The Two-Echelon Capacitated Vehicle Routing Problem

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Abstract

Multi-echelon distribution systems are quite common in supply-chain and logistic systems. They are used by public administrations in their transportation and traffic planning strategies as well as by companies to model their distribution systems. Unfortunately, the literature on combinatorial optimization methods for multi-echelon distribution systems is very poor.

The aim of this paper is twofold. First, the new family of Multi-Echelon Vehicle Routing Problems is introduced. Second, their simplest version, the Two-Echelon Capacitated Vehicle Routing Problem, is presented.

The Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP) is an extension of the classical CVRP where the delivery depot-customers passes through intermediate depots (called satellites). As in CVRP, the goal is to deliver goods to customers with known demands, minimizing the total delivery cost in the respect of vehicle capacity constraints.

We introduce a mathematical model for 2E-CVRP and give some valid inequalities able to significantly improve the results on benchmark tests up to 50 customers and 4 satellites. Computational results under different realistic scenarios are presented.

Keywords: Vehicle Routing, Multi-echelon systems, City Logistics.

1 Introduction

The freight transportation industry is a major source of employment and supports the country's economic improvement. However, freight transportation is also a disturbing activity, due to congestion and environmental nuisances, that negatively affects the quality of life, in particular in urban areas.

In freight transportation there are two main distribution strategies: direct shipping and multi-echelon distribution. In the direct shipping, vehicles, starting from a depot, bring their freight directly to the destination, while in the multi-echelon systems, freight is delived from the depot to the customers through intermediate points. The freight volume growing as well as the necessity of taking into account factors as the environmental impact and the traffic congestion focused the research on multi–echelon distribution systems, and on two-echelon systems in particular (Crainic et al., 2004) . In two-echelon distribution systems, freight is delivered to an intermediate depot and, from this depot, to the customers.

Multi-echelon systems presented in the literature usually explicitly consider the routing problem at the last level of the transportation system, while a simplified routing problem is considered at higher levels. In such a way, the routing costs fn the higher levels are often underestimated and decision-makers cannot directly use the solutions obtained from the models (Sand, 1981; Svoronos, 1988; Verrijdt and de Kok, 1995) . Moreover, in the past decade multi-echelon systems have been introduced by practitioners in different areas:

- *Logistics enterprises and express delivery service companies.* These operators are usually in a two-echelon system. Their offices are used as intermediate points for organizing the freight to be delivered and compose vehicles that will transport the freight to another intermediate point (airport, regional center, etc.) or to the final destination. (http://www.tntlogistics.com)

- *Multimodal freight transportation.* In the past decade, the number of intermodal logistics centers in the countries of central and south-west Europe increased. This is a good example of freight distribution involving two or more echelons (Ricciardi et al., 2002) . In a classical road-train multimodal distribution the freight goes from the producer to a logistic center by road and then it is loaded on a train that goes to another logistics center. The train is unloaded and the freight goes by road to its final destination.

- *Grocery and hypermarkets products distribution.* Large companies use hypermarkets as intermediate storage points to serve smaller stores and supermarkets of the same brand in urban areas.

- *Spare parts distribution in the automotive market.* Some companies uses couriers and other actors to deliver their spare parts. This is the case of FIAT and General motors, which spare parts are distributed by TNT (http://www.tntlogistics.com) from their factories to the garages. Similarly, Bridgeston uses an organization of the distribution system in zones and sub-zones, to decrease the transportation times and reduce the size of the storage areas.

- *E-commerce and home delivery services.* The new possibilities given by the development of e-commerce and the home delivery services offered by some supermarkets and other stores like SEARS (http://www.sears.com), in some large cities implies the presence of intermediate depots used to optimize the delivery process.

- *City logistics.* In the past decade, researchers started to investigate the urban areas as a system, stopping to consider each shipment, firm, and vehicle individually. Rather, one should consider that all stakeholders and movements are components of an integrated logistics system (Crainic et al., 2004) . This implies the coordination of shippers, carriers, and movements as well as the consolidation of loads of several customers and carriers into the same "green" vehicles and the adopted distribution system is typically a two-echelon system.

The main contribution of this paper is the introduction of the *Multi-Echelon Vehicle Routing Problems*, a new family of routing problems where the routing and the freight management are explicitly considered at the different levels. Moreover, we introduce and study one of the simplest types of Multi-Echelon Vehicle Routing Problem, the *Two-Echelon Capacitated Vehicle Routing Problem* (2E-CVRP). In 2E-CVRP, the freight delivery from the depot to the customers is managed by shipping the freight through intermediate depots. Thus, the transportation network is decomposed into two levels: the 1st level connecting the depot to intermediate depots and the 2nd one connecting the intermediate depots to the customers. The objective is the minimization of the total transportation cost of the vehicles involved in both levels. Constraints on the maximum capacity of the vehicles and the intermediate depots are considered, while the timing of the deliveries are ignored.

The paper is organized as follows. In Section 2 we recall the literature related to Multi-Echelon Vehicle Routing Problems. In Section 3 we give a general description of Multi-Echelon Vehicle Routing Problems. Section 4 is dedicated to introduce 2E-CVRP and give a mathematical model, which is strengthened by means of valid inequalities in Section 5. Finally test instances for 2E-CVRP are introduced and some computational results are discussed in Section 6.

2 Literature review

In freight distribution there are different distribution strategies. The most developed strategy is based on the direct shipping: freight starts from a depot and arrives directly to customers. In many applications, this strategy is not the best one and the usage of a two-echelon distribution system can optimize several features as the number of the vehicles, the transportation costs and their loading factor.

In the literature the multi-echelon system, and the two-echelon system in particular, refer mainly to supply chain and inventory problems (Sand, 1981; Svoronos, 1988; Verrijdt and de Kok, 1995) . These problems do not use an explicit routing approach for the different levels, focusing more on the production and supply chain management issues.

The first real application of a two-tier distribution network optimizing the global transportation costs is due to Crainic, Ricciardi and Storchi and is related to the city logistics area (Crainic et al., 2004) . They developed a two-tier freight distribution system for congested urban areas, using small intermediate platforms, called satellites, as intermediate points for the freight distribution. This system is developed for a specific case study and a generalization of such a system has not already been formulated.

To our knowledge, even if VRP contains a large class of combinatorial problems, VRP variants are studied in their single-level version. In the following, we present the main results in this area.

The Capacitated VRP (CVRP) is the case in which we consider a fleet of identical vehicles. The CVRP objective is the minimization of the transportation costs under the capacity constraints on the maximum freight that can be loaded on each vehicle. A variant of the problem is the Distance Constrained VRP (DVRP), where an additional constraint on the maximum distance that each vehicle can cover is added. These two groups of constraints can be combined in the Distance Constrained Capacitated VRP (DCVRP). This variant of VRP is the most commonly studied, and recent studies have developed good heuristic methods. Exact algorithms can solve relatively small instances and their computational times are highly variable (Cordeau et al., 2004) . Exact methods are mainly used to determine optimal solutions of the test instances, while heuristic methods are used for practical applications. One of the first and most used heuristics is the savings algorithm of Clarke and Wright (Clarke and Wright, 1964) . One of the best methods to solve CVRP is Tabu Search (Cordeau et al., 1997, 2001) , which has been improved in recent studied using hybrid algorithms, usually combining Tabu Search with Genetic Algorithms (Perboli et al., 2006; Prins, 2004; Mester and Bräysy, 2005) . For a detailed survey about the exact and heuristic methods see (Cordeau et al., 2007; Toth and Vigo, 2002; Cordeau et al., 2004) .

VRP has become a central problem in the fields of logistics and freight transportation. In some market sectors, transportation means a high percentage of the value added to goods. Therefore, the utilization of computerized methods for transportation often results in significant savings ranging from 5% to 20% in the total costs, as reported at Toth and Vigo, 2002. Usually, in real world applications, the problem can be different to the one presented here. For representing these applications, many variants of VRP have been developed. The most known variants are VRP with time windows (VRP-TW), multi-depot VRP (MDVRP) and VRP with pickups and deliveries (VRP-PD) (for a survey, see Toth and Vigo, 2002). We note only one variant of VRP where satellites facilities are explicitly considered, the VRP with Satellites facilities (VRPSF). In this variant, the network presents facilities that are used to replenish vehicles during a route. When possible, satellite replenishment allows the drivers to continue making deliveries without necessarily returning to the central depot. This situation arises primarily in the distribution of fuels and some other retail items and the satellites are not used as depots to reduce the transportation costs (Angelelli and Speranza, 2002; Bard et al., 1998) .

3 The Multi-Echelon Vehicle Routing Problems

The freight consolidation from different shippers and carriers associated to some kind of coordination of operations is among the most important ways to achieve a rationalization of the distribution activities. Intelligent Transportation Systems technologies and operations research-based methodologies enable to optimize the design, planning, management, and operation of City Logistics systems (Crainic and Gendreau, forthcoming; Taniguchi et al., 2001) .

Consolidation activities take place at so-called *Distribution Centers* (DCs). The DCs, when their size is smaller than the depot and the freight can be stored for a short time, are also called *satellite platforms*, or simply *satellites*. Long-haul transportation vehicles dock at a Satellite to unload their cargo. Loads are then consolidated into smaller vehicles that will deliver them to their final destinations. Of course, a similar system can be defined to address the reverse movements, from origins within an area to destinations outside it.

As stated in the introduction, in the Multi-Echelon Vehicle Routing Problems the delivery from the depot to the customers is managed by rerouting and consolidating the freight through different intermediate satellites. The general goal of the process is to ensure an efficient and low-cost operation of the system, while the demand is delivered on time and the total cost of the traffic on the overall transportation network is minimized. Usually, capacity constraints on the vehicles are considered.

More precisely, in the Multi-Echelon Vehicle Routing Problems the overall transportation network can be decomposed into $k \geq 2$ levels:

- the 1st level, which connects the depots to the 1st-level satellites;
- $k-2$ intermediate levels interconnecting the satellites;
- the last level, where the freight is delivered from the satellites to the customers.

Each transportation level has its own fleet to manage the delivery and the vehicles assigned to a level cannot be reassigned to another one.

The most common version of Multi-Echelon Vehicle Routing Problem used in practice is the *Two-Echelon Vehicle Routing Problem*, where just two levels are considered. From a physical point of view, a Two-Echelon Capacitated Vehicle Routing system operates as follows:

- freight arrives at an external zone, the depot, where it is consolidated into the 1st-level vehicles, unless it is already carried into a fully-loaded 1st-level truck;
- Each 1st-level vehicle travels to a subset of satellites that will be determined by the model and then it will return to the depot;
- At a satellite, freight is transferred from 1st-level vehicles to 2nd-level vehicles;

- Each 2nd-level vehicle performs a route to serve the designated customers, and then travels to a satellite for its next cycle of operations. The 2nd-level vehicles return to their departure satellite.

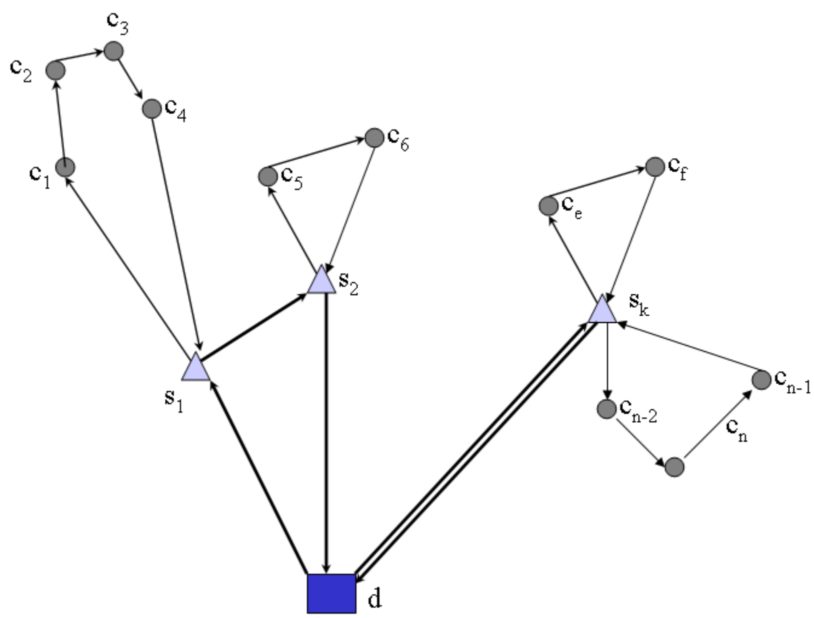

Figure 1: Example of 2E-CVRP transportation network

In the following, we will focus on the Two-Echelon Vehicle Routing Problems, using it to illustrate the different type of constraints that are commonly defined on Multi-Echelon Vehicle Routing Problems. We can define three groups of variants:

Basic variants with no time dependence:

- Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP). This is the simplest version of Multi-Echelon Vehicle Routing Problems. At each level, all vehicles belonging to that level have the same fixed capacity. The objective is to serve customers by minimizing the total transportation cost, and satisfying the capacity constraints of the vehicles. There is a single depot and a fixed number of capacitated satellites. All customer demands are fixed and known in advance. Moreover, no time window is defined for the deliveries and the satellite operations. All customer demands must be satisfied. For the 2nd level, the demand of each customer is smaller than each vehicle capacity and the demands cannot be split into multiple routes of the same level. For the 1st level we can consider two complementary distribution strategies. In the first case, each satellite is served by just one 1st-level vehicle and the demand passing through the satellite cannot be split into different 1st-level vehicles. This strategy is similar to the CVRP one, and the capacity of 1st-level vehicles has to be greater than the demand of each satellite. In the second case, a satellite can be served by more of one 1st-level vehicle, so each satellite demand can be split. This strategy has some analogies to the VRP with split deliveries and allow to have 1st-level vehicles with capacity which is lower than each satellite demand.

Basic variants with time dependence:

- Two-Echelon Capacitated VRP with Time Windows (2E-CVRP-TW). This problem is the extension of 2E-CVRP where time windows on the arrival or departure time at the satellites and/or at the customers are considered. The time windows can be hard or soft. In the first case the time windows cannot be violated, while in the second one if they are violated a penalty cost is paid.

- Two-Echelon Capacitated VRP with Satellites Synchronization (2E-CVRP-SS). In this problem, time constraints on the arrival and the departure of the vehicles at the satellites are considered. In fact, the vehicles arriving in a satellite unload their cargo, which must be immediately loaded into a 2nd-level vehicle. Also this kind of constraints can be of two types: hard and soft. In the hard case, every time a 1st-level vehicle unloads its freight, 2nd-level vehicles must be ready to deliver it (this constraint is formulated through a very small hard time window). In the second case, if 2nd-level vehicles are not available, the demand is lost and a penalty is paid.

Other 2E-CVRP variants are:

- Multi-depot problem. In this problem the satellites are served by more than one depot. A constraint forcing to serve each customer by only one 2nd-level vehicle can be considered. In this case, we have a Multi-Depot Single-Delivery Problem.

- 2E-CVRP with Pickup and Deliveries (2E-CVRP-PD). In this case we can consider the satellites as intermediate depots to store both the freight that has been picked-up from or must be delivered to the customers.

- 2E-CVRP with taxi services (2E-CVRP-TS). In this variant, direct shipping from the depot to customers is allowed if it helps to decrease the cost, or to satisfy time and/or synchronization constraints.

4 The Two-Echelon Capacitated Vehicle Routing Problem

As stated in Section 3, 2E-CVRP is the two-echelon extension of CVRP. In this section we formalize 2E-CVRP and introduce a mathematical formulation able to solve small and medium-sized instances. We do not consider any time windows and satellite synchronization constraints.

Let us denote the depot v_0, the set of intermediate depots called satellites V_s and the set of customers V_c. Let n_s be the number of satellites and n_c the number of customers. The depot is the starting point of the freight. The satellites may be capacitated. The customers are the destinations of the freight and each customer i has associated a demand d_i, i.e. the quantity of freight that has to be delivered to that customer. The demand of each customer cannot be split among different vehicles, neither at the 1st nor at the 2nd level.

The distribution of the freight cannot be managed by direct shipping from the depot to the customers, but the freight must be consolidated from the depot to a satellite and then, from the satellite, is delivered to the desired customer. This implicitly defines a two-level transportation system: the 1st level interconnecting the depot to the satellites and the 2nd one the satellites to the customers (see Figure 1).

Define the arc (i, j) as the direct route connecting node i to node j. If both nodes are satellites or one is the depot and the other is a satellite, we define the arc as belonging to the 1st-level network, while if both nodes are customers or one is a satellite and the other is a customer, the arc belongs to the 2nd-level network.

We consider only one type of freight, i.e. the volumes of freight belonging to different customers can be stored together and loaded in the same vehicle, for both the 1st and the 2nd-level vehicles. Moreover, the vehicles belonging to the same level have the same capacity.

We define as *1st-level route* a route made by a 1st-level vehicle which starts from the depot, serves one or more satellites and ends to the depot. A *2nd-level route* is a route made by a 2nd-level vehicle which starts from a satellite, serves one or more customers and ends to the same satellite.

The problem is easily seen to be NP-Hard via a reduction to CVRP, which is a special case of 2E-CVRP arising when just one satellite is considered.

4.1 A Flow-based model for 2E-CVRP

According to the definition of 2E-CVRP, if the assignments between customers and satellites are determined, the problem reduces to $1 + n_s$ CVRP (1 for the 1st-level and n_s for the 2nd-level).

The main question when modeling 2E-CVRP is how to connect the two levels and manage the dependence of the 2nd-level from the 1st one.

The freight must be delivered from the depot v_0 to the customers set $V_c = \{v_{c_1}, v_{c_2}, ..., v_{c_{n_c}}\}$. Let d_i the demand of the customer c_i. The number of 1stlevel vehicles available at the depot is m_1. These vehicles have the same given capacity K^1. The total number of 2nd-level vehicles available in the satellites is equal to m_2. We do not introduce a constraint on the number of vehicles available at each single satellite. The 2nd-level vehicles have the same given capacity K^2.

In our model we will not consider the fixed costs of the vehicles, since they are available in fixed number. We consider only the travel costs c_{ij} on the arcs of the graph, which are of two types:

- costs of the arcs traveled by 1st-level vehicles, i.e. arcs connecting the depot to the satellites and the satellites between them;
- costs of the arcs traveled by 2nd-level vehicles, i.e. arcs connecting the satellites to the customers and the customers between them.

Another cost that can be used is the cost of loading and unloading operations at the satellites. Supposing that the number of workers in each satellite v_{s_k} is fixed, we consider only the cost due to the management of the freight and we define S_k as the unit cost of freight handling at the satellite v_{s_k}.

The formulation we present derives from the multi-commodity network design and uses the flow of the freight passing through each arc.

We define five sets of variables, that can be divided in three groups:

- The first group represents the arc usage variables. We define two sets of such variables, one for each level. The variable x_{ij} is an integer variable of the 1st-level routing and is equal to the number of 1st-level vehicles using

$V_0 = \{v_0\}$	Depot
$V_s = \{v_{s_1}, v_{s_2}, ..., v_{s_{n_s}}\}$	Set of satellites
$V_c = \{v_{c_1}, v_{c_2}, ..., v_{c_{n_c}}\}$	Set of customers
n_s	number of satellites
n_c	number of customers
m_1	number of the 1st-level vehicles
m_2	number of the 2nd-level vehicles
K^1	capacity of the vehicles for the 1st level
K^2	capacity of the vehicles for the 2nd level
d_i	demand required by customer i
c_{ij}	cost of the arc (i, j)
S_k	cost for loading/unloading operations of a unit of freight in satellite k

Table 1: Definitions and notations

arc (i,j). The variable y_{ij}^k is a binary variable of the 2nd-level routing and is equal to 1 if a 2nd-level vehicle makes a route starting from satellite k and goes from node i to node j and 0 otherwise.

- The second group of variables represents the assignment of each customer to one satellite and are used to link the two transportation levels. More precisely, we define z_{kj} as a binary variable that is equal to 1 if the freight to be delivered to customer j is consolidated in satellite k and 0 otherwise.

- The third group of variables, split into two subsets, one for each level, represents the freight flow passing through each arc. We define the freight flow as a variable Q_{ij}^1 for the 1st-level and Q_{ijk}^2 for the 2nd level, where k represents the satellite where the freight is passing through. Both variables are continuous.

In order to lighten the model formulation, we define the auxiliary quantity

$$D_k = \sum_{j \in V_c} d_j z_{kj}, \forall k \in V_s, \tag{1}$$

which represents the freight passing through each satellite k.

The model to minimize the total cost of the system may be formulated as follows:

$$\min \sum_{i,j \in V_0 \cup V_s} c_{ij} x_{ij} + \sum_{k \in V_s} \sum_{i,j \in V_s \cup V_c} c_{ij} y_{ij}^k + \sum_{k \in V_s} S_k D_k$$

Subject to

$$\sum_{i \in V_s} x_{0i} \leq m_1 \tag{2}$$

$$\sum_{j \in V_s \cup V_0, j \neq k} x_{jk} = \sum_{i \in V_s \cup V_0, i \neq k} x_{ki} \quad \forall k \in V_s \cup V_0 \tag{3}$$

$$\sum_{k \in V_s} \sum_{j \in V_c} y_{kj}^k \leq m_2 \tag{4}$$

$$\sum_{i \in V_s, j \in V_c} y_{ij}^k = \sum_{i \in V_s, j \in V_c} y_{ji}^k \quad \forall k \in V_s \tag{5}$$

$$\sum_{i \in V_s, i \neq j} Q_{ij}^1 - \sum_{i \in V_s, i \neq j} Q_{ji}^1 = \begin{cases} D_j & j \text{ is not the depot} \\ \sum_{i \in V_c} -d_i & \text{otherwise} \end{cases} \quad \forall j \in V_s \cup V_0 \tag{6}$$

$$Q_{ij}^1 \leq K^1 x_{ij} \quad \forall i, j \in V_s \cup V_0, i \neq j \tag{7}$$

$$\sum_{i \in V_c, i \neq j} Q_{ijk}^2 - \sum_{i \in V_c, i \neq j} Q_{jik}^2 = \begin{cases} z_{kj} d_j & j \text{ is not a satellite} \\ -D_j & \text{otherwise} \end{cases} \quad \forall j \in V_c \cup V_s, \forall k \in V_s \tag{8}$$

$$Q_{ijk}^2 \leq K^2 y_{ij}^k \quad \forall i, j \in V_s \cup V_c, i \neq j, \forall k \in V_s \tag{9}$$

$$\sum_{i \in V_s} Q^1_{iv_0} = 0 \tag{10}$$

$$\sum_{j \in V_c} Q^2_{jkk} = 0 \;\; \forall k \in V_s \tag{11}$$

$$y^k_{ij} \leq z_{kj} \;\; \forall i \in V_s \cup V_c, \forall j \in V_c, \forall k \in V_s \tag{12}$$

$$y^k_{ji} \leq z_{kj} \;\; \forall i \in V_s, \forall j \in V_c, \forall k \in V_s \tag{13}$$

$$\sum_{i \in V_s \cup V_c} y^k_{ij} = z_{kj} \;\; \forall k \in V_s, \forall j \in V_c, i \neq k \tag{14}$$

$$\sum_{i \in V_s} y^k_{ji} = z_{kj} \;\; \forall k \in V_s, \forall j \in V_c, i \neq k \tag{15}$$

$$\sum_{i \in V_s} z_{ij} = 1 \;\; \forall j \in V_c \tag{16}$$

$$y^k_{ij} \leq \sum_{l \in V_s \cup V_0} x_{kl} \;\; \forall k \in V_s, \forall i, j \in V_c \tag{17}$$

$$y^k_{ij} \in \{0, 1\}, z_{kj} \in \{0, 1\}, \;\; \forall k \in V_s \cup V_0, \forall i, j \in V_c \tag{18}$$

$$x_{kj} \in \mathbb{Z}^{+}, \quad \forall k, j \in V_s \cup V_0 \tag{19}$$

$$Q^1_{ij} \geq 0, \forall i, j, \in V_s \cup V_0, \; Q^2_{ijk} \geq 0, \; \forall i, j \in V_s \cup V_c, \forall k \in V_s. \tag{20}$$

The objective function minimizes the sum of the traveling and handling operations costs. Constraints (3) show, for $k = v_0$, that each 1st-level route begins and ends at the depot, while when k is a satellite, impose the balance of vehicles entering and leaving that satellite. Constraints (5) force each 2nd-level route to begin and end to one satellite and the balance of vehicles entering and leaving each customer. The number of the routes in each level must not exceed the number of vehicles for that level, as imposed by constraints (2) and (4).

Constraints (6) and (8) indicate that the flows balance on each node is equal to the demand of this node, except for the depot, where the exit flow is equal to the total demand of the customers, and for the satellites at the 2nd-level, where the flow is equal to the demand (unknown) assigned to the satellites. Moreover, constraints (6) and (8) forbid the presence of subtours not containing the depot or a satellite, respectively. In fact, each node receives an amount of flow equal to its demand, preventing the presence of subtours. Consider, for example, that a subtour is present between the nodes i, j and k at the 1st level. It is easy to check that, in such a case, does not exist any value for the variables Q^1_{ij}, Q^1_{jk} and Q^1_{ki} satisfying the constraints (6) and (8). The capacity constraints are formulated in (7) and (9), for the 1st-level and the 2nd-level, respectively.

Constraints (10) and (11) do not allow residual flows in the routes, making the returning flow of each route to the depot (1st-level) and to each satellite (2nd-level) equal to 0.

Constraints (12) and (13) indicate that a customer j is served by a satellite k ($z_{kj} = 1$) only if it receives freight from that satellite ($y_{ij}^k = 1$). Constraint (16) assigns each customer to one and only one satellite, while constraints (14) and (15) indicate that there is only one 2nd-level route passing through each customer and connect the two levels. Constraints (17) allow to start a 2nd-level route from a satellite k only if a 1st-level route has served it.

5 Valid inequalities for 2E-CVRP

In order to strengthen the continuous relaxation of the flow model, we introduce cuts derived from CVRP formulations. In particular, we use two families of cuts, one applied to the assignment variables derived from the subtour elimination constraints (edge cuts) and the other based on the flows.

The *edge cuts* explicitly introduce the well-known subtours elimination constraints derived from the TSP. They can be expressed as follows:

$$\sum_{i,j \in S_c} y_{ij}^k \leq |S_c| - 1, \forall S_c \subset V_c,\ 2 \leq |S_c| \leq |V_c| - 2 \tag{21}$$

The inequalities explicitly forbid the presence in the solution of subtours not containing the depot, already forbidden by the constraints (8). The number of potential valid inequalities are exponential, so we need a separation algorithm to add them.

Flow cuts aim to reduce the splitting of the values of the binary variables when the continuous relaxation is performed, strengthening the BigM constraints (9). The idea is to reduce the constant K^2 by considering that each customer reduces the flow of an amount equal to its demand d_i. Thus the following inequalities are valid:

$$\begin{cases} Q^2_{ijk} \leq (K^2 - d_i) y^k_{ij}, \forall i,j \in V_c \;\; \forall k \in V_s \\ Q^2_{ijk} - \sum_{l \in V_s} Q^2_{jlk} \leq (K^2 - d_i) y^k_{ij} \;\; \forall i,j \in V_c, \forall k \in V_s. \end{cases} \tag{22}$$

Constraints (22) are of the same order of magnitude of (9), so they can be directly introduced into the model.

6 Computational tests

In this section, we analyze the behavior of the model using a commercial solver. Being 2E-CVRP introduced for the first time in this paper, in Subsection 6.1 we define some benchmark instances, extending the instance sets from the CVRP literature. In Subsection 6.2 we present the results of the models on a set of small-sized instances highlighting the properties of 2E-CVRP and the cost distribution according to the geographic distribution of the satellites. Finally, Section 6.3 is devoted to present the computational results on all the benchmark instances and the impact on the computational results of the valid inequalities of Section 5.

6.1 Construction of the instance sets

In this section we introduce different instance sets for 2E-CVRP. The instances cover up to 51 nodes (1 depot and 50 customers), built from the existing instances for CVRP by Christofides and Elion denoted as E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5 (Christofides and Elion, 1969) . All the instance sets can be downloaded from the web site *http://www.orgroup.polito.it/*.

The first instance set is made by 66 small-sized instances with 1 depot, 12 customers and 2 satellites. All the instances have the same position for depot and customers, whose coordinates are the same of the instance E-n13-k4. The two satellites are placed over two customers in all the $\begin{pmatrix} 12 \\ 2 \end{pmatrix} = 66$ possible ways (the case where some customers are used as satellites is quite common for different kinds of distribution, e.g. grocery distribution). When i is both a customer and a satellite, the arc cost c_{ii} is set equal to 0. The number of vehicles for the 1st-level is set to 2, while the 2nd-level vehicles are 4, as in the original CVRP instance. The capacity of the 1st-level vehicles is 2.5 times the capacity of the 2nd-level vehicles, to represent cases in which the 1st-level is made by trucks and the 2nd-level is made by vehicles of weight inferior to 3.5 t. The capacity of the 2nd-level vehicles is equal to the capacity of the vehicles of the CVRP instance.

The second set of instances is obtained in a similar way from the instances E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5. The instances are obtained by considering only 6 pairs of satellites randomly generated. For the instance E-

CVRP Instances	n_s	n_c	m_1	m_2	K^1	K^2	Satellite location					
							1	2	3	4	5	6
E-n13-k4	2	12	3	4	15000	6000	2,3	2,5	3,6	4,8	5,10	7,9
E-n22-k4	2	21	3	4	15000	6000	6,17	8,14	9,19	10,14	11,12	12,16
E-n33-k4	2	32	3	4	2000	8000	1,9	2,13	3,17	4,5	7,25	14,22
E-n51-k5	2	50	3	5	400	160	2,17	4,46	6,12	11,19	27,47	32,37
E-n51-k5	4	50	4	5	400	160	2,4,17,46	6,12,32,37	11,19,27,47			

Table 2: Characteristics of the benchmarks for 2E-CVRP

n51-k5, which has 50 customers, we build an additional group of 3 instances obtained randomly placing 4 satellites instead of 2.

In all the instances, the cost due to loading/unloading operations is set equal to 0, while the arc costs are the same of the CVRP instances.

A summary of the main features of the second set of instances is reported in Table 2. In the first column we report the name of the original CVRP instance. Columns 2 and 3 contain the number of satellites and customers, respectively. The number of vehicles for the 1st and the 2nd level can be read in columns 4 and 5, while columns 6 and 7 give the capacity of the vehicles of the two levels. In the remaining columns the customers hosting the satellites are reported.

6.2 Small-sized instances results

In this section, we report the results obtained by solving to optimality all the 66 instances of the first set (12 customers and 2 satellites) by means of XPress 2003 solver. The objective function values are reported in Table 3. The table contains, in the first column, the customer's number of the CVRP instance E-n13-k4 where the satellites are placed. Column 2 reports the value of the optimum. Column 3 contains the percentage variation of the optimum of the 2E-CVRP compared to 247, the optimum of the original CVRP instance. Column 4 shows the mean value of the accessibility index (Hansen, 1959) computed on each satellite as

$$A_k = \frac{1}{|V_c|} \sum_{i \in V_c} \frac{d_i - d_{min}}{d_{max} - d_{min}} e^{-\beta \frac{c_{ki} - c^2_{min}}{c^2_{max} - c^2_{min}}}, \tag{23}$$

where d_i is the demand of the customer i, d_{min} and d_{max} the minimum and the maximum demands overall the customers, respectively, c_{ki} the transportation cost between the satellite k and the customer i, c^2_{min} and c^2_{max} the minimum and maximum values of the transportation costs at the 2nd-level, respectively, and $\beta > 0$ is a given parameter (we have assumed $\beta = 0.1$). Finally, Column 5 reports the mean normalized transportation cost of the satellites with respect to the depot, where the normalized transportation cost of each satellite k is given by:

$$\bar{c}_k = \frac{c_{0k} - c^1_{min}}{c^1_{max} - c^1_{min}}, \tag{24}$$

where c_{0k} is the transportation cost between the depot and the satellite k and c^1_{min} and c^1_{max} are the minimum and the maximum values of the transportation costs of the 1st-level. By considering all the pairs of customers as possible satellite location and comparing the results with the optimal solution of the original CVRP instance with optimum 247, in the following we discuss advantages and disadvantages of the proposed two-level distribution system.

From the results, it is clear the advantage of using the 2E-CVRP distribution model instead of the CVRP one. Indeed, the former is able to achieve a smaller cost in 42 instances, while the decreasing/increasing of the costs is, except for satellites $11, 12$ with $+24\%$, in the range $[-15\%, +12\%]$ of the corresponding CVRP instance. The mean decrease in the 42 instances with a reduced transportation cost is 8.5%, which could be used to balance the costs due to the loading/unloading operations at the satellites. In the city logistics field, this means that the 2E-CVRP distribution model could be introduced without rising the total transportation cost, while obtaining indirect advantages, such as the reduction of the traffic flows and pollution level.

In Figure 2 we reported the dispersion of the optima of the 66 instances with respect to the mean transportation cost of the satellites from the depot. These costs have been categorized in three sets: low (L), medium (M) and high (H) with the following meaning:

- Low: mean transportation cost of the satellites in the interval $[0, 50]$;
- Medium: mean transportation cost of the satellites in the interval $[50, 67]$;
- High: mean transportation cost of the satellites in the interval $[67, 100]$.

On the X axis the mean transportation cost is reported, while on Y we report the ratio between the optimum of the 2E-CVRP instance and the optimum of the CVRP instance. Thus, a ratio greater than 1 means that the optimum of the 2E-CVRP instance is worse than the CVRP one.

According to the figure, it is clear that the instances with an optimum better than the CVRP one are characterized by a low mean transportation cost from depot to satellites. The greater the mean transportation cost the less likely to improve the optimum. On the other hand, it is possible to obtain a gain even with an high mean transportation cost, which means that the mean transportation cost depot to satellites is not the only parameter to be taken into account.

In Figure 3 we report the dispersion of the optima for the 66 instances with respect to the mean accessibility index of the satellites. As for the mean transportation cost, we split the mean accessibility values into three sets, low (L), medium (M) and high (H) mean accessibility, with the following meaning:

- Low: mean accessibility in the interval $[A_{min}, 33\% \ of \ [A_{min}, A_{max}]]$;
- Medium: mean accessibility in the interval $[33\% \ of \ [A_{min}, A_{max}], 66\% \ of \ [A_{min}, A_{max}]]$;
- High: mean accessibility in the interval $[66\% \ of \ [A_{min}, A_{max}], A_{max}]$;

where $A_{min} = \min_k\{A_k\}$ and $A_{max} = \max_k\{A_k\}$.

On the X axis the mean accessibility index is reported, while on Y we report the ratio between the optimum of the 2E-CVRP instance and the optimum of the CVRP one.

According to the figure, when accessibility increases the number of the 2E-CVRP instances with a gain does increase. However, even in the instances with a high mean accessibility, it is possible to have a deterioration of their optimum.

Table 4 reports a resume of the instances where on the rows we considered the mean accessibility and on the columns the mean transportation cost values. Each cell contains a ratio y/z, where y is the number of instances with an optimum better than the CVRP one and z with an optimum worse than the

CVRP one. According to the resume, 2E-CVRP gives its best performance when the mean transportation cost of the satellites is low (less than 50% of the maximum transportation cost), with the ratio between gain and loss which is decreasing with the accessibility index. When the mean transportation cost is medium and the mean accessibility is high or medium, the optimum using the satellites is better than the CVRP one, while with a low mean accessibility it is difficult to improve the optimum. Finally, with a high mean transportation cost it becomes hard to obtain an optimum with satellites better than the CVRP one, even with a high accessibility index. This is mainly due to the fact that even if satellites are placed in the customers' neighborhood, they are usually near to the border of the customers' area, so the trasportation cost paid at the 1st level to reach the satellites is not compensated by the gain due to the proximity of the satellites to the customers.

6.3 Valid inequalities and overall computational results

In this section we present the computational results of the first and the second set of instances for 2E-CVRP using the valid inequalities introduced in 5 within a computation time limit of 10000 seconds.

In Table 5 the results of the 66 instances corresponding to the problem with 12 customers and 2 satellites are given. The optimum is reported in the second column, while columns 3 and 4 contain the time in seconds needed to solve the instances with and without the valid inequalities introduced in Section 5. Finally, the last column presents the percentage of decreasing/increasing of

Satellites	OPT	Variation (%)	Mean acc	Mean transp. Cost
1,2	262	6.07	62	6
1,3	274	10.93	66	14
1,4	274	10.93	50	16
1,5	208	-15.79	57	15
1,6	212	-14.17	76	19
1,7	220	-10.93	88	27
1,8	214	-13.36	102	31
1,9	226	-8.50	89	34
1,10	234	-5.26	101	38
1,11	258	4.45	67	48
1,12	266	7.69	64	50
2,3	262	6.07	52	20
2,4	262	6.07	36	22
2,5	208	-15.79	43	21
2,6	212	-14.17	62	24
2,7	218	-11.74	74	33
2,8	214	-13.36	88	37
2,9	230	-6.88	75	40
2,10	234	-5.26	87	44
2,11	256	3.64	53	53
2,12	262	6.07	50	56
3,4	278	12.55	40	30
3,5	218	-11.74	48	29
3,6	226	-8.50	67	33
3,7	226	-8.50	79	41
3,8	228	-7.69	93	45
3,9	244	-1.21	79	48
3,10	236	-4.45	91	52
3,11	256	3.64	57	62
3,12	266	7.69	54	64
4,5	218	-11.74	31	31
4,6	226	-8.50	50	35
4,7	228	-7.69	63	43

Satellites	OPT	Variation (%)	Mean acc	Mean transp. Cost
4,8	228	-7.69	76	48
4,9	244	-1.21	63	50
4,10	236	-4.45	75	55
4,11	254	2.83	41	64
4,12	258	4.45	38	66
5,6	210	-14.98	58	34
5,7	210	-14.98	70	42
5,8	214	-13.36	84	47
5,9	218	-11.74	70	49
5,10	218	-11.74	82	53
5,11	218	-11.74	48	63
5,12	218	-11.74	45	65
6,7	234	-5.26	89	45
6,8	230	-6.88	103	50
6,9	230	-6.88	89	52
6,10	230	-6.88	101	57
6,11	230	-6.88	67	66
6,12	230	-6.88	64	69
7,8	234	-5.26	115	58
7,9	246	-0.40	101	60
7,10	240	-2.83	114	65
7,11	246	-0.40	80	74
7,12	246	-0.40	77	77
8,9	254	2.83	115	65
8,10	254	2.83	127	70
8,11	254	2.83	93	79
8,12	254	2.83	90	81
9,10	270	9.31	114	72
9,11	274	10.93	80	81
9,12	274	10.93	77	84
10,11	274	10.93	92	86
10,12	274	10.93	89	88
11,12	308	24.70	55	98

Table 3: 12 customers and 2 satellites instances: detailed results

Mean accessibility			
H	9/0	5/2	0/6
M	10/3	3/0	1/4
L	9/5	2/6	0/1
Mean transportation cost	L	M	H

Table 4: 12 customers instances: resume of the results

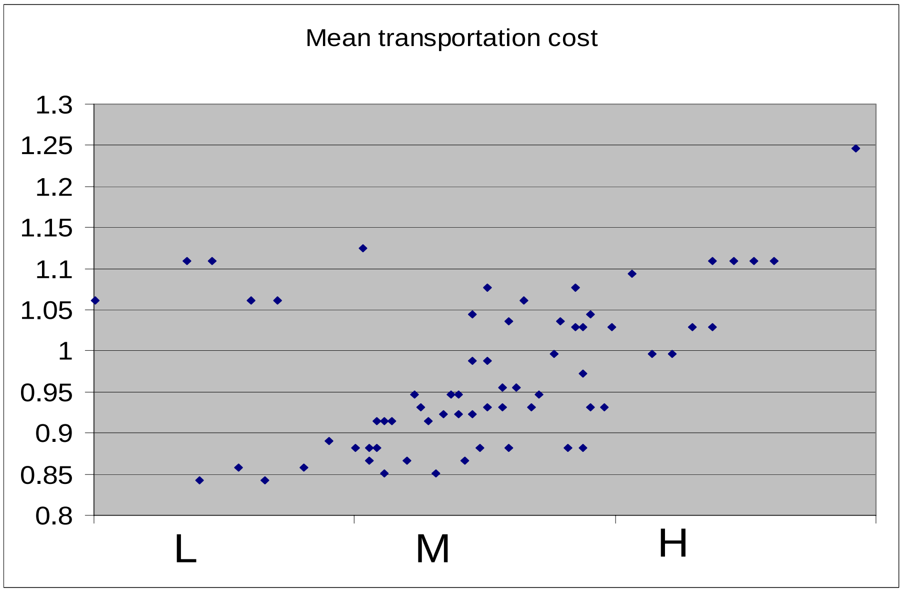

Figure 2: 12 customers instances: dispersion of the mean transportation cost

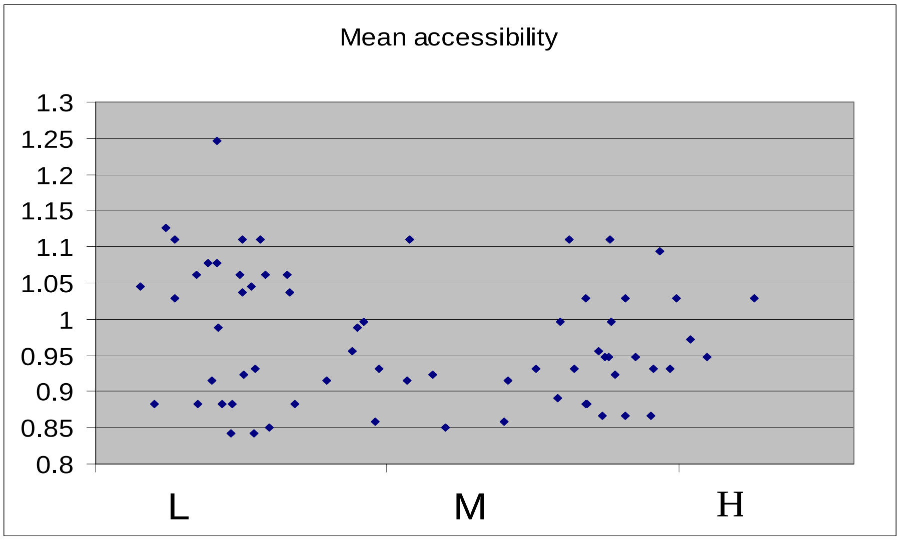

Figure 3: 12 customers instances: dispersion of the mean accessibility

computational time due to the usage of the valid inequalities.

According to the results most instances are solved in less than one minute, and only 10 of them need more than 2 minutes to be solved. There are however seven instances which computational times are greater than 10 minutes. This gap is mostly related to the satellite location. In fact, the greatest computational times are related to the situation where choosing which satellite to use has mostly no effect on the final solution. In this situation, the model finds quickly an optimal solution, but spends a lot of time to close the nodes of the decision tree due to the poor quality of the lower bound obtained by the continuous relaxation of the model. A better behavior is obtained with the valid inequalities. As a counter effect, on some instances, the computational time still increases, but this is mainly due to the fact that the management of the additional inequalities can affect the computational times on small-sized instances, which show a rather small computational time without the cuts.

The results on the second set of instances are presented in Tables 6 and 7.

Table 6 presents the behavior of the lower bound computed with a continuous relaxation of the model found with and without the valid inequalities. More precisely, columns 1 to 4 contain, respectively, the number of customers in the original Christofides and Elion's instance, the position of the satellites given as customer number, the mean accessibility as defined by (23) and the mean distance of the satellites computed according to (24). The best lower bound, the values and the gap with the best lower bound of the lower bound with and without the valid inequalities at the root node are reported in column 5, while the lower bound at the end of the optimization process, increased by letting the solver to apply lift-and-project cuts during the optimization is presented in column 6. The last column summarizes the best lower bound obtained for each instance (bold values mean optimal values).

From these results, we can notice how the usage of the cuts helps the model to reduce the gap up to 30%. The behaviour is confirmed by considering, in Table 7, the values of the feasible solutions found by the model with and without the valid inequalities. More precisely, columns 1 to 6 contain, respectively, the name of the original Christofides and Elion's instance, the position of the satellites given as customer number, the mean accessibility as defined by (23), the mean distance of the satellites computed according to (24), the best solution found and

Satellites	Cost	Time		% Time
		Without cuts	With cuts	
1,2	262	8507	3590.9	-57.79%
1,3	274	3337.2	915.9	-72.55%
1,4	274	3274.3	3922.1	19.78%
1,5	208	16.4	6.2	-62.20%
1,6	212	9.6	10.3	7.29%
1,7	220	24.7	15.4	-37.65%
1,8	214	8.5	4.4	-48.24%
1,9	226	13.8	9.2	-33.33%
1,10	234	24.1	20.2	-16.18%
1,11	258	83.3	178.1	113.81%
1,12	266	348.2	375.7	7.90%
2,3	262	1526.4	749.5	-50.90%
2,4	262	693.6	940.2	35.55%
2,5	208	8.4	5.4	-35.71%
2,6	212	11.1	7.1	-36.04%
2,7	218	10.1	10.4	2.97%
2,8	214	10.2	6	-41.18%
2,9	230	13.1	14.9	13.74%
2,10	234	25.7	21.5	-16.34%
2,11	256	207.5	109.8	-47.08%
2,12	262	672.6	197.5	-70.64%
3,4	278	5393.8	3307	-38.69%
3,5	218	5.2	6.3	21.15%
3,6	226	23.4	11.3	-51.71%
3,7	226	8.7	9	3.45%
3,8	228	7.4	7.8	5.41%
3,9	244	13.2	15	13.64%
3,10	236	28.5	23.1	-18.95%
3,11	256	12.1	11.6	-4.13%
3,12	266	40.3	15.6	-61.29%
4,5	218	8.6	6	-30.23%
4,6	226	30.4	28.5	-6.25%
4,7	228	16.3	14	-14.11%

Satellites	Cost	Time		% Time
		Without cuts	With cuts	
4,8	228	7	4.2	-40.00%
4,9	244	14.6	14.2	-2.74%
4,10	236	18.3	23.2	26.78%
4,11	254	18.5	19.2	3.78%
4,12	258	15.1	15.4	1.99%
5,6	210	10.5	6.3	-40.00%
5,7	210	4.6	5.2	13.04%
5,8	214	12.7	10.9	-14.17%
5,9	218	9.2	5.1	-44.57%
5,10	218	14.4	5.6	-61.11%
5,11	218	5.7	2.6	-54.39%
5,12	218	2.6	3	15.38%
6,7	234	4.8	4.6	-4.17%
6,8	230	23.4	13.3	-43.16%
6,9	230	14.3	16.4	14.69%
6,10	230	32.9	11	-66.57%
6,11	230	11.8	13.5	14.41%
6,12	230	11	7.2	-34.55%
7,8	234	6.1	9.6	57.38%
7,9	246	95.4	56.9	-40.36%
7,10	240	12.4	14.1	13.71%
7,11	246	42.4	33.4	-21.23%
7,12	246	19.9	19.2	-3.52%
8,9	254	64.1	37.5	-41.50%
8,10	254	43.5	37.2	-14.48%
8,11	254	8.4	6	-28.57%
8,12	254	12	12.1	0.83%
9,10	270	286.2	148.3	-48.18%
9,11	274	31.5	23.3	-26.03%
9,12	274	23.5	12.2	-48.09%
10,11	274	84.1	75.9	-9.75%
10,12	274	64.3	73.5	14.31%
11,12	308	234.8	78.5	-66.57%

Table 5: 12 customers and 2 satellites instances: valid inequalities improvements

				First Bound				Final Bound				Best
				Without cuts		With cuts		Without cuts		With cuts		Bound
CVRP Instance	Satellites	Accessibility	Distance	Bound	Gap	Bound	Gap	Best Bound	Gap	Best Bound	Gap	Best Sol.
E-n13-k4	2,3	52	20	184.13	-29.72%	228.69	-12.72%	262	0.00%	262	0.00%	**262.00**
	2,5	43	21	151.84	-27.00%	158.23	-23.93%	208	0.00%	208	0.00%	**208.00**
	3,6	67	33	155.12	-31.36%	182.73	-19.15%	226	0.00%	226	0.00%	**226.00**
	4,8	76	48	158.37	-30.54%	212.69	-6.72%	228	0.00%	228	0.00%	**228.00**
	5,10	82	53	201.42	-7.60%	201.76	-7.45%	218	0.00%	218	0.00%	**218.00**
	7,9	101	60	166.92	-32.15%	171.37	-30.34%	246	0.00%	246	0.00%	**246.00**
E-n22-k4	7,18	69	41	374.27	-10.26%	403.86	-3.17%	417.07	0.00%	417.07	0.00%	**417.07**
	9,15	30	31	338.12	-12.17%	360.16	-6.44%	384.96	0.00%	384.96	0.00%	**384.96**
	10,20	31	30	403.89	-14.14%	425.95	-9.45%	470.42	0.00%	470.42	0.00%	**470.42**
	11,15	38	25	332.02	-10.63%	349.89	-5.82%	371.50	0.00%	371.50	0.00%	**371.50**
	12,13	45	53	354.05	-17.13%	387.03	-9.41%	427.22	0.00%	427.22	0.00%	**427.22**
	13,17	33	64	334.34	-14.88%	355.73	-9.43%	392.78	0.00%	392.78	0.00%	**392.78**
E-n33-k4	2,10	28	41	567.30	-19.35%	613.01	-12.85%	668.63	-4.94%	703.39	0.00%	703.39
	3,14	19	31	593.12	-12.66%	609.36	-10.27%	665.35	-2.02%	679.08	0.00%	679.08
	4,18	31	30	580.23	-11.65%	605.81	-7.75%	637.09	-2.99%	656.74	0.00%	656.74
	5,6	28	25	61.18	-91.59%	633.37	-12.96%	710.06	-2.43%	727.71	0.00%	727.71
	8,26	30	53	615.51	-16.20%	632.68	-13.87%	699.74	-4.74%	734.54	0.00%	734.54
	15,23	18	64	626.80	-16.60%	663.38	-11.74%	717.27	-4.57%	751.59	0.00%	751.59
E-n51-k5	3.18	38	41	530.43	-2.57%	524.29	-3.70%	527.36	-3.13%	544.41	0.00%	544.41
	5.47	48	18	489.86	-2.89%	499.81	-0.92%	495.14	-1.85%	504.44	0.00%	504.44
	7,13	36	18	487.18	-4.64%	502.69	-1.61%	494.88	-3.14%	510.91	0.00%	510.91
	12,20	32	47	534.95	-2.27%	539.46	-1.44%	539.07	-1.52%	547.37	0.00%	547.37
	28,48	37	16	490.65	-2.52%	495.33	-1.59%	490.66	-2.51%	503.32	0.00%	503.32
	33,38	29	28	488.94	-5.61%	507.99	-1.93%	502.80	-2.93%	517.99	0.00%	517.99
E-n51-k5	3,5,18,47	43	29	422.45	-11.60%	468.84	-1.89%	459.01	-3.95%	477.87	0.00%	477.87
	7,13,33,38	32	23	432.68	-10.28%	471.37	-2.25%	445.88	-7.54%	482.24	0.00%	482.24
	12,20,28,48	34	31	446.96	-8.18%	466.89	-4.09%	462.27	-5.03%	486.78	0.00%	486.78

Table 6: Lower bound for the instances E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5

the best lower bound. The other columns contain the values and the gap with the best lower bound of the first feasible solution, the best solutions after 100, 1000 and 5000 seconds, and the best solution, respectively. For each column, the results with and without the cuts are given.

According to these results, the model is able to find good quality solutions in at most 5000 seconds up to 32 customers, while, when the number of customers increases to 50, more than 5000 seconds are required to find a good solution. Moreover, the usage of the cuts increases in average the quality of the model both of the initial solution and the lower bound. The gaps between the best solutions and the best bounds are rather small up to 32 customers, but increasing with 50 customers, with a gap up to 54% in the 4 satellites ones.

CVRP Instance	Satellites	Accessibility	Distance	Final solution	Best Bound	First Solution				Solution after 100 s			
						Without cuts		With cuts		Without cuts		With cuts	
				Best Sol.		Solution	Gap	Solution	Gap	Solution	Gap	Solution	Gap
E-n13-k4	2,3	52	20	**262**	**262.00**	328.00	25.19%	310.00	18.32%	262.00	0.00%	262.00	0.00%
	2,5	43	21	**208**	**208.00**	292.00	40.38%	228.00	9.62%	208.00	0.00%	208.00	0.00%
	3,6	67	33	**226**	**226.00**	242.00	7.08%	230.00	1.77%	226.00	0.00%	226.00	0.00%
	4,8	76	48	**228**	**228.00**	276.00	21.05%	254.00	11.40%	228.00	0.00%	228.00	0.00%
	5,10	82	53	**218**	**218.00**	238.00	9.17%	338.00	55.05%	218.00	0.00%	218.00	0.00%
	7,9	101	60	**246**	**246.00**	392.00	59.35%	386.00	56.91%	246.00	0.00%	246.00	0.00%
E-n22-k4	7,18	69	41	**417.07**	**417.07**	426.75	2.32%	435.54	4.43%	420.14	0.74%	417.07	0.00%
	9,15	30	31	**384.96**	**384.96**	521.77	35.54%	691.53	79.64%	446.57	16.01%	392.42	1.94%
	10,20	31	30	**470.42**	**470.42**	841.17	78.81%	1027.18	118.35%	505.33	7.42%	507.86	7.96%
	11,15	38	25	**371.50**	**371.50**	754.83	103.19%	631.89	70.09%	378.42	1.86%	403.17	8.53%
	12,13	45	53	**427.22**	**427.22**	909.98	113.00%	782.06	83.06%	451.93	5.78%	482.46	12.93%
	13,17	33	64	**392.78**	**392.78**	552.39	40.63%	603.67	53.69%	408.66	4.04%	431.65	9.89%
E-n33-k4	2,10	28	41	749.36	703.39	1078.47	53.32%	1148.15	63.23%	1078.47	53.32%	844.21	20.02%
	3,14	19	31	751.74	679.08	954.72	40.59%	1060.75	56.20%	954.72	40.59%	818.52	20.53%
	4,18	31	30	729.91	656.74	903.22	37.53%	1287.94	96.11%	903.22	37.53%	937.39	42.74%
	5,6	28	25	851.78	727.71	1065.18	46.37%	951.83	30.80%	854.36	17.40%	873.40	20.02%
	8,26	30	53	766.94	734.54	956.46	30.21%	1359.81	85.12%	932.53	26.95%	1359.81	85.12%
	15,23	18	64	787.31	751.59	1048.22	39.47%	1198.34	59.44%	1048.22	39.47%	1017.29	35.35%
E-n51-k5	3.18	38	41	644.96	544.41	1103.07	102.62%	1166.36	114.24%	No solution found		No solution found	
	5.47	48	18	621.00	504.44	998.23	97.89%	No solution found		No solution found		No solution found	
	7,13	36	18	672.84	510.91	1018.26	99.30%	1134.54	122.06%	No solution found		No solution found	
	12,20	32	47	643.25	547.37	835.94	52.72%	814.91	48.88%	No solution found		No solution found	
	28,48	37	16	619.19	503.32	876.10	74.06%	1166.87	131.83%	No solution found		1166.87	131.83%
	33,38	29	28	606.84	517.99	984.21	90.01%	606.84	17.15%	984.21	90.01%	No solution found	
E-n51-k5	3,5,18,47	43	29	700.63	477.87	778.37	62.88%	700.63	46.61%	No solution found		No solution found	
	7,13,33,38	32	23	744.73	482.24	1110.92	130.37%	1204.20	149.71%	No solution found		No solution found	
	12,20,28,48	34	31	775.70	486.78	1163.71	139.06%	1240.07	154.75%	No solution found		No solution found	

CVRP Instance	Satellites	Accessibility	Distance	Final solution	Best Bound	Solution after 1000 s				Solution after 5000 s				Best solution			
						Without cuts		With cuts		Without cuts		With cuts		Without cuts		With cuts	
				Best Sol.		Solution	Gap	Solution	Gap	Solution	Gap	Solution	Gap	Solution	Gap	Solution	Gap
E-n13-k4	2,3	52	20	**262**	262.00	262.00	0.00%	262.00	0.00%	262.00	0.00%	262.00	0.00%	262.00	0.00%	262.00	0.00%
	2,5	43	21	**208**	208.00	208.00	0.00%	208.00	0.00%	208.00	0.00%	208.00	0.00%	208.00	0.00%	208.00	0.00%
	3,6	67	33	**226**	226.00	226.00	0.00%	226.00	0.00%	226.00	0.00%	226.00	0.00%	226.00	0.00%	226.00	0.00%
	4,8	76	48	**228**	228.00	228.00	0.00%	228.00	0.00%	228.00	0.00%	228.00	0.00%	228.00	0.00%	228.00	0.00%
	5,10	82	53	**218**	218.00	218.00	0.00%	218.00	0.00%	218.00	0.00%	218.00	0.00%	218.00	0.00%	218.00	0.00%
	7,9	101	60	**246**	246.00	246.00	0.00%	246.00	0.00%	246.00	0.00%	246.00	0.00%	246.00	0.00%	246.00	0.00%
E-n22-k4	7,18	69	41	**417.07**	417.07	417.07	0.00%	417.07	0.00%	417.07	0.00%	417.07	0.00%	417.07	0.00%	417.07	0.00%
	9,15	30	31	**384.96**	384.96	386.94	0.51%	384.96	0.00%	384.96	0.00%	384.96	0.00%	384.96	0.00%	384.96	0.00%
	10,20	31	30	**470.42**	470.42	472.71	0.49%	476.81	1.36%	471.18	0.16%	470.42	0.00%	470.42	0.00%	470.42	0.00%
	11,15	38	25	**371.50**	371.50	371.50	0.00%	371.50	0.00%	371.50	0.00%	371.50	0.00%	371.50	0.00%	371.50	0.00%
	12,13	45	53	**427.22**	427.22	432.37	1.21%	433.46	1.46%	427.22	0.00%	427.22	0.00%	427.22	0.00%	427.22	0.00%
	13,17	33	64	**392.78**	392.78	408.66	4.04%	392.78	0.00%	392.78	0.00%	392.78	0.00%	392.78	0.00%	392.78	0.00%
E-n33-k4	2,10	28	41	749.36	703.39	798.85	13.57%	783.64	11.41%	774.73	10.14%	752.80	7.02%	761.39	8.25%	749.36	6.54%
	3,14	19	31	751.74	679.08	888.47	30.83%	803.97	18.39%	758.80	11.74%	803.97	18.39%	751.74	10.70%	769.96	13.38%
	4,18	31	30	729.91	656.74	808.39	23.09%	852.36	29.79%	808.39	23.09%	729.91	11.14%	780.17	18.79%	729.91	11.14%
	5,6	28	25	851.78	727.71	854.36	17.40%	873.40	20.02%	851.78	17.05%	866.30	19.04%	851.78	17.05%	866.30	19.04%
	8,26	30	53	766.94	734.54	932.53	26.95%	1061.66	44.53%	819.16	11.52%	779.99	6.19%	804.14	9.47%	766.94	4.41%
	15,23	18	64	787.31	751.59	856.04	13.90%	792.02	5.38%	816.71	8.66%	788.88	4.96%	816.71	8.66%	787.31	4.75%
E-n51-k5	3.18	38	41	644.96	544.41	No solution found		742.07	36.31%	1103.07	102.62%	644.96	18.47%	1021.88	87.70%	644.96	18.47%
	5.47	48	18	621.00	504.44	No solution found		No solution found		930.73	84.51%	621.27	23.16%	930.73	84.51%	621.27	23.16%
	7,13	36	18	672.84	510.91	861.92	68.70%	No solution found		861.92	68.70%	812.28	58.99%	816.19	59.75%	672.84	31.70%
	12,20	32	47	643.25	547.37	No solution found		814.91	48.88%	835.94	52.72%	797.93	45.78%	835.94	52.72%	643.25	17.52%
	28,48	37	16	619.19	503.32	No solution found		1166.87	131.83%	No solution found		619.19	23.02%	876.10	74.06%	619.19	23.02%
	33,38	29	28	606.84	517.99	861.50	66.32%	606.84	17.15%	861.50	66.32%	606.84	17.15%	861.50	66.32%	606.84	17.15%
E-n51-k5	3,5,18,47	43	29	700.63	477.87	No solution found		700.63	46.61%	778.37	62.88%	700.63	46.61%	727.20	52.17%	700.63	46.61%
	7,13,33,38	32	23	744.73	482.24	1107.04	129.56%	744.73	54.43%	767.34	59.12%	744.73	54.43%	767.34	59.12%	744.73	54.43%
	12,20,28,48	34	31	775.70	486.78	1106.09	127.23%	No solution found		808.42	66.08%	955.71	96.33%	775.70	59.36%	713.26	46.53%

Table 7: Solutions for the instances E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5

The model, however, is able to find solution with an average gap of 18%, which is quite large, but understandable considering that the lower bound comes from the simple continuos relaxation of the model with cuts and the original 50 customers instance is still considered a difficult one for Branch & Cut and Branch & Bound algorithms developed for CVRP.

7 Conclusions

In this paper, we introduced a new family of VRP models, the Multi-Echelon VRP. In particular, we considered the 2-Echelon Capacitated VRP, giving a MIP formulation and valid inequalities for it. The model and the inequalities have been tested on new benchmarks derived from the CVRP instances of the literature, showing a good behavior of the model for small and medium sized instances.

Moreover, a first analysis to find a priori conditions on the solution quality of 2E-CVRP has been performed, letting us to introduce a classification of the problems according to the combination of easy-to-compute instance parameters, such as satellite accessibility and distance.

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