

# Multiple Model Predictive Control (MMPC) for Nonlinear Systems and Improved Disturbance Rejection

- Motivation & Tutorial Overview
- Multiple Model Predictive Control
  - Nonlinear Processes
  - Disturbance Rejection
- Summary

B. Wayne Bequette

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for Energy and Sustainability

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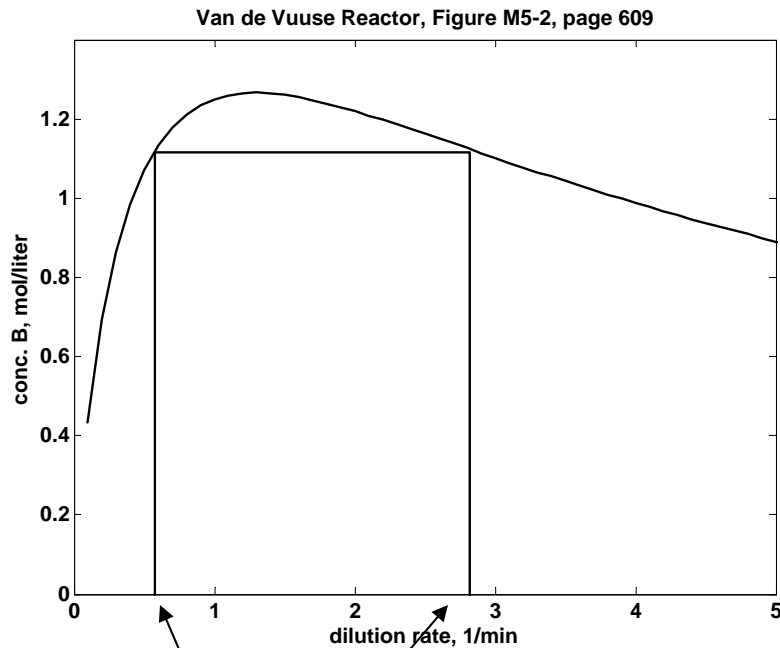


**Rensselaer**

Chemical and Biological Engineering

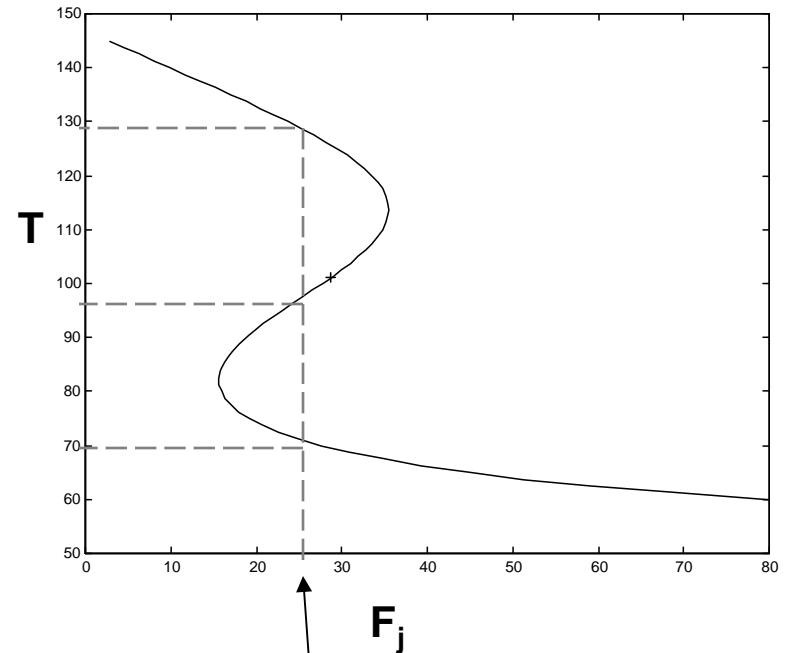
# Nonlinear Behavior: Steady-state

## Input Multiplicity



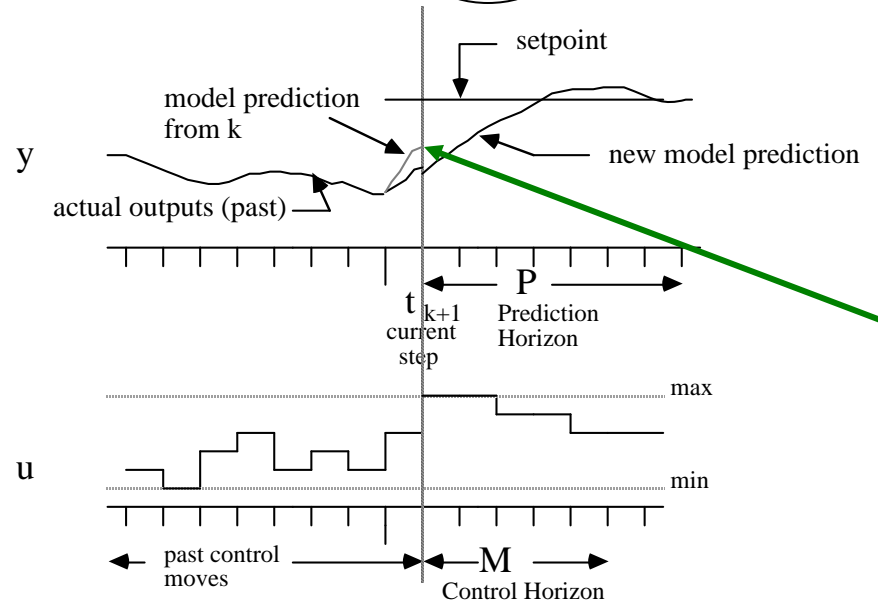
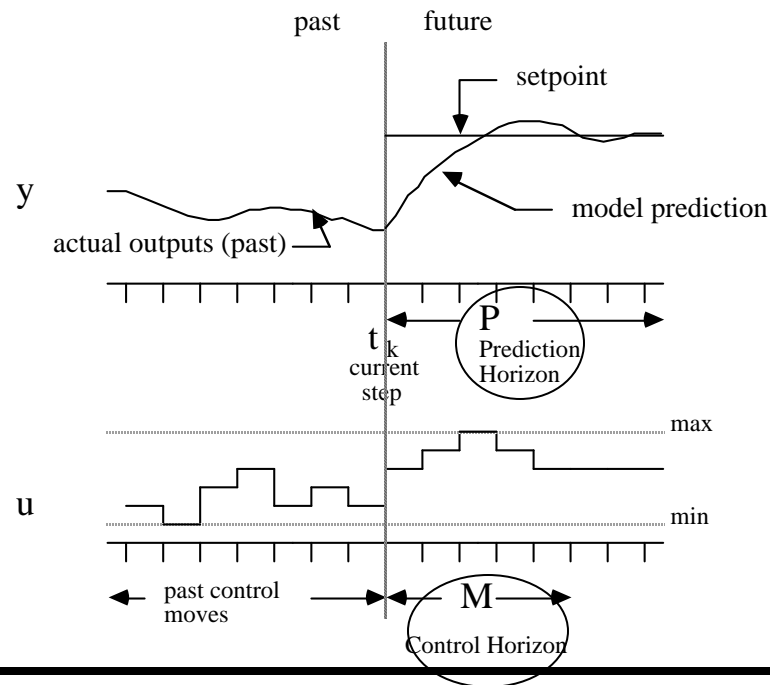
Two different input values  
yield the same output

## Output Multiplicity (hysteresis)



One input can yield three  
different output values

# MPC



- Constraints
- Multivariable
- Time-delays

- Objective function?
- Optimization technique?
- Model type?
- Disturbances/mismatch?
  - Current and Future
- Initial cond./state est.?

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# Our Approaches to Nonlinear MPC

- Quadratic Objective Function
- Models
  - Fundamental: numerical integration or collocation
  - Fundamental with linearization at each time step
  - **Multiple model**
  - Artificial neural network
- State Estimates/Initial Conditions
  - **Additive output disturbance (e.g. DMC)**
  - Estimation horizon (optimization)
  - **Extended/appended state Kalman Filter**
    - **Importance of stochastic states**

# Intuitive Nonlinear Model-based Strategy

Model equations

$$\dot{x} = f(x, u)$$
$$y = g(x)$$

Integrate model from time step  $k-1$  to  $k$

$$\hat{x}_k = F_{t_s}(\hat{x}_{k-1}, u_{k-1})$$
$$\hat{y}_{k|k-1} = g(\hat{x}_k)$$

Obtain plant measurement

$$y_k$$

Calculate model error

**(additive output disturbance)**

$$d_k = y_k - \hat{y}_{k|k-1}$$

Choose hypothetical set of current and future control moves

$$u_k, u_{k+1}, \dots, u_{k+P-1}$$

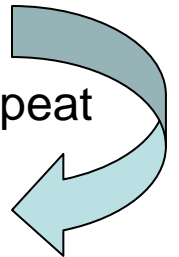
Integrate model from time step  $k$  to  $k+P$   
(based on hypothetical control moves)

$$\hat{x}_{k+1} = F_{t_s}(\hat{x}_k, u_k)$$
$$\hat{y}_{k+1|k} = g(\hat{x}_{k+1}) + d_k$$

to

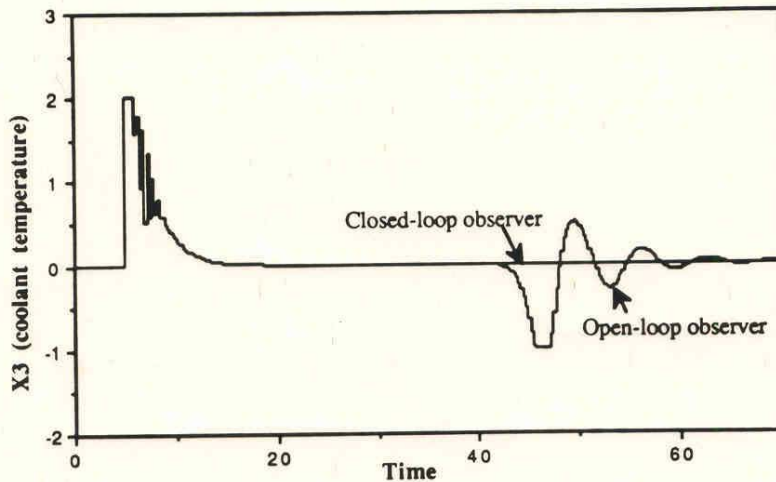
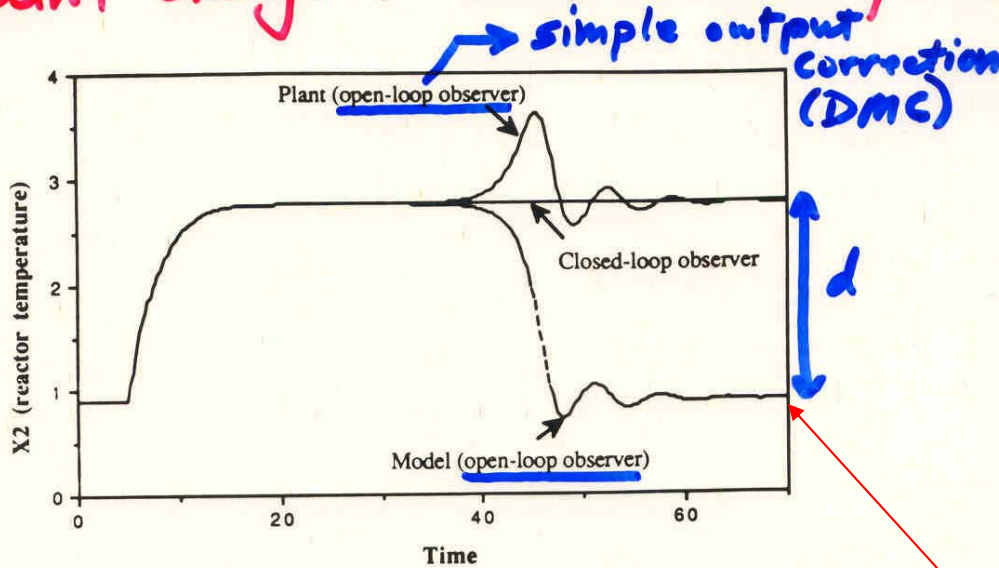
$$\hat{x}_{k+P} = F_{t_s}(\hat{x}_{k+P-1}, u_{k+P-1})$$
$$\hat{y}_{k+P|k} = g(\hat{x}_{k+P}) + d_k$$

Evaluate objective function and repeat until optimum is obtained



# Exothermic CSTR

Setpoint change to Unstable Steady-State



Response to a setpoint change to an open-loop unstable operating point with a perfect model.  $P = 10$ ,  $M = 1$ .

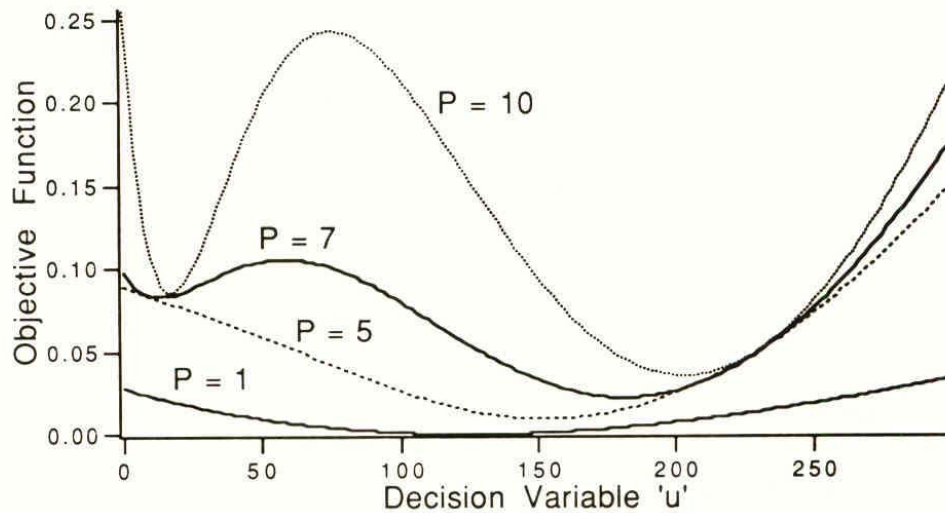
Additive  
Disturbance  
Assumption

Model converges to different steady-state than plant (compensated by additive disturbance term)

# Non-Convex Problem

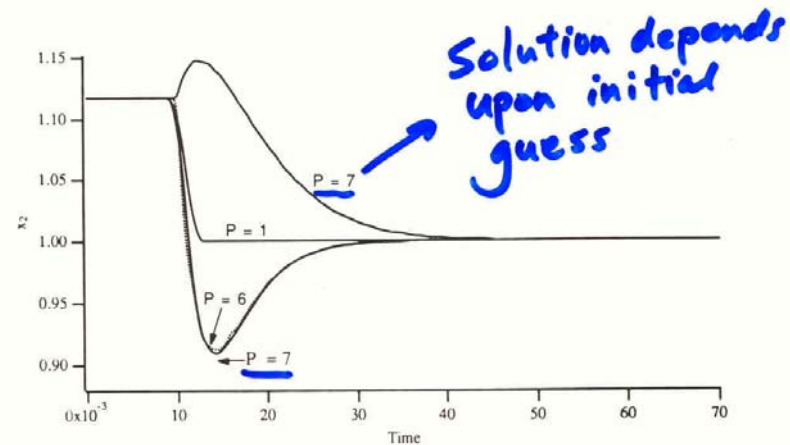
Objective Function  
(Decrease in Setpoint)

$M = 1$ , different values of  $P$

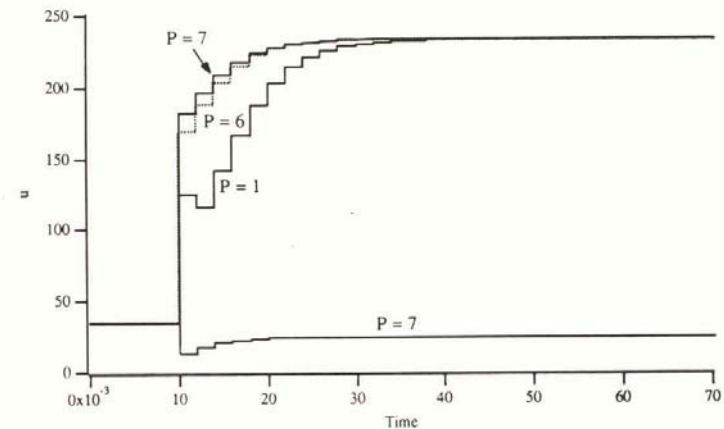


Input Multiplicity  $\dashrightarrow$  Multiple Minima

Decrease in Setpoint



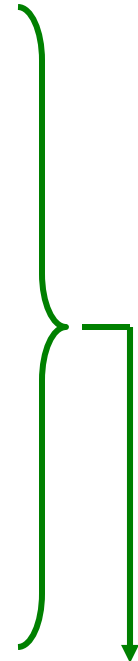
a) Output Variable



b) Manipulated Variable

# EKF-based NMPC (Lee & Ricker, 1994)

- Nonlinear Model, integrate from step  $k-1$  to  $k$
- State Estimation: Extended Kalman Filter
  - Linearized at each time step
  - Find best state estimate at time step  $k$
- Prediction
  - One integration of NL ODEs based on set of control moves (**unforced** or **“free response”**) from step  $k$  to  $k+P$
  - Perturbation (linear) model - effect of changes in control moves (**forced**)
- Optimization
  - QP, since linear model is used



Can use linear state-space KF-MPC code!

## Extended Kalman Filter Based Nonlinear Model Predictive Control

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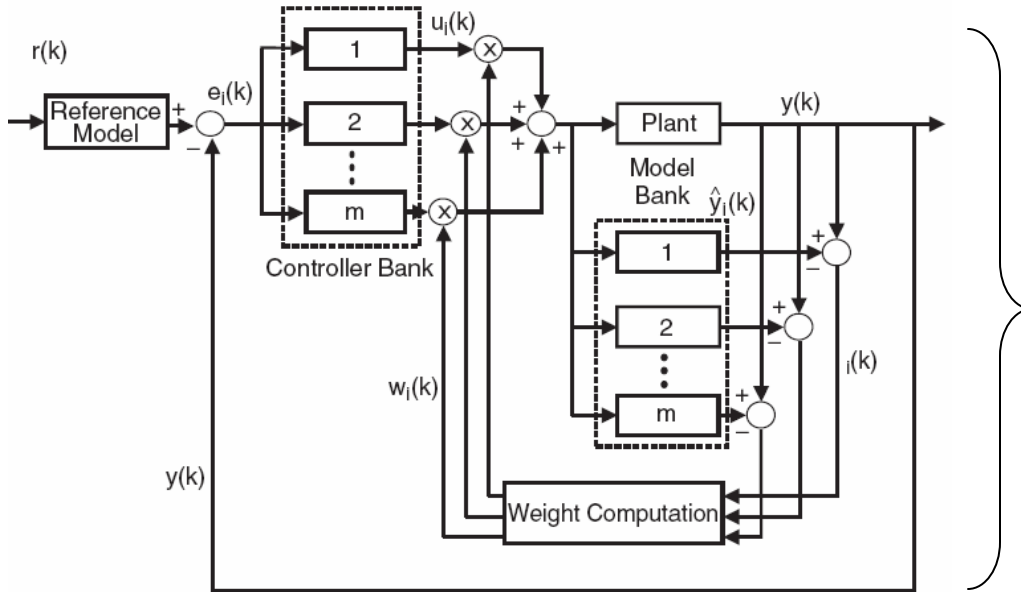
*Department of Chemical Engineering, University of Washington, Seattle, Washington 98195*



# Motivation for Multiple Linear Models

- Development time for fundamental models
  - Difficulty with physiological systems
- Much data required for artificial neural networks
  - Problems with “overfitting” and extrapolation
- At particular operating points, linear models are often a good description
  - How to switch between models?

# Multiple Model-based Control



## Multiple Model Adaptive Control (MMAC)

Athans et al. (1977) – LQG, Jet aircraft control

Roy, Kaufman - Drug infusion control

Schott, Bequette (1997) – PI, CSTR

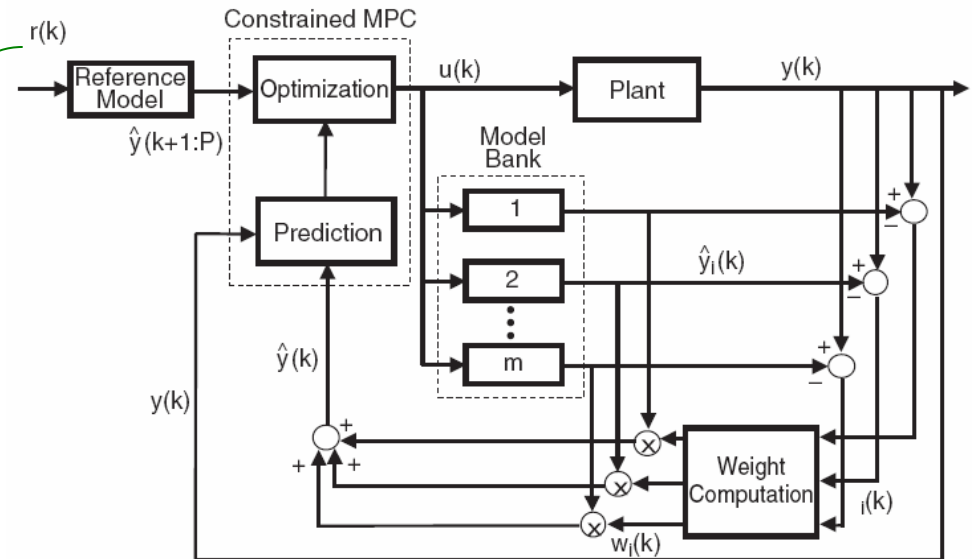
## Multiple Model Predictive Control (MMPC)

Rao et al. (2001, 2003)

Drug Infusion Control

Aufderheide & Bequette (2003)

Nonlinear CSTR



# First, a Concise Review of Linear MPC

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# Step Response & Additive Output

**Correction term**  $\rightarrow d_k = y_k - \hat{y}_{k|k-1}$

measured output  $\nearrow$   $\nwarrow$  model predicted output

The “corrected prediction” is set equal to the measured output

$$\hat{y}_{k|k} = \hat{y}_{k|k-1} + d_k \quad \longleftrightarrow \quad \hat{y}_{k|k} = y_k \quad \text{“deadbeat” observer}$$

The “corrected prediction” for jth future step is (**step response form**)

$$\hat{y}_{k+j|k} = \underbrace{\sum_{i=1}^j s_i \Delta u_{k-i+j}}_{\text{effect of future control moves}} + \underbrace{\sum_{i=j+1}^{N-1} s_i \Delta u_{k-i+j} + s_N u_{k-N+j}}_{\text{effect of past control moves}} + \underbrace{\hat{d}_{k+j}}_{\text{correction term}}$$

**forced response** **free response**

$$\hat{d}_{k+j} = \hat{d}_{k+j-1} = \dots = d_k = y_k - \hat{y}_{k|k-1} \quad \text{constant additive disturbance}$$

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# Problems with “Classical MPC” (e.g. DMC)

- Finite Step/Impulse Models Limited
  - Many parameters (~50 for each input-output relationship)
  - Limited to open-loop stable processes (there is no corrective feedback to model states)
- Additive Output Disturbance Assumption
  - Poor performance for input step disturbances
  - No explicit measurement noise trade-off



The most common criticism of MPC (Shinskey, 2002)

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# State Space Models & State Estimation

For perturbations  
to manipulated inputs

$$x_{k+1} = \Phi x_k + \Gamma u_k + \Gamma^d d_k$$

$$y_k = C x_k$$

$$\Gamma^d = \Gamma$$

**Assume disturbance propagation**

$$d_{k+1} = d_k$$

**Appended state formulation**

$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x_{k+1}^a} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k$$

$$y_k = \underbrace{[C \quad 0]}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a}$$

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# State Estimation Problem

$$d_{k+1} = d_k + w_k \quad \text{Random walk}$$

$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x_{k+1}^a} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k + \underbrace{\begin{bmatrix} 0 \\ \Gamma^w \end{bmatrix}}_{\Gamma^{w,a}} w_k$$
$$y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a} + v_k$$

## Kalman Filter

$$\hat{x}_{k|k-1}^a = \Phi^a \hat{x}_{k-1|k-1}^a + \Gamma^a u_{k-1} \quad \text{Prediction}$$

$$\hat{x}_{k|k}^a = \hat{x}_{k|k-1}^a + L_k (y_k - C^a \hat{x}_{k|k-1}^a) \quad \text{Correction}$$

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# Offset-Free Performance

- Next few slides present conditions for offset-free performance
- Unmeasured disturbances estimated as either state or output disturbances
- State observer techniques can then be used

Ref: Muske & Badgwell, J. Proc. Cont., 12: 617-632 (2002)

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# Disturbance Models

## State or Input Disturbance

$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x_{k+1}^a} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k$$

$$y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a}$$

## Additive Output Disturbance

$$\underbrace{\begin{bmatrix} x_{k+1} \\ p_{k+1} \end{bmatrix}}_{x_{k+1}^a} = \underbrace{\begin{bmatrix} \Phi & 0 \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ p_k \end{bmatrix}}_{x_k^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k$$

$$y_k = \underbrace{\begin{bmatrix} C & G_p \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ p_k \end{bmatrix}}_{x_k^a}$$



DMC:  $G_p = I$

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# Disturbance Models

## General Input and Output Disturbances

$$\begin{bmatrix} x_{k+1} \\ d_{k+1} \\ p_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}}_{\Phi^a} \begin{bmatrix} x_k \\ d_k \\ p_k \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \\ 0 \end{bmatrix}}_{\Gamma^a} u_k \quad y_k = \underbrace{\begin{bmatrix} C & 0 & G_p \end{bmatrix}}_{C^a} \begin{bmatrix} x_k \\ d_k \\ p_k \end{bmatrix}$$

Ref: Muske & Badgwell, J. Proc. Cont., 12: 617-632 (2002)

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# State Estimator

$$\begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \\ \hat{p}_{k|k-1} \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}}_{\Phi^a} \begin{bmatrix} \hat{x}_{k-1|k-1} \\ \hat{d}_{k-1|k-1} \\ \hat{p}_{k-1|k-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \\ 0 \end{bmatrix}}_{\Gamma^a} u_{k-1}$$

$$\begin{bmatrix} \hat{x}_{k|k} \\ \hat{d}_{k|k} \\ \hat{p}_{k|k} \end{bmatrix} = \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \\ \hat{p}_{k|k-1} \end{bmatrix} + \begin{bmatrix} L_x \\ L_d \\ L_p \end{bmatrix} \left( y_k - C^a \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \\ \hat{p}_{k|k-1} \end{bmatrix} \right)$$

Deterministic or  
stochastic  
observer design

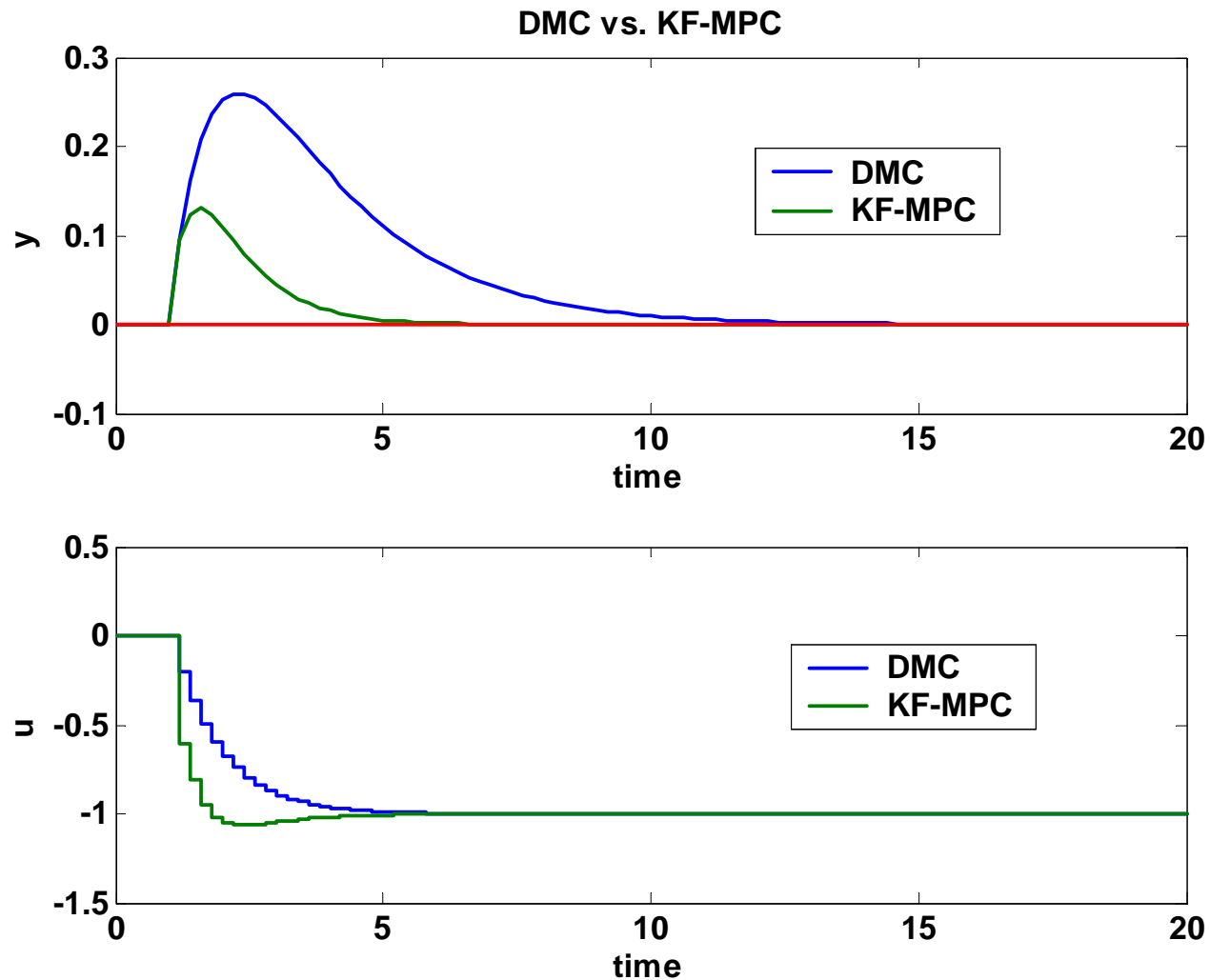
For offset-free performance:

# disturbances = # outputs, and augmented system  
must be detectable

Ref: Muske & Badgwell, J. Proc. Cont., 12: 617-632 (2002)

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# Problem: Unmeasured Step Input Disturbance

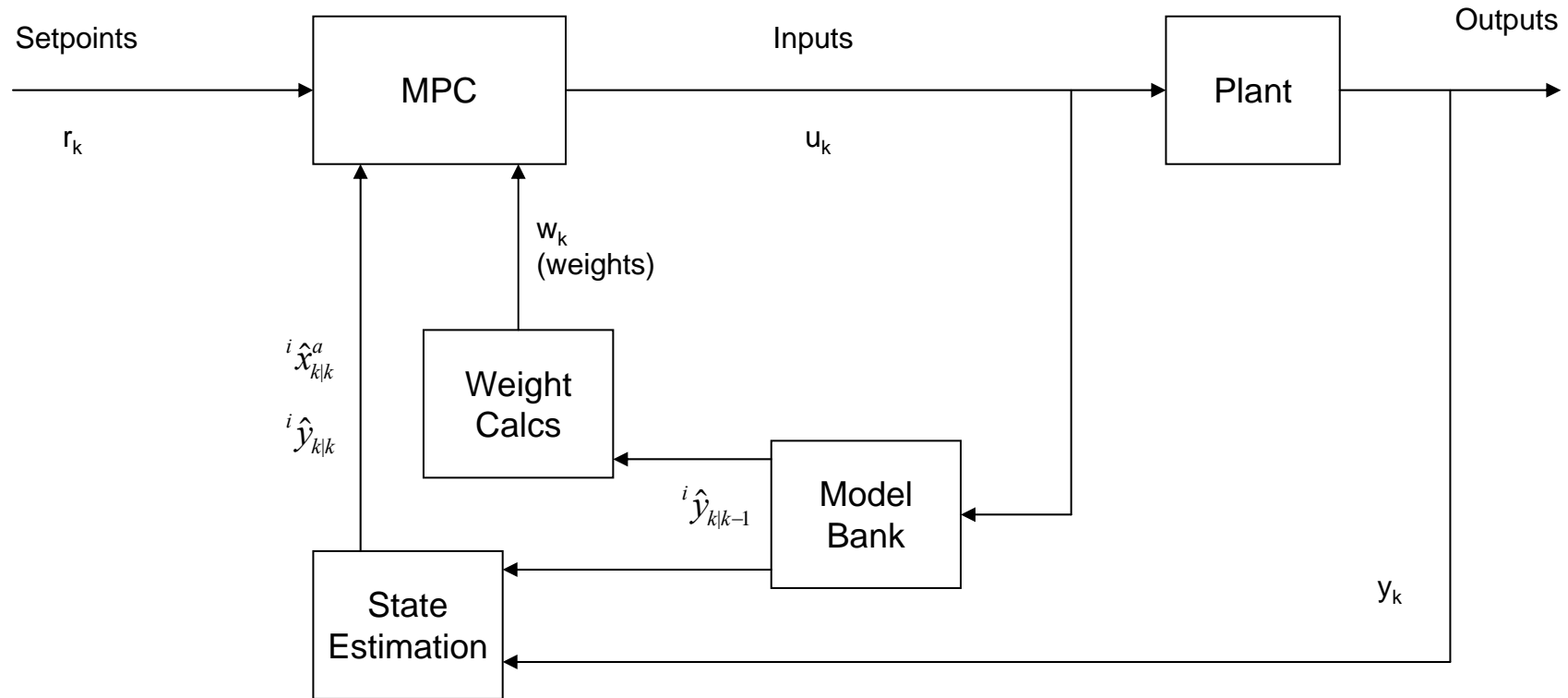
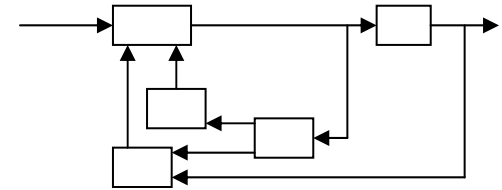


DMC: additive output disturbance assumption (bias)

KF-MPC: appended state, estimated step input disturbance

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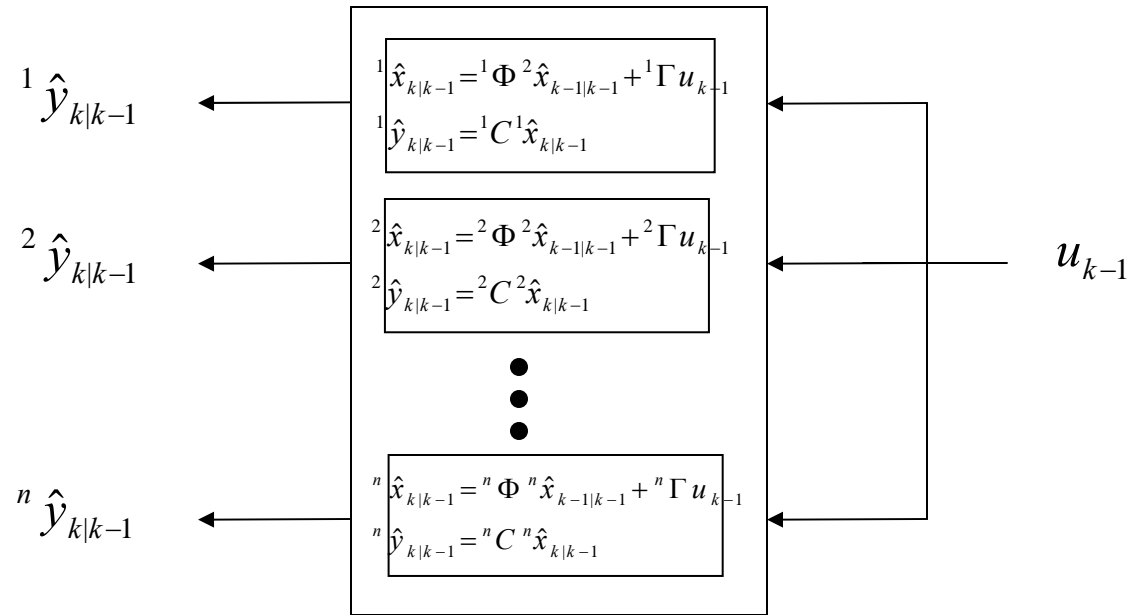
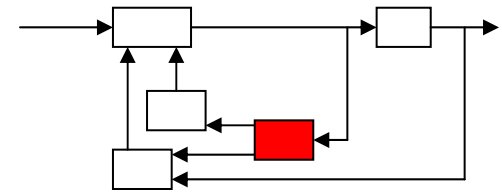
# Structure of MMPC



## Multiple Model Predictive Control of Nonlinear Systems

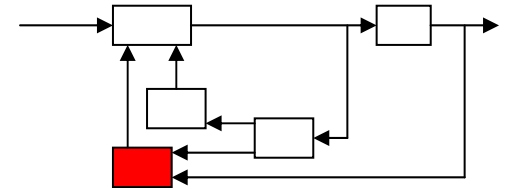
Matthew Kuure-Kinsey<sup>†</sup> and B. Wayne Bequette<sup>‡</sup>

# Model Bank Details



- Each of the n linear models represents the system at a specified operating condition
- A model predicted output is calculated for each model using the current manipulated input

# State Estimation



- Need a way to account for model uncertainty and mismatch.

$${}^i x_{k+1} = {}^i \Phi {}^i x_k + {}^i \Gamma u_k$$

$${}^i y_k = {}^i C {}^i x_k$$

Additive output disturbance

$${}^i x_{k+1} = {}^i \Phi {}^i x_k + {}^i \Gamma u_k$$

$${}^i d_{k+1} = {}^i d_k$$

$${}^i y_k = {}^i C {}^i x_k + {}^i d_k$$

Step input disturbance

$${}^i x_{k+1} = {}^i \Phi {}^i x_k + {}^i \Gamma u_k + {}^i \Gamma^d {}^i d_k$$

$${}^i d_{k+1} = {}^i d_k$$

$${}^i y_k = {}^i C {}^i x_k$$

$$\begin{bmatrix} {}^i x_{k+1} \\ {}^i d_{k+1} \end{bmatrix} = \begin{bmatrix} {}^i \Phi & G_x \\ 0 & I \end{bmatrix} \begin{bmatrix} {}^i x_k \\ {}^i d_k \end{bmatrix} + \begin{bmatrix} {}^i \Gamma \\ 0 \end{bmatrix} u_k$$

$${}^i y_k = \begin{bmatrix} {}^i C & G_y \end{bmatrix} \begin{bmatrix} {}^i x_k \\ {}^i d_k \end{bmatrix}$$

$$G_x = {}^i \Gamma^d$$

$$G_x = 0$$

$$G_y = 0$$

$$G_y = I$$

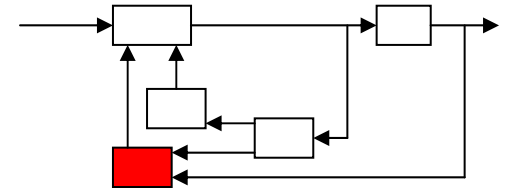
Step input

Additive output

Step input

Additive output

# State Estimation



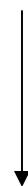
- Kalman predictor/corrector equations

$$\begin{bmatrix} {}^i x_{k+1} \\ {}^i d_{k+1} \end{bmatrix} = \begin{bmatrix} {}^i \Phi & G_x \\ 0 & I \end{bmatrix} \begin{bmatrix} {}^i x_k \\ {}^i d_k \end{bmatrix} + \begin{bmatrix} {}^i \Gamma \\ 0 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ {}^i \Gamma^w \end{bmatrix} w_k$$

$${}^i y_k = \begin{bmatrix} {}^i C & G_y \end{bmatrix} \begin{bmatrix} {}^i x_k \\ {}^i d_k \end{bmatrix} + v_k$$

$${}^i x_{k+1}^a = {}^i \Phi^a {}^i \hat{x}_k^a + {}^i \Gamma^a u_k + {}^i \Gamma^{1,w} w_k$$

$${}^i y_k = {}^i C^a {}^i x_k^a + v_k$$



$${}^i \hat{x}_{k|k-1}^a = {}^i \Phi^a {}^i \hat{x}_{k-1|k-1}^a + {}^i \Gamma^a u_{k-1} \quad \text{Prediction}$$

$${}^i \hat{x}_{k|k}^a = {}^i \hat{x}_{k|k-1}^a + L (y - {}^i C^a {}^i \hat{x}_{k|k-1}^a) \quad \text{Correction}$$

$${}^i \hat{y}_{k|k} = {}^i C^a {}^i \hat{x}_{k|k}^a \quad \text{Updated output prediction}$$

$${}^i L = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

Additive output

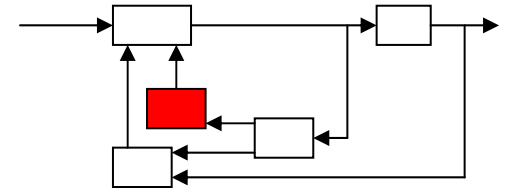
${}^i L =$  Kalman gain

Step input



# Weight Calculation

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$${}^i \varepsilon_k = y_k - {}^i \hat{y}_{k|k} \quad \text{Model residual}$$

$${}^i p_k = \frac{\exp(-0.5 {}^i \varepsilon_k^T {}^i \Lambda {}^i \varepsilon_k) {}^i p_{k-1}}{\sum_{j=1}^n \exp(-0.5 {}^j \varepsilon_k^T {}^i \Lambda {}^j \varepsilon_k) {}^j p_{k-1}}$$

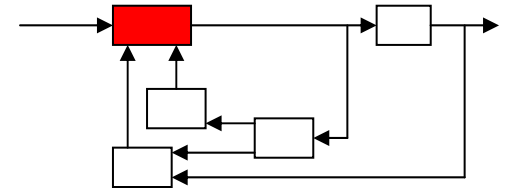
Bayesian probability,  
i<sup>th</sup> model

$${}^i w_k = \left\{ \begin{array}{ll} \frac{{}^i p_k}{\sum_{j=1}^n {}^j p_k} & {}^i p_k > \delta \\ 0 & {}^i p_k \leq \delta \end{array} \right\}$$

Weight, i<sup>th</sup> model

# Model Predictive Control

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- “Model average” for the output prediction

$$\bar{y}_{k+j|k} = \sum_{i=1}^n w_k^i \hat{y}_{k+j|k}^i$$

- Quadratic objective function

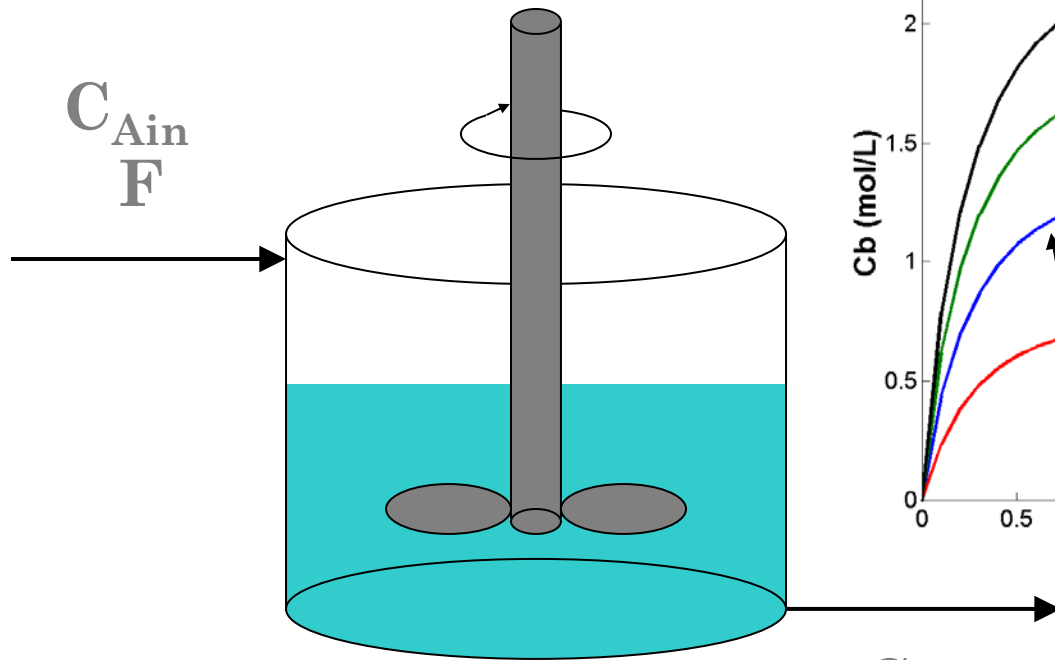
$$\min \Phi = (R - \bar{Y})^T W_y (R - \bar{Y}) + \Delta U^T W_u \Delta U$$

## Analytical Solution

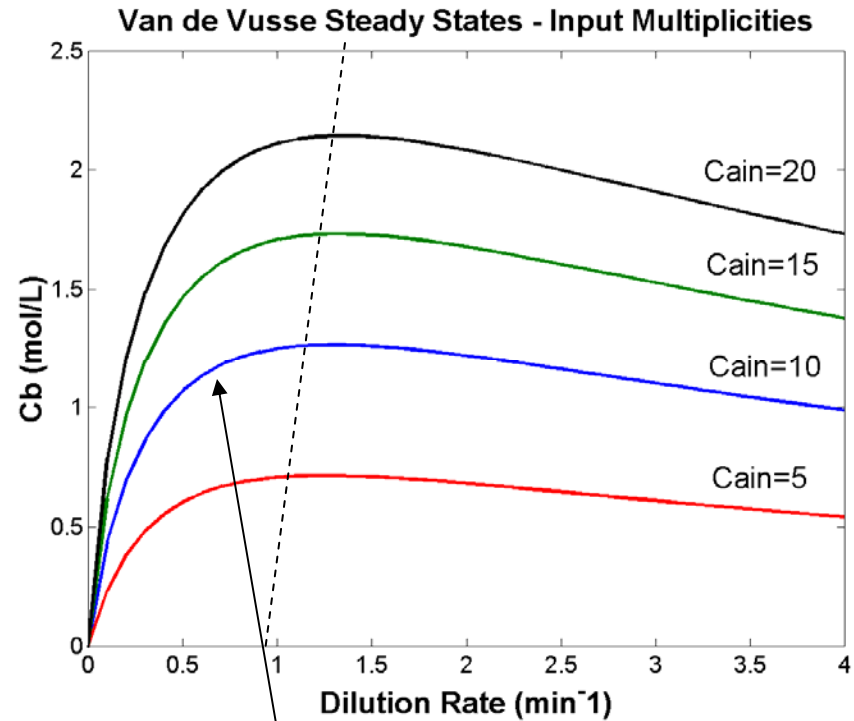
- Constraints on inputs and outputs

## Quadratic Program (QP)

# Van de vuisse isothermal reactor



Constant  $V, T, \rho$



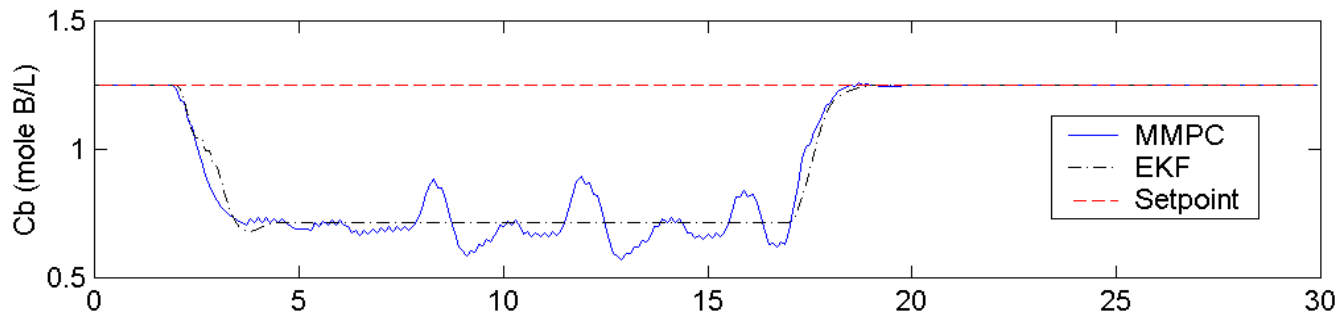
$C_B$   
 $F$

Region with RHP zeros  
(nonminimum phase behavior)

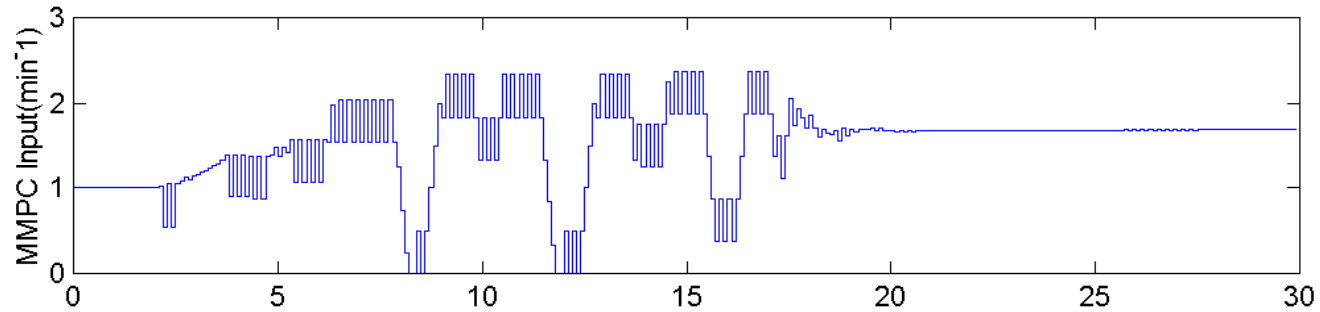
Connection between IM and RHP zeros:  
Sistu & Bequette, *Chem. Eng. Sci.* (1996)

# Feed Concentration Disturbance

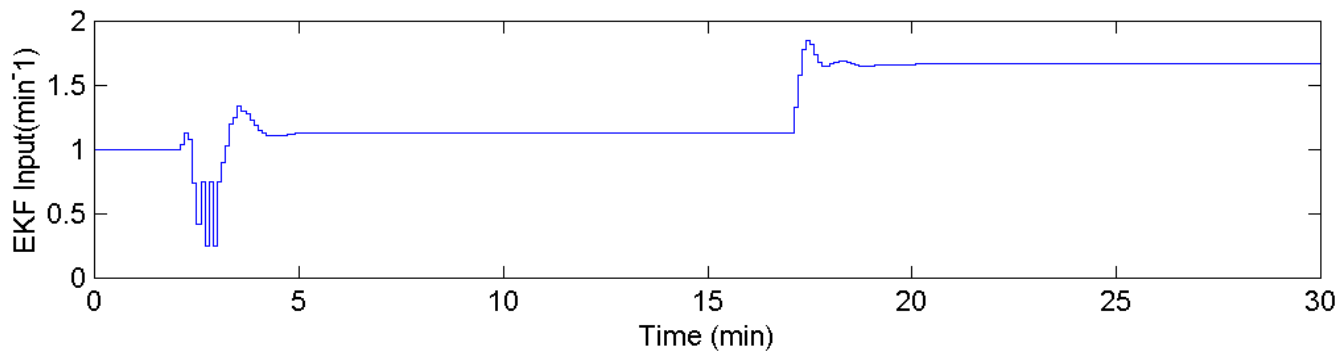
$C_B (y)$



MMPC  
(u)



EKF-  
based  
NL-MPC  
(u)



# Disturbances & Propagation Into Future

- Output step
  - Generally poor assumption for chemical processes
- Input step
  - Improved performance for many processes
- Input ramp
  - Motivated by experience with diabetes problems
- Pulse
  - Duties performed infrequently (shift change, etc.)
- Periodic
  - Poorly tuned upstream controllers, diurnal variations

# Step, Ramp, Generic

$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x_{k+1}^a} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & 1 \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k + \underbrace{\begin{bmatrix} 0 \\ \Gamma^w \end{bmatrix}}_{\Gamma^{w,a}} w_k$$

$$y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a} + v_k$$

$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \\ \Delta d_{k+1} \end{bmatrix}}_{x_{k+1}^a} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ 0 & 0 & 1 \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \\ \Delta d_k \end{bmatrix}}_{x_k^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \\ 0 \end{bmatrix}}_{\Gamma^a} u_k + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\Gamma^{w,a}} w_k$$

$$y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \\ \Delta d_k \end{bmatrix}}_{x_k^a} + v_k$$

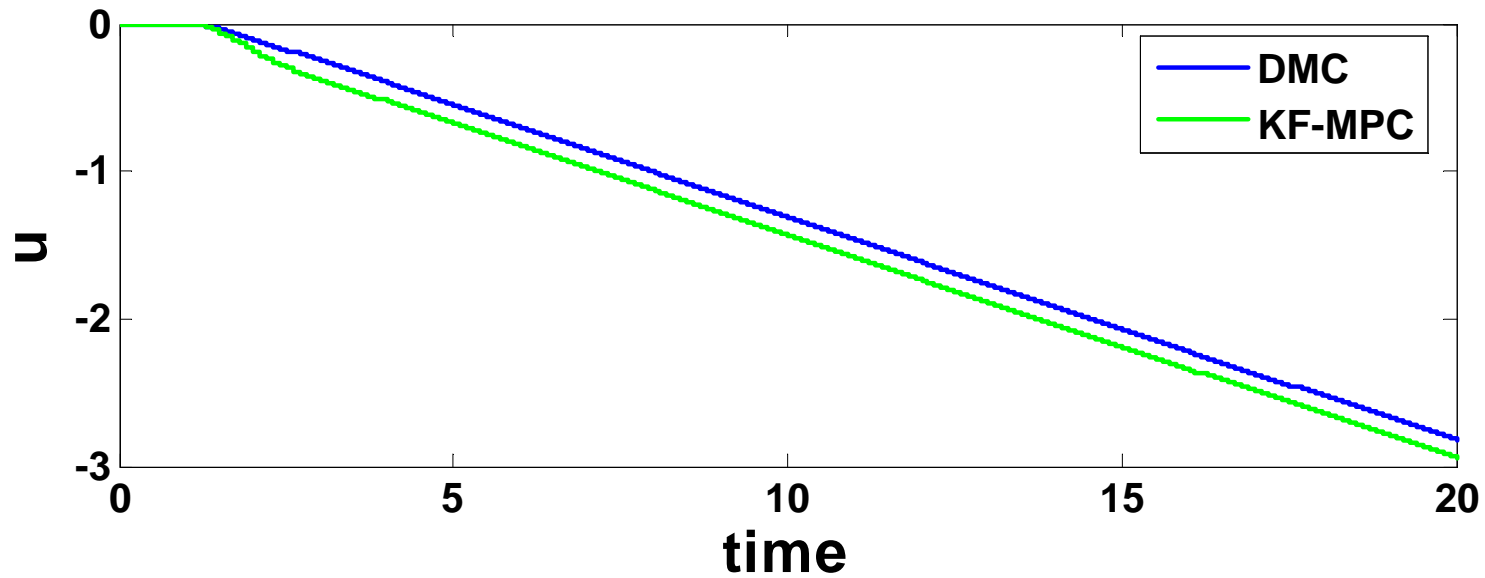
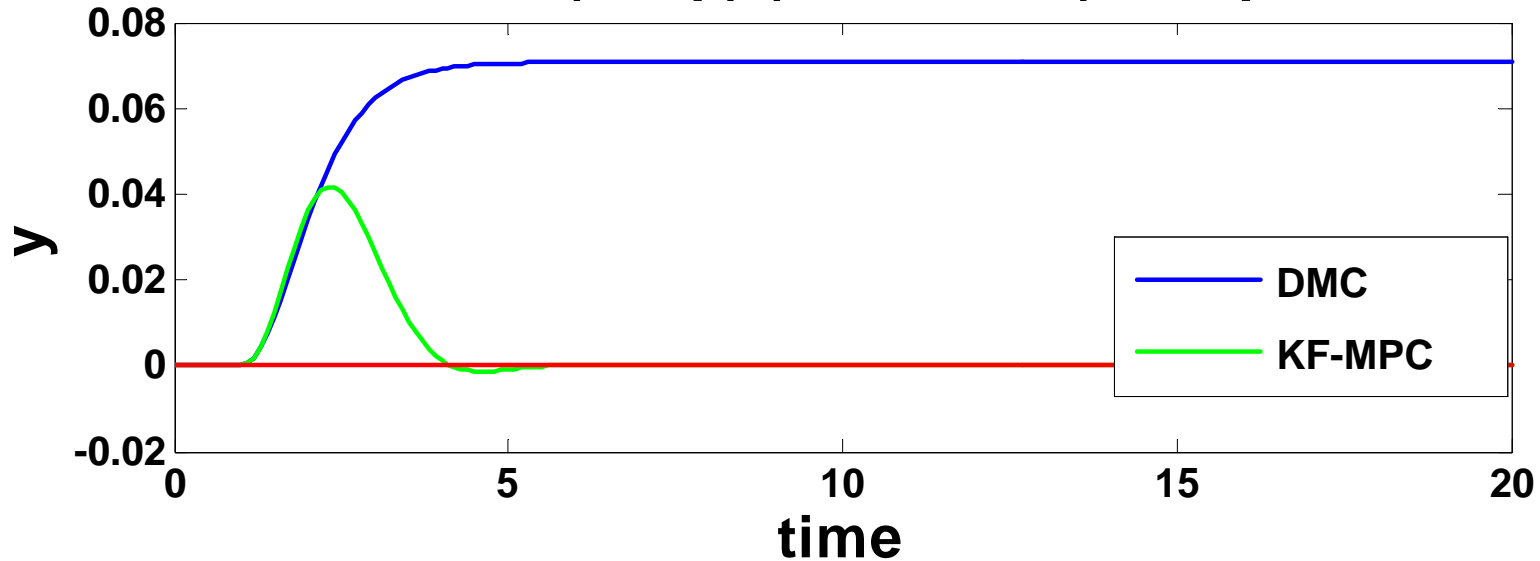
$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x_{k+1}^a} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & \Phi^w \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k + \underbrace{\begin{bmatrix} 0 \\ \Gamma^w \end{bmatrix}}_{\Gamma^{w,a}} w_k$$

$$y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a} + v_k$$

# Ramp Disturbance

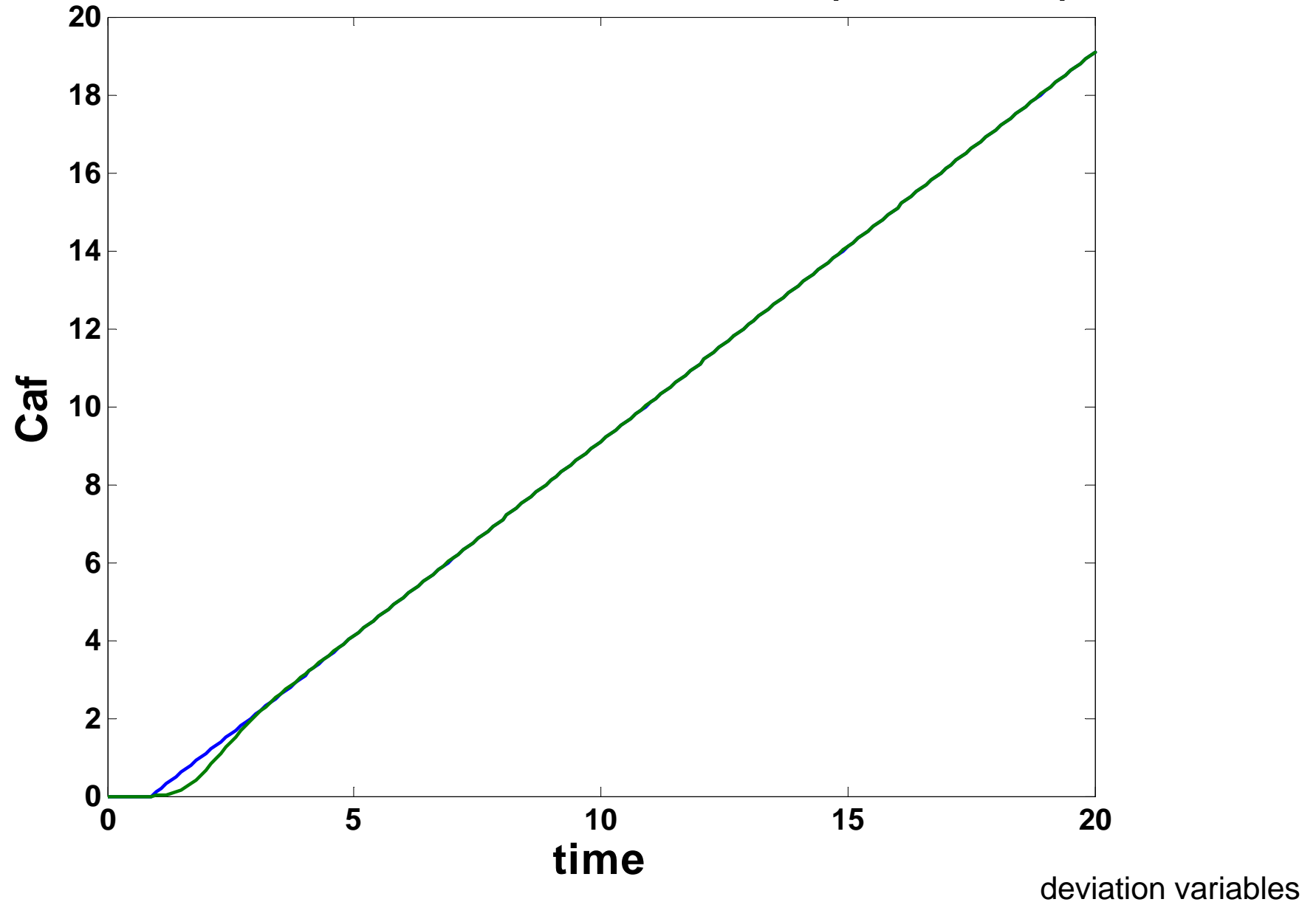
deviation variables

## DMC vs. KF-MPC (ramp) ( $Q=100, R=1$ ), ramp disturbance



# Unmeasured Feed Concentration

feed conc, actual & estimated, KF (Q=100,R=1)





# Periodic Disturbance

$$\begin{bmatrix} \dot{C}_A \\ \dot{C}_B \end{bmatrix} = \begin{bmatrix} -2.4048 & 0 \\ 0.8333 & -2.2381 \end{bmatrix} \begin{bmatrix} C_A \\ C_B \end{bmatrix} + \begin{bmatrix} 7 & 0.5714 \\ -1.117 & 0 \end{bmatrix} \begin{bmatrix} F/V \\ C_{Af} \end{bmatrix}$$

$$\begin{bmatrix} \dot{C}_{Af} \\ \ddot{C}_{Af} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} C_{Af} \\ \dot{C}_{Af} \end{bmatrix} + \begin{bmatrix} 0 \\ d \end{bmatrix}$$

Poles on imaginary axis



Results in sin variation in feed concentration

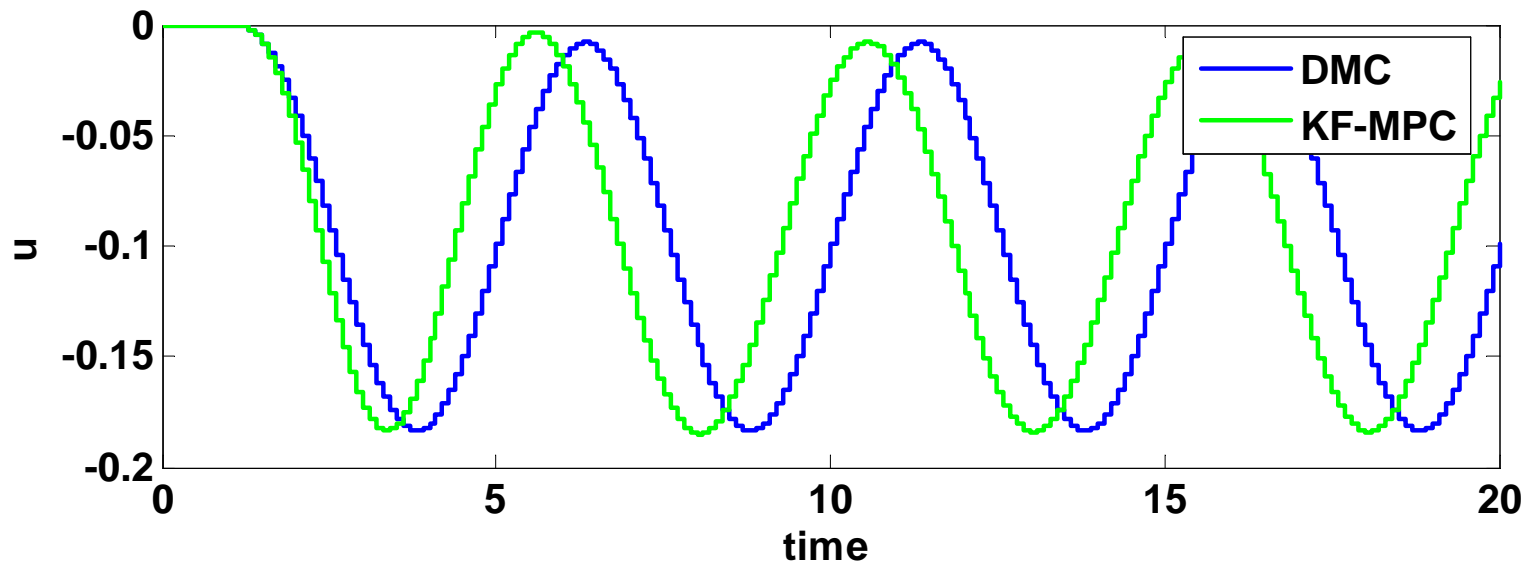
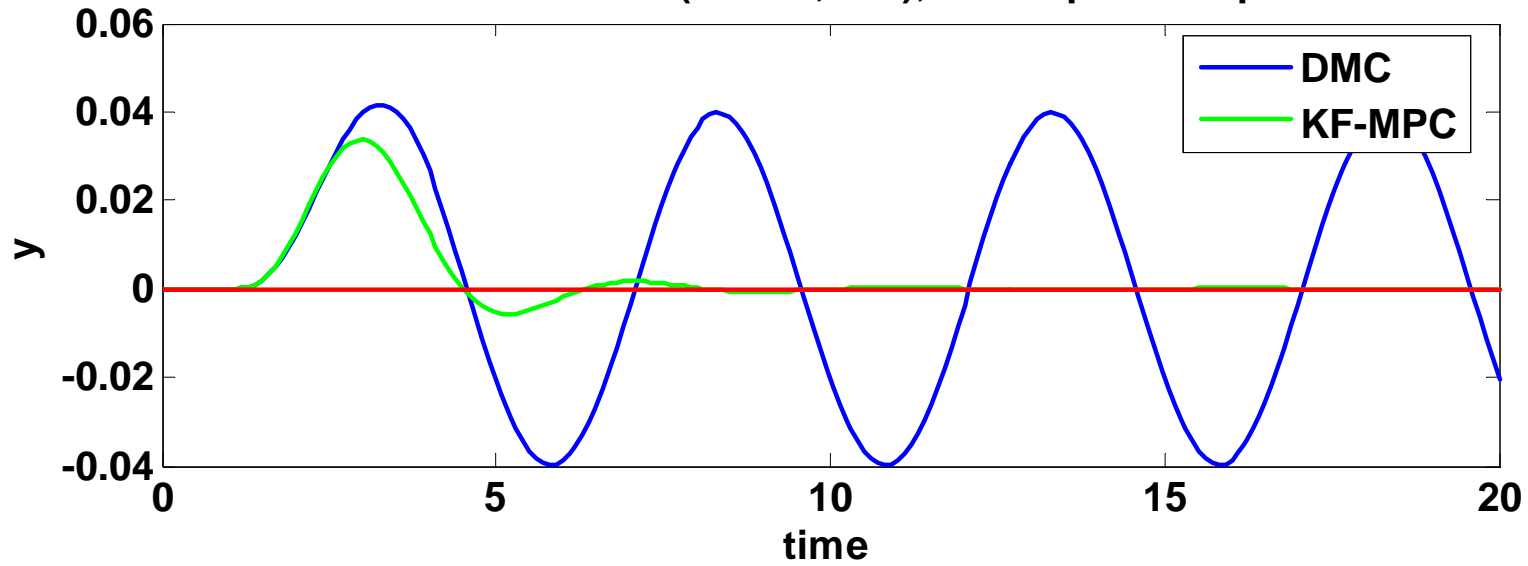
$$\begin{bmatrix} \dot{C}_A \\ \dot{C}_B \\ \dot{C}_{Af} \\ \ddot{C}_{Af} \end{bmatrix} = \begin{bmatrix} -2.4048 & 0 & 0.5714 & 0 \\ 0.8333 & -2.2381 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} C_A \\ C_B \\ C_{Af} \\ \dot{C}_{Af} \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ -1.117 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F/V \\ d \end{bmatrix}$$

Appended state form

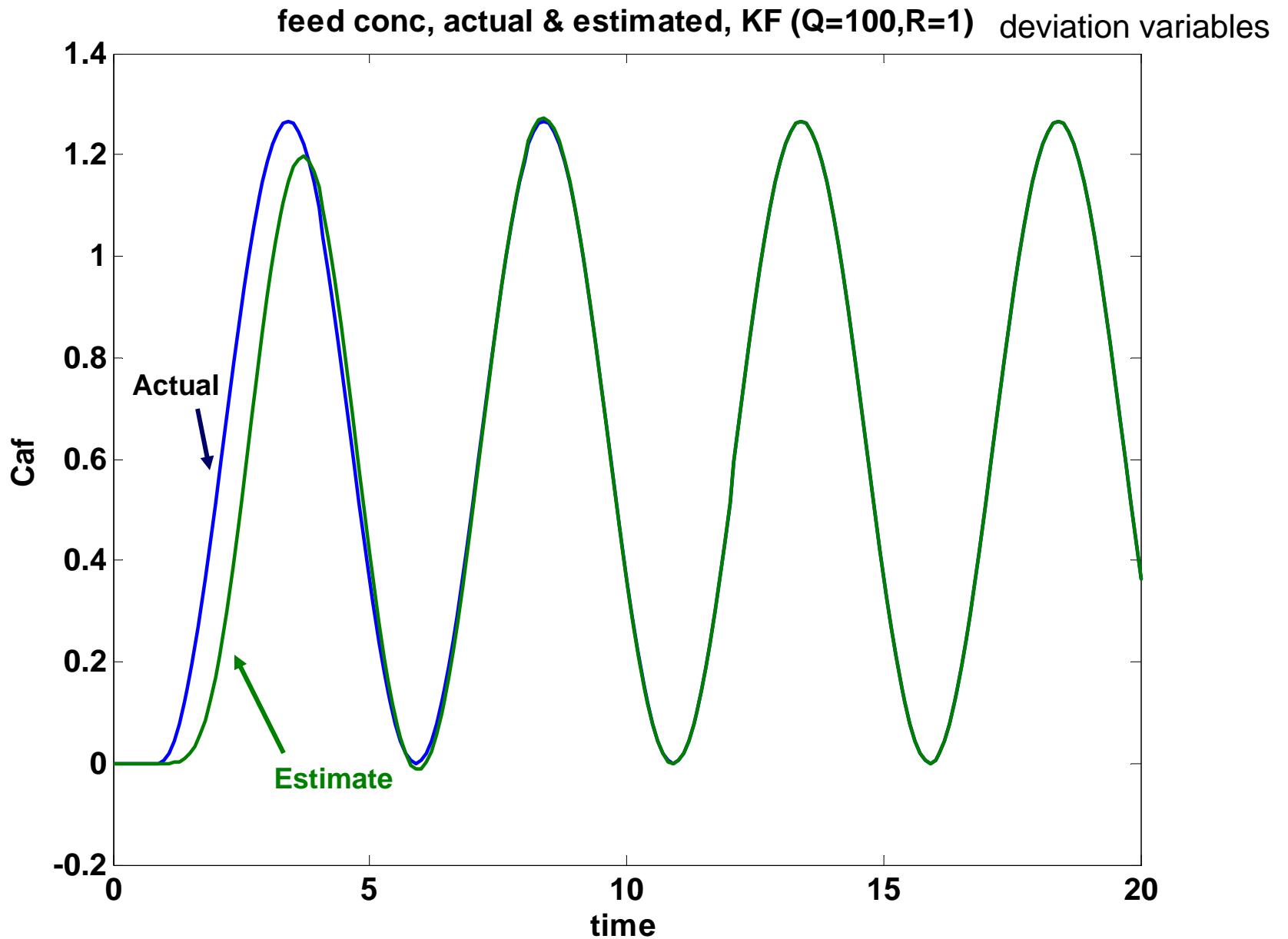
# DMC vs. KF-MPC

deviation variables

DMC vs. KF-MPC (Q=100,R=1), actual plant outputs



# Feed Concentration Estimate



# Numerous Potential Disturbances

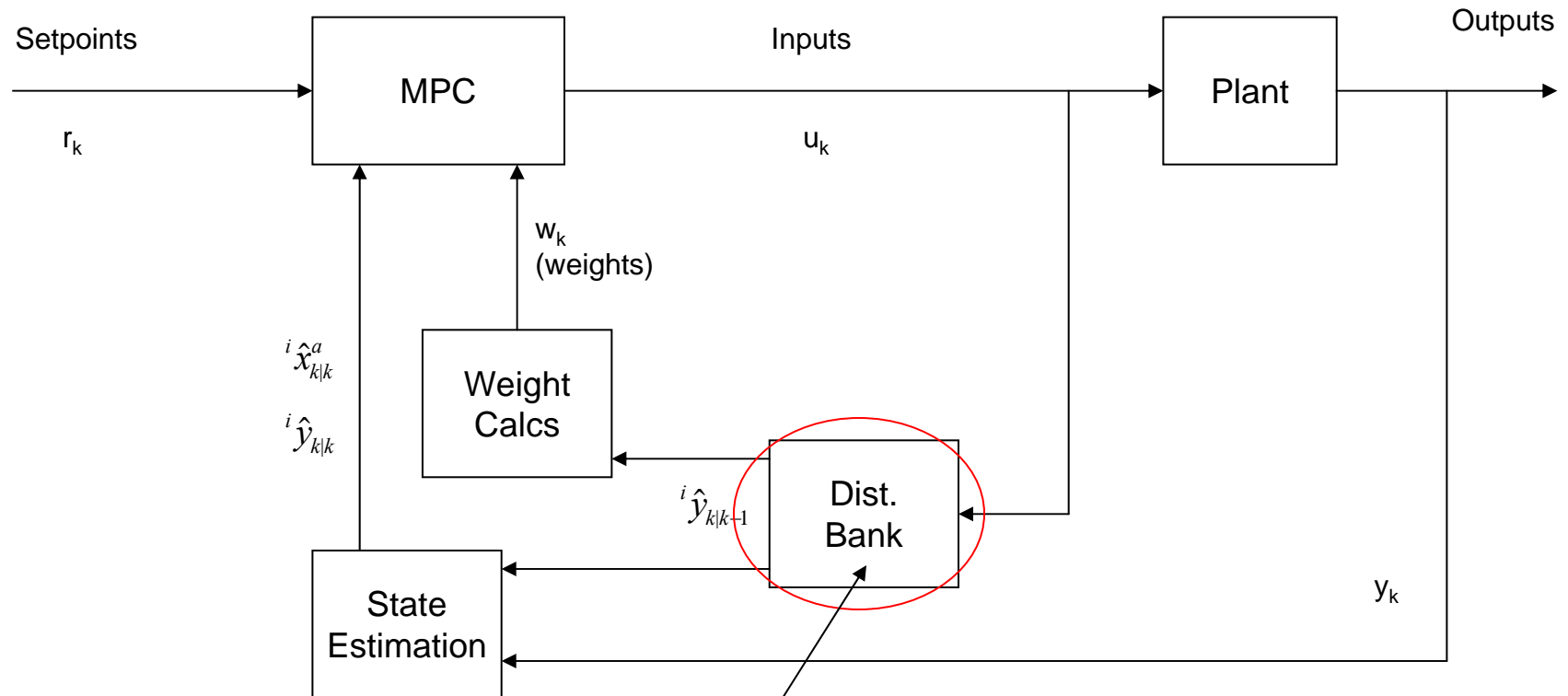
- Current Practice
  - Choose most important disturbance(s) to estimate & reject (number of disturbances = number of measurements)
- Model Bank
  - Each model associated with a different type of disturbance
  - Weighting/Blending or Switching between models

*Ind. Eng. Chem. Res.* 2010, 49, 7983–7989

**Multiple Model Predictive Control Strategy for Disturbance Rejection<sup>†,‡</sup>**

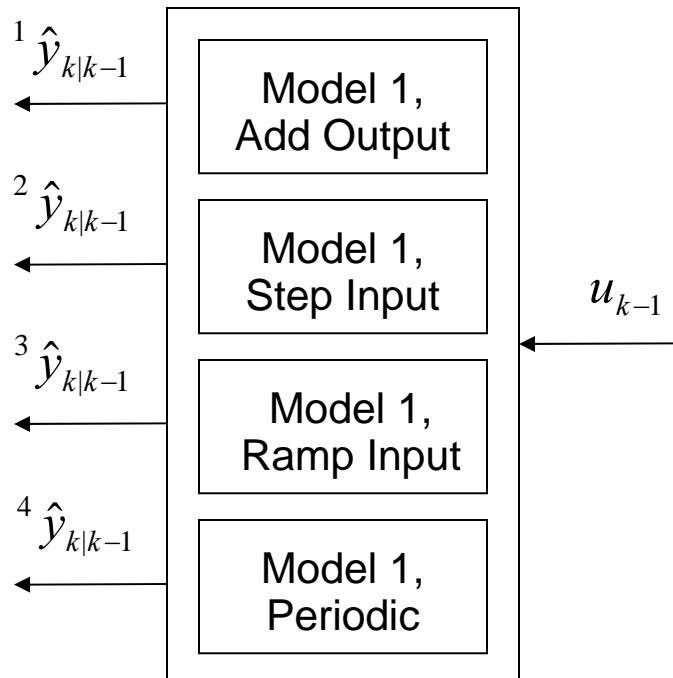
Matthew Kuure-Kinsey<sup>§</sup> and B. Wayne Bequette\*

# Disturbance Estimation - Framework



Primary difference is in the “disturbance bank”

# Disturbance Bank Structure

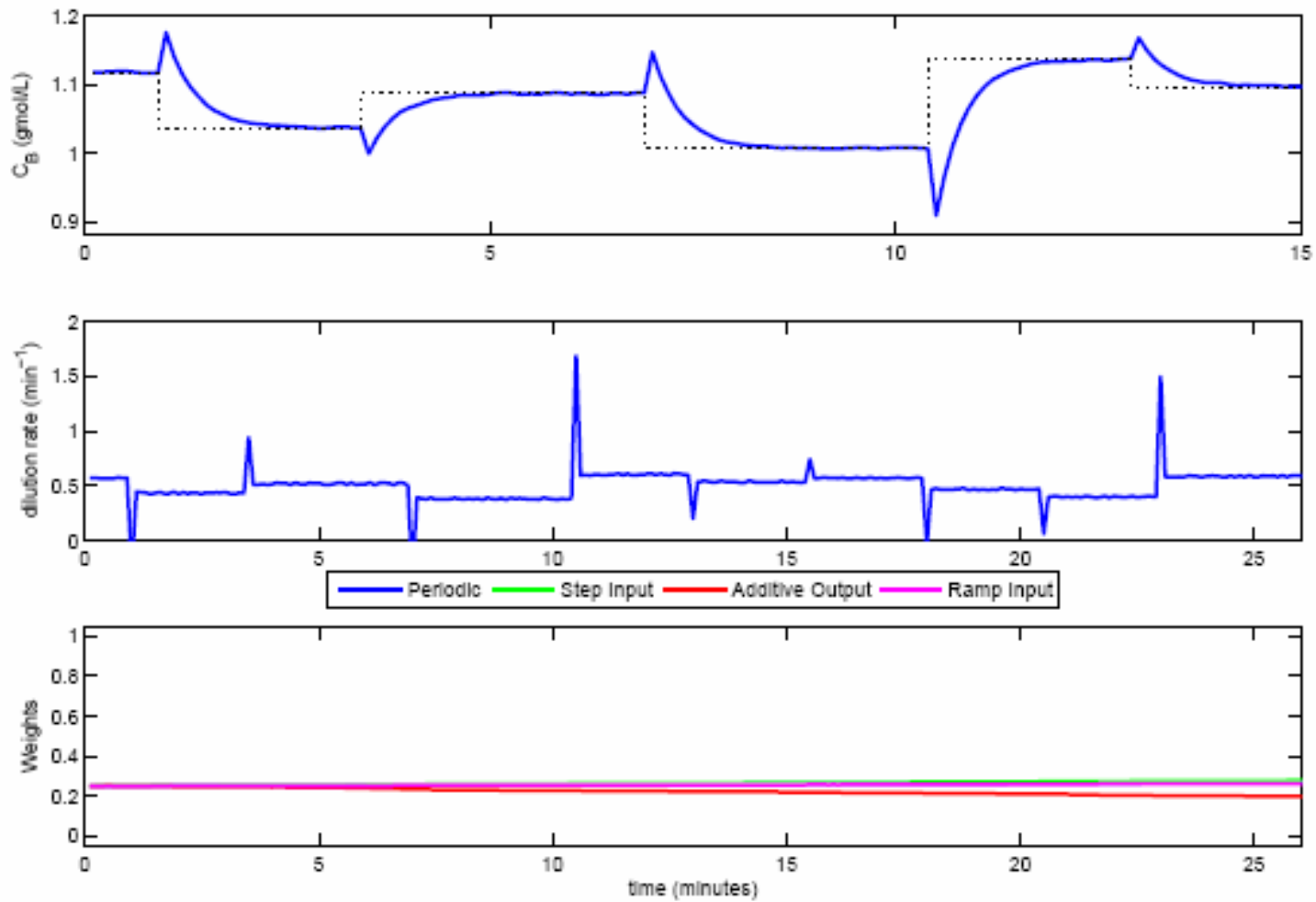


$$\underbrace{\begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix}}_{\hat{x}_{k+1|k}^a} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & \Phi^w \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} \hat{x}_{k-1|k-1} \\ \hat{d}_{k-1|k-1} \end{bmatrix}}_{\hat{x}_{k-1|k-1}^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_{k-1}$$

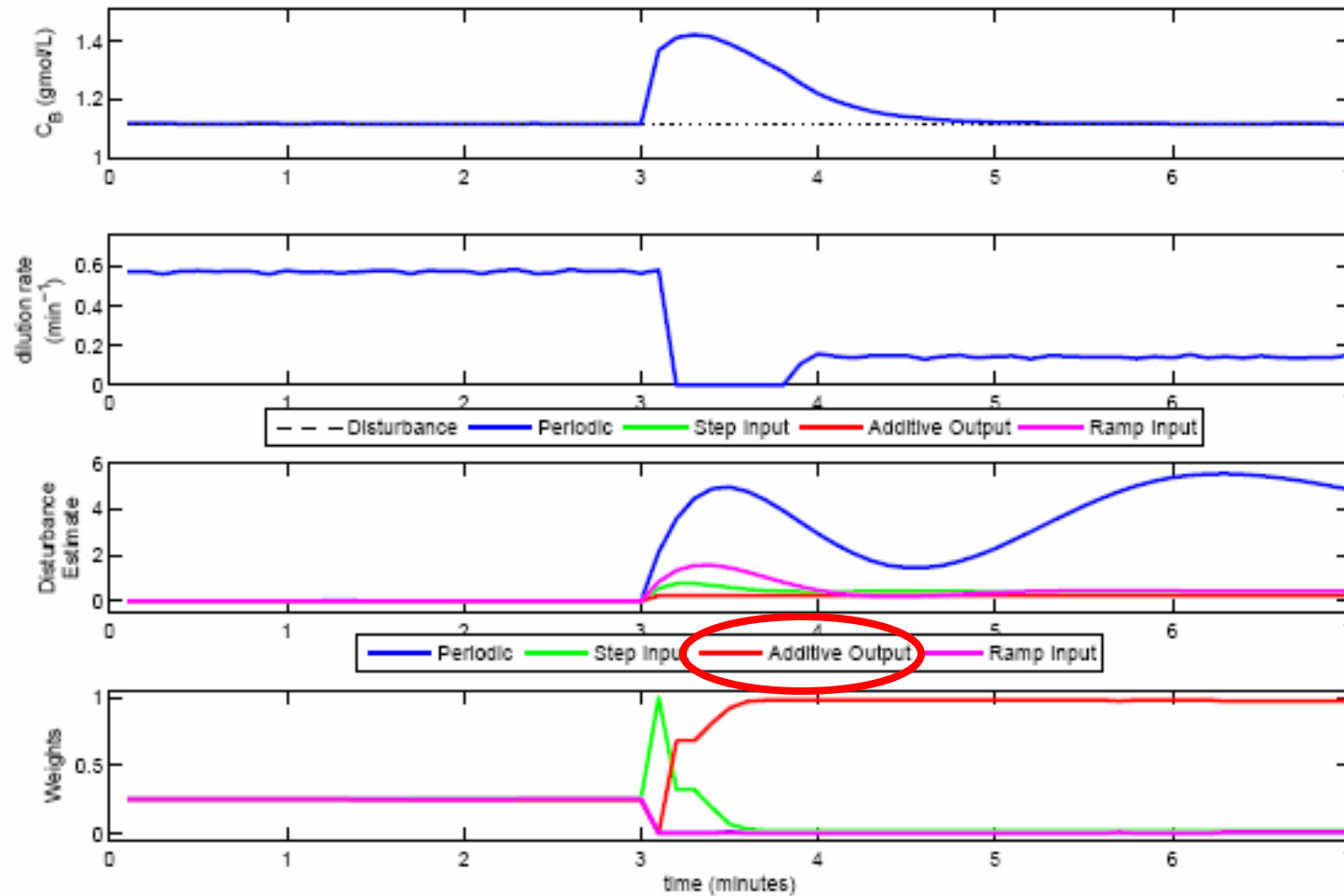
$$\hat{y}_{k|k-1} = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} \hat{x}_{k-1|k} \\ \hat{d}_{k-1|k} \end{bmatrix}}_{x_{k-1|k}^a}$$

# Setpoint Tracking (No Disturbances)

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# Additive Output Disturbance

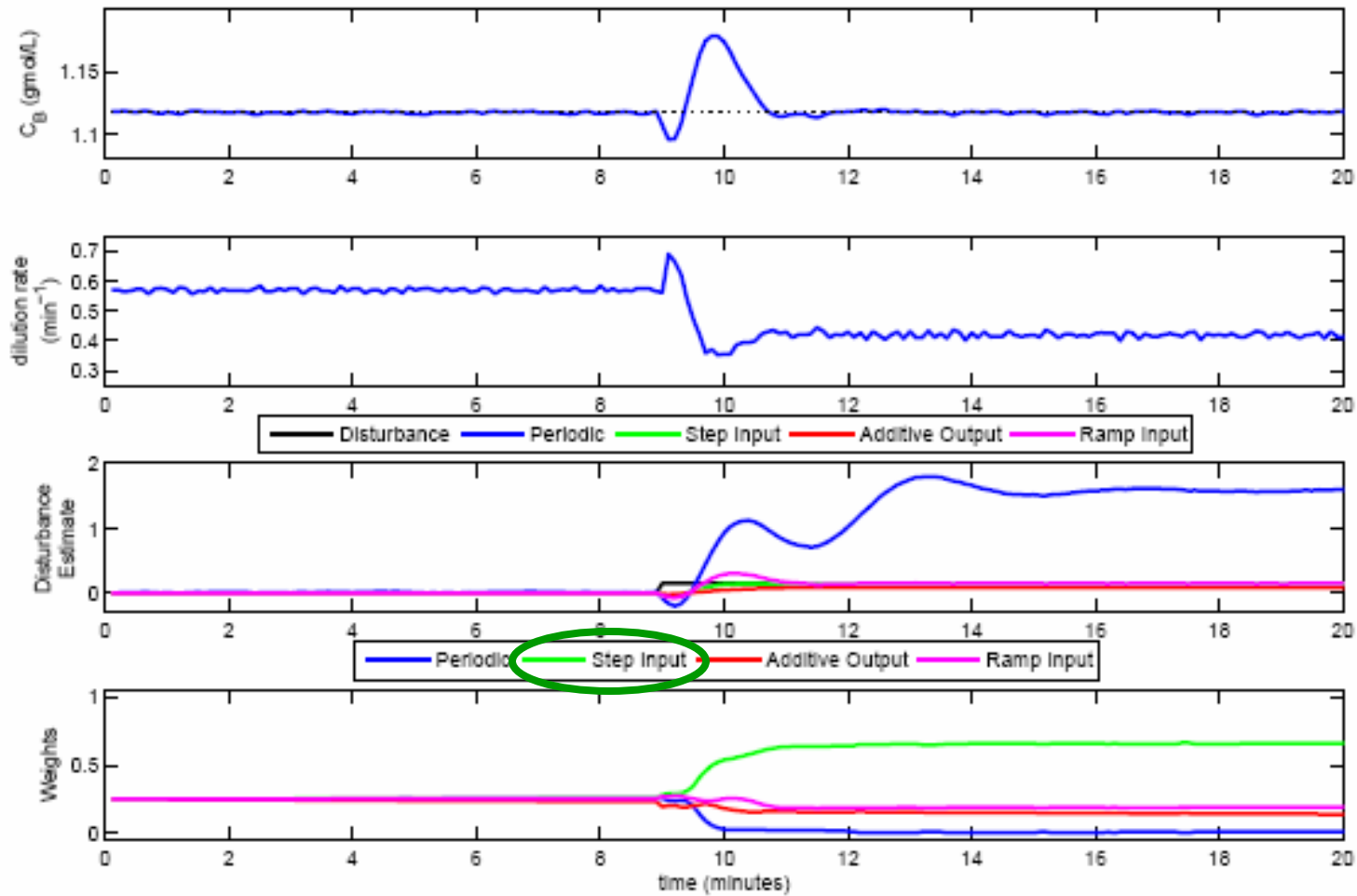


Measured output

Weights



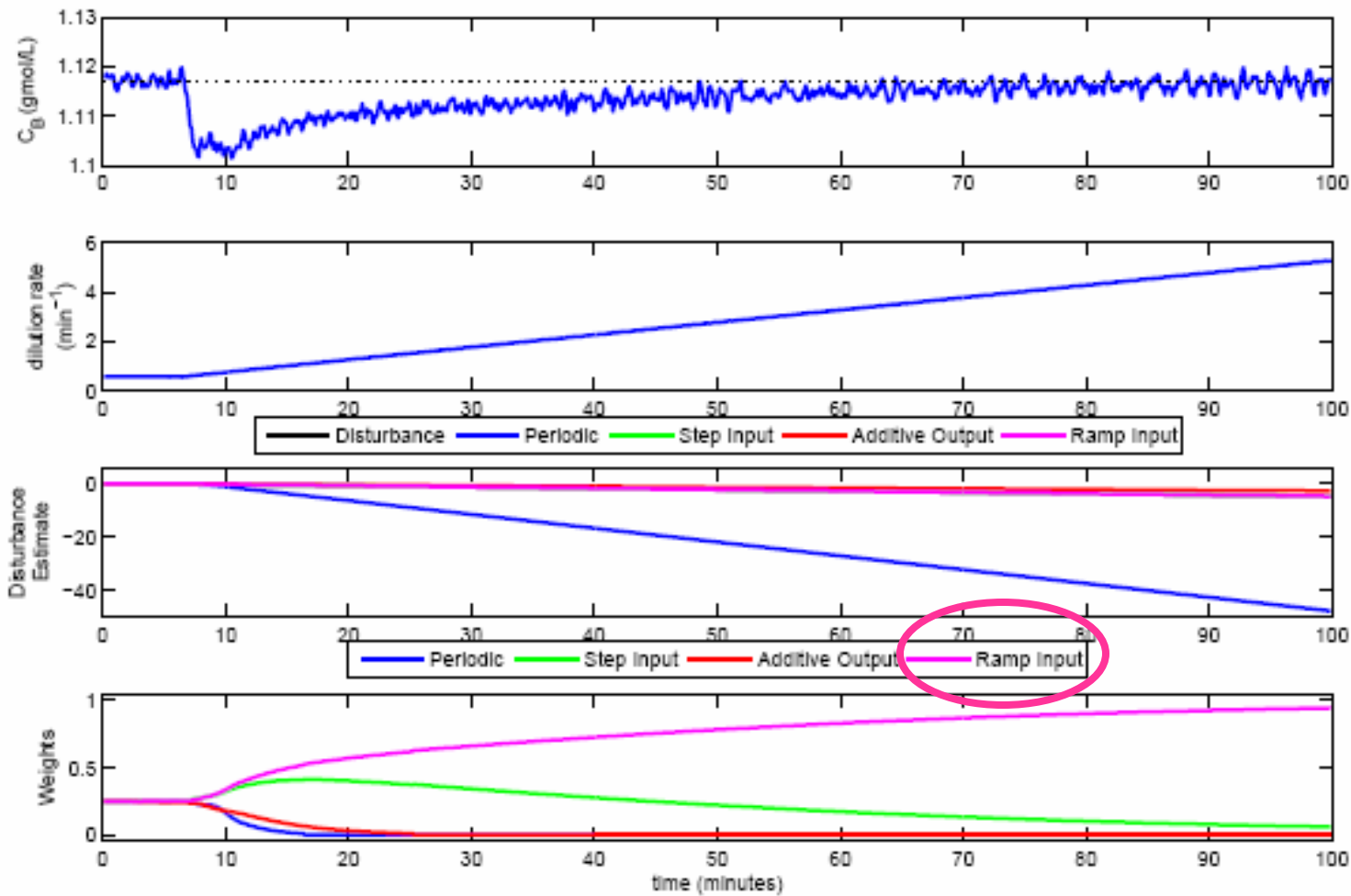
# Step Input Disturbance



Measured output

Weights

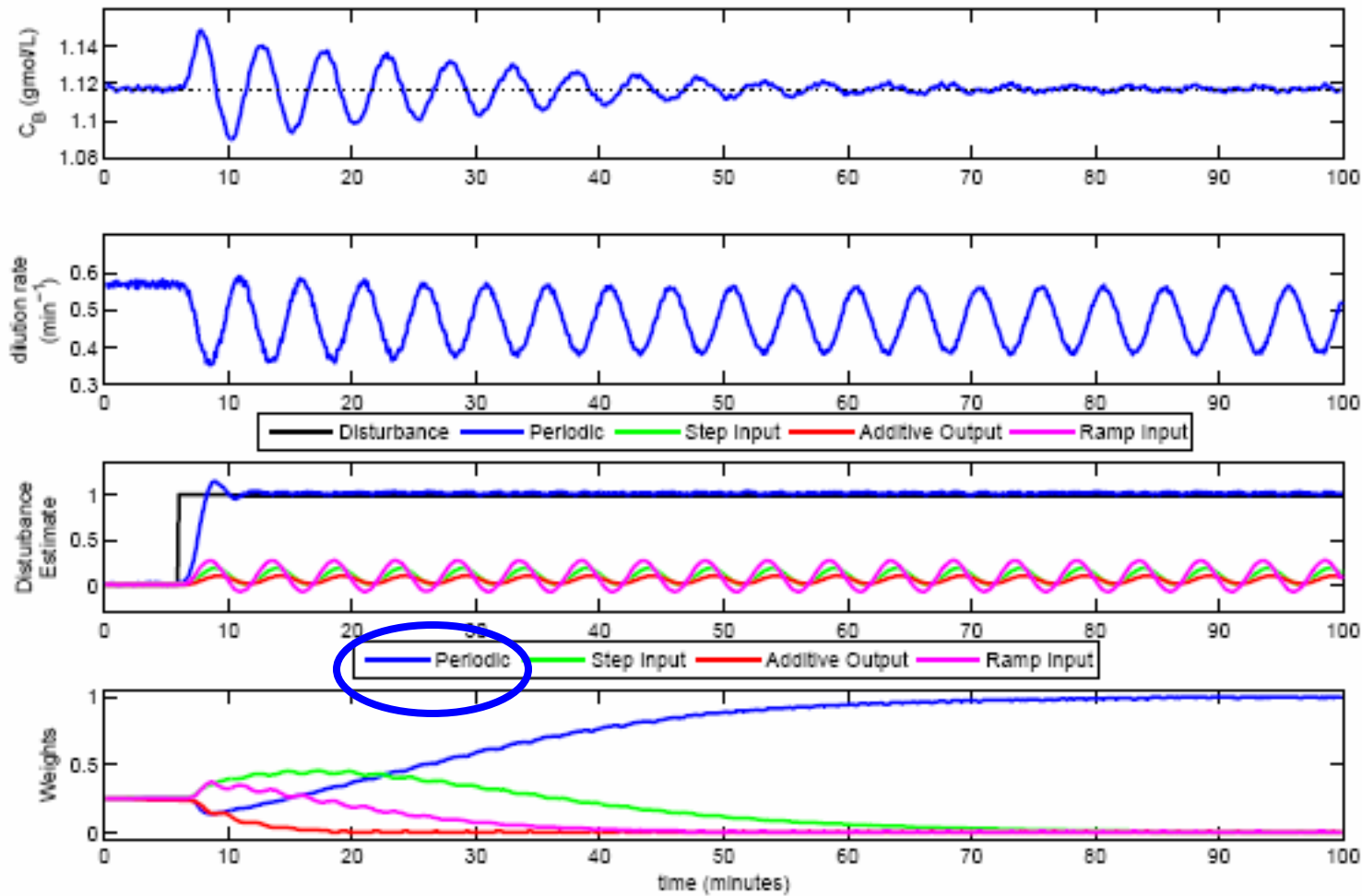
# Ramp Input Disturbance



Measured output

Weights

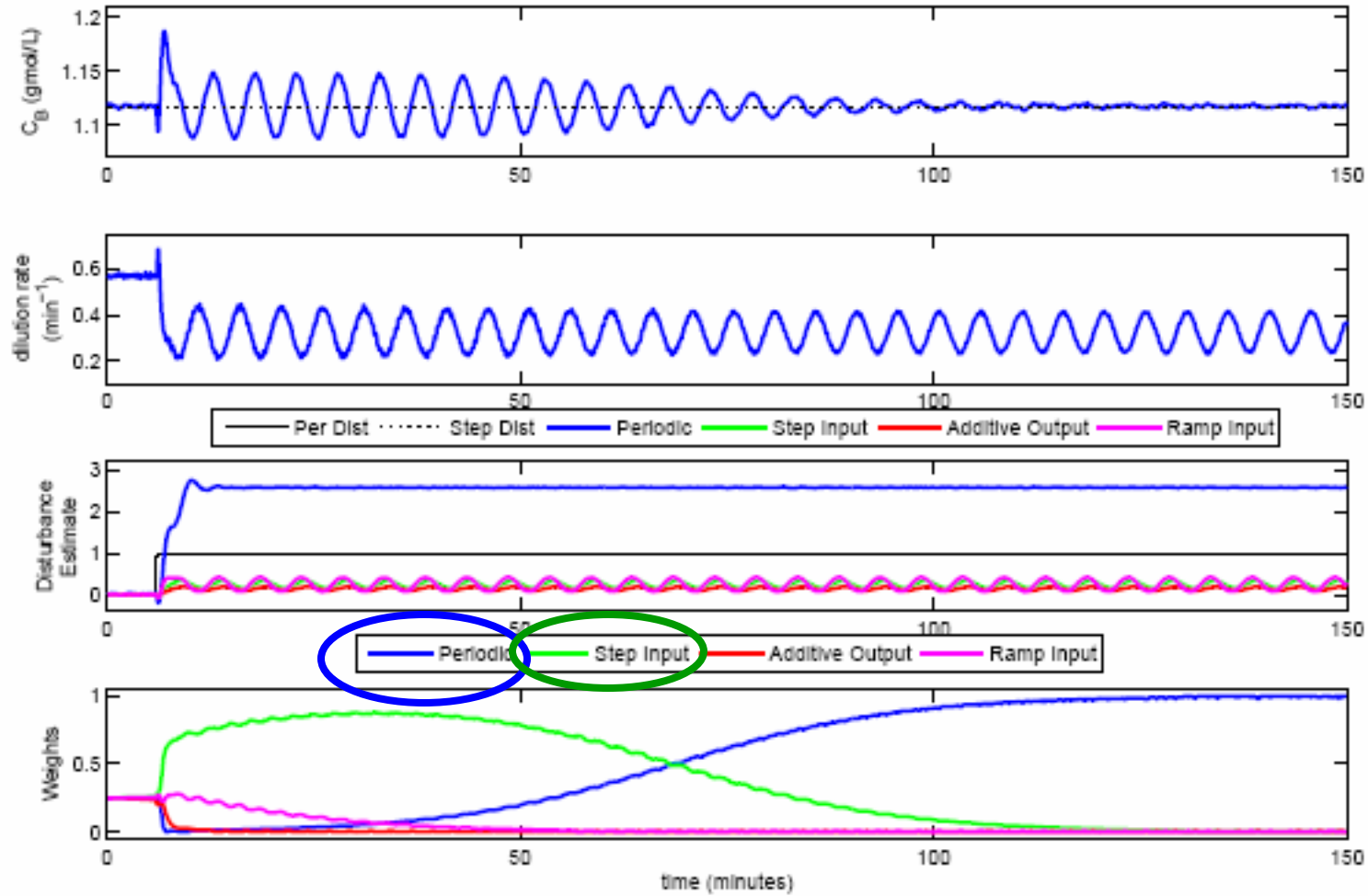
# Periodic Input Disturbance



Measured output

Weights

# Step + Periodic Input Disturbances



Measured output

Weights

# Presentation Summary

- Nonlinear Model Predictive Control
  - Limitations to DMC formulation
  - State estimation-based approaches
  - Multiple linear model-based approaches
  - Formulation for different disturbances

# Acknowledgments

- Matthew Kuure-Kinsey (2008)



- Brian Aufderheide (2002)



- Ramesh Rao (2000)  
– ASPENTECH



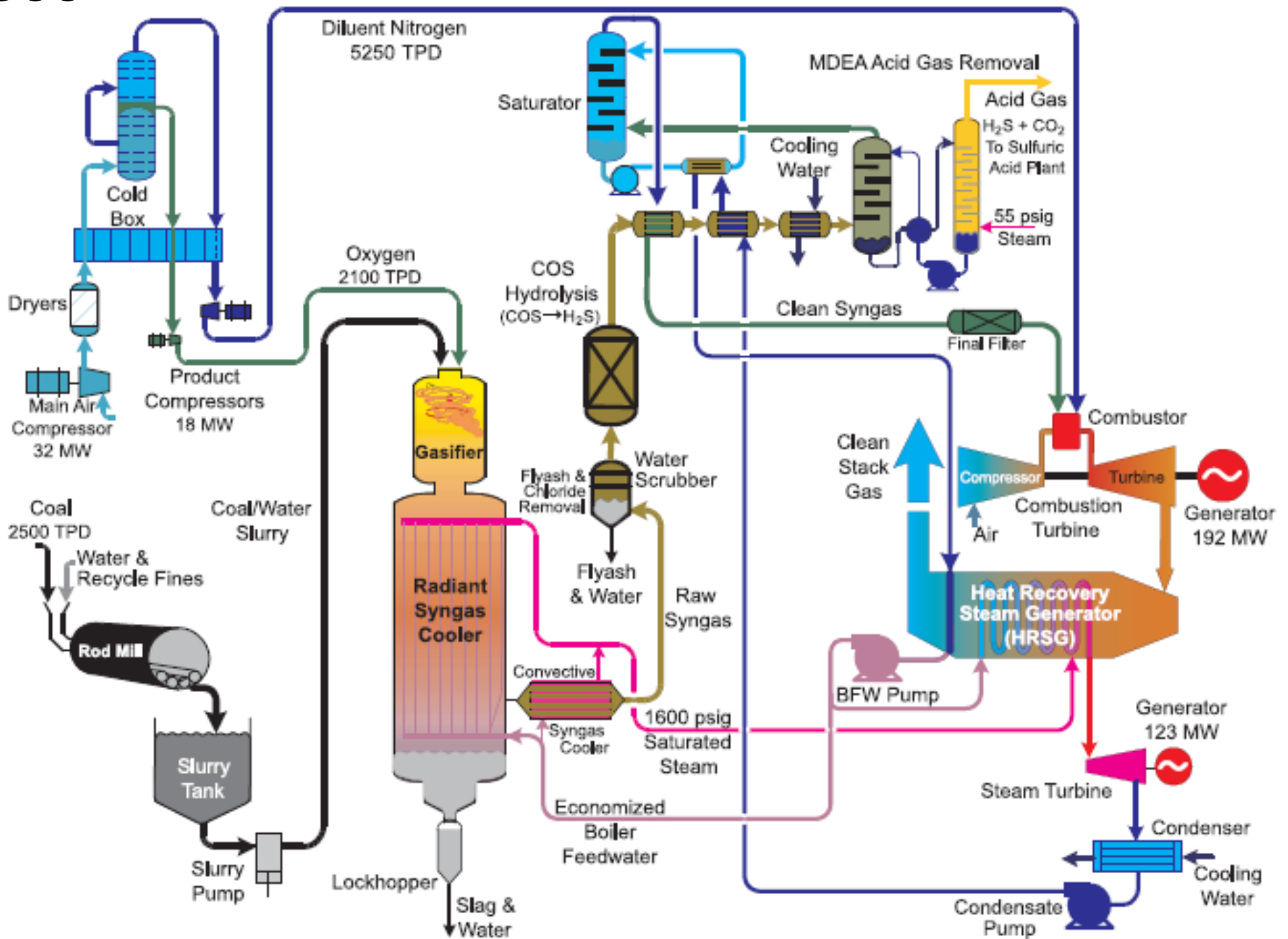
- Clem Yu (1992)  
– Baxter



# Current Applications of MPC

- Integrated Gasification Combined Cycle (IGCC) Power Plants
  - Carbon Capture case
- Fuel Cell Systems
  - Smart Grid
- Closed-loop Artificial Pancreas for Type 1 Diabetes

# IGCC





# IGCC Summary

- **IGCC Power Plant is a complex system to simulate**
  - Significant reworking of steady-state flowsheet required adding plant details and dynamic behavior
  - 300 Units & 450 Streams
  - 80,000 equations solved – dynamic simulations
- **Linear MPC is a powerful tool to control complex plants**
  - Hierarchical Control Structure Design
  - Load Following / Disturbance Rejection
  - Centralized Approach is better suited for following load-demand
  - Negative effect on controllability and robustness w/ increasing level of interactions
- **Limitations of this research**
  - Simplifying assumptions on some units
  - Proprietary equipment details/ lack of validation data
  - Workaround for software bugs/limitations



**Priyadarshi Mahapatra**  
Ph.D., 2010; NETL

# Fuel Cell Systems

- Combined heat & power
- Nonlinear dynamics & control
- Stack condition monitoring
- Real-time optimization
- Nature-inspired membrane and catalyst design (Marc-Olivier Coppens)



Matthew Kuure-Kinsey (Dec '08)



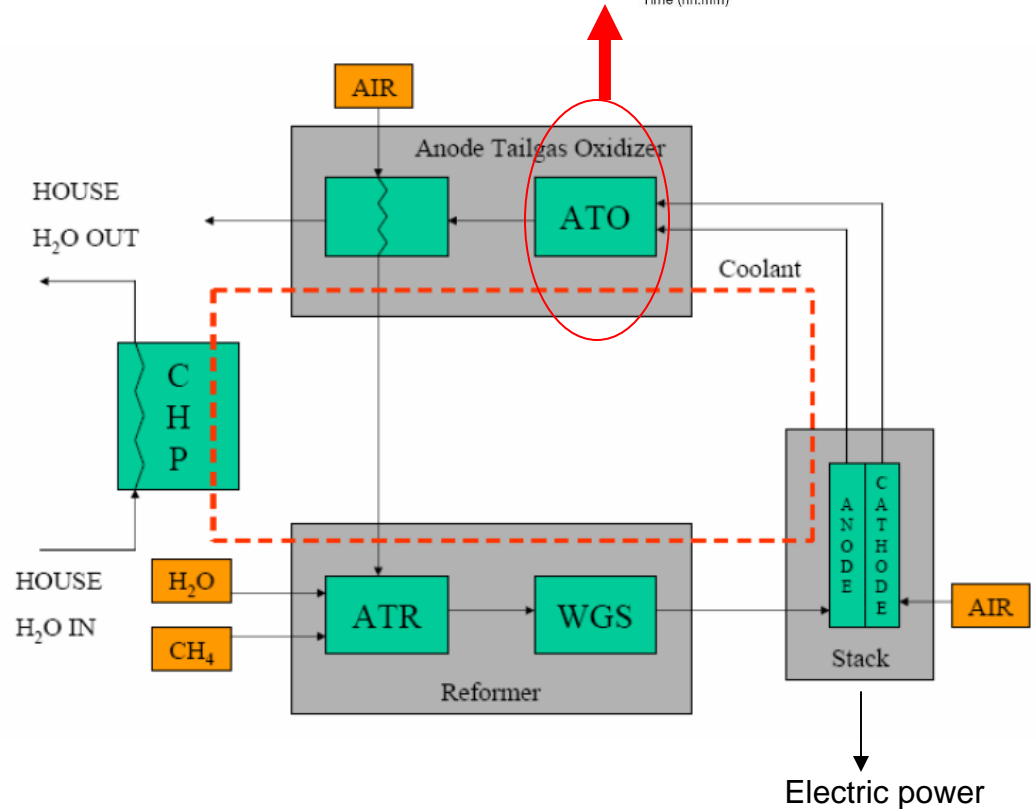
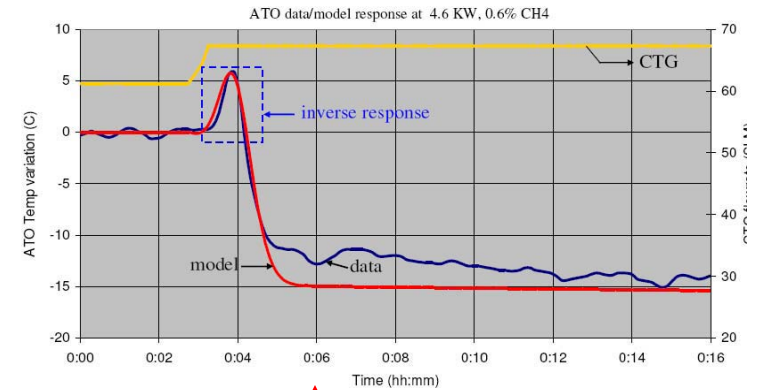
Judy O'Rourke (Dec 2008)



Jeff Marquis  
(w/Coppens)



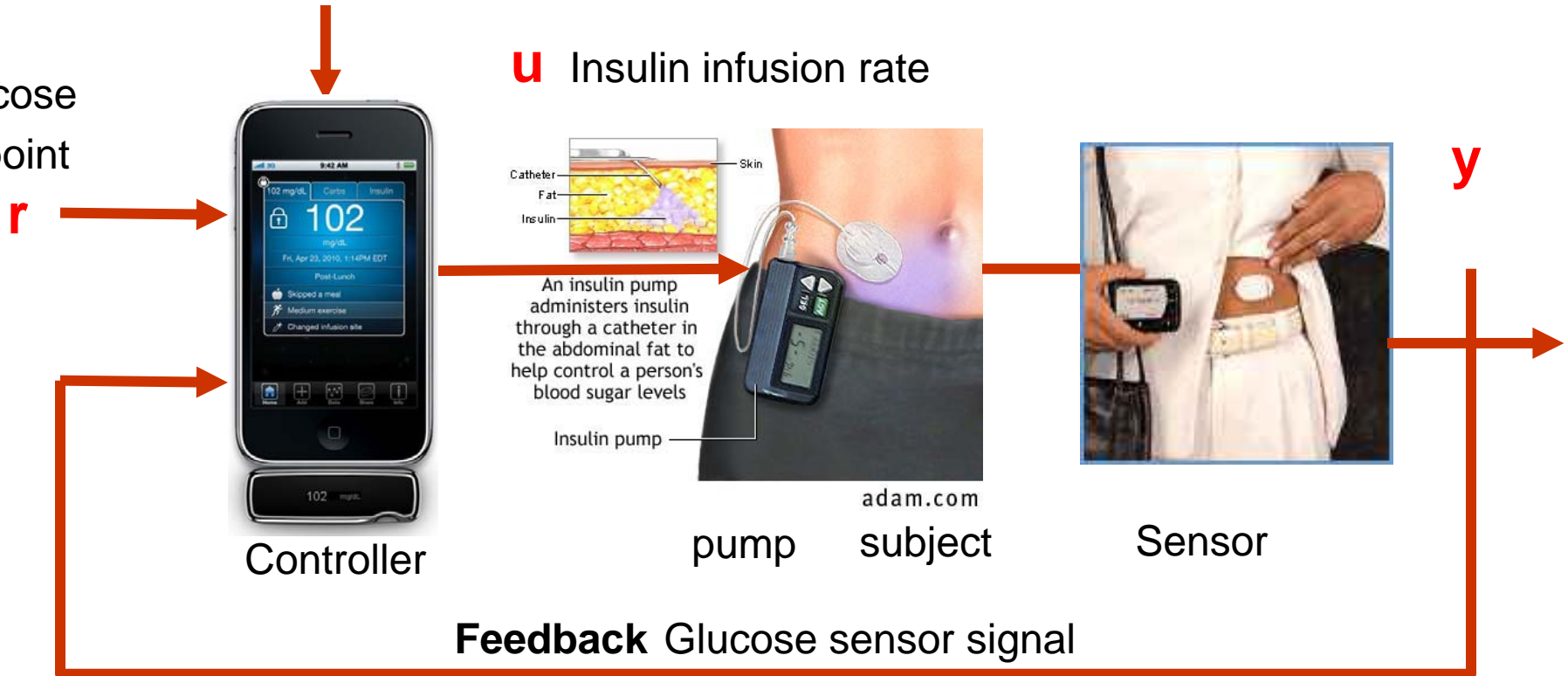
Matt Titus



# Closed-loop Artificial Pancreas

Meal knowledge: **Feedforward**

Glucose setpoint  
 $r$



Hyunjin Lee  
Now at Shell  
Development



Jing  
Sun



Ruben Rojas  
Fulbright Scholar  
Venezuela



Fraser  
Cameron