Disjunctive Optimization Tools

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Motivation

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- Discrete/Continuous Optimization
  - **Nonlinear models**
  - **0-1 and continuous decisions**
- Optimization Models
  - **Mixed-Integer Linear Programming (MILP)**
  - **Mixed-Integer Nonlinear Programming (MINLP)**
- Alternative approaches
  - **Logic-based: Generalized Disjunctive Programming (GDP)**
  - **Constraint Programming (CP)**
- Challenges
  - **How to develop "best" model?**
  - **How to improve relaxation?**
  - **How to solve nonconvex GDP problems to global optimality?**
  - **How to overcome computational complexity?**

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Solving models with discrete decisions
Mixed-Integer Linear-Nonlinear Programming Codes

NON-LINEAR Codes:
SBB *GAMS simple B&B*
MINLP-BB (AMPL)*Fletcher and Leyffer (1999)*
Bonmin (COIN-OR) *Bonami et al (2006)*
FilMINT Linderoth and Leyffer (2006)
DICOPT (GAMS) *Viswanathan and Grossman (1990)*
AOA (AIMSS)
α–ECP *Westerlund and Peterssson (1996)*
MINOPT *Schweiger and Floudas (1998)*
BARON *Sahinidis et al. (1998)*

For LINEAR many B&B and its variants:
CPLEX, XPRESS, OSL, etc.

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Extended Modeling Systems

- Identification of specific problem structures within a model ***(Ferris et. al FOCAPO 2008 meeting)***
- Modeling Systems like AIMMS, AMPL, GAMS, ILOG, etc. have language extensions to include special features
- Examples:
  - Complementarity constraints
  - Variational constraints
  - Bilevel Programs
  - Non-linear extensions
  - Indicator constraints (ILOG) – similar to disjunctions

Logic-based Optimization

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- **Facilitate modeling of discrete/continuous optimization problems through use symbolic expressions**
- **Reduce combinatorial search effort**
- **Improve handling nonlinearities**

Emerging techniques

- ❑ **Constraint Programming**
  Van Hentenryck (1989)
- ❑ **Generalized Disjunctive Programming**
  Raman and Grossmann (1994)
- ❑ **Mixed-Logic Linear Programming**
  Hooker and Osorio (1999)

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LogMIP
Logical Mixed Integer Programming

Features

System linked to GAMS

Problems can be formulated as disjunctive programs

Problems can be linear or nonlinear with discrete decisions

Provides:

- **Language to write disjunctions**
- **Operators and sentences for logic propositions**
- **Linear and nonlinear solvers**

http://www.ceride.gov.ar/logmip

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LogMIP: Modeling two terms disjunction

$$\begin{bmatrix} \text{condition} \\ \text{constraint set t} \end{bmatrix} \vee \begin{bmatrix} \neg\text{condition} \\ \text{constraint set f} \end{bmatrix}$$

Conditions in this LogMIP version are Boolean (binary) variables

Declaration sentence: **Disjunction TTD;**

TTD is
IF (condition) THEN
constraints set (names) to satisfy when condition is TRUE;
ELSE
constraints set (names) to satisfy when condition is FALSE;
END IF;

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LogMIP: Modeling a multi-term disjunction

$$\begin{bmatrix} \text{condition 1} \\ \text{constraints set 1} \end{bmatrix} \vee \begin{bmatrix} \text{condition 2} \\ \text{constraints set 2} \end{bmatrix} \vee .. \vee \begin{bmatrix} \text{condition } N \\ \text{constraints set } N \end{bmatrix}$$

Declaration sentence:

Disjunction MTD;

Definition sentence:

MTD is

IF (condition$_1$) THEN

constraints set 1 (names) to be satisfied when condition$_1$ is True;

ELSIF (condition$_2$) THEN

constraints set 2 (names) to be satisfied when condition$_2$ is True;

ELSIF (condition$_3$) THEN

…

ELSIF (condition$_N$) THEN

constraints set N (names) to be satisfied when condition$_n$ is True;

END IF

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LogMIP: Posing logic propositions

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Operands : Boolean (binary) variables (must correspond to disjunctions conditions)

Operators: and, or, not , -> (implication), <-> equivalence

Y('2') -> Y('3') or Y('4') or Y('5');
Y('1') and not Y('2') -> not Y('3');
Y('2') -> not Y('3') ;
Y('3') -> Y('8');

Special Sentences:

Syntax: [*atmost*]
[*atleast*] *(<list of boolean variables>, n)*
[*exactly*]

Parameter n indicates how many variables must comply the sentence (default = 1)

atmost(Y('1'), Y('2'));
atleast(Y('4'), Y('5'));

MODELING MIPs

A Job Shop Scheduling Problem

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John Hooker. "Logic-Based Methods for Optimization". John Wiley & Sons, Inc. (2000)

Objective: How shops may be scheduled on machines to optimize makespan or total tardiness

Constraints:

- Each job is processed on a certain subset of the machines in a certain order
- A machine can work on only one job at a time
- Not all jobs can be processed in one machine

Nomenclature:

- Index k ➔ task of processing any given job on any of its machines
- D_k task duration
- $k = p_{j1},\ldots, p_{jnj}$ sequence of task of processing job j in the order they must occur
- $Q_i = \{ q_{i1},\ldots,q_{imi}\}$ set of task that are done on machine i. The order depends on how are they scheduled.
- R_j Job j release time
- S_j Job j due time

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MODELING MIPs

A Job Shop Scheduling Problem

Given two consecutive tasks in the processing of job j:

$t_{pj,k+1} \geq t_{pj,,k} + D_k$ the second task cannot start until the previous is done

Discrete decision:
Given a pair of task k, ℓ taking place on the same machine

$$t_\ell \geq t_k + D_k \quad \text{or} \quad t_k \geq t_\ell + D_k$$

By using of a Big-M formulation we can transform the discrete decision into inequalities:

$$t_\ell \geq t_k + D_k - M(1-y_{k\ell})$$

$$t_k \geq t_\ell + D\ell - My_{k\ell} \quad , \; y_{k\ell} \in \{0,1\}$$

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MODELING MIPs

A Job Shop Scheduling Problem: Formulation

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minimize T

subject to $T \geq t_{pjnj} + D_{pjnj} \quad , \ \forall_j$

$$\left.\begin{aligned} t_\ell &\geq t_k + D_k - M(1-y_{k\ell}) \\ t_k &\geq t_\ell + D_\ell - My_{k\ell} \end{aligned}\right\} \quad \forall\ k,\ell \in Q_i \text{ with } k < \ell\ ,\ \forall\ i$$

$$y_{k\ell} \in \{0,1\}$$

M is a large number making the constraint redundant

MODELING with DISJUNCTION

A Job Shop Scheduling Problem: Formulation

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minimize T

subject to

$T \geq t_{pjnj} + D_{pjnj} \quad , \quad \forall_j$

$t_\ell \geq t_k + D_k - M(1-y_{k\ell})$

$t_k \geq t_\ell + D_\ell - M y_{k\ell}$

$y_{k\ell} \in \{0,1\}$

minimize T

subject to

$T \geq t_{pjnj} + D_{pjnj} \quad , \quad \forall_j$

$$\begin{bmatrix} y_{kl} \\ t_l \geq t_k + \mathrm{D}_k \end{bmatrix} \vee \begin{bmatrix} \neg y_{kl} \\ t_k \geq t_l + \mathrm{D}_l \end{bmatrix}$$

$y_{k\ell} \in \{\text{True},\text{False}\}$

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MODELING with DISJUNCTION

A Job Shop Scheduling Problem

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Job/stage	1	2	3
A	5	-	3
B	-	3	2
C	2	4	-

$$\min Z = T$$

$$\textit{subject to} \quad T \geq x_A + 8$$

$$T \geq x_B + 5$$

$$T \geq x_C + 6$$

$$\begin{bmatrix} Y_{AC} \\ x_A - x_C + 5 \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{AC} \\ x_C - x_A + 2 \leq 0 \end{bmatrix}$$

$$\begin{bmatrix} Y_{BC} \\ x_B - x_C + 1 \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{BC} \\ x_C - x_B + 6 \leq 0 \end{bmatrix}$$

$$\begin{bmatrix} Y_{AB} \\ x_A - x_B + 5 \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{AB} \\ x_B - x_A \leq 0 \end{bmatrix}$$

$$T, x_A, x_B, x_C \geq 0$$

$$Y_k \in \{true, false\}, k = 1,2,3.$$

MODELING with DISJUNCTION

Strip Packing Problem

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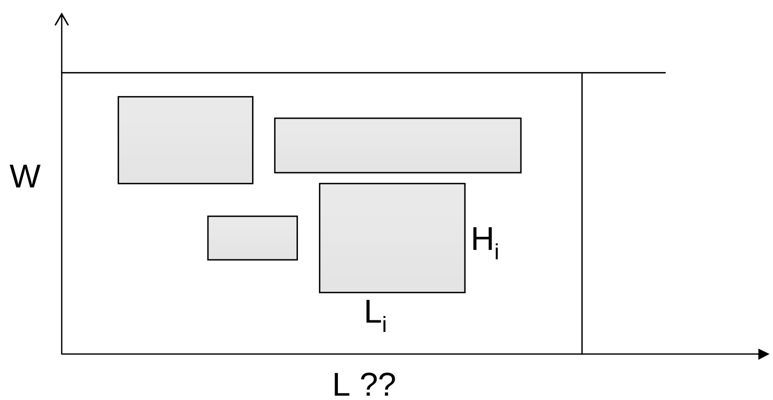
Problem statement: ***Hifi (1998)***

- **Objective is to fit small rectangles onto strip without overlap and rotation while minimizing length L of the strip**

Given:

- **A set of small rectangles with width H_i and length L_i**
- **Large rectangular strip of fixed width W and unknown length L.**



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MODELING with DISJUNCTIONS

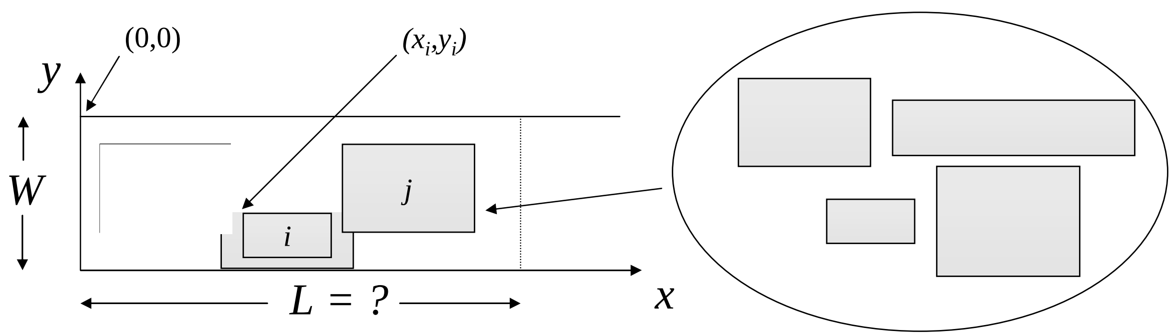
Strip Packing Problem

Discrete decision:

Given a pair of rectangles i, j then :

[i left j] or [i right j] or [i above j] or [i below j]

[$x_i + L_i \le x_j$] or [$x_j + L_j \le x_i$] or [$y_i - H_i \ge y_j$] or [$y_j - H_j \ge y_i$]



Set of small rectangles

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MODELING with DISJUNCTIONS

Strip Packing Problem

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Objective function
Minimize length

$$Min \quad lt$$

$$s.t. \quad lt \geq x_i + L_i \qquad \forall i \in N$$

Disjunctive constraints
No overlap between rectangles

$$\begin{bmatrix} Y^1_{ij} \\ x_i + L_i \leq x_j \end{bmatrix} \vee \begin{bmatrix} Y^2_{ij} \\ x_j + L_j \leq x_i \end{bmatrix} \vee \begin{bmatrix} Y^3_{ij} \\ y_i - H_i \geq y_j \end{bmatrix} \vee \begin{bmatrix} Y^4_{ij} \\ y_j - H_j \geq y_i \end{bmatrix} \qquad \forall i, j \in N, i < j$$

Bounds on variables

$$x_i \leq UB_i - L_i \qquad \forall i \in N$$

$$H_i \leq y_i \leq W \qquad \forall i \in N$$

$$lt, x_i, y_i \in \mathbf{R}^1_+ \ , \ Y^1_{ij}, Y^2_{ij}, Y^3_{ij}, Y^4_{ij} \in \{True, False\} \qquad \forall i, j \in N, i < j$$

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Linear Disjunction Relaxations

Linear disjunction $\bigvee_{i\in D}\left[a_i^T x \le b_i\right] \quad x \in R^n$

Big-M relaxation

$$\begin{cases} a_i^T x \le b_i + M_i(1-y_i) \\ \sum_{i\in D} y_i = 1 \quad , \quad 0 \le y_i \le 1, \; i \in D \\ M_i = \max\{a_i^T x - b_i \mid x^{lo} \le x \le x^{up}\} \end{cases}$$

Convex-Hull relaxation

$$\begin{cases} x - \sum_{i\in D} v_i = 0 \qquad x, v_i \in R^n \\ a_i^T v_i - b_i y_i \le 0 \\ \sum_{i\in D} y_i = 1 \quad , 0 \le y_i \le 1, \; i \in D \\ 0 \le v_i \le v_i^{up} y_i \end{cases}$$

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Linear disjunction relaxations

	$\begin{bmatrix} Y_{AC} \\ x_A - x_C + 5 \le 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{AC} \\ x_C - x_A + 2 \le 0 \end{bmatrix}$	$\begin{bmatrix} Y_{AC} \\ x_A - x_C + 5 \le 0 \end{bmatrix} \vee \begin{bmatrix} Y_{CA} \\ x_C - x_A + 2 \le 0 \end{bmatrix}$
Big-M	$x_A - x_C + 5 \le M(1 - Y_{AC})$ $x_C - x_A + 2 \le M(1 - (1 - Y_{AC}))$	$x_A - x_C + 5 \le M(1 - Y_{AC})$ $x_C - x_A + 2 \le M(1 - Y_{CA})$ $Y_{AC} + Y_{CA} = 1$
Convex-Hull	$x_A = v_{A1} + v_{A2}$ $x_C = v_{C1} + v_{C2}$ $v_{A1} - v_{C1} + 5\,Y_{AC} \le 0$ $v_{A2} - v_{C2} + 2(1 - Y_{AC}) \le 0$ $0 \le v_{A1} \le v_{A1}^{up}\, Y_{AC}$ $0 \le v_{A2} \le v_{A2}^{up}\, (1 - Y_{AC})$ $0 \le v_{C1} \le v_{C1}^{up}\, Y_{AC}$ $0 \le v_{C2} \le v_{C2}^{up}\, (1 - Y_{AC})$	$x_A = v_{A1} + v_{A2}$ $x_C = v_{C1} + v_{C2}$ $v_{A1} - v_{C1} + 5\,Y_{AC} \le 0$ $v_{A2} - v_{C2} + 2\,Y_{CA} \le 0$ $Y_{AC} + Y_{CA} = 1$ $0 \le v_{A1} \le v_{A1}^{up}\, Y_{AC}$ $0 \le v_{A2} \le v_{A2}^{up}\, Y_{CA}$ $0 \le v_{C1} \le v_{C1}^{up}\, Y_{AC}$ $0 \le v_{C2} \le v_{C2}^{up}\, Y_{CA}$

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Defining linear disjunctions in LOGMIP

$$\begin{bmatrix} Y_{AC} \\ x_A - x_C + 5 \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{AC} \\ x_C - x_A + 2 \leq 0 \end{bmatrix}$$

DISJUNCTION D1;

D1 IS
IF Y('AC') THEN
$x_A - x_C + 5 \leq 0$ **;**
ELSE
$x_C - x_A + 2 \leq 0$ **;**
ENDIF;

$$\begin{bmatrix} Y_{AC} \\ x_A - x_C + 5 \leq 0 \end{bmatrix} \vee \begin{bmatrix} Y_{CA} \\ x_C - x_A + 2 \leq 0 \end{bmatrix}$$

DISJUNCTION D1;

D1 IS
IF Y('AC') THEN
$x_A - x_C + 5 \leq 0$ **;**
ELSIF Y('CA')
$x_C - x_A + 2 \leq 0$ **;**
ENDIF;

Linear disjunction relaxations

several terms disjunctions

$$\begin{bmatrix} Y_{ij}^1 \\ x_i + L_i \le x_j \end{bmatrix} \vee \begin{bmatrix} Y_{ij}^2 \\ x_j + L_j \le x_i \end{bmatrix} \vee \begin{bmatrix} Y_{ij}^3 \\ y_i - H_i \ge y_j \end{bmatrix} \vee \begin{bmatrix} Y_{ij}^4 \\ y_j - H_j \ge y_i \end{bmatrix} \forall i, j \in N, i < j$$

Big-M

$$x_i + L_i \le x_j + M(1\text{-}Y_{ij}^1)$$
$$x_j + L_j \le x_i + M(1\text{-}Y_{ij}^2)$$
$$y_i - H_i \le y_j + M(1\text{-}Y_{ij}^3)$$
$$y_j - H_j \le y_i + M(1\text{-}Y_{ij}^4)$$
$$\sum_{ijk} Y_{ij}^k = 1$$

Convex hull

$$x_j = v_{j1} + v_{j2} \qquad 0 \le v_{i1} \le v_{i1}^{up}\, Y_{ij}^1$$
$$x_i = v_{i1} + v_{i2} \qquad 0 \le v_{j1} \le v_{j1}^{up}\, Y_{ij}^1$$
$$y_j = v_{j3} + v_{j4} \qquad 0 \le v_{i2} \le v_{i2}^{up}\, Y_{ij}^2$$
$$y_i = v_{i3} + v_{i4} \qquad 0 \le v_{j2} \le v_{j2}^{up}\, Y_{ij}^2$$
$$v_{i1} + L_i Y_{ij}^1 \le v_{j1} \qquad 0 \le v_{i3} \le v_{i3}^{up}\, Y_{ij}^3$$
$$v_{j2} + L_j Y_{ij}^2 \le v_{i2} \qquad 0 \le v_{j3} \le v_{j3}^{up}\, Y_{ij}^3$$
$$v_{i3} - H_i Y_{ij}^3 \le v_{j3} \qquad 0 \le v_{i4} \le v_{i4}^{up}\, Y_{ij}^4$$
$$v_{j4} - H_i Y_{ij}^3 \le v_{i4} \qquad 0 \le v_{j4} \le v_{j4}^{up}\, Y_{ij}^4$$
$$\sum_{ijk} Y_{ij}^k = 1$$

Defining linear disjunctions in LOGMIP

several terms disjunctions

$$\begin{bmatrix} Y_{ij}^1 \\ x_i + L_i \le x_j \end{bmatrix} \vee \begin{bmatrix} Y_{ij}^2 \\ x_j + L_j \le x_i \end{bmatrix} \vee \begin{bmatrix} Y_{ij}^3 \\ y_i - H_i \ge y_j \end{bmatrix} \vee \begin{bmatrix} Y_{ij}^4 \\ y_j - H_j \ge y_i \end{bmatrix} \forall i, j \in N, i < j$$

```
DISJUNCTION D(I,J);

D(I,J) with (ord(I) < ord(J)) is
IF Y(I,J,'1') THEN
        x_i + L_i <= x_j ;
ELSIF Y(I,J,'2') THEN
        x_j + L_j <= x_i ;
ELSIF Y(I,J,'3') THEN
        y_i - H_i >= y_j ;
ELSIF Y(I,J,'4') THEN
        y_j - H_j >= y_i ;
ENDIF;
```

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LogMIP Linear Algorithms

LINEAR DISJUNCTIVE PROGRAM

↓

MIP Reformulation by "Big-M" or Convex hull Relaxation

LMBigM ***LMChull***

↓

MILP PROGRAM

↓

B&B (OSL, CPLEX, etc)

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Generalized Disjunctive Programming (GDP)

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- Raman and Grossmann (1994) *(Extension Balas, 1979)*

$$\min \ Z = \sum_k c_k + f(x) \qquad \textbf{Objective Function}$$

$$s.t. \quad r(x) \le 0 \qquad \textbf{Common Constraints}$$

OR operator ⟶

$$\bigvee_{j \in J_k} \begin{bmatrix} Y_{jk} \\ g_{jk}(x) \le 0 \\ c_k = \gamma_{jk} \end{bmatrix}, k \in K \qquad \begin{matrix} \textbf{Disjunction} \\ \textbf{Constraints} \\ \textbf{Fixed Charges} \end{matrix}$$

$$\Omega(Y) = true \qquad \textbf{Logic Propositions}$$

$$x \in R^n, c_k \in R^1 \qquad \textbf{Continuous Variables}$$

$$Y_{jk} \in \{true, \ false\} \qquad \textbf{Boolean Variables}$$

- **Multiple Terms / Disjunctions**

Nonlinear Disjunction Relaxations

Non-linear disjunction $\underset{i \in D}{\vee} [g_i(x) \le 0] \quad x \in R^n$

Big-M relaxation

$$\begin{cases} g_i(x) \le M_i(1-\lambda_i) \\ \sum_{i \in D} \lambda_i = 1 \ ; 0 \le \lambda_i \le 1 \\ M_i = \max\{g(x) \mid x^{lo} \le x \le x^{up}\} \end{cases}$$

Convex-Hull relaxation

$$\begin{cases} x - \sum_{i \in D} v_i = 0 \qquad x, v_i \in R^n \\ (\lambda_i)\, g_i(v_i / (\lambda_i)) \le 0 \\ \sum_{i \in D} \lambda_i = 1 \quad , 0 \le \lambda_i \le 1, \ i \in D \\ 0 \le v^i \le v_i^{up} \lambda_i \end{cases}$$

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Implementation of nonlinear Convex-Hull relaxation

Implementation $$\lambda_{jk} g_{jk}(\nu^{jk}/(\lambda_{jk}+\varepsilon)) \le 0$$

ε **tolerance (e.g. 0.0001) should be smaller than integer tolerance**

Note: rigorous Sawaya & Grossmann (2005)

$$((1-\varepsilon)\lambda_{jk}+\varepsilon)(g_{jk}(\nu_{jk}/((1-\varepsilon)\lambda_{jk}+\varepsilon)))-\varepsilon g_{jk}(0)(1-\lambda_{jk}) \le 0$$

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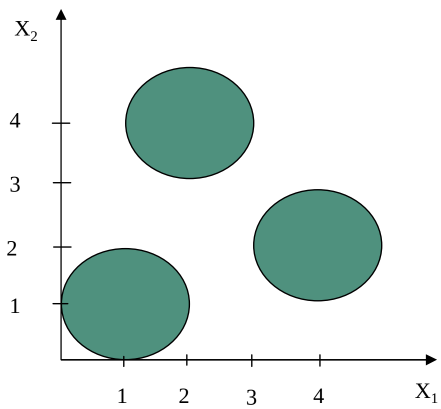
Nonlinear Disjunction Relaxations

$[\ (x_1-1)^2+(x_2-1)^2 \leq 1\ ] \vee [\ (x_1-4)^2+(x_2-2)^2 \leq 1\ ] \vee [\ (x_1-2)^2+(x_2-4)^2 \leq 1\ ]$

$0 \leq x_1 \leq 5\ ;\ 0 \leq x_2 \leq 5$



Big-M

$$(x_1-1)^2 + (x_2-1)^2 \leq 1 + 31(1-\lambda_1)$$
$$(x_1-4)^2 + (x_2-2)^2 \leq 1 + 24(1-\lambda_2)$$
$$(x1-2)^2 + (x2-4)^2 \leq 1 + 24(1-\lambda_3)$$
$$\sum_j \lambda_j = 1 \quad j=1,2,3\ ;\ \lambda_j \in \{0, 1\}$$

Convex hull

$$x_1 = v_{11} + v_{12} + v_{13}$$
$$x_2 = v_{21} + v_{22} + v_{23}$$
$$(\lambda_1+\varepsilon)[(v_{11}/(\lambda_1+\varepsilon)-1)^2 + (v_{21}/(\lambda_1+\varepsilon)-1)^2 - 1] \leq 0$$
$$(\lambda_2+\varepsilon)[(v_{12}/(\lambda_2+\varepsilon)-4)^2 + (v_{22}/(\lambda_2+\varepsilon)-2)^2 - 1] \leq 0$$
$$(\lambda_3+\varepsilon)[(v_{13}/(\lambda_3+\varepsilon)-2)^2 + (v_{23}/(\lambda_3+\varepsilon)-1)^2 - 1] \leq 0$$
$$\sum_j \lambda_j = 1 \quad j=1,2,3$$
$$v_{ij} = 5\lambda_j \qquad \forall i, \forall j,\ i=1,2;\ j=1,2,3;\ \lambda_j \in \{0, 1\}$$

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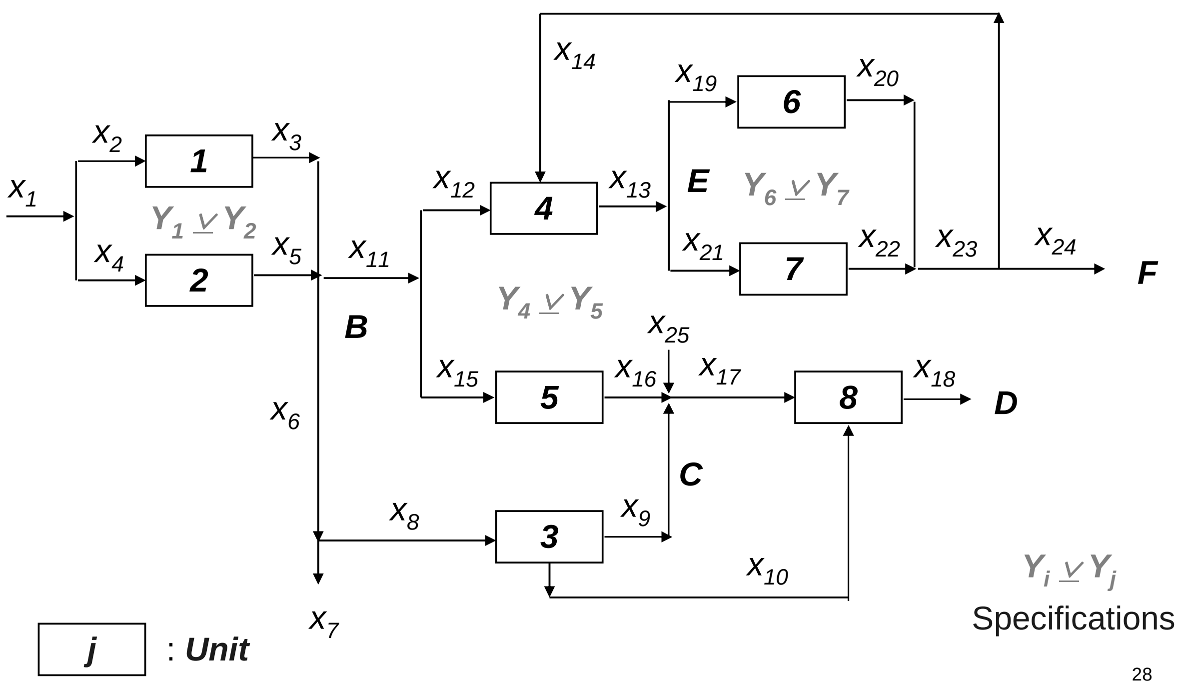
Process Network with Fixed Charges

- *Türkay and Grossmann (1997)*
  - **Superstructure of the process**



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Process Network with Fixed Charges

Disjunctions have the following form

$$\begin{bmatrix} Y_k \\ h_{ik}(x) = 0 \\ c_k = \gamma_k \end{bmatrix} \vee \begin{bmatrix} \neg Y_k \\ B_{ik} x = 0 \\ c_k = 0 \end{bmatrix} \qquad i \in D_k,\ k \in K$$

The following properties apply to the disjunctions of this problem

i. In the second term of the disjunctions a subset of the continuous variables *x* are zero

ii. No continuous variable *x* is repeated in the second term ($\neg Y_k$) of the disjunctions.

Because of this properties Logic-Based Outer Approximation algorithm can be applied

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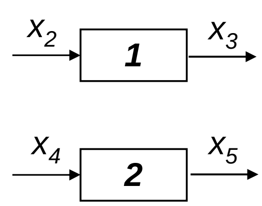
LogMIP specifiying special two terms disjunction

```
disjunction   d1, d2;

d1 is
      if Y('1') then
         EXP(X('3')) -1. =E= X('2');
         CF('1') =E= 5             ;
      else
         X('2') =E= 0             ;
         X('3') =E= 0             ;
      endif;

d2 is
      if Y('2') then
        EXP(X('5')/1.2) -1. =E= X('4');
        CF('2') =E= 8                ;
      else
        X('4') =E= 0                ;
        X('5') =E= 0                ;
      endif;
```

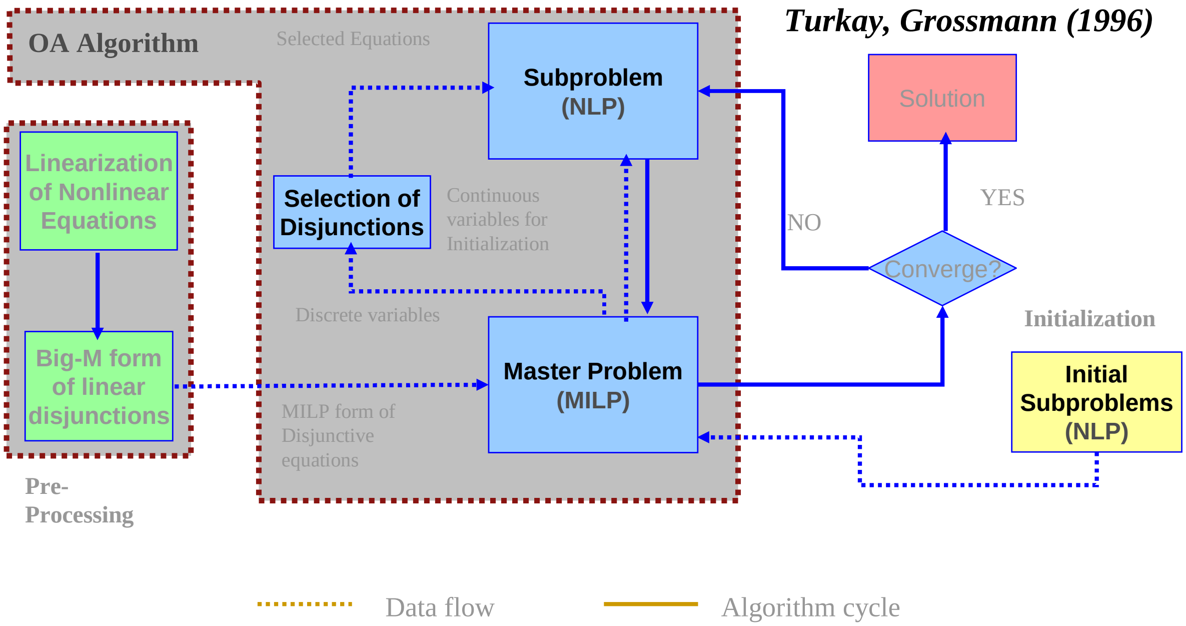


$$Y_1 \veebar Y_2$$

Propositional logic

Logic-based OA Algorithm

Turkay, Grossmann (1996)



Implemented in LogMIP

Methods Generalized Disjunctive Programming

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GDP *Key: relaxation*

- **Logic based methods**
  - **Branch and bound**
    (Lee & Grossmann, 2000)
  - **Decomposition**
    Outer-Approximation
    Generalized Benders
    (Turkay & Grossmann, 1997)
- **Reformulation MINLP**
  Outer-Approximation
  Generalized Benders
  Extended Cutting Plane
  - **Convex-hull**
    Cutting plane
    (Sawaya & Grossmann, 2004)
  - **Big-M**

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LogMIP NonLinear Algorithms

NONLINEAR DISJUNCTIVE PROGRAM

Special two terms disjunction

LMLBOA

Logic Based OA

MINLP Reformulation by “Big-M” or Convex hull Relaxation

MINLP PROGRAM

B&B (SBB)

OA (DICOPT++)

SOME EXPERIMENTS DONE NOT IMPLEMENTED YET!

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CONCLUSIONS

- Extensions to mathematical models based on logic
- Easier modeling techniques involving discrete decisions
- Language sentences to pose disjunctions and logic propositions
- Several algorihms to choose depends on the model show better performance

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