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Five Formulations of Extended Kalman Filter: Which is the best for D-RTO?

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Abstract

The Extended Kalman Filter (EKF) application encloses four important areas, related directly with control based on the real-time dynamic optimization strategies implantation to an industrial process: state estimation, unknown process parameters estimation, dynamic data reconciliation, and data filtering. The main goal of this paper is to evaluate the quality of five different EKF formulations for these four areas. The filters are applied in a studied case: the Sextuple Tank-Process. This process presents a high non-linearity degree and a RHP transmission zero, with multivariable gain inversion.

Keywords

D-RTO, EKF, MPC, Model Updating, Parameter Estimation.

1. Introduction

Real-time control based on the optimization of nonlinear dynamic process models has attracted increasing attention over the past decade, e.g. in chemical engineering. One important precondition of this methodology has a connection with approximate models used in the Dynamic Real-Time Optimization (D-RTO) and in Model-Based Predictive Controller (MPC) since these models require frequent updates. Optimization approaches with very different modeling fidelity has been investigated since the ability of a D-RTO system to track the changing optimum closely relies on an accurate model for representing the plant behavior [1, 2]. Moreover, the use of data preprocessing and dynamic data reconciliation techniques can considerably reduce the inaccuracy of process data due to measurement errors, improving the overall performance of the MPC when the data is first reconciled prior to being fed to the controller [3]. Hence, in order to take into account the requirements of D-RTO and MPC, in this work is used some formulations of EKF. Moreover, many physical systems exhibit nonlinear dynamics and have states subject to hard constraints, such as nonnegative concentrations or pressures [4-7]. As a result, many different types of nonlinear state estimators have been proposed, being the EKF the most popular because its application encloses four important areas related directly with advanced control strategies for industrial processes: state estimation, unknown process parameters estimation, dynamic data reconciliation, and data filtering.

2. Five Formulations of Extended Kalman Filter

- Extended Kalman Filter (EKF): The Continuous-Discrete Extended Kalman-Bucy Filter [9, 10] is used. The prediction stage consists basically of the integration of differential equations, from the dynamic model, and the differential equations related to the covariance matrix P, which are made between the sampling times.
- Continuous Extended Kalman Filter (CtEKF): The error covariance matrix is not updated in the correction stage. During the prediction stage, like in EKF, the integration of error covariance matrix is carried through with the differential equations of the system in a different proposed way [11].
- Discrete Extended Kalman Filter (DEKF): The basic difference between the DEKF and the EKF is that this formulation uses only the discrete form. The matrices Q and R are, respectively, the covariances of the process and the measurements noises (random errors). In order to make the transition from the discrete to continuous case, relations between Q_k and R_k and the corresponding Q and R for a small step size were used [11].
- Constrained Extended Kalman Filter (CEKF): The basic equations of CEKF can be divided, like in the EKF, in prediction and correction stages [4]. However, the integration of error covariance matrix is not carried through with the differential equations of the system. In correction stage, the system constrains directly appears in the optimization problem.
- Modified Discrete Extended Kalman Filter (MDEKF): The error covariance matrix is not updated in the correction stage. The prediction stage is similar to CEKF and the error covariance is estimated and updated in discrete form using the correction equation of CEKF.

The aforementioned process is non-linear and continuous. The prediction stage of the states $\hat{x}_k^-$ is carried out in the same way for all the formulations. It is gotten from the system states integration in the time interval $[t_{k-1}, t_k]$, according to the Equation 1. z, w and v are, respectively, vectors of measured variables, modeling and measurement errors. The filters formulations additional equations are showed in Table 1, where F and H are the Jacobian matrices of the functions f and h related to the $\hat{x}_k^-$ in the model provided by Equation 1.

$$\begin{aligned} \dot{x} &= f(x,u) + w(t) \\ x(t_{k-1}) &= \hat{x}_{k-1} \\ z_k &= h(\hat{x}_k^-) + v_k \end{aligned} \quad (1)$$

Table 1. Five Formulations of Extended Kalman Filter

Filter	*Prediction of P_k^-	K_k Gain	Correction of P_k
EKF	$\dot{P}=FP+PF^T+Q$ $P(t_{k-1})=P_{k-1}$	$K_k=P_k^- H_k^T (H_k P_{k-1}^- H_k^T+R_k)^{-1}$	$P_k=(I-K_kH_k)P_k^-(I-K_kH_k)^T+K_kR_kK_k^-$
CtEKF	$\dot{P}=FP+PF^T-PH^TR^{-1}HP+Q$ $P(t_{k-1})=P_{k-1}$	$K_k=P_k^- H_k^T (H_k P_{k-1}^- H_k^T+R_k)^{-1}$	-
DEKF	$P_k^-=\varphi_k P_{k-1}^- \varphi_k^T+Q_k$	$K_k=P_k^- H_k^T (H_k P_{k-1}^- H_k^T+R_k)^{-1}$	$P_k=(I-K_kH_k)P_k^-$
CEKF	$P_k^-=P_{k-1}$	$\min_{w_{k-1,k}} \Psi_k = \hat{w}_{k-1,k}^T (P_k^-)^{-1} \hat{w}_{k-1,k} + \hat{v}_{k,k}^T R_k^{-1} \hat{v}_{k,k}$ $\hat{x}_k = \hat{x}_k^- + \hat{w}_{k-1}$, $z_k = H\hat{x}_k^- + \hat{v}_k$	$P_k^-=Q_k+\varphi_k P_{k-1}^- \varphi_k^T - \varphi_k P_{k-1}^- H_k^T [H_k P_{k-1}^- H_k^T+R_k]^{-1} H_k P_{k-1}^- \varphi_k^T$
MDEKF	$P_k^-=Q_k+\varphi_k P_{k-1}^- \varphi_k^T - \varphi_k P_{k-1}^- H_k^T [H_k P_{k-1}^- H_k^T+R_k]^{-1} H_k P_{k-1}^- \varphi_k^T$	$K_k=P_k^- H_k^T (H_k P_{k-1}^- H_k^T+R_k)^{-1}$	-

* φ_k is the discrete states transition, carried through the discrete Jacobian matrix F_k.

3. Case Study

The proposed unit [12], depicted in Figure 1, consists of six interacting spherical tanks with different diameters D_i. The objective consists in controlling the levels of the lower tanks (h_1 and h_2), using as manipulated variables the flow rates (F_1 and F_2) and the valve distribution flow factors of these flow rates ($0 \leq x_1 \leq 1$, $0 \leq x_2 \leq 1$) that distribute the total feed among the tanks 3, 4, 5 and 6. The complemental flow rates feed the intermediary tank on the respective opposite side. The levels of the tanks 3 and 4 are controlled by means of SISO

PI controllers around the set-points given by h_{3s} and h_{4s}. The manipulated variable in each loop is the discharge coefficients R_i of the respective valve. Under these assumptions, the system can be described by equations showed in Table 2.

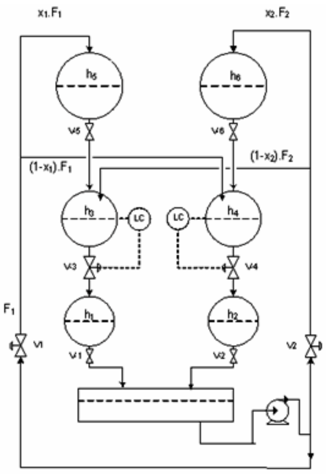

Fig:1 The Sextuple-Tank Process

Table 2. Model Equations

Tanks Levels	Control Actions	Supporting Equations
$A_5(h_5)\frac{dh_5}{dt} = x_1F_1 - R_5\sqrt{h_5}$	$\frac{dI_3}{dt} = \frac{1}{T_{I3}}(h_{3s} - h_3)$	$R_3 = R_{3s} + K_{P3}(h_{3s} - h_3) + K_{P3}I_3$
$A_3(h_3)\frac{dh_3}{dt} = R_5\sqrt{h_5} + (1 - x_2)F_2 - R_3\sqrt{h_3}$	$\frac{dI_4}{dt} = \frac{1}{T_{I4}}(h_{4s} - h_4)$	$R_4 = R_{4s} + K_{P4}(h_{4s} - h_4) + K_{P4}I_4$
$A_1(h_1)\frac{dh_1}{dt} = R_3\sqrt{h_3} - R_1\sqrt{h_1}$		$R_{3s} = \frac{x_{1s}F_{1s} + (1 - x_{2s})F_{2s}}{\sqrt{h_{3s}}}$
$A_6(h_6)\frac{dh_6}{dt} = x_2F_2 - R_6\sqrt{h_6}$		$R_{4s} = \frac{x_{2s}F_{2s} + (1 - x_{1s})F_{1s}}{\sqrt{h_{4s}}}$
$A_4(h_4)\frac{dh_4}{dt} = (1 - x_1)F_1 + R_6\sqrt{h_6} - R_4\sqrt{h_4}$		$A_i(h_i) = \pi(D_ih_i - h_i^2)$
$A_2(h_2)\frac{dh_2}{dt} = R_4\sqrt{h_4} - R_2\sqrt{h_2}$		$i = 1,2,3,4,5,6$

4. Results and Discussions

The EKF formulations were implemented in MATLAB and applied in the process dynamic model, previously presented, using SIMULINK [13]. The goal is the estimation of intermediary tanks level, since these levels are the controlled variables of this process. The estimation of superior tanks levels also

is verified. In this case, the inferior tanks levels are measured directly in the process, and the filtration of these variables is carried out by the Kalman filters. All the evaluated simulations were made in 100 minutes and the system initial condition is an operating point that presents a minimum-phase behavior ($1<x_1+x_2<2$). However, due to step changes in the valve distribution flow factors during the process simulation the system moves to an operating region presenting non-minimum phase behavior ($1<x_1+x_2<0$) in t = 50 minutes.
A studied case had been simulated to evaluate the quality of prediction and robustness for all the filters formulations, considering the aforementioned important areas related directly with the advanced control strategies. Furthermore, the following conditions and settings were imposed:

1. A PRBS (Pseudo-Random Binary Signal) is used in the manipulated inlet flow rates (F_1 and F_2), which is initialized at the simulation beginning. Hence, this situation characterizes that different initial conditions are used in the filters.
2. Q is considered as a diagonal matrix: $Q=I_{nxn}$, where n is the states number and R is considered as a diagonal matrix with an uncertainty in the measurements: $R= (10)I_{mxm}$, where m is the measured variables number.
3. The measured variables are the inferior tanks levels. These variables are generated from the model simulation with a band-limited white noise addition.

Supposing a leak in the process, it was considered an error of 1000 $cm^3.min^{-1}$ in the manipulated inlet flow rate 1 (Δ_1). Errors of 10% in the outlet flow coefficients of tank 1 (R_1) and tank 2 (R_2) were also considered. The filters performances are evaluated using an error criterion: Integral Time Absolute Error (ITAE) and are shown in Table 3.

Table 3. Filters Performance - ITAE values

	h_1	h_2	h_5	h_6	R_1	R_2	Δ_1	Δ_2
EKF	1023.6	1137.5	4070.2	222.4	-	-	-	-
CEKF	2185.8	2489.0	4040.7	83.5	-	-	-	-
DEKF	2185.8	2488.8	4040.6	83.5	-	-	-	-
MDEKF	2185.9	2489.3	4040.6	83.3	-	-	-	-
CtEKF	795.5	864.0	4074.5	266.4	-	-	-	-
EKF_{est}	81.3	91.6	112.7	24.5	157009	13972	19654	23733
$CEKF_{est}$	151.8	123.1	547.1	39.9	668100	35161	113071	97109
$DEKF_{est}$	151.7	123.2	547.4	40.0	668511	35276	113111	97197
$MDEKF_{est}$	151.1	122.7	545.5	39.6	666250	34903	112818	96851
$CtEKF_{est}$	95.1	107.5	28.2	34.6	37947	23168	10147	11143

The EKF continuous formulations have presented the best performance in inferior tanks levels estimation and the EKF discrete formulations have presented the best performance in superior tanks levels estimation when only state estimation was applied. On the other hand, when unknown process parameters estimation and dynamic data reconciliation, considering the state observability analysis, were applied (subscript “est” in Table 1), all filters performances were improved. Furthermore, the EKF continuous formulations have presented the best performance in all state, parameter and leak estimation.

5. Conclusions and future work

The performances of different Extended Kalman Filter formulations were evaluated, not only for the state estimation, but also for unknown process parameters estimation and dynamic data reconciliation. It was shown that when the unknown process parameters estimation and dynamic data reconciliation were implemented together with the state estimation, the EKF continuous formulations always have presented the best performance. In these case studies no constrains were imposed to the state variables, which could improve the relative performance of CEKF against the others evaluated filters. Moreover, the effects of the filters design parameters (Q and R matrices) were evaluated through some different values and the results have not presented a consistent conclusion because the results for the filters performance were very similar for any chosen filters design parameters. Hence, a deeper analysis of this parameters effect need to be performed in a further work.

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