18th European Symposium on Computer Aided Process Engineering – ESCAPE 18
Bertrand Braunschweig and Xavier Joulia (Editors)

Comprehensive evaluation of EKF, CEKF, and Moving Horizon estimators for on-line processes applications

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Abstract

In this work we performed a comprehensive evaluation between MHE and Kalman filter-based estimators, where the estimators were applied to the classical four-cylindrical tanks model and two nonlinear reaction systems. From the results, the advantages and drawbacks of the moving horizon formulation are brought up to justify the high effort spent in the design and evaluation phases, compared to the EKF and CEKF estimators, when the system has relatively high nonlinearities and the disturbances are bounded.

Keywords: state estimation, EKF, CEKF, moving horizon.

1. Introduction

The estate estimation is an integral part o many process modeling, monitoring, and control strategies [1]. The success of many estimation methods depends on the accuracy of the process models. Take in account that many process uncertainties, such as model parameters and process disturbances are bounded, including constraints may improve the state estimation by the reconciliation of the approximated model with the process measurements [2].

Most of the process models has non-measured parameters and variables and/or present process-model mismatches, bringing difficulties to control and monitor the process by the available model-based techniques. Furthermore, nonlinear systems and gain sign inversion still represent challenging problems in that scenery. Using examples of classical control problems typically involving approximate models, high nonlinearity, gain sign inversion, uncertain initial estimate, and bounded disturbances, we want to evaluate the Kalman-based estimators and the moving horizon formulation, with respect to accurate and meaningful state estimates. At once, we rely that MHE formulation is a superior estimation technique, due its capability to deal with constraints, nonlinear systems, and gain sign inversion. At same time we expect the classical estimators (Kalman-based), like the Extended Kalman Filter (EKF) and the Constrained Extended Kalman Filter (CEKF), may fail when they are in face with this class of control problems.

2. Estimators formulation

The mathematical model of dynamic systems often yields in nonlinear differential-algebraic equations of the form [3]:

$$
\begin{aligned}
\dot{x} &= f(x,u) + \xi \\
x_0 &= x_0 + \xi_0 \\
y &= g(x) + \varphi
\end{aligned}
\tag{1}
$$

where x, u, y, ξ and φ are the state vector, vector of controlled variables, measurement vector and the model and measurement disturbances, respectively. In sampled systems these continuous equations have to be transformed to a discrete time form [3]. Also here all the estimators assume the process or system model form described by Eq. (1).

2.1. Extended Kalman Filter (EKF)

The EKF can be divided in two parts in each sample time: the observation/correction (given $\hat{x}_{k|k-1}$, its error covariance ($\hat{P}_{k|k-1}$) and the current measurement y_k, find $\hat{x}_{k|k}$ and $\hat{P}_{k|k}$), and the prediction (given $\hat{x}_{k|k}$ and $\hat{P}_{k|k}$, find $\hat{x}_{k+1|k}$ and $\hat{P}_{k+1|k}$).
The EKF considered here is described by the following discrete-time system:
Observation/correction:

$$
\begin{aligned}
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + L_k\left(y_k - g\left(\hat{x}_{k|k-1}\right)\right) \\
L_k &= \hat{P}_{k|k-1}\mathcal{G}_k^T\left(\mathcal{G}_k\hat{P}_{k|k-1}\mathcal{G}_k^T + R\right)^{-1} \\
\hat{P}_{k|k} &= (I - L_k\mathcal{G}_k)\hat{P}_{k|k-1} \\
\mathcal{G}_k &= \frac{\partial g(x_k)}{\partial x_k}\Big|_{x_k=\hat{x}_{k|k-1}}
\end{aligned}
\tag{2}
$$

Prediction:

$$
\begin{aligned}
\hat{x}_{k+1|k} &= f\left(\hat{x}_{k|k}, u_k\right) + \xi_k \\
\dot{\hat{P}}_{k+1|k} &= \mathcal{F}_k\hat{P}_{k|k} + \hat{P}_{k|k}\mathcal{F}_k^T + Q \\
\mathcal{F}_k &= \frac{\partial f(x_k,u_k)}{\partial x}\Big|_{x_k=\hat{x}_{k|k}}
\end{aligned}
\tag{3}
$$

2.2. Moving Horizon Estimation (MHE)

The Moving Horizon Estimator (MHE) was introduced by [4] and [1]. The advantages of MHE compared to the classical estimators are the possibility to incorporate physical constraints on the states, e.g. mass fraction that ranges between 0 and 1, and the fact that over the considered horizon no information is lost. According to [3], the disadvantage comes to the necessity to solve a nonlinear dynamic optimization problem.
Observation/correction:

$$
\begin{aligned}
&min_{\hat{\xi}_{k-N-1|k},\dots,\hat{\xi}_{k-1|k},\hat{\varphi}_{k-N|k},\dots,\hat{\varphi}_{k|k}}\Big(\Psi_k^N = \\
&\hat{\xi}_{k-N-1|k}^T\,\hat{P}_{k-N|k-1}^{-1}\hat{\xi}_{k-N-1|k} + \textstyle\sum_{j=k-N}^{k-1}\hat{\xi}_{j|k}^T\,Q^{-1}\hat{\xi}_{j|k} + \sum_{j=k-N}^{k}\hat{\varphi}_{j|k}^T\,R^{-1}\hat{\varphi}_{j|k}\Big)
\end{aligned}
\tag{4}
$$

subject to,

$$
\begin{aligned}
&\hat{x}_{k-N|k} = \hat{x}_{k-N|k-1} + \hat{\xi}_{k-N-1|k} \\
&\hat{x}_{j+1|k} = f\left(\hat{x}_{j|k}, u_j\right) + \hat{\xi}_{j|k}, \quad j = k-N,\dots,k-1 \\
&y_j = g\left(\hat{x}_{j|k}\right) + \hat{\varphi}_{j|k}, \;\; j = k-N,\dots,k \\
&\qquad x_{min} \le \hat{x}_{j|k} \le x_{max}, \xi_{min} \le \hat{\xi}_{j|k} \le \xi_{max}, \varphi_{min} \le \hat{\varphi}_{j|k} \le \varphi_{max}
\end{aligned}
\tag{5}
$$

The covariance matrix P_{k-N+1} is up-dated by the solution of the discrete-time Ricatti equation:

$$\hat{P}_{i+1|k} = Q + \mathcal{F}_i \hat{P}_{i|k-1} \mathcal{F}_i^T - \mathcal{F}_i \hat{P}_{i|k-1} \mathcal{G}_i^T \left[\mathcal{G}_i \hat{P}_{i|k-1} \mathcal{G}_i^T + R\right]^{-1} \mathcal{G}_i \hat{P}_{i|k-1} \mathcal{F}_i^T$$
$$\mathcal{G}_i = \frac{\partial g(x_i)}{\partial x}\Big|_{x_i=\hat{x}_{i|k}}, \mathcal{F}_i = \frac{\partial f(x_i,u_i)}{\partial x}\Big|_{x_i=\hat{x}_{i|k}}, i = k - N \quad (6)$$

Prediction:

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k) + \xi_k \quad (7)$$

Following the notation by [4], N represent the length of the horizon. The last $N + 1$ measurements are considered. $\hat{x}_{j|k}$ is the estimate of the state at time $t = t_j$ based on the measurements up to time $t = t_k$.

2.3. Constrained Extended Kalman Filter (CEKF)

The CEKF formulation described below is basically the MHE resolution with an estimation horizon of length zero ($N = 0$) and is similar to the conventional EKF that also contains a zero-length estimation horizon in the correction stage.
Observation/correction:

$$min_{\hat{\xi}_{k-1|k}, \hat{\varphi}_{k|k}} (\Psi_k^0 = \hat{\xi}_{k-1|k}^T \hat{P}_{k|k-1}^{-1} \hat{\xi}_{k-1|k} + \hat{\varphi}_{k|k}^T R^{-1} \hat{\varphi}_{k|k}) \quad (8)$$

subject to,

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + \hat{\xi}_{k-1|k}$$
$$y_k = g(\hat{x}_{k|k}) + \hat{\varphi}_{k|k}$$
$$\xi_{min} \le \hat{\xi}_{k-1|k} \le \xi_{max}, \varphi_{min} \le \hat{\varphi}_{k|k} \le \varphi_{max} \quad (9)$$

The covariance matrix P_{k+1} also is up-dated by the solution of the discrete-time Ricatti equation:

$$\hat{P}_{k+1|k} = Q + \mathcal{F}_k \hat{P}_{k|k-1} \mathcal{F}_k^T - \mathcal{F}_k \hat{P}_{k|k-1} \mathcal{G}_k^T \left[\mathcal{G}_k \hat{P}_{k|k-1} \mathcal{G}_k^T + R\right]^{-1} \mathcal{G}_k \hat{P}_{k|k-1} \mathcal{F}_k^T$$
$$\mathcal{G}_k = \frac{\partial g(x_k)}{\partial x}\Big|_{x_k=\hat{x}_{k|k}}, \mathcal{F}_k = \frac{\partial f(x_k,u_k)}{\partial x}\Big|_{x_k=\hat{x}_{k|k}} \quad (10)$$

Prediction:

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k) + \xi_k \quad (11)$$

The understanding of $\hat{P}_k, \hat{P}_{k-N}, Q$ and R is identical for the EKF, CEKF and MHE estimators, where they are the basic parameters of the filter adjustment.

3. Application Examples

3.1. Four cylindrical tanks model

Presented by [5], this plant has gain sign inversion, and we modified it by adding a nonlinear measurement equation provided by Eq. (12), where n is the coefficient of nonlinearity and C is a constant. Thus, our goal here is to estimate the four tank levels by the measurement of the flows (y) of the inferior tanks (1 and 2). In order to include a process-model mismatch, we also considered a different measurement equation for the real process ($n = 0.3$) and for the estimators ($n = 0.5$). This mismatch may compromise the state estimation, and we want to show that the addition of constraints on the state disturbance ($\xi_k \geq 0$) can improve it. Once, for simulations a white noise (φ_k) of amplitude or covariance (NA) was added on the measurements.

$$y = Cx^n \tag{12}$$

The estimated initial conditions for the states x_1 and x_2 were given far from the real initial conditions (a large initial disturbance $\xi_{0|0}$) to investigate the estimators capability to track the real state value since the first estimations.
A comparison of the estimators, for the four cylindrical tanks model, is shown in Fig. (1) below (only the states x_1 and x_3 are shown).

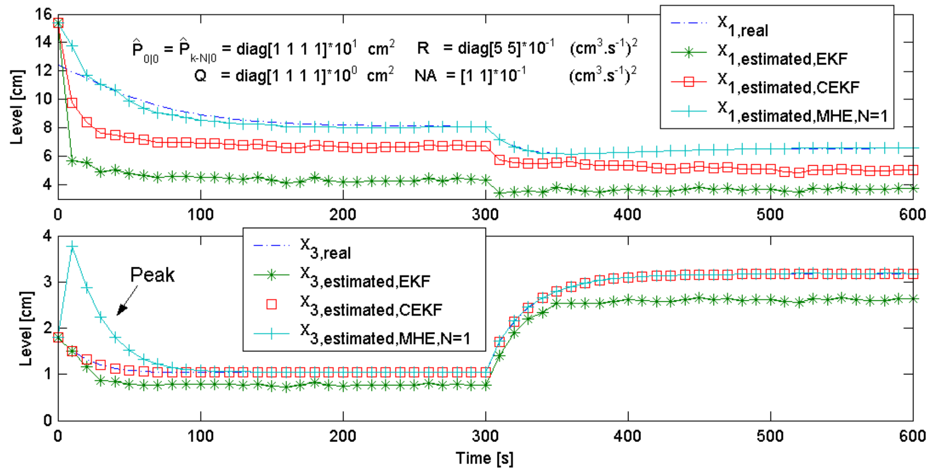

Figure 1. Simulation results to EKF, CEKF and MHE estimators, for the four cylindrical tanks model.

Except for the "peak" that appeared in estimation of the state x_1, as a consequence of the notable initial disturbance ($\xi_{0|0}$) and by the fact it is constrained, MHE performance was superior. As expect the performance of the constrained estimators is superior to the unconstrained estimators (e.g. EKF), because the constrained estimators acquired, with the addition of inequality constraints, the proper statistics of the disturbance sequence ξ_k [6]. Although the constrained CEKF estimator also incorporates constraints explicitly ($\xi_k \geq 0$), the estimation of the state x_1 failed, because quadratic problem (QP) did not provide a feasible solution for the optimization problem involved in this application example.

3.2. Isothermal CSTR reactor – van de Vusse reaction

Aiming to explore a more nonlinear system, we investigated the peculiar behavior of the classical van de Vusse reaction accomplished in an isothermal CSTR reactor, where simulations changing the operation points were carried out, in order to obtain the gain sign inversion, based on work of [7]. Our goal here is to estimate the B product concentration (x_2), by the measurement of the A reactant concentration (x_1). Once again in this example, the estimated initial condition for the state x_1 ($\hat{x}_{0|0}$) was given far from the real initial condition to investigate the estimators capability to track the real state value and to eliminate the initial disturbance ($\xi_{0|0}$) since at the first estimations. The measured variable is the state x_1, with a white noise (φ_k) of amplitude (NA).

A comparison of the estimators, for the isothermal CSTR model, is shown in Fig. (2) below. Differently of what we had been previously relied, this example show that the Kalman-based estimators can also deal with gain sign inversion, even for systems with reasonable nonlinearity. In spite of $\xi_{0|0}$ being large, all estimators brought the estimation, in a fast way, near to the real state value since the first estimations.

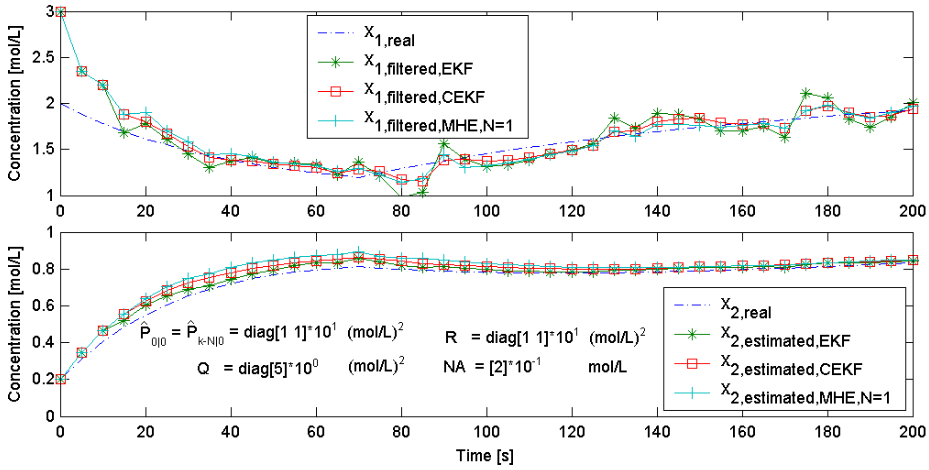

Figure 2. Simulation results to EKF, CEKF and MHE estimators, for the isothermal CSTR.

3.3. Exothermic CSTR reactor - Irreversible First-Order Reactions

Here we have a complex nonlinear behavior of a CSTR presented by [8]. The states x_1 and x_2 are the measured states with a white noise of amplitude (NA), and their initial conditions were given near to the real initial conditions (small $\xi_{0|0}$) to avoid that the estimation loses or goes toward to another one of the several steady states that concerning this model.

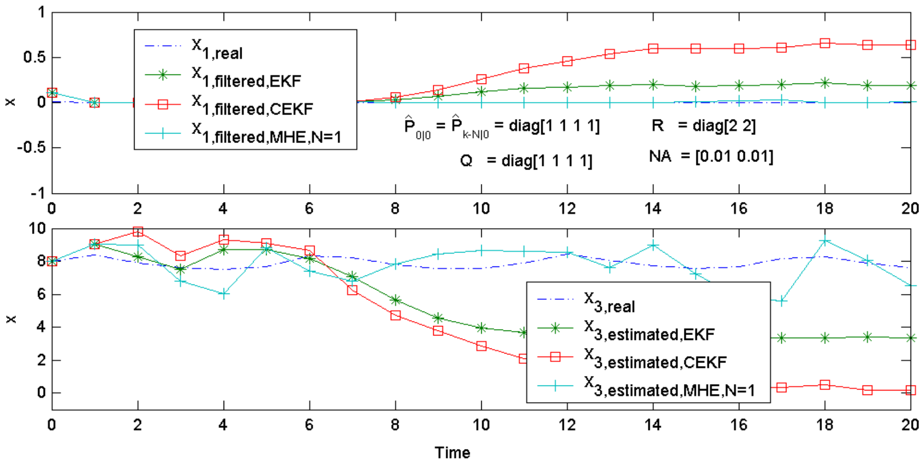

Figure 3. Simulation results to the EKF, CEKF and MHE estimators, exothermic CSTR.

In Fig. (3) we showed only the states x_1 and x_3 from the estimation using the exothermic CSTR model. Once again, the Kalman-based estimators failed. Note that the real state x_3 has an oscillatory behavior, and for the estimation it is a hard task to track the real profile. This effect was felt more by the CEKF than the EKF, but both estimators diverged in their estimation, even for the filtered state (x_1). Although MHE results are equal to the CEKF estimator when performed with a zero horizon ($N = 0$), by increasing the horizon length to $N = 1$ the MHE was successful (when compared with EKF and CEKF) on tracking the real state value, even for the estimated state x_3.

4. Conclusions

Kalman-based estimators (EKF and CEKF) and MHE formulation were evaluated. From the application of the considered estimation to the industrial importance of the examples studied here, it can be concluded that the high computational efforts of moving horizon formulation, when compared with EKF and CEKF estimators, are justified since the problems have high nonlinearities associated and bounded disturbances. Also we concluded that Kalman-based estimators still lead to good results when we have models that present gain sign inversion. The other point that deserves to be emphasized is that MHE can lead to good results, when compared with EKF and CEKF, even for short horizon lengths reducing its CPU-time consuming. Although MHE demands higher computational effort, the computing power advances, as well as the development of efficient methods for solving nonlinear optimization problems, encourage us to say that MHE is presently an interesting advanced tool to use in on-line processes applications.

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