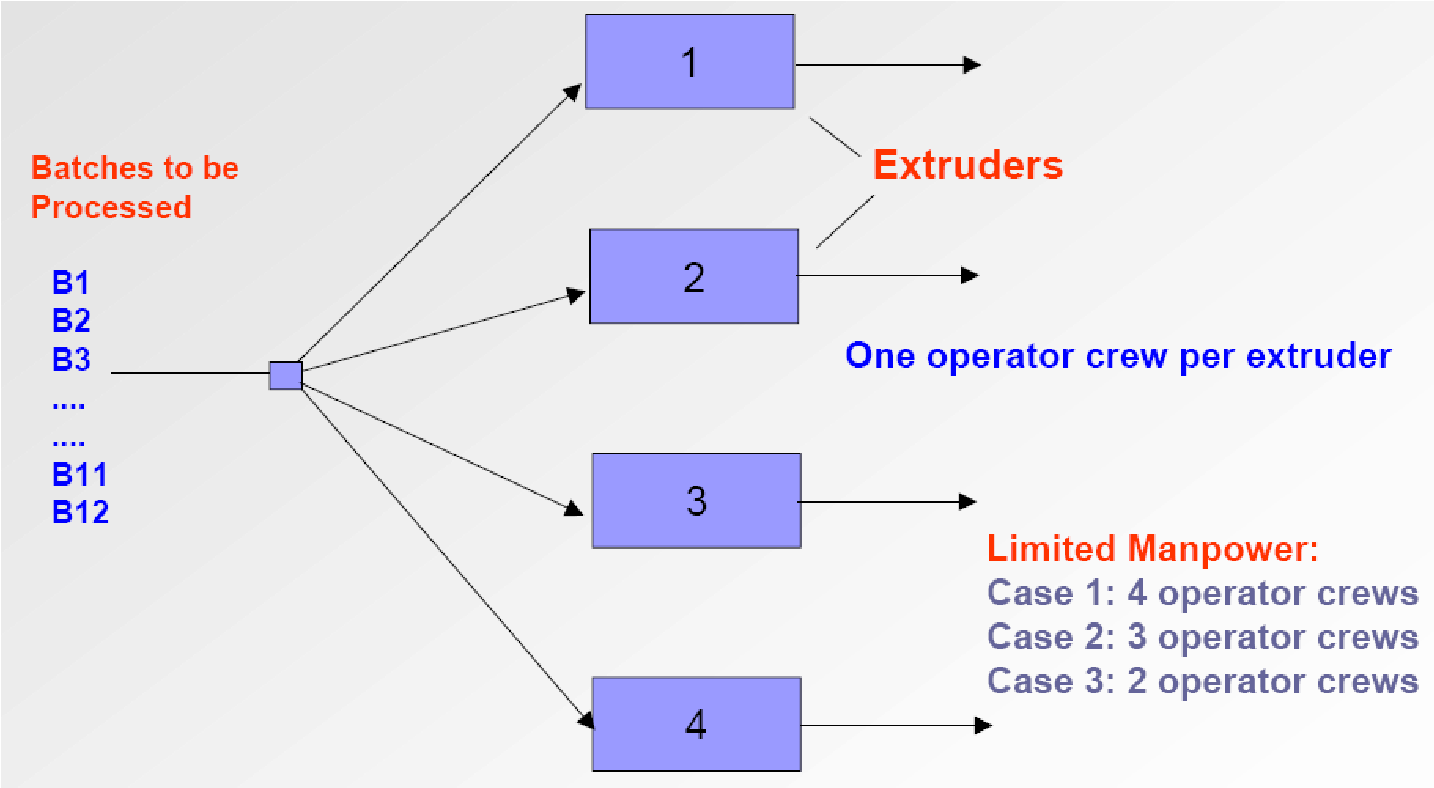
STAGE PRECEDENCE

SUMMARY OF OPTIMIZATION APPROACHES

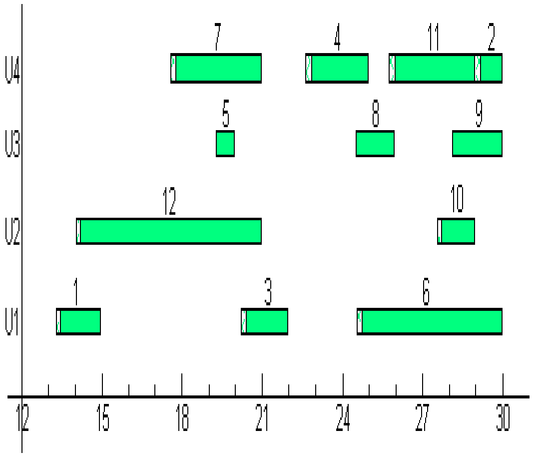
Time representation	DISCRETE	CONTINUOUS					
Event representation	Global time intervals	Global time points	Unit-specific time events	Time slots*	Unit-specific immediate precedence*	Immediate precedence*	General precedence*
Main decisions	------------------ Lot-sizing, allocation, sequencing, timing -----------------				-------- Allocation, sequencing, timing ----------		
Key discrete variables	W_{ijt} defines if task *I* starts in unit *j* at the beginning of time interval *t*.	Ws_{in} / Wf_{in} define if task *i* starts/ends at time point *n*. $W_{inn'}$ defines if task *i* starts at time point *n* and ends at time point *n'*.	Ws_{in} / W_{in} / Wf_{in} define if task *i* starts/is active/ends at event point *n*.	W_{ijk} define if unit *j* starts task *i* at the beginning of time slot *k*.	$X_{ii'j}$ defines if batch *i* is processed right before of batch *i'* in unit *j*. XF_{ij} defines if batch *i* starts the processing sequence of unit *j*.	$X_{ii'}$ defines if batch *i* is processed right before of batch *i'*. XF_{ij} / W_{ij} defines if batch *i* starts/is assigned to unit *j*.	$X'_{ii'}$ define if batch *i* is processed before or after of batch *i'*. W_{ij} defines if batch *i* is assigned to unit *j*
Type of process	-------------------------------- General network --------------------------------				---------------------- Sequential --------------------		
Material balances	Network flow equations (STN or RTN)	Network flow equations (STN or RTN)	--- Network flow equations --- (STN)		------------------- Batch-oriented -----------------		
Critical modeling issues	Time interval duration, scheduling period (data dependent)	Number of time points (iteratively estimated)	Number of time events (iteratively estimated)	Number of time slots (estimated)	Number of batch tasks sharing units (lot-sizing) and units	Number of batch tasks sharing units (lot-sizing)	Number of batch tasks sharing resources (lot-sizing)
Critical problem features	Variable processing time, sequence-dependent changeovers	Intermediate due dates and raw-material supplies	Intermediate due dates and raw-material supplies	Resource limitations	Inventory, resource limitations	Inventory, resource limitations	Inventory

RESOURCE-CONSTRAINED EXAMPLE

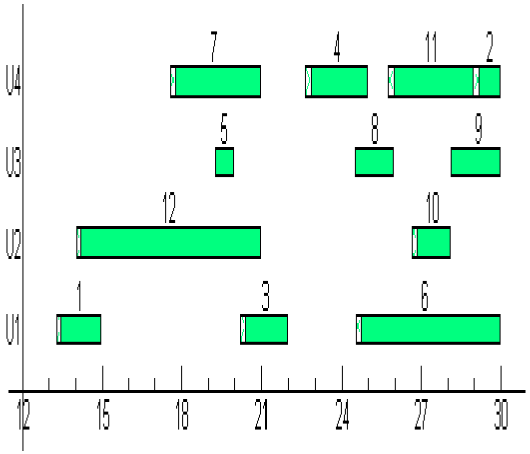
- 12 batches and 4 processing units in parallel
- Manpower limitations (4 , 3 , 2 operators crews)
- Specific batch due dates
- Total earliness minimization
- Three approaches: time-slots, general precedence and event times



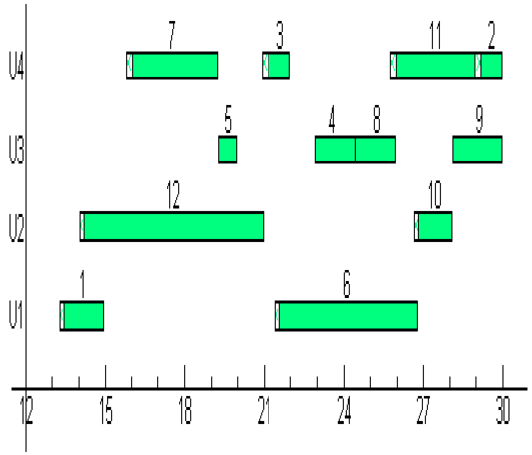
COMPUTATIONAL RESULTS



(a) without manpower limitation



(b) 3 operator crews



(c) 2 operators crews

Case Study	Event representation	Binary vars, cont. vars, constraints	Objective function	CPU time	Nodes
2.a	Time slots & preordering	100, 220, 478	1.581	67.74^a	456
	General precedence	82, 12, 202	1.026	0.11^b	64
	Unit-based time events (4)	150, 513, 1389	1.026	0.07^c	7
2.b	Time slots & preordering	289, 329, 1156	2.424	2224^a	1941
	General precedence	127, 12, 610	1.895	7.91^b	3071
	Unit-based time events (12)	458, 2137, 10382	1.895	6.53^c	1374
2.c	Time slots & preordering	289, 329, 1156	8.323	76390^a	99148
	General precedence	115, 12, 478	7.334	35.87^b	19853
	Unit-based time events (12)	446, 2137, 10381	7.909	178.85^c	42193

TIGHTENING CONSTRAINTS

MAJOR GOAL

Use additional constraints to

- **Obtain a good estimation of problem variables related to the objective function (makespan, tardiness, earliness)**

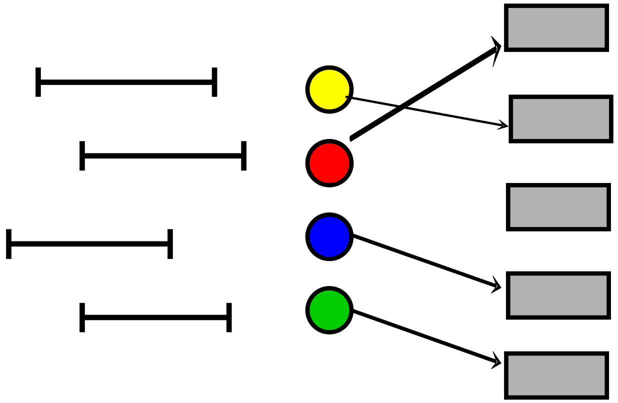
- **Accelerate the pruning process by producing a better estimation on the RMIP solution value at each node**

- **Exploit the information provided by 0-1 decision variables**

- **Reduce computational effort**

SCHEDULING OF A SINGLE-STAGE BATCH PLANT

OBJECTIVE: MINIMUM MAKESPAN



If unit-dependent setup times are required

$$ru_j^* + \sum_{i \in I_j} \left(su_{ij} + pt_{ij}\right) Y_{ij} \leq MK \qquad \forall\, j \in J$$

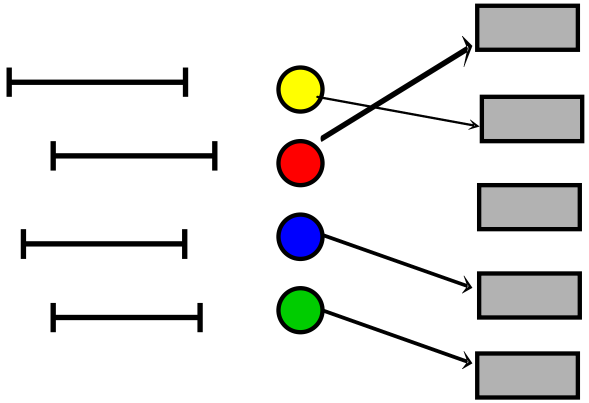
where $$ru_j^* = \mathrm{Max}\left[ru_j, \mathrm{Min}_{i \in I_j} \left[rt_i - su_{ij} \right] \right]$$

is a better estimation of the *j*th-unit ready time because it also considers the release times of the candidate tasks for unit *j*.

The estimation for makespan is based only on assignment variables.

SCHEDULING OF A SINGLE-STAGE BATCH PLANT

OBJECTIVE: MINIMUM MAKESPAN



If sequence-dependent setup times are required

$$ru_j^* - \underset{i \in I_j}{\text{Max}}\,[\sigma_{ij}^{Min}] + \sum_{i \in I_j} \left(\sigma_{ij}^{Min} + su_{ij} + pt_{ij}\right) Y_{ij} \leq MK \qquad \forall j \in J$$

where

$$ru_j^* = \text{Max}\left[ru_j , \underset{i \in I_j}{\text{Min}} \left[rt_i - su_{ij} \right] \right]$$

$$\sigma_{ij}^{Min} = \underset{i' \in I_j : i' \neq i}{\text{Min}} \left[\tau_{i'ij} \right]$$

The estimation for makespan is based only on assignment variables.

COMPUTATIONAL RESULTS

WITHOUT TIGHTENING CONSTRAINTS

n	Binary vars, Continuous vars, Constraints	Example 1A: Sequence independent setup times				Example 1B: Sequence dependent setup times			
		Objective Function	Relative Gap (%)	CPU time (sec.)	Nodes	Objective Function	Relative Gap (%)	CPU time (sec.)	Nodes
12	**82, 25, 214**	**8.428**	**-**	**19.03**	**94365**	**8.645**	**-**	**8.36**	**39350**
16	**140, 33, 382**	**12.353**	**2.43**	**3600 †**	**8893218**	**12.854**	**-**	**1188.50**	**3421982**
18	**161, 37, 444**	**13.985**	**-**	**2872.81**	**7166701**	**14.633**	**27.07**	**3600 †**	**8708577**
20	**201, 41, 558**	**15.268**	**22.62**	**3600 †**	**6282059**	**15.998**	**21.95**	**3600 †**	**6570231**

WITH TIGHTENING CONSTRAINTS

12	**82, 25, 218**	**8.428**	**-**	**0.05**	**12**	**8.645**	**-**	**0.05**	**15**
16	**140, 33, 386**	**12.353**	**-**	**0.03**	**1**	**12.854**	**-**	**0.09**	**44**
18	**161, 37, 448**	**13.985**	**-**	**0.11**	**27**	**14.611**	**-**	**40.36**	**116413**
20	**201, 41, 562**	**15.268**	**-**	**0.14**	**21**	**15.998**	**-**	**183.56**	**417067**
22	**228, 45, 622**	**15.794**	**-**	**0.20**	**49**	**16.396**	**-**	**167.09**	**359804**
25	**286, 51, 792**	**18.218**	**-**	**0.42**	**110**	**19.064 ***	**-**	**79.25**	**109259**
29	**382, 59, 1064**	**23.302**	**-**	**0.61**	**82**	**24.723 ***	**-**	**5.92**	**5385**
35	**532, 71, 1430**	**26.683**	**-**	**0.97**	**90**				
40	**625, 81, 1656**	**28.250**	**-**	**0.91**	**34**				

Marchetti, P. A. and Cerdá, J., Submitted 2007

SOLUTION OF A LARGE-SCALE MULTISTAGE PROCESS

MULTISTAGE MULTIPRODUCT BATCH PROCESS

Major problem features (Pharmaceutical industry)

17 processing units

5 processing stages

30 to 300 production orders per week (thousands of batch operations)

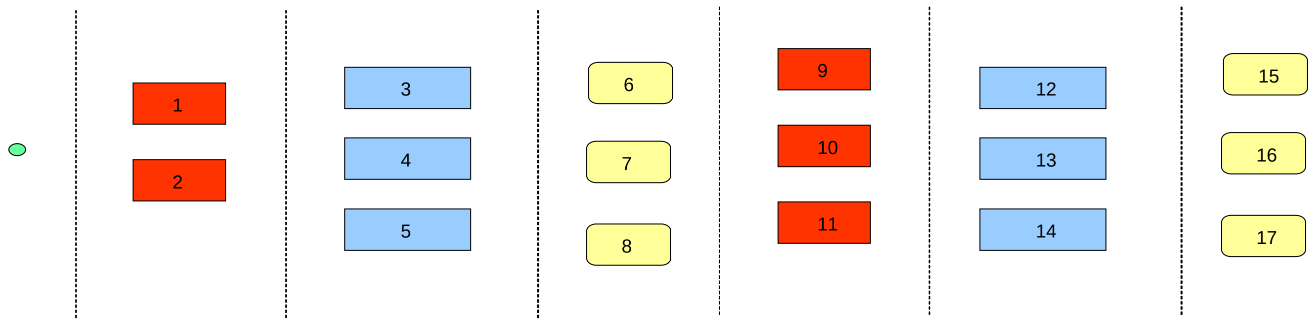
Different processing times (0.2 h to 3 h)

Sequence-dependent changeovers (0.5 h to 2 h)

Allocation restrictions

Few minutes to generate the schedule

Rescheduling on a daily basis



SOLUTION STRATEGY

PROPOSED TWO-STAGE SOLUTION STRATEGY

FIRST STAGE: CONSTRUCTIVE STAGE

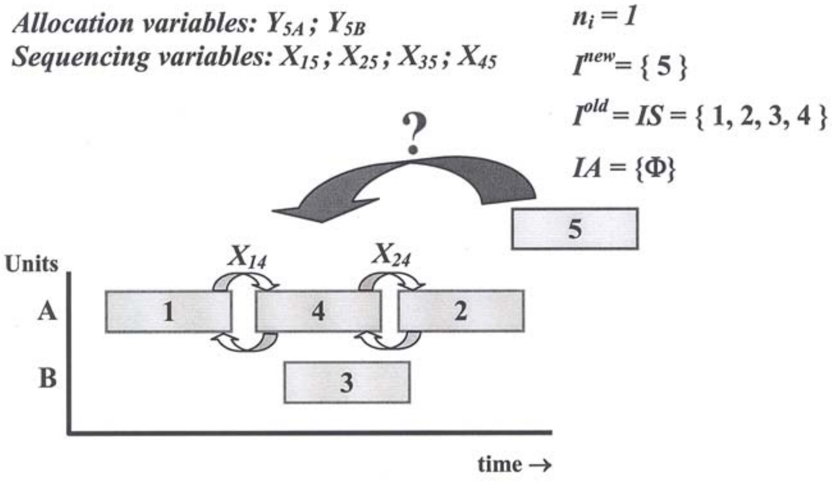
BASED ON A REDUCED MILP-BASED MODEL

GENERATE THE BEST POSSIBLE SCHEDULE IN A SHORT-TIME
OPTION: GENERATE A FULL SCHEDULE BY INSERTING ORDERS ONE BY ONE

SECOND STAGE: IMPROVEMENT STAGE

BASED ON A REDUCED MILP-BASED MODEL

IMPROVE THE INITIAL SCHEDULE BY LOCAL RE-ASSIGNMENTS AND RE-SEQUENCING



GENERAL MILP MODEL

(Méndez and Cerdá, 2003)

Multistage multipurpose batch plant

$$\sum_{j \in J_{il}} W_{ilj} = 1 \quad \forall i, l \in L_i$$

ALLOCATION CONSTRAINT

$$Tf_{il} = Ts_{il} + \sum_{j \in J_{il}} tp_{ilj} W_{ilj} \quad \forall i, l \in L_i$$

PROCESSING TIME

$$Ts_{i'l'} \geq Tf_{il} + cl_{il,i'l'} + su_{i'l'} - M\left(1 - X_{il,i'l'}\right) - M\left(2 - W_{ilj} - W_{i'l'j}\right) \quad \forall i, i', l \in L_i, l' \in L_{i'}, j \in J_{il,i'l'}$$

SEQUENCING CONSTRAINTS

$$Ts_{il} \geq Tf_{i'l'} + cl_{i'l',il} + su_{il} - M\, X_{il,i'l'} - M\left(2 - W_{ilj} - W_{i'l'j}\right) \quad \forall i, i', l \in L_i, l' \in L_{i'}, j \in J_{il,i'l'}$$

$$Ts_{il} \geq Tf_{i(l-1)} \quad \forall i, l \in L_i, l > 1$$

STAGE PRECEDENCE

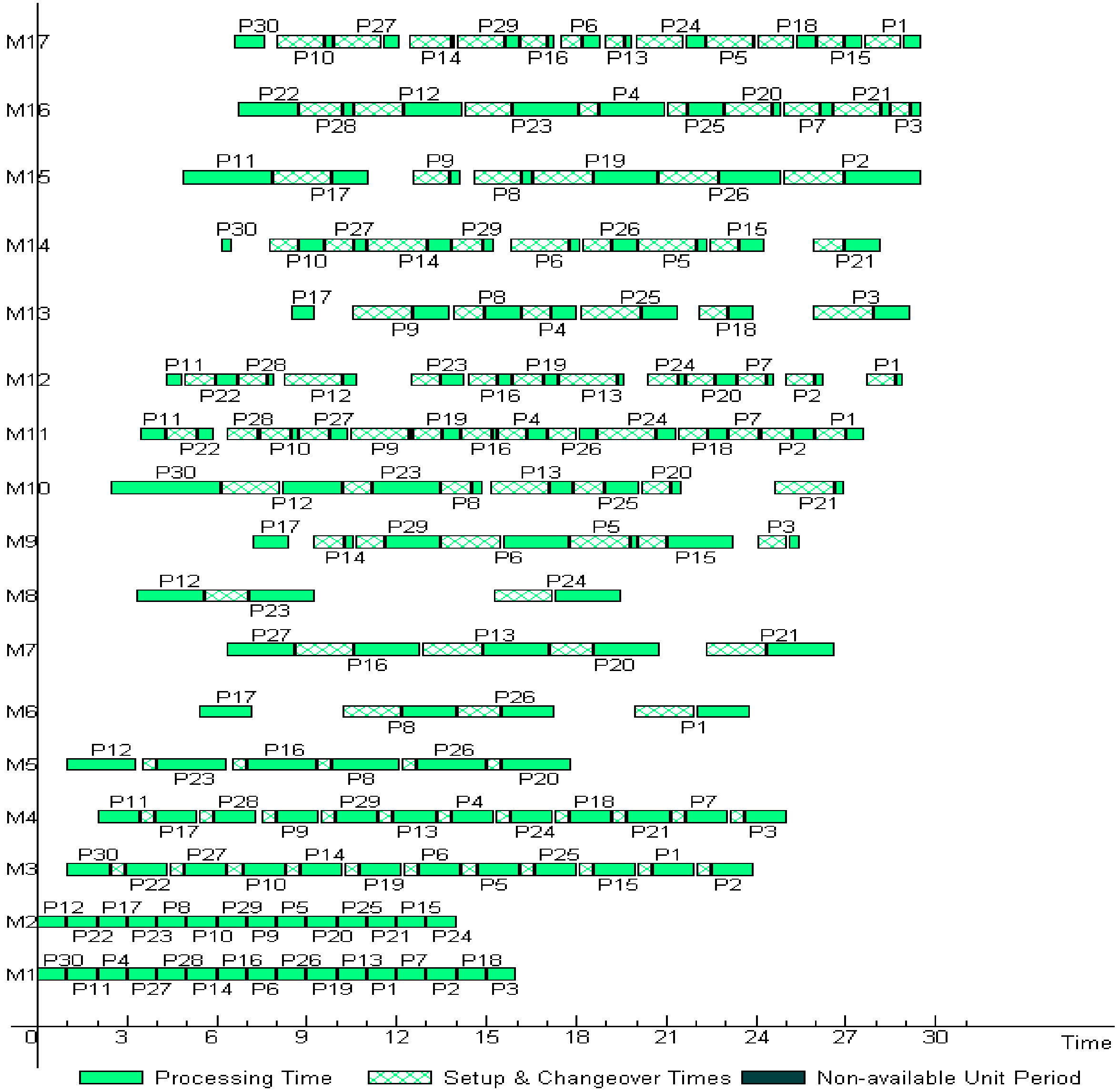
General problem representation

BEST SCHEDULE

SCHEDULE FOR 30-ORDER PROBLEM

TOTAL COMPUTATIONAL EFFORT

FEW MINUTES



COMPUTATIONAL RESULTS

Pure optimization approaches

Model	Bin. vars.	Cont. vars.	Cons.	RMIP	MIP	Best possible	CPUs	Nodes
MILP	2521	2708	10513	7.449	47.982	13.325	3600	62668
CP	-	1050	1300	-	79.989	-	3600	1432

Proposed solution strategy

			CPU-limit (s)	Makespan (h)		CPUs	
Orders	Ap.	NPS	Per phase	Constructive Stage	Improvement stage	Scheduling	Total
30	AP2	1	(10);(10)	33.149	31.175	86.4	188
30	AP2	2	(20);(10)	32.523	31.007	175	275
30	AP2	3	(30);(10)	34.447	31.787	255	355
50	AP2	1	(10);(10)	52.911	51.275	240	342
50	AP2	2	(15);(10)	52.964	51.080	321	429
50	AP3	3	(20);(10)	55.705	52.960	306	407

REMARKS

- Current optimization models are able to solve complex scheduling problems
- Small examples can be solved to optimality
- Discrete-time models may be computationally more effective than continuous-time
- Discrete-time models are usually more flexible than continuous-time models
- Difficult selection of the number of time or event points in the general continuous-time formulation.
- General continuous-time models become quickly computationally intractable for scheduling of medium complexity process networks.
- Problems with more than 150 time intervals are usually difficult to solve by using discrete time models.
- Problems with more than 15 time or event points appear intractable for continuous time models.
- Different performance depending on the objective function.

CONCLUSIONS

- Batch-oriented continuous models are more efficient for sequential processes and larger number of batches

- Batch-oriented models can incorporate practical process knowledge in a more natural way

- Resource constraints can be efficiently addressed without point references

- Inventory constraints seem very difficult to address without point references

- Combine other approaches with mathematical programming for solving large scale problems looks very promising