19th European Symposium on Computer Aided Process Engineering – ESCAPE19
J. Jeżowski and J. Thullie (Editors)

The supply-chain pick-up and delivery problem with transshipments

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Abstract

In this work, a strict MILP formulation for the supply-chain pick-up and delivery problem with transshipments is presented (SC-PDP-T). This problem adds the option for transferring goods from one vehicle to another. This additional flexibility is very attractive for the optimal management of supply chains. Based on the novel features of the SC-PDP-T, this work addresses the major modeling and solution issues related to this problem by presenting a new MILP-based strategy to find the set of decisions to optimally manage complex multi-site distribution systems of moderate-size.

Keywords: Supply-chain, pick-up and delivery, transshipments.

1. Introduction

The cost-effective management of multisite production system is a complex task that needs to be aided by efficient computational tools. In this direction, the pickup and delivery problem (PDP) has been one of the most studied network logistic problems in the transportation-research literature [1, 2]. In the emerging area of enterprise-wide optimization, a great deal of effort is focussed on the optimization of complex supply chains. So, in the last years, the inherent features of real-world supply chains have motivated the development of numerous and more realistic variations of the classical PDP. Following this trend, the so-called supply-chain PDP (SC-PDP) have been defined by [3] aiming at generalizing the typical PDP by considering alternative supply sites, inventory constraints, multiple visits to the same site and multiple commodities. Sometimes, forcing each load to be directly transported from its source to its final destination by using a single vehicle is a hard assumption. This limitation explains why the PDP, a widely studied problem in the transportation research area, has not been widely used in supply chains applications. The possibility for goods to be transferred from one vehicle to another adds a higher flexibility to the operation that could improve the overall productivity by exploiting the interaction between vehicles at specific transfer points. Obviously, if the added flexibility reduces the total load-transported and travel times, one could try to find favorable conditions for implementing a support system that allows transfers instead of rigid PDP formulations. So, the PDP is here extended to consider transfer points, i.e. distribution centers, where some vehicles can drop its load to allow others to pick up it later, as defined by [4]. The modeling of this new feature generates a new logistic problem called the SC-PDP with transshipment (SC-PDP-T).

2. Problem statement

Consider a routes-network represented by a graph $\boldsymbol{G}[\{p \cup I \cup T\}; A]$, where p is the vehicles base; $I = \{I^+ \cup I^-\}$ is the set of customers, being $I^+ = \{i_1, ..., i_n: l_i > 0$ for all $i_i\}$ the set of pick-up customers; $I^- = \{j_1, ..., j_{n'}: l_i > 0$ for all $j_i\}$ the set of delivery clients; and T is a transshipment base, i.e a distribution center, defined as $T = \{T^+ \cup T^-\}$, where T^+ represents the set of reception nodes and T^- the set of delivery nodes. Both sets are linked by a set of transfer operations R. Therefore, if the load delivered to node $i \in T^-$ must be transferred to the node $j \in T^+$, then operation $r \in R$ is defined by the couple $r = \{i, j\}$. Finally, $A = \{a_{ij} / i, j \in I^+ \cup I^- \cup T^+ \cup T^- \cup p\}$ defines the net of minimum cost arcs among customers $i \in I$; the warehouse p and the transshipment base T. A distance-based traveling-cost matrix $C = \{c_{ij}\}$ and a travel-time matrix $\Gamma = \{t_{ij}\}$ are associated to the net A. The service-times to pickup/deliver the load l_i from (to) nodes $i \in (I \cup T)$ are denoted by st_i and they must be fulfilled by some truck of vehicles fleet $V = \{v_1, v_2, ..., v_m\}$. The solution must provide a finite sequence of arcs, called *route*, for some of the vehicles of V such that: (i) each vehicle starts and ends the trip at p; (ii) each site $i \in I$ is assigned to exactly one route; (iii) a delivery transfer-node $i \in T^-$ may be used or not. If used, its paired pick-up transfer node(s) must also be visited by some vehicle; (iv) the actual load carried by a vehicle must never exceed its transport capacity q_v; (v) the service for any node $i \in I$ must start within the time-window $[a_i, b_i]$; (vi) the duration of the vehicle-v trip must be shorter than a maximum routing time tv_v^{max}. The problem goal is to minimize the total cost for providing pickup/ delivery service to every node $i \in I$.

3. The MILP model

Objective function. The objective aims at minimizing the total routing cost.

$$Min \quad \sum_{v \in V} CV_v \tag{1}$$

Assignment constraints. According to eq. (2.a), each customer $i \in I$ must be visited by a single vehicle, while eq. (2.b) states that a transshipment node $i \in T^-$ may be used ($Y_{iv} = 1$) or not ($Y_{iv} = 0$). Finally, if the delivery node $i \in T^-$ is used, eq. (2.c) forces that its paired node $j \in T^+$: $r = (i,j) \in R$ to be also visited.

$$\sum_{v \in V} Y_{iv} = 1 \quad \forall i \in I \quad (2.a) \qquad \sum_{v \in V} Y_{iv} \leq 1 \quad \forall i \in T^- \quad (2.b)$$

$$\sum_{v \in V} Y_{iv} = \sum_{v \in V} Y_{jv} \quad \forall i \in T^-, j \in T^+, r = \{i, j\} \in R \quad (2.c)$$

Cost-based constraints. Eq. (3.a) computes the least travelling cost (C_i) from the vehicle base to any location $i \in I \cup T$. Eqs. (3.b)-(3.c) sequence customers and transfer nodes in the cost dimension. Thus, if nodes i and j are allocated to the vehicle-v tour ($Y_{iv} = Y_{jv} = 1$), the ordering of both nodes is determined by the value of the sequencing variable S_{ij}. So, if location i is visited before j ($S_{ij} = 1$), according constraints (3.b), the travel cost up to location j (C_j) must be larger than C_i by at least c_{ij}. In case node j is visited earlier ($S_{ij} = 0$), the reverse statement hold and constraint (3.c) becomes active. If one or both nodes are not allocated to the vehicle-v tour, eqs. (3.b)-(3.c) become redundant. Finally, eq. (3.d) states that the overall cost of using any vehicle $v \in V$ (CV_v) must be larger than the travelling cost up to any site i (C_i), by at least the cost of the arc i-p (c_{ip}). M_c is an upper bound for variables C_i and CV_v.

$$C_i \geq c_{pi} \qquad \forall i \in (I \cup T) \tag{3.a}$$

$$\left\{\begin{array}{l} C_j \geq C_i + c_{ij} - M_C(1 - S_{ij}) - M_C(2 - Y_{iv} - Y_{jv}) \\ C_i \geq C_j + c_{ij} - M_C S_{ij} - M_C(2 - Y_{iv} - Y_{jv}) \end{array}\right\} \qquad \forall v \in V, i, j \in (I \cup T): i < j \qquad \begin{array}{r}(3.b)\\(3.c)\end{array}$$

$$CV_v \geq C_i + c_{ip} - M_C(1 - Y_{iv}) \qquad \forall v \in V, i \in (I \cup T) \tag{3.d}$$

Time-based constraints. Eq. (4) forces the service time on any customer $i \in I$ to start at a time T_i bounded by the interval [a_i, b_i]. Also, due to eq. (5), the total routing time TV_v for vehicle $v \in V$ must be lower than the bound t_v^{max}. Eqs (6.a) to (6.d) state visiting-time constraints that are similar to eqs (3.a)-(3.d) but apply to the time dimension. M_T is an upper bound for variables T_i and TV_v.

$$a_i \leq T_i \leq b_i \quad \forall i \in I \quad (4) \qquad\qquad TV_v \leq t_v^{max} \quad \forall v \in V \quad (5)$$

$$T_i \geq t_{pi} \qquad \forall i \in (I \cup T) \tag{6.a}$$

$$\left\{\begin{array}{l} T_j \geq T_i + st_i + t_{ij} - M_T(1 - S_{ij}) - M_T(2 - Y_{iv} - Y_{jv}) \\ T_i \geq T_j + st_j + t_{ji} - M_T S_{ij} - M_T(2 - Y_{iv} - Y_{jv}) \end{array}\right\} \qquad \forall v \in V, i, j \in (I \cup T): i < j \qquad \begin{array}{r}(6.b)\\(6.c)\end{array}$$

$$TV_v \geq T_i + st_i + t_{ip} - M_T(1 - Y_{iv}) \qquad \forall v \in V, i \in (I \cup T) \tag{6.d}$$

Cargo constraints. Eq. (7.a) states that the cargo α_i to be picked-up from any customer $i \in I^+$ must be equal to l_i, while the cargo β_i to be delivered to this client must be zero. Conversely, the cargo β_i to be delivered to any customer $i \in I^-$ must be l_i while the cargo α_i to be picked-up from this customer must be zero (eq. 7.b). Eq. (8.a) states that the quantity of goods picked from a transfer node $i \in T^+$ is equal to the quantity previously delivered to the paired delivery-node $j \in T^-$: $(i,j) \in R$ and constraint (8.b) states that the quantity of goods γ_{iv} delivered to a transfer node $i \in T^-$ by the vehicle v must be at least the collected quantity γ_{iv}.

$$\left\{\begin{array}{l} \alpha_i = l_i \\ \beta_i = 0 \end{array}\right\} \quad \forall i \in I^+ \quad (7.a) \qquad\qquad \left\{\begin{array}{l} \alpha_i = 0 \\ \beta_i = l_i \end{array}\right\} \quad \forall i \in I^- \quad (7.b)$$

$$\left\{\begin{matrix}\alpha_i = \beta_j \\ \beta_i = 0\end{matrix}\right\} \; i, j \in T : \{i, j\} \in R \qquad 8.(a) \qquad \left\{\begin{matrix}\alpha_i = 0 \\ \beta_i \geq \gamma_{iv} - M_L(1 - Y_{iv})\end{matrix}\right\} \forall v \in V, i \in T^- \qquad (8.b)$$

Vehicle constrains on the transshipment base. Eq. (9.a) states that the quantity γ_{iv} to be delivered to transfer node $i \in T^-$ must be zero if the vehicle v does not visit such a node. Otherwise, it must be smaller that the total quantity collected on pick-up sites. Conversely, according to eq. (9.b), the quantity δ_{iv} picked-up by the vehicle v from the transfer node $i \in T^+$ must be zero if v does not visit such a node. Otherwise, it must be equal to the quantity of goods to load from this transfer point.

$$\left\{\begin{matrix}\gamma_{iv} \leq M_L Y_{iv} \\ \gamma_{iv} \leq \sum_{j \in I^+} l_j Y_{jv}\end{matrix}\right\} \forall v \in V, i \in T^- \qquad (9.a) \qquad \left\{\begin{matrix}\delta_{iv} \leq M_L Y_{iv} \\ \delta_{iv} \leq \alpha_i + M_L(1 - Y_{iv}) \\ \delta_{iv} \geq \alpha_i - M_L(1 - Y_{iv})\end{matrix}\right\} \forall v \in V, i \in T^+ \qquad (9.b)$$

Vehicle-cargo constraints. Variables L_i and U_i compute respectively the total cargo loaded and unloaded by a visiting vehicle up to the node $i \in I \cup T$. So, eq. (10.a) states that the current load transported on the visiting vehicle up to the node $i \in I \cup T$, computed as the difference (L_i - U_i) must be larger than zero and smaller than the vehicle capacity q_v. Constraints (10.b)-(10.e) determines the accumulated loaded and unloaded cargo in a similar way to eqs. (3.a)-(3.d). Eq. (11.a) states that the load available on vehicle v after visiting node i (L_i) must be larger than the quantity α_i to be picked-up from the node and smaller than the total quantity of goods collected by the visiting vehicle in case vehicle v visits the site i ($Y_{iv} = 1$). Constraint (11.b) is similar to (11.a) but for the cargo unloaded after servicing node i (U_i). M_L is an upper bound for L_i and U_i.

$$0 \leq L_i - U_i \leq \sum_{v \in V} Y_{iv} q_v \qquad \forall i \in (I \cup T) \qquad (10.a)$$

$$\left\{\begin{matrix}L_j \geq L_i + \alpha_j - M_L(1 - S_{ij}) - M_L(2 - Y_{iv} - Y_{jv}) \\ U_j \geq U_i + \beta_j - M_L(1 - S_{ij}) + M_L(2 - Y_{iv} - Y_{jv}) \\ L_i \geq L_j + \alpha_i - M_L S_{ij} - M_L(2 - Y_{iv} - Y_{jv}) \\ U_i \geq U_j + \beta_i - M_L S_{ij} + M_L(2 - Y_{iv} - Y_{jv})\end{matrix}\right\} \quad \forall v \in V, i, j \in (I \cup T) : i < j \qquad \begin{matrix}(10.b) \\ (10.c) \\ (10.d) \\ (10.e)\end{matrix}$$

$$\left\{\begin{matrix}\alpha_i \leq L_i \leq \sum_{j \in I^+} Y_{jv} l_j + \sum_{j \in T^+} \delta_{jv} + M_L(1 - Y_{iv}) \\ \beta_i \leq U_i \leq \sum_{j \in I^-} Y_{jv} l_j + \sum_{j \in T^-} \gamma_{jv} + M_L(1 - Y_{iv})\end{matrix}\right\} \quad \forall v \in V, i \in (I \cup T) \qquad \begin{matrix}(11.a) \\ (11.b)\end{matrix}$$

Time and inventory constraints on transshipment bases. As the number of pick-up/delivery tasks within the nodes of a transfer base is known beforehand and can be pre-ordered, eq. (12) fixes the vehicles visiting times to the transfer base and eqs. (13.a)-(13.b) track of the cargo inventoried in the base by sequencing variables I_i (unloaded cargo) and D_i (delivered cargo). The actual inventory of goods, computed by the difference (I_i - D_i), is bounded by the interval [0, Q^{max}] enforced in Eq. (14).

$$T_i \geq T_j + st_i \qquad \forall i \in T^+, j \in T^- : \{i, j\} \in R \tag{12}$$

$$\begin{Bmatrix} I_j \geq L_i + \beta_j \\ D_j \leq U_i + \alpha_j \end{Bmatrix} \qquad \forall (i, j) \in T : i < j \qquad \begin{matrix} (13.a) \\ (13.b) \end{matrix}$$

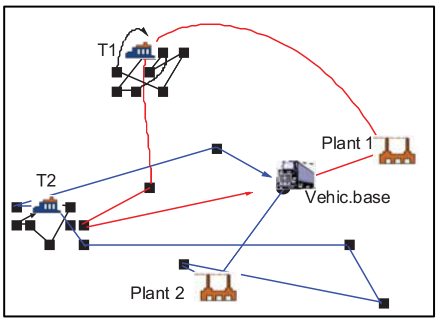
$$0 \leq I_i - D_i \leq Q^{\max}{}_v \qquad \forall i \in T \tag{14}$$

4. An Illustrative example

The proposed MILP model was developed in ILOG OPL Studio 3.7 and solved in a 2 Ghz 2 MB RAM Pentium IV PC. The logistics problem addressed comprises twenty geographically dispersed customers with known demands of a single commodity produced in two different production plants. Demands can be directly satisfied from the plants or from two distribution centers located in urban areas by using two vehicles (*VL1, VL2*). In addition, each distribution center hosts a small vehicle that can subsequently deliver a given quantity of a commodity to neighboring clients. Delivery tasks must be fulfilled within given time windows and the problem solution must provide the optimal sequence of clients to be visited by each vehicle while minimizing the total travelling cost. Cartesian coordinates that specify locations for the vehicle-base, the plants, the distribution-centers and the clients as well as their respective time-windows are presented in Table 1. Vehicles characteristics are also presented in this Table. Inter-nodal Euclidean distances are computed from the reported coordinates and the arc cost/travelling-times are numerically equal to those distances as defined by Solomon (1987). The optimal solution, found in 536 s CPU time, implies a total travelled distance of 399.9 units and is explicitly illustrated in Figure 1.

Table 1: Problem data

Nodes	Vbase; Plant1; Plant2; Ssite1;Ssite2;1;2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20
X coord	40;55;30;20;5; 25;22;22;20;18;15;15;10;10;8;8;5;2;0;0;55;25;20;30;50
Y coord	50;60;25;85;45; 85;75;85;80;75;75;80;35;40;40;45;35;40;40;45;20;30;50;60;35
Readytime	-;-;-;-;-; 145;0;109;141;95;79;0;0;119;0;0;0;0;0;0;83;52;91;140;130
Duedate	-;-;-;-;-;255;300;139;171;125;109;300;300;149;300;300;300;300;300;300;113;82;121;300;160
Demand	-;-;-;-;-;20;30;10;40;20;20;10;20;30;40;20;10;20;20;20;19;3;5;16;19
Vehicles	Large (*VL1, VL2*): q_{VL} = 300; $t_{VL}{}^{max}$ = 300/ Small (*VS1, VS2*): q_{VS} = 150; $t_{VS}{}^{max}$ = 300



DETAILED SOLUTION
- *VL1*: Vbase; Plant2; 19; 18; 23; 9; 11; T2; 17; 22; Base. (TV $_{VL1}$ = 293.7; CV $_{VL1}$= 185.0)
- *VL2*: Vbase; Plant1; T1; 20; 10; Base (TV $_{VL2}$ = 181.7; CV $_{VL2}$= 141.7)
- *VS1*: T1; 7; 6; 2; 8; 4; 3; 1; T1 (TV $_{VS1}$ = 206.0; CV $_{VS1}$ = 45.0)
- *VS2*: T2; 12; 13; 15; 16; T2 (TV$_{VS2}$ = 399.3; CV $_{VS2}$ = 28.2)

Figure 1. Optimal solution for the SC-PDP-T problem

5. **Conclusions**

This work has introduced a novel MILP formulation for the SC-PDP-T. Its applicability has been successfully illustrated by solving a typical supply chain problem. This problem fully exploits the possibility of cargo-transfers between vehicles in order to find flexible supply programs. This feature is of utmost importance for saving transportation cost while keeping a high level of supply efficiency. Future work should include the possibility of considering alternative supply sites, inventory constraints; multiple visits to the same customer and multiple commodities.

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