OPERATIONAL PLANNING OF MULTI-ECHelon TRANSPORTATION NETWORKS

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The distribution function
Types of transportation modes and costs
Distribution network design and planning
Brief review of important routing and scheduling problems (TSP, VRP, PDP)
The PDP with transshipment (PDPT)
The vehicle routing problem in multi-echelon networks with and without crossdocking (VRP-SCM & VRPCD-SCM)
Conclusions
Logistics function: provision of goods and services from supply points to demand points.

It involves the management of a wide range of business operations: acquisition, production, storage, transportation and delivery of goods along the supply chain.
It comprises all movements and storage of goods “downstream” from the manufacturing plants.

- **Inbound transportation**: The movement of product from manufacturing plants to various warehouses or depots.
- **Intermediate storage**: The product storage at intermediate facilities.
- **Outbound transportation**: The delivery of products from intermediate facilities to the final customers.
Transportation play a key role because products are rarely produced and consumed at the same location.

The last transportation step from distribution centers to customers (i.e. the outbound transportation), is usually the most costly link of the distribution chain.

Distribution costs accounts for about 16% of the sale value of an item; approximately one fourth is due to the outbound transportation.
There are two key players in any transportation that takes place within a supply chain:

- **THE SHIPPER**
  - It is the party requiring the movement of products between two points in the supply chain.
  - He seeks to minimize the total cost, while providing an appropriate level of responsiveness to the customer.

- **THE CARRIER**
  - It is the party that moves the products.
  - He makes operating decisions trying to maximize the return from its assets.

For example, DELL uses UPS as the carrier to ship its computers from the factory to the customer.
## Types of transportation costs

<table>
<thead>
<tr>
<th>Cost Type</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td><strong>Vehicle-related cost</strong></td>
<td>It is the cost a carrier incurs for the purchase or lease of the vehicle used to transport goods.</td>
</tr>
<tr>
<td><strong>Fixed operating cost</strong></td>
<td>It includes the cost associated with terminals, airport gates that are incurred whether the vehicles are used or not.</td>
</tr>
<tr>
<td><strong>Trip-related variable cost</strong></td>
<td>This cost is incurred each time a vehicle leaves on a trip and includes the price of labour and fuel.</td>
</tr>
<tr>
<td><strong>Quantity related cost</strong></td>
<td>This category includes loading and unloading costs and a portion of the fuel cost that varies with the quantity being transported.</td>
</tr>
<tr>
<td><strong>Overhead cost</strong></td>
<td>It includes the cost of planning and scheduling a transportation network as well as any investment in information technology.</td>
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Listed by decreasing freight market share, the modes of transportation include:

- **TRUCK**
  - TRUCKLOAD (TL)
  - LESS THAN TRUCKLOAD (LTL)
- **WATER**
- **RAIL**
- **AIR**
- **PIPELINE**

or a combination of them,

- **INTERMODAL TRANSPORTATION**
Dominant mode of transportation

Two major segments:
- **full truckload (TL):** charge for the full truck, and rates vary with the distance travelled.
- **less-than truckload (LTL):** charge based on the quantity loaded and the distance travelled.

Trucking is more expensive than rail but offers the advantage of:
- door-to-door shipment
- shorter delivery time
- no transfer between pickup and delivery points
OTHERS MODES OF TRANSPORTATION

Rail
• High fixed cost in terms of rails, locomotives, cars, and yards
• Large, heavy, low-value shipments that are not very time-sensitive over long distances

Water
• Global Trade
• Very large loads
• Low costs
• Delays at ports and terminal

Air
• Very fast and fairly expensive
• Small, high-value items or time-sensitive emergency shipments

Pipeline
• Crude petroleum, refined petroleum products, and natural gas

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Response Time (RT). The time between the placement and the delivery of a customer order.

- Location of warehouses closer to the market reduce RT.

- Trade-off between response time and inventory costs. A decrease of RT produces an increase of the number and fixed-cost of facilities, and the inventory costs.

- Major issue to solve the trade-off: Level of inventory aggregation
Transportation costs

- A higher number of warehouses lowers both average distance to customers and outbound transportation costs.
- Warehousing allows consolidation of shipments from multiple suppliers in the same truck, to get lower inbound costs.
- Product customization at the delivery stage is postponed until receiving customer orders at the warehouse.
- **Major issues:** Consolidation of inbound shipments and temporal order aggregation at the delivery stage (frequency of visits vs. full truckload).
- Trade-off between customer service level and outbound transportation costs.
Product Availability (PA). It is the probability of having the requested product in stock when a customer order arrives.

- **Direct shipping centralizes inventories** at the manufacturer site, and guarantees a high level of PA with lower amounts of inventories.

- **Warehousing disaggregates inventories at intermediate facilities**, to lower response time and transportation costs, but decreasing the product availability.

- For products with low/uncertain demand, or high-value products, all inventories are better aggregated at the manufacturer storage.

- For low value, high-demand products, all inventories are better disaggregated and hold close to the customers.
A well-designed transportation network allows to achieve the desired degree of responsiveness at a low cost.

According to the number of stocking levels, transportation network designs can be classified into two categories:

1. Single-echelon networks: Goods are directly shipped from suppliers to retail stores or customers.

2. Multi-echelon networks: Goods are shipped from suppliers to retail stores via intermediate stocking points.
Direct shipping network
- All shipments come directly from suppliers to retail stores

Direct shipping with milk runs (A)
- A truck conveys lots of products from a supplier to multiple retail stores

Direct shipping with milk runs (B)
- A truck conveys lots of products from multiple suppliers to a retail store

Consolidation of shipments from a supplier to different destinations in a single truck

Consolidation of shipments from different suppliers in a truck
All shipments via a central DC

All shipments via distribution center using milk runs
Consolidation operations combine shipments from different suppliers and destined for multiple customers in the same truck.

Break-bulk operations at warehouses to customize a large shipment from various origins into multiple, smaller shipments to customers.

Cross-docking operations to perform break-bulk operations over ingoing, consolidated shipments right after their arrival at the intermediate facility, and immediately dispatch the customized parcels to their destinations.
Distribution management involves a variety of decision-making problems at three levels: strategic, tactical, and operational planning levels.

Strategic decisions deal with the distribution network design, including the number, location and size of facilities.

Tactical decisions include: a) the area served by each depot; b) the fleet size and composition; c) inventory policies at each facility; d) customer service levels.

Operational decisions are concerned with the routing and scheduling of vehicles on a day-to-day basis.
## Major Vehicle Routing and Scheduling Problems

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>TSP</td>
<td>Travelling salesman problem</td>
</tr>
<tr>
<td>MTSP</td>
<td>M-travelling salesman problem</td>
</tr>
<tr>
<td>VRP/VRPTW</td>
<td>Single depot, vehicle routing problem without or with time windows</td>
</tr>
<tr>
<td>m-VRPTW</td>
<td>Multi-depot, vehicle routing problem with time windows</td>
</tr>
<tr>
<td>PDPTW</td>
<td>Pickup and delivery problem with time windows</td>
</tr>
<tr>
<td>VRPCD</td>
<td>Pickup and delivery problem with cross-docking at a central depot</td>
</tr>
<tr>
<td>VRP-SCM</td>
<td>Vehicle routing and scheduling problem in a multi-echelon supply chain</td>
</tr>
<tr>
<td>VRPCD-SCM</td>
<td>Vehicle routing and scheduling problem with cross-docking in a multi-echelon supply chain</td>
</tr>
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</table>
Given a set of cities and the distances between them, determine the shortest path starting from a given city, passing through all the other cities and returning to the first town.

It can be represented by an undirected weighted graph, at which cities are the vertices, and paths are the edges.
It is a generalization of the travelling salesman problem where there is a need to account for more than one salesman.

Given a set of cities and a set of salesmen, find the set of routes for the salesmen with a minimum total length so that:

a) each salesman visits a unique set of cities and completes the route by returning to the starting city
b) each city is visited by exactly one salesman.
1. Model-based exact approaches
2. Heuristic approaches
   - Tour construction procedures. Generate a feasible tour from the distance and saving matrices.
   - Tour improvement procedures. Find a better tour assuming that an initial tour is given, and perform arc/node moves.
   - Composite procedures. Construct a starting tour using a tour construction procedure and find a better tour using a tour improvement procedure.
- Nearest neighbour
Generalization of the m-TSP, where a demand is associated to each node and every vehicle has a finite capacity.

In VRP, the sum of fixed costs (associated to the number of used vehicles) and variable costs (associated to the total distance traveled) is minimized.

Generate optimal routes for the vehicle fleet based on a given road network so as to meet customers demands while satisfying capacity and time constraints at minimum transportation cost
The demand at each node is assumed to be deterministic.
Each vehicle has a known capacity that cannot be exceeded.
Each vehicle route must start and end at the central depot.
Each node must be visited only once by exactly one vehicle.
Only a single type of task is performed by each vehicle (pickup or delivery task).
All routing problems (TSP, m-TSP, VRP) are NP hard problems.

Computational effort to solve these problems increase exponentially with the problem size.
1. **Routing Problem**
   - A spatial problem. Temporal considerations are ignored. No a priori restrictions on delivery times (i.e. no TW constraints) and goods can be delivered within a short period of time (i.e. maximum service time constraints can be ignored).

2. **Routing and Scheduling Problem**
   - The movement of each vehicle must be traced through both time and space. Visiting times to various locations are of primary importance. Temporal considerations may no longer be ignored and time restrictions guide the routing and scheduling activities.
SOLUTION TECHNIQUES FOR VRP PROBLEMS

- Heuristics
- Metaheuristics

Heuristic Techniques

Exact Optimization

- Branch-and-price
- Branch-and-cut
- Lagrangian relaxation
METAHEURISTIC TECHNIQUES FOR THE VRP

- Simulated annealing
- Tabu search
- Genetic algorithms
- Neural networks
- Model-based large-scale neighborhood search methods
- Threshold algorithms

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A generalization of the VRP where both **pickup and delivery operations** are done by a fleet of vehicles.

It is called the **multi-vehicle pickup and delivery problem with time-windows (m-PDPTW)**.

It is a **single-echelon transportation problem** involving pickup and delivery tasks, **but accomplishing a single type of operation at each node**.

It involves a set of **transportation requests** \( r \in R \), each one satisfied by a single vehicle and defined by:
- a pickup location (the origin),
- a delivery location (the destination),
- a freight to be delivered from one to the other site, and
- the time required to complete the service at each location.

All the requests are known in advance.
ILLUSTRATING THE m-PDPTW PROBLEM
Vehicles depart and return to the central depot (tour constraint).

Each transportation request must be serviced by a single vehicle. Pickup and delivery locations related to the same request are visited by the same vehicle (pairing constraint).

Each pickup location has to be visited prior to the associated delivery location (precedence constraint).

Each vehicle can satisfy one or more customer requests (a “composite” milk run).
Vehicle capacity must never be exceeded after visiting a pickup node (capacity constraint at pickup nodes).

A vehicle must transport enough load to meet customer demand when servicing a delivery node (capacity constraint at delivery nodes). Only important for requests involving multiple pickup points.

The service at each node must be started within the specified time window (time window constraints).

The total time/distance travelled from the depot to a certain node must be greater than the one required to reach a preceding node on the tour (compatibility between routes and schedules).
The alternative problem goals aim to minimize:

- the total distance travelled
- the total travel time required to service all customer requests
- the total customers’ inconvenience
- a weighted combination of total service time and customers’ inconvenience.

Customer inconvenience is usually a linear function of a customer’s waiting time (because of late arrivals).
m-PDPTW SOLUTION METHODS

- Metaheuristic methods
  - Construction procedures
- Heuristic techniques
  - Insertion procedures
- Exact optimization approaches
  - Improvement procedures
1. Dynamic programming

2. Branch-and-price

3. Branch-and-cut
PDPTWT

The PDPTW problem with transshipment
Loads are transported from suppliers (pickup nodes) to customers (delivery nodes) via a single cross-dock.

Each request is split into two sub-requests, i.e. a pickup and a delivery sub-request that may be handled by two different vehicles.

The incorporation of transshipment points may yield solutions with shorter travel distances or fewer vehicles.

Loads from suppliers are picked up by a fleet of vehicles, consolidated at the cross-dock facility, and immediately delivered to customers by the same set of vehicles, without intermediate storage.
ILLUSTRATING THE PDPTW PROBLEM WITH TRANSSHIPMENT
Each node must be visited only once by a single vehicle.

Each vehicle can pick up loads from more than one supplier and deliver loads to more than one customer. Milk runs are allowed.

Pickup and delivery routes start and end at the cross-dock facility.

Loads to pickup/deliver at problem nodes are known data.

The total amounts unloaded at the receiving dock and loaded into trucks at the shipping dock should be equal. There is no end inventory at the cross-dock facility.
- Service time windows for the nodes are usually specified.

- The problem goal is to minimize the total transportation cost while satisfying all customer requests.
VRPTW-SCM

The vehicle routing problem with time windows in the supply chain management
PROBLEM GOAL: Determine the best operational planning of multi-echelon transportation networks comprising factories, warehouses, and customers (the problem nodes).

Different types of distribution strategies like direct shipping, shipping via DC or regional warehouses, or a mix of them (hybrid strategies) can be implemented.

To resemble the logistics activities at multi-site distribution networks, multiple events at every node can occur.
ILLUSTRATING THE VRPTW-SCM PROBLEM
Types of nodes

- **“Pure” source nodes (IS)**, usually manufacturer storages, where vehicles carry out only pickup operations
- **Mixed nodes (IM)**, like DCs, where visiting vehicles can accomplish pickup and/or delivery operations.
- **Destination nodes (ID)**, like consumer zones, where visiting trucks just perform delivery operations

Number of events. The number of events for a location must be at least equal to the number of vehicles stops for performing pickup/delivery operations at the optimum.

Global precedence. For each vehicle stop, the model provides all the visits the vehicle has made before.
Multiple products are distributed from manufacturing plants and warehouses to customers.

Customer requests may involve several products (multi-commodity distribution) and do not generally have predefined suppliers.

The amounts of products to pick up at source nodes by a vehicle are not predefined but chosen by the model.

Multiple partial shipments to a customer location are allowed (order splitting).

Milk runs are performed on both sides, i.e. inbound and outbound sides. Then, several customers can be serviced by the same truck.
Problem events are the vehicle stops at DCs and customer locations.

A location can be visited either by multiple trucks or several times by the same vehicle. Then, several events can sequentially occur at any site.

A set of pre-specified, timely-ordered events for each site are defined. Its number is chosen by the user.

A vehicle can accomplish pickup and delivery operations during a stop at DCs.

The magnitude and composition of the freight transported by a vehicle at any stop must be traced in order to meet:

- capacity constraints at pickup locations
- product availability constraints at delivery points
Each vehicle must finally return to the assigned depot (tour constraint).

Customer time windows and the specified maximum service time for each vehicle must be respected.

Finite inventories at manufacturer storage and distribution centers at the initial time are known.

In addition to customer demands, specific replenishment orders from DCs are to be fulfilled.

Cross-docking at DCs is not permitted. Customer demands should be satisfied using the initial inventories.

At the tactical level, each DC may have been predefined as the exclusive supplier for some nearby customer nodes.
The problem goal is to minimize the total transportation cost while satisfying customer requests and meeting the service-level requirements.

Transportation costs include:
- fixed expenses incurred by used vehicles,
- distance-based variable costs, mainly fuel costs
- time-based variable costs, mainly driver wages.
MODEL VARIABLES IN VRPTW-SCM

- **Binary variables**
  - **Assignment variables** $Y_{niv}$: Event $n$ at location $i$ has been allocated to vehicle $v$ whenever $Y_{niv} = 1$. If so, vehicle $v$ stops at node $i$ at time event $n$.
  - **Sequencing variable** $X_{ni,n'i'}$: Vehicle stop $(n,i)$ will occur earlier than the vehicle stop $(n', i')$ whenever both nodes $(i,i')$ nodes are visited by the same vehicle and $X_{ni,n'i'} = 1$.

- **Continuous variables**
  - **Cost-variable** $C_{ni}$: distance-based transportation cost incurred by the visiting vehicle to move from the base up to node $i$ along the assigned route.
  - **Time-variable** $T_{ni}$: total time required by the assigned vehicle to travel from the base up to stop $(n,i)$.
  - **Pickup-Load** $L_{ni,pv}$: amount of product $p$ to pick up by vehicle $v$ during stop $(n,i)$.
  - **Delivery-Load** $U_{ni,pv}$: amount of product $p$ to deliver by vehicle $v$ during stop $(n,i)$.
  - **Accumulated-Pickup & Delivery Loads** $AL_{ni,pv}$ / $AU_{ni,pv}$: accumulated amount of product $p$ to pick up/deliver by vehicle $v$ along the assigned route from the base up to stop $(n,i)$. 
ILLUSTRATING THE VRPTW-SCM PROBLEM EVENTS

<table>
<thead>
<tr>
<th>Node</th>
<th>Delivery</th>
<th>Pickup</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>n1</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>n1</td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>n1</td>
<td></td>
</tr>
<tr>
<td>C4</td>
<td>n1</td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>n1</td>
<td></td>
</tr>
<tr>
<td>C6</td>
<td>n1</td>
<td></td>
</tr>
<tr>
<td>C7</td>
<td>n1</td>
<td></td>
</tr>
<tr>
<td>W1</td>
<td>n1</td>
<td>n2</td>
</tr>
<tr>
<td>S1</td>
<td></td>
<td>n1</td>
</tr>
</tbody>
</table>

\[
Y_{n_1W_1V_1} = 1 \\
Y_{n_2W_1V_2} = 1 \\
Y_{n_1C_1V_2} = 1 \\
X_{n_2W_1,n_1C_1} = 1
\]
Route building constraints assigning a particular stop \((n,i)\) to at most a single vehicle, and ordering vehicle stops \((n,i)\) on the same route.

Product inventory constraints restraining the maximum amount of every product to pick up by visiting vehicles at source nodes accounting for the current product stocks.

Additional inventory constraints monitoring the amount of every product received at each warehouse over the planning horizon.

Product demand constraints ensuring that customer requests are satisfied.
Null in-transit inventory constraints requiring that every product unit picked up by a vehicle must be delivered to a customer before the end of the vehicle trip.

Loading/unloading constraints monitoring the total amount of products transported by each vehicle to prevent from overcapacity or product shortages.

Time window and maximum service time constraints ensuring that the customer service begins within the specified TW, and each vehicle returns to its base within the allowed working period.
**Allocating vehicles to depots**

\[ \sum_{i \in B_v} W_{iv} \leq 1 \quad \forall v \in V \]

**Assigning the event n at node i to vehicle v**

\[ \sum_{v \in V_i} Y_{iv} \leq 1 \quad \forall n \in N_i, i \in I \]

**Preordering of time events predefined for node i**

\[ \sum_{v \in V_i} Y_{iv} \geq \sum_{v \in V_i} Y_{iv'} \quad \forall (n, n') \in N_i, i \in I : n < n' \]

**Activated vehicle condition**

\[ \sum_{i \in I_v} \sum_{n \in N_i} Y_{iv} \leq M \sum_{i \in B_v} W_{iv} \quad \forall v \in V \]

**Travel cost and time from the vehicle depot to the first visited node**

\[ \begin{cases} C_v \geq \sum_{i \in I_v} d_{iv} - M^c (1 - X_{iv}) - M^c (2 - Y_{iv} - Y_{iv}) \\ T_v \geq T_n + \sum_{i \in I_v} (L_{i,v} + U_{i,v}) + \frac{d_{iv}}{sp_v} - M^t (1 - X_{iv}) - M^t (2 - Y_{iv} - Y_{iv}) \end{cases} \quad \forall n \in N_i, i \in I, v \in V_i : i < i' \]

**Travel cost and time from the base to vehicle stop (i,n)**

\[ \begin{cases} C_v \geq C_v + dc_v d_{iv} - M^c (1 - X_{iv}) - M^c (2 - Y_{iv} - Y_{iv}) \\ T_v \geq T_n + \sum_{i \in I_v} (L_{i,v} + U_{i,v}) + \frac{d_{iv}}{sp_v} - M^t (1 - X_{iv}) - M^t (2 - Y_{iv} - Y_{iv}) \end{cases} \quad \forall n \in N_i, i \in I, v \in V_i \]

**Bound on the routing cost and time for the tour assigned to vehicle v**

\[ \begin{cases} CV_v \geq C_v + \sum_{i \in I_v} d_{iv} W_{iv} - M^c (1 - Y_{iv}) \\ TV_v \geq T_n + \sum_{i \in I_v} (L_{i,v} + U_{i,v}) + \sum_{i \in I_v} \frac{d_{iv}}{sp_v} W_{iv} - M^t (1 - Y_{iv}) \end{cases} \quad \forall n \in N_i, i \in I, v \in V_i \]

**Time-window and maximum service time constraints**

\[ a_i \leq T_n \leq b_i \quad n \in N_i, i \in I D \]

\[ TV_v \leq t_v \text{ max} \quad v \in V \]

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Product availability constraints
\[ \sum_{v \in V_i} \sum_{n \in N_i} L_{npv} \leq INV_{ip} \quad \forall i \in \{IS_p \cap IM_p\}, p \in P \]

Product demand constraints
\[ \sum_{v \in V_i} \sum_{n \in N_i} U_{npv} = DEM_{ip} - BL_{ip} \quad \forall i \in \{ID_p \cap IM_p\}, p \in P \]

Null in-transit inventory constraints
\[ \sum_{n \in N_i} \sum_{i \in IS, i \in ID} L_{npv} = \sum_{n \in N_i} \sum_{i \in ID} U_{npv}, p \in P, v \in V \]

Vehicle loading/unloading operation constraints
\[
\begin{align*}
L_{npv} & \leq INV_{ip} Y_{nv} \quad \forall n \in N_i, i \in \{IS_p \cap IM_p\}, p \in P, v \in V_i \\
U_{npv} & \leq DEM_{ip} Y_{nv} \quad \forall n \in N_i, i \in \{ID_p \cap IM_p\}, p \in P, v \in V_i 
\end{align*}
\]

Accumulated amount of product p picked-up by vehicle v up to the stop (i,n)
\[
\begin{align*}
AL_{npv} & \geq AL_{npv} + L_{npv} - M_L(1-X_{nv}) - M_L(2-Y_{nv} - Y_{nv'}) \\
AL_{npv} & \geq AL_{npv} + L_{npv} - M_L X_{nv} - M_L(2-Y_{nv} - Y_{nv'}) \\
\forall n \in N_i, n' \in N_{i'}, i, i' \in I, p \in P_i, v \in V_i, (i,n) < (i',n')
\end{align*}
\]

Accumulated amount of product p delivered by vehicle v up to the stop (i,n)
\[
\begin{align*}
AU_{npv} & \geq AU_{npv} + U_{npv} - M_L(1-X_{nv}) - M_L(2-Y_{nv} - Y_{nv'}) \\
AU_{npv} & \geq AU_{npv} + U_{npv} - M_L X_{nv} - M_L(2-Y_{nv} - Y_{nv'}) \\
\forall n \in N_i, n' \in N_{i'}, i, i' \in I, p \in P_i, v \in V_i 
\end{align*}
\]

Maximum volumetric and weight vehicle capacity constraints
\[
\begin{align*}
\sum_{p \in P} uv_p \left(AL_{npv} - AU_{npv}\right) & \leq vq_v Y_{nv} \\
\sum_{p \in P} uv_p \left(AL_{npv} - AU_{npv}\right) & \leq wq_v Y_{nv} \\
\forall n \in N_i, i \in I, p \in P, v \in V_i
\end{align*}
\]

Lower bounds on the cargo transported by vehicle v after stop (i,n)
\[
\begin{align*}
AL_{npv} & \geq AU_{npv} \\
AU_{npv} & \geq L_{npv} \\
AU_{npv} & \geq U_{npv} \\
\forall n \in N_i, i \in I, p \in P, v \in V_i
\end{align*}
\]

Upper bounds on the accumulated amount of product p loaded/unloaded by vehicle v after stop (i,n)
\[
\begin{align*}
AL_{npv} & \leq \sum_{\bar{p} \in P} \sum_{\bar{p} \in N_i} L_{np\bar{p}} \\
AU_{npv} & \leq \sum_{\bar{p} \in P} \sum_{\bar{p} \in N_i} U_{np\bar{p}} \\
\forall n \in N_i, i \in I, p \in P, v \in V_i
\end{align*}
\]
Alternative Objective Functions

The sum of distance-based travel costs and vehicle fixed costs

$$\text{Min } \sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in B_v} f c_v W_{lv}$$

The weighted sum of distance-based and time-based travel costs plus vehicle fixed costs

$$\text{Min } \sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in B_v} f c_v W_{lv} + \sum_{v \in V} utc_v TV_v$$

Fixed and variable transportation costs plus the penalties for unsatisfied demands, late services and overtime journeys

$$\text{Min } \sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in B_v} f c_v W_{lv} + \sum_{v \in V} utc_v TV_v + \sum_{v \in V} (c o_v OVT_v + \sum_{i \in ID_v n \in N_i} c l_i TD_n ) + \sum_{p \in P i \in I} c_{B,i} B_{ip}$$
VRPCD-SCM

The VRPCD-SCM problem with cross-docking
Generalization of the VRP-SCM problem to consider the possibility of cross-docking.

Intermediate depots may keep finite stocks of fast-moving products (warehousing) and act as cross-dock platforms for slow-moving, high-value items.

Replenishment orders and cross-docking operations are triggered when the initial stock in a warehouse is insufficient to meet the demand of the assigned customers.

Inbound and outbound vehicles must stay at receiving/shipping docks of DCs until they complete their delivery/pickup tasks.

Target product inventories at the end of the planning horizon may be specified.
Product inventories at cross-dock facilities must be traced over the planning horizon.

Sequencing of pickup and delivery operations by different vehicles at the same warehouse now become important.

The problem goal aims to minimize fixed and variable transportation costs.
Allocating base nodes to vehicles

\[
\sum_{l \in L_B} \sum_{n \in N_l} Y_{nlv} \leq 1 \quad \forall v \in V
\]

Allocating events at every node to vehicles

\[
\sum_{n' \in V_l} Y_{niv} \leq 1 \quad n \in N_i, i \in I
\]

Pre-ordering events occurring at the same node

\[
\sum_{n' \in V_l} Y_{niv} \geq \sum_{n'' \in V_l} Y_{n''iv} \quad (n, n') \in N_i : n' > n, i \in I
\]

Used vehicle condition

\[
\sum_{l \in L} \sum_{n \in N_l} Y_{niv} \leq M_v \left( \sum_{l \in L_B} \sum_{n \in N_l} Y_{nlv} \right) \quad \forall v \in V
\]

Travelling cost constraints

Travelling cost from the base node \( l \) to the first serviced node \( i \) for vehicle

\[
C_{ni} \geq \sum_{l \in L_B} \sum_{n' \in N_l} c_{li} Y_{n'iv} - M_C (1 - Y_{niv}) \quad n \in N_i, i \in I, v \in V_i
\]

Accumulated travelling cost for vehicle \( v \) up to the stop \((n,i)\)

\[
\begin{cases}
C_{n'i'} \geq C_{ni} + c_{i'i'} - M_C \left( 1 - X_{ni,n'i'} \right) - M_C \left( 2 - Y_{niv} - Y_{n'i'v} \right) \\
C_{ni} \geq C_{n'i'} + c_{i'i} - M_C X_{ni,n'i'} - M_C \left( 2 - Y_{niv} - Y_{n'i'v} \right)
\end{cases}
\]

\[
\begin{cases}
C_{n'i'} \geq C_{ni} + c_{i'i'} - M_C \left( 1 - X_{ni,n'i'} \right) - M_C \left( 2 - Y_{niv} - Y_{n'i'v} \right) \\
C_{ni} \geq C_{n'i'} + c_{i'i} - M_C X_{ni,n'i'} - M_C \left( 2 - Y_{niv} - Y_{n'i'v} \right)
\end{cases}
\]

- For multiple stops of vehicle \( v \) at the same node \( l \) :

\[
C_{n'i'} \geq C_{ni} - M_C \left( 2 - Y_{niv} - Y_{n'i'v} \right) \quad (n, n') \in N_i, i \in I, v \in V_i : n < n'
\]

Overall travelling cost for vehicle \( v \)

\[
CV_v \geq C_{ni} + \sum_{l \in L_B} \sum_{n' \in N_l} c_{li} Y_{n'iv} - M_C (1 - Y_{niv}) \quad n \in N_i, i \in I, v \in V
\]
Travelling time constraints

Travelling cost from the assigned base node \( I \in B_v \) to the first serviced node for vehicle \( v \)

\[
T_{n,i} \geq \sum_{n' \in B_v, n' \neq n} t_{n,n'} + f_{t_i} + v_t \sum_{p \in P} L_{n,p} - M_c (1 - Y_{nv})
\]
\( n \in N_i, i \in I, v \in V \)

Travelling time for vehicle \( v \) from the assigned base node to the stop \((n,i)\)

\[
\begin{align*}
T_{n,i} & \geq T_{n,i} + f_{t_i} + v_t \left( \sum_{p \in P} L_{n,p,v} + U_{n,p} \right) + t_{v,i} - M_c (1 - S_{n,v}) - M_c (2 - Y_{nv} - Y_{n'i'}) \\
T_{n,i} & \geq T_{n,i} + f_{t_i} + v_t \left( \sum_{p \in P} L_{n,v,i'} + U_{n,v,i'} \right) + t_{v,i} - M_c S_{n,v} - M_c (2 - Y_{nv} - Y_{n'i'}) \\
& \quad n \in N_i, n' \in N_i, (i,i') \in I, v \in V_{v'}: i < i'
\end{align*}
\]

- In case of multiple stops of the same vehicle \( v \) at node \( i \) at different time events

\[
T_{n,i} \geq T_{n,i} + f_{t_i} + v_t \left( \sum_{p \in P} L_{n,p,v} + U_{n,p} \right) - M_c (2 - Y_{nv} - Y_{n'i'}) \quad (n,n') \in N_i, i \in I, v \in V_i : n < n'
\]

- In case of consecutive stops of two different vehicles \( v \) and \( v' \) at the same warehouse \( i \)

\[
T_{n,i} \geq T_{n,i} + f_{t_i} + v_t \left( \sum_{p \in P} L_{n,p,v} + U_{n,p} \right) - M_c (2 - Y_{nv} - Y_{n'i'}) \quad (n,n') \in N_i, i \in I, v,v' \in V_i : n < n'
\]

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Overall travelling time for vehicle $v$

$$OT_v \geq T_{ni} + ft_i + vt\left(\sum_{p \in P_v} L_{ni_{pv}} + U_{ni_{pv}}\right) + \sum_{l \in B_i} \sum_{n \in N_i} t_{il} Y_{nl} - M_c (1 - Y_{niv}) \quad n \in N_i, i \in I, v \in V$$

Time window and maximum service-time constraints

$$a_i \leq T_{ni} \leq b_i \quad n \in N_i, i \in I$$

$$OT_v \leq t_v^{\max} \quad v \in V$$

Product availability constraints

For pure sources:

$$\sum_{v \in V_i} \sum_{n \in N_i} L_{ni_{pv}} \leq H_{ip} \quad i \in IS, p \in P_i$$

For warehouses:

$$\sum_{v \in V_i} \sum_{n \in N_i} L_{n_{i_{1},pv}} \leq H_{ip} + AINV_{ap} \quad n \in N_i, i \in IM, p \in P_i$$

Product demand constraints at customer nodes

$$\sum_{v \in V_i} \sum_{n \in N_i} U_{ni_{pv}} \geq D_{ip} \quad i \in ID, p \in P_i$$

Overall product balance at each intermediate facility

$$\sum_{v \in V_i} \sum_{n \in N_i} L_{ni_{pv}} \leq H_{ip} + \sum_{v \in V_i} \sum_{n \in N_i} U_{ni_{pv}} - FINV_{ip} \quad p \in P, i \in IM$$

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Overall product balance for every vehicle

\[
\sum_{i \in IS, j \in JM} \sum_{n \in N_i} L_{ni, pv} = \sum_{i \in (IM \cup ID)} \sum_{n \in N_i} U_{ni, pv} \quad p \in P, v \in V
\]

Accumulated amount of product \( p \) picked up by vehicle \( v \) up to the stop \( (n, i) \)

\[
\begin{cases}
AL_{n'i, pv} \geq AL_{ni, pv} + L_{n'i, pv} - M_L \left( 1 - S_{ni, n'i} \right) - M_L \left( 2 - Y_{niv} - Y_{n'i v} \right) \\
AL_{ni, pv} \geq AL_{n'i, pv} + L_{ni, pv} - M_L \left( 1 - S_{ni, n'i} \right) - M_L \left( 2 - Y_{niv} - Y_{n'i v} \right)
\end{cases}

n \in N_i, n' \in N_i', (i, i') \in I, p \in P, v \in V_v : n < n', i \neq i'

- In case of multiple stops of the same vehicle \( v \):

\[
UL_{n'i, pv} \geq UL_{ni, pv} + U_{n'i, pv} - M_L \left( 1 - S_{ni, n'i} \right) - M_L \left( 2 - Y_{niv} - Y_{n'i v} \right)
\]

\[
UL_{ni, pv} \geq UL_{n'i, pv} + U_{ni, pv} - M_L \left( 1 - S_{ni, n'i} \right) - M_L \left( 2 - Y_{niv} - Y_{n'i v} \right)
\]

n \in N_i, n' \in N_i', (i, i') \in I, p \in P, v \in V_v : n < n', i \neq i'

- In case of multiple stops of the same vehicle \( v \):

Vehicle capacity constraints

\[
\begin{cases}
\sum_{p \in P} uw_p \left( AL_{ni, pv} - AU_{ni, pv} \right) \leq qw_v \\
\sum_{p \in P} uv_p \left( AL_{ni, pv} - AU_{ni, pv} \right) \leq qv_v \\
AL_{ni, pv} - AU_{ni, pv} \geq 0
\end{cases}

n \in N_i, i \in I_v, v \in V
\]

Relationship between variables \( L_{ni, pv} / U_{ni, pv} \) with \( Y_{niv} \)

\[
L_{ni, pv} \leq M_L \ Y_{niv} \quad n \in N_i, i \in (IS \cup IM), p \in P, v \in V_i
\]

\[
U_{ni, pv} \leq M_L \ Y_{niv} \quad n \in N_i, i \in IM \cup ID, p \in P, v \in V_i
\]

Bounds on variables \( AU_{ni, pv} \) and \( AL_{ni, pv} \)

\[
\begin{cases}
L_{ni, pv} \leq AL_{ni, pv} \leq \sum_{i \in IS, j \in JM} \sum_{n \in N_i} L_{n'i, pv} \\
U_{ni, pv} \leq AU_{ni, pv} \leq \sum_{i \in IS, j \in JM} \sum_{n \in N_i} U_{n'i, pv}
\end{cases}

n \in N_i, i \in I_v, v \in V, p \in P
\]
Additional inventory received at cross-docking facilities from other sources

\[ AI_{n',i,p} \geq AI_{n,i,p} + \sum_{v \in V} U_{n'i,pv} \]  
\((n,n') \in N_i,i \in IM, p \in P : n < n'\)

\[ \sum_{v \in V} U_{n'i,pv} \leq AI_{n,i,p} \leq \sum_{n \in N_i,v \in V} U_{n'i,pv} \]  
\(n \in N_i,i \in IM, p \in P\)

Alternative Objective Functions

Min \( \left[ \sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in IB_v} \sum_{n \in N_i} fc_v Y_{nlv} \right] \)

Min \( \left[ \sum_{v \in V} OT_v \right] \)

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### AVAILABLE INVENTORIES AT SOURCE LOCATIONS

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### PRODUCT DEMANDS AT DESTINATION NODES

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(*) Bilbao is a special case with demands in P1, P2, and P3.
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<td>28</td>
<td>22</td>
<td>15</td>
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</table>

**Fixed Cost**
- $ 5000 (V1-V4)
- $ 4000 (V5-V6)

**Variable Cost**
- 3 $/km (V1-V4)
- 2.5 $/km (V5-V6)

**Loading/unloading times**
- Fixed 1 h
- Variable 250 units/h
- Average Speed 70 km/h
### Detailed schedule of vehicle activities

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<thead>
<tr>
<th>Vehicle</th>
<th>Site</th>
<th>Arrival time</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>Used Capacity</th>
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<td>Zaragoza</td>
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CASE STUDY II
(ONE-FACTORY, THREE-WAREHOUSES, 28 CUSTOMERS)
Transportation is a significant link between different stages in a global supply chain. Small reductions in transportation expenses could result in substantial total savings over a number of years. The use of vehicle routing and scheduling models and techniques can be instrumental in realizing those savings. Different types of vehicle routing problems have studied over the years; most of them dealing with single-echelon networks and a single type of operation (pickup or delivery) at every location. Since they are NP-hard, solution methods based on metaheuristic techniques are generally applied. Recently, new model-based approaches have been developed for the operational planning of multi-echelon distribution networks. The so-called VRPCD-SCM problem includes many features usually arising in the operation of real-world distribution networks. Further work on this area is still under way.
THANKS FOR YOUR ATTENTION

QUESTIONS?