OPERATIONAL PLANNING OF MULTI-ECHELON TRANSPORTATION NETWORKS

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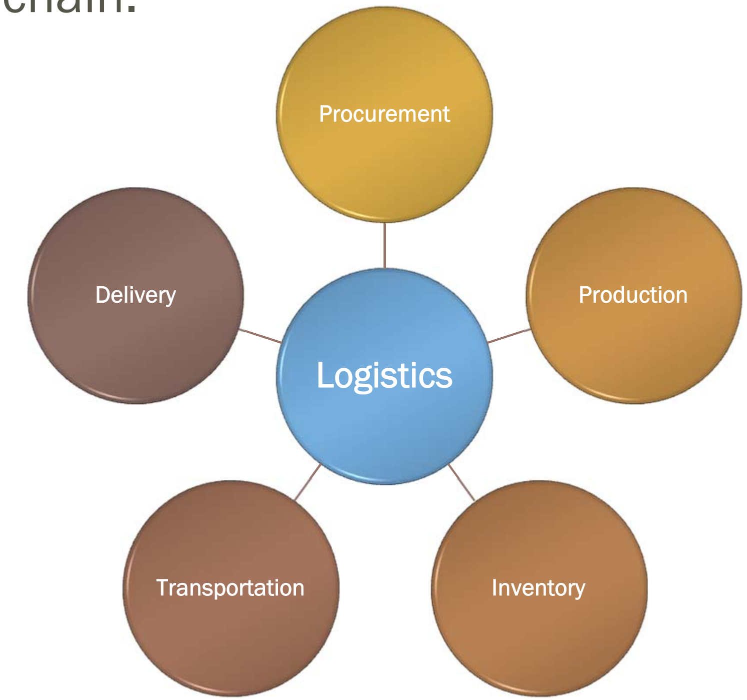
OUTLINE

- The distribution function
- Types of transportation modes and costs
- Distribution network design and planning
- Brief review of important routing and scheduling problems (TSP, VRP, PDP)
- The PDP with transshipment (PDPT)
- The vehicle routing problem in multi-echelon networks with and without crossdocking (VRP-SCM & VRPCD-SCM)
- Conclusions

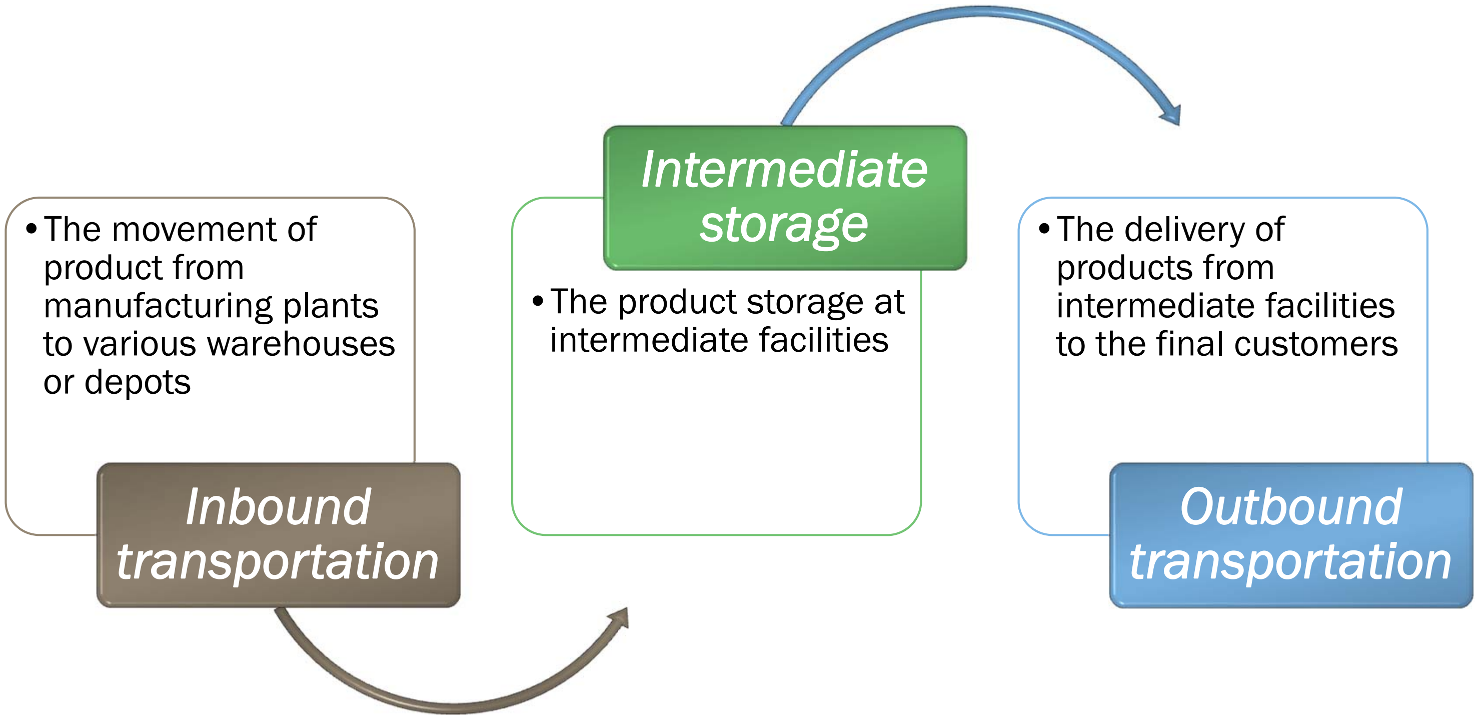
WHAT IS LOGISTICS ?

- Logistics function: provision of goods and services from supply points to demand points.
- It involves the management of a wide range of business operations: acquisition, production, storage, transportation and delivery of goods along the supply chain.



PHYSICAL DISTRIBUTION

- It comprises all movements and storage of goods "downstream" from the manufacturing plants.



THE ROLE OF TRANSPORTATION

- Transportation play a key role because products are rarely produced and consumed at the same location.

- The last transportation step from distribution centers to customers (i.e. the outbound transportation), is usually the most costly link of the distribution chain.

- Distribution costs accounts for about 16% of the sale value of an item; approximately one fourth is due to the outbound transportation.

KEY PLAYERS IN TRANSPORTATION

- There are two key players in any transportation that takes place within a supply chain:

THE SHIPPER	THE CARRIER
✓It is the party requiring the movement of products between two points in the supply chain. ✓He seeks to minimize the total cost, while providing an appropriate level of responsiveness to the customer.	✓It is the party that moves the products. ✓He makes operating decisions trying to maximize the return from its assets

- For example, DELL uses UPS as the carrier to ship its computers from the factory to the customer.

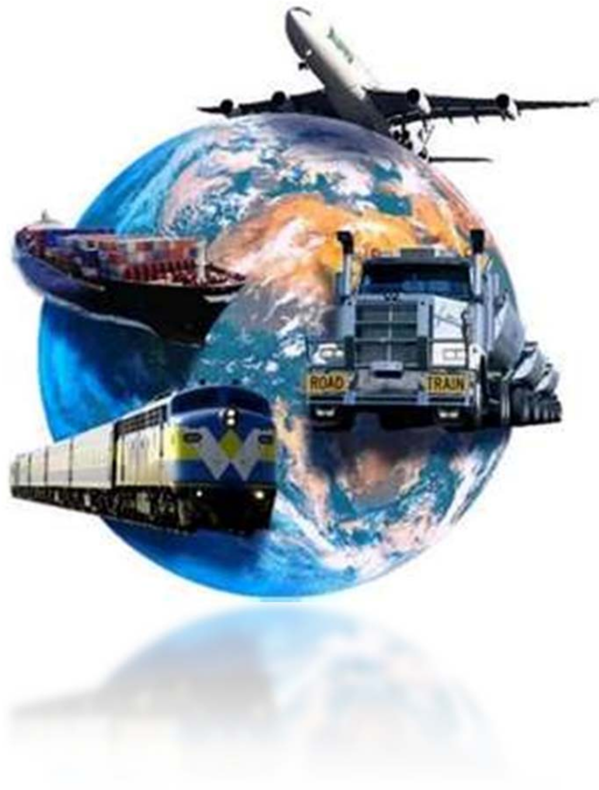
TRANSPORTATION COSTS

- Types of transportation costs

Vehicle-related cost	It is the cost a carrier incurs for the purchase or lease of the vehicle used to transport goods.
Fixed operating cost	It includes the cost associated with terminals, airport gates that are incurred whether the vehicles are used or not.
Trip-related variable cost	This cost is incurred each time a vehicle leaves on a trip and includes the price of labour and fuel.
Quantity related cost	This category includes loading and unloading costs and a portion of the fuel cost that varies with the quantity being transported.
Overhead cost	It includes the cost of planning and scheduling a transportation network as well as any investment in information technology.

MODES OF TRANSPORTATION

- Listed by decreasing freight market share, the modes of transportation include:
  - *TRUCK*
    - TRUCKLOAD (TL)
    - LESS THAN TRUCKLOAD (LTL)
  - *WATER*
  - *RAIL*
  - *AIR*
  - *PIPELINE*



or a combination of them,

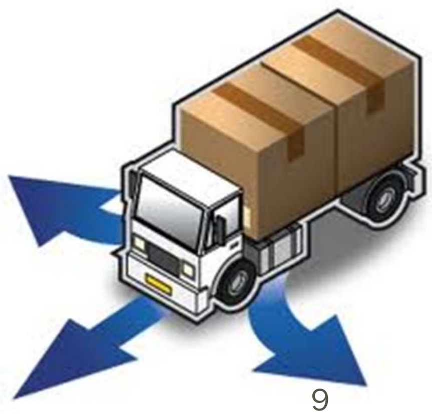
- *INTERMODAL TRANSPORTATION*

TRUCKING

- Dominant mode of transportation

- Two major segments:
  - full truckload (TL): charge for the full truck, and rates vary with the distance travelled.
  - less-than truckload (LTL): charge based on the quantity loaded and the distance travelled.

- Trucking is more expensive than rail but offers the advantage of :
  - door-to-door shipment
  - shorter delivery time
  - no transfer between pickup and delivery points



OTHERS MODES OF TRANSPORTATION

Rail

- High fixed cost in terms of rails, locomotives, cars, and yards
- Large, heavy, low-value shipments that are not very time-sensitive over long distances

Water

- Global Trade
- Very large loads
- Low costs
- Delays at ports and terminal

Air

- Very fast and fairly expensive
- Small, high-value items or time-sensitive emergency shipments

Pipeline

- Crude petroleum, refined petroleum products, and natural gas

PERFORMANCE MEASURES OF DISTRIBUTION NETWORKS

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- Response Time (RT). The time between the placement and the delivery of a customer order.
  - Location of warehouses closer to the market reduce RT.
  - *Trade-off between response time and inventory costs.* A decrease of RT produces an increase of the number and fixed-cost of facilities, and the inventory costs.
  - Major issue to solve the trade-off: *Level of inventory aggregation*

PERFORMANCE MEASURES OF DISTRIBUTION NETWORKS

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- Transportation costs
  - A higher number of warehouses lowers both average distance to customers and outbound transportation costs.
  - Warehousing allows consolidation of shipments from multiple suppliers in the same truck, to get lower inbound costs.
  - Product customization at the delivery stage is postponed until receiving customer orders at the warehouse.
  - *Major issues*: Consolidation of inbound shipments and temporal order aggregation at the delivery stage (frequency of visits vs. full truckload) .
  - Trade-off between customer service level and outbound transportation costs.

PERFORMANCE MEASURES OF DISTRIBUTION NETWORKS

- **Product Availability (PA).** It is the probability of having the requested product in stock when a customer order arrives.

  - Direct shipping centralizes inventories at the manufacturer site, and guarantees a high level of PA with lower amounts of inventories.
  - Warehousing disaggregates inventories at intermediate facilities, to lower response time and transportation costs, but decreasing the product availability.
  - For products with low/uncertain demand, or high-value products, all inventories are better aggregated at the manufacturer storage.
  - For low value, high-demand products, all inventories are better disaggregated and hold close to the customers.

DISTRIBUTION NETWORK DESIGN

- A well-designed transportation network allows to achieve the desired degree of responsiveness at a low cost.

- According to the number of stocking levels, transportation network designs can be classified into two categories:

1. Single-echelon networks: Goods are directly shipped from suppliers to retail stores or customers.

2. Multi-echelon networks: Goods are shipped from suppliers to retail stores via intermediate stocking points

SINGLE-ECHELON TRANSPORTATION NETWORKS

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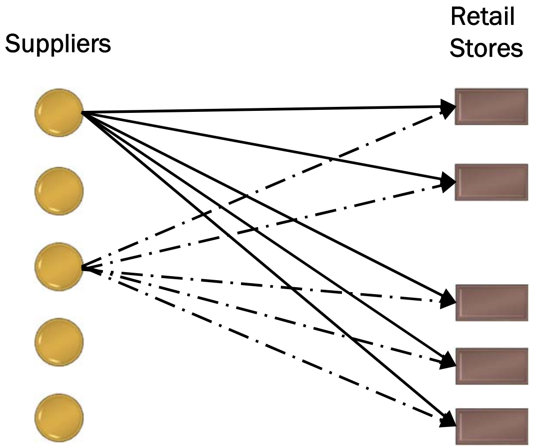
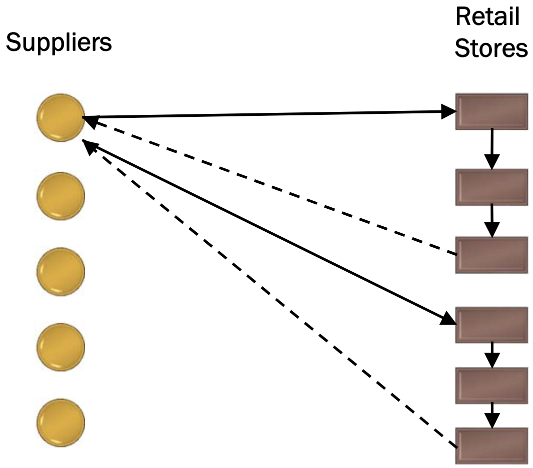


Direct shipping network

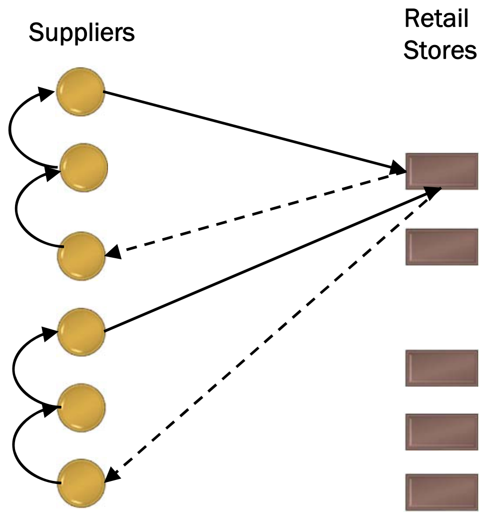
All shipments come directly from suppliers to retail stores



Direct shipping with milk runs (A)

A truck conveys lots of products from a supplier to multiple retail stores

Consolidation of shipments from a supplier to different destinations in a single truck

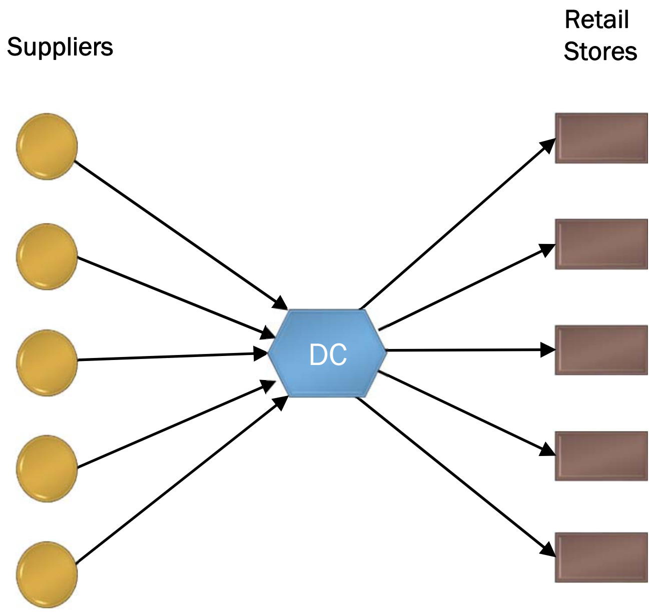


Direct shipping with milk runs (B)

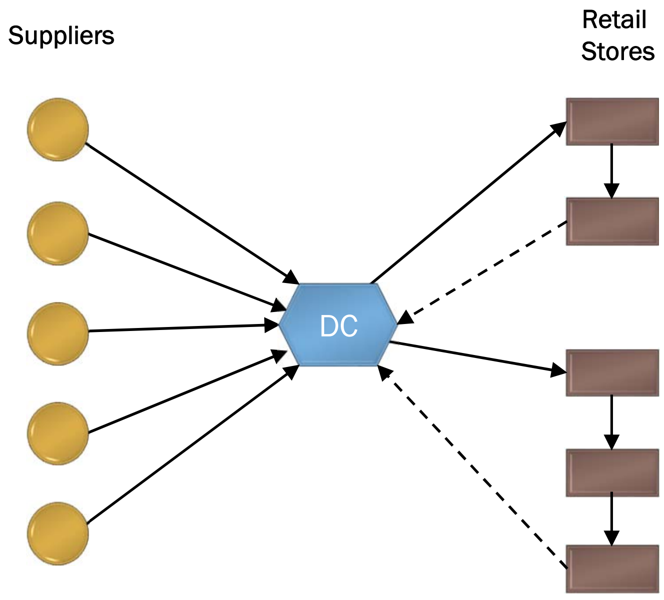
A truck conveys lots of products from multiple suppliers to a retail store

Consolidation of shipments from different suppliers in a truck

MULTI-ECHELON TRANSPORTATION NETWORKS



All shipments via a central DC



All shipments via distribution center using milk runs

OPERATIONS IN MULTI-ECHELON NETWORKS

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- Consolidation operations combine shipments from different suppliers and destined for multiple customers in the same truck.
- Break-bulk operations at warehouses to customize a large shipment from various origins into multiple, smaller shipments to customers.
- Cross-docking operations to perform break-bulk operations over ingoing, consolidated shipments right after their arrival at the intermediate facility, and immediately dispatch the customized parcels to their destinations.

DISTRIBUTION PLANNING LEVELS

- Distribution management involves a variety of decision-making problems at three levels: *strategic*, *tactical*, and *operational planning levels*.

- Strategic decisions deal with the distribution network design, including the number, location and size of facilities.

- Tactical decisions include: a) the area served by each depot; b) the fleet size and composition; c) inventory policies at each facility; d) customer service levels.

- Operational decisions are concerned with the routing and scheduling of vehicles on a day-to-day basis.

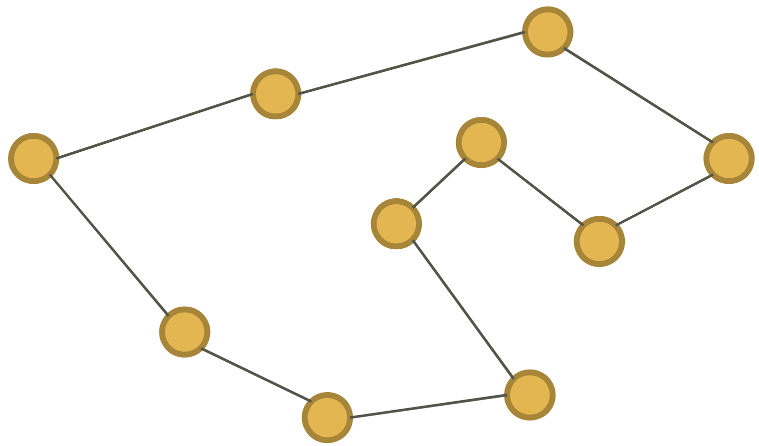
MAJOR VEHICLE ROUTING AND SCHEDULING PROBLEMS

TSP	Travelling salesman problem
MTSP	M-travelling salesman problem
VRP/VRPTW	Single depot, vehicle routing problem without or with time windows
m-VRPTW	Multi-depot, vehicle routing problem with time windows
PDPTW	Pickup and delivery problem with time windows
VRPCD	Pickup and delivery problem with cross-docking at a central depot
VRP-SCM	Vehicle routing and scheduling problem in a multi-echelon supply chain
VRPCD-SCM	Vehicle routing and scheduling problem with cross-docking in a multi-echelon supply chain

TRAVELLING SALESMAN PROBLEM (TSP)

- Given a set of cities and the distances between them, determine the shortest path starting from a given city, passing through all the other cities and returning to the first town.

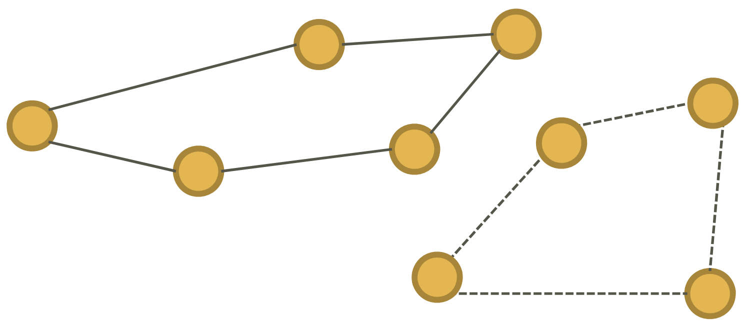
- It can be represented by an undirected weighted graph, at which cities are the vertices, and paths are the edges.



MULTIPLE TRAVELING SALESMAN PROBLEM (m-TSP)

- It is a generalization of the travelling salesman problem where there is a need to account for more than one salesman.

- Given a set of cities and a set of salesmen, find the set of routes for the salesmen with a minimum total length so that:
  a) each salesman visits a unique set of cities and completes the route by returning to the starting city
  b) each city is visited by exactly one salesman.



TYPES OF APPROACHES FOR THE TSP

1. Model-based exact approaches
2. Heuristic approaches

- Tour construction procedures. Generate a feasible tour from the distance and saving matrices.

- Tour improvement procedures. Find a better tour assuming that an initial tour is given, and perform arc/node moves.

- Composite procedures. Construct a starting tour using a tour construction procedure and find a better tour using a tour improvement procedure.

A TOUR CONSTRUCTION PROCEDURE FOR THE TSP

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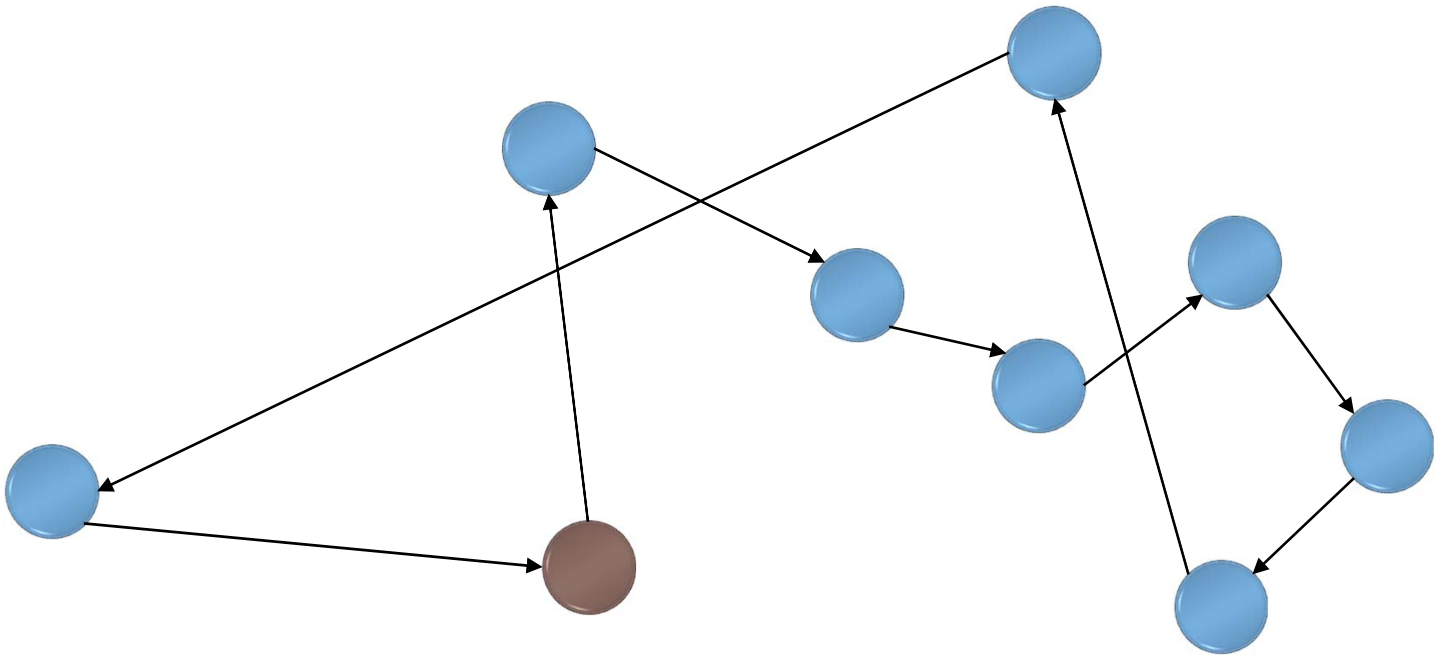
- Nearest neighbour



THE VEHICLE ROUTING PROBLEM (VRP)

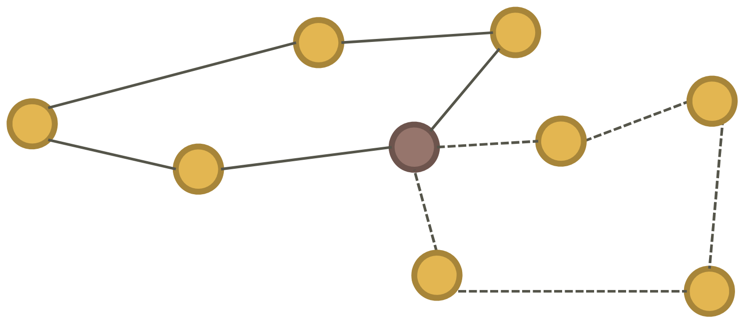
- Generalization of the m-TSP, where a demand is associated to each node and every vehicle has a finite capacity.
- In VRP, the sum of fixed costs (associated to the number of used vehicles) and variable costs (associated to the total distance traveled) is minimized.

GOAL

Generate optimal routes for the vehicle fleet based on a given road network so as to meet customers demands while satisfying capacity and time constraints at minimum transportation cost

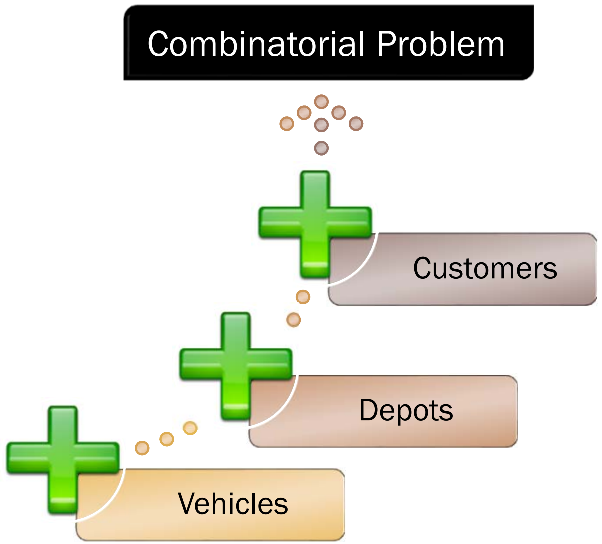
MAJOR FEATURES OF THE CLASSICAL VRP

- The demand at each node is assumed to be deterministic.
- Each vehicle has a known capacity that cannot be exceeded.
- Each vehicle route must start and end at the central depot.
- Each node must be visited only once by exactly one vehicle.
- Only a single type of task is performed by each vehicle (pickup or delivery task).



NP HARD PROBLEMS

- All routing problems (TSP, m-TSP, VRP) are NP hard problems
- Computational effort to solve these problems increase exponentially with the problem size



CLASSIFICATION OF VRP PROBLEMS

1. Routing Problem
   - A spatial problem. Temporal considerations are ignored. No a priori restrictions on delivery times (i.e. no TW constraints) and goods can be delivered within a short period of time (i.e. maximum service time constraints can be ignored).
2. Routing and Scheduling Problem
   - The movement of each vehicle must be traced through both time and space. Visiting times to various locations are of primary importance. Temporal considerations may no longer be ignored and time restrictions guide the routing and scheduling activities.

SOLUTION TECHNIQUES FOR VRP PROBLEMS

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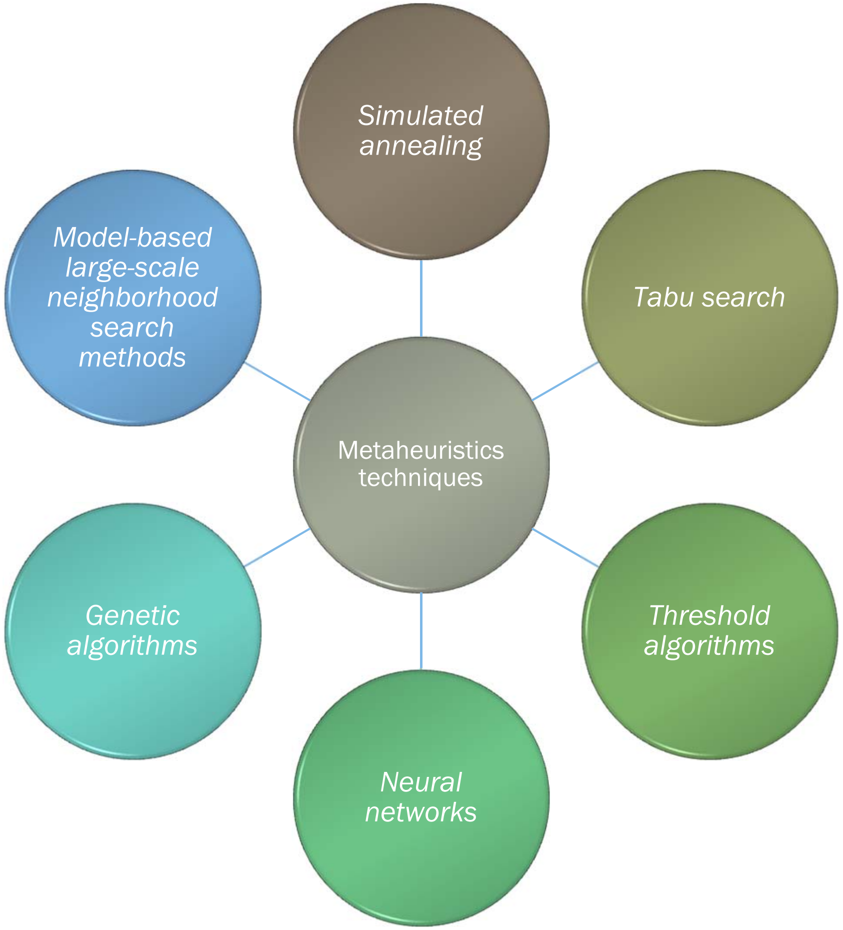
- ✓ *Heuristics*
- ✓ *Metaheuristics*

Heuristic Techniques

Exact Optimization

- ✓ *Branch-and-price*
- ✓ *Branch-and-cut*
- ✓ *Lagrangian relaxation*

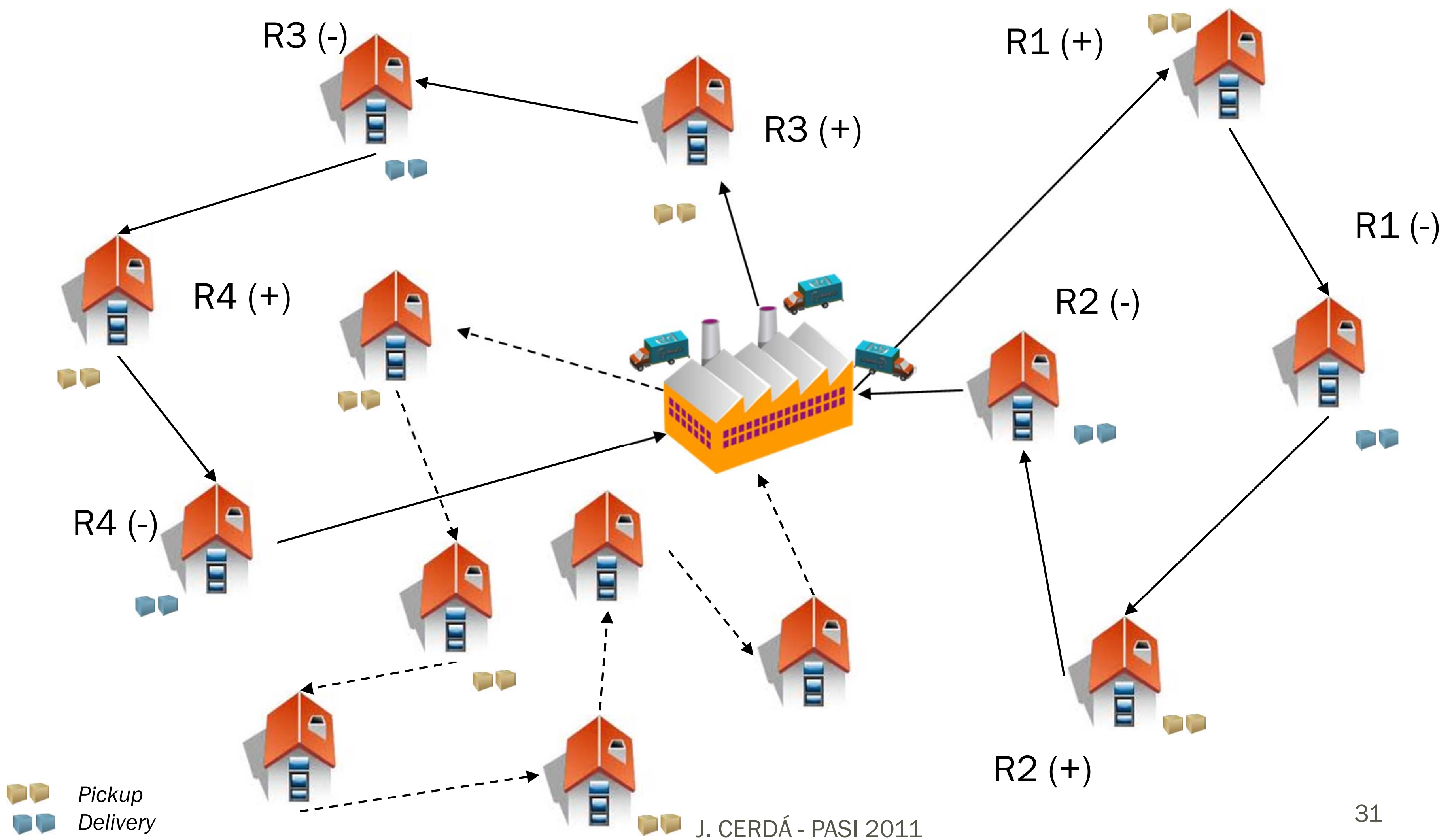
METAHEURISTIC TECHNIQUES FOR THE VRP



THE PICKUP & DELIVERY PROBLEM (PDP)

- A generalization of the VRP where both ***pickup and delivery operations*** are done by a fleet of vehicles.
- It is called the multi-vehicle pickup and delivery problem with time-windows (m-PDPTW).
- It is a single-echelon transportation problem involving pickup and delivery tasks, but accomplishing a single type of operation at each node.
- It involves a set of transportation requests $r \in R$, each one satisfied by a single vehicle and defined by:
  - a pickup location (the origin),
  - a delivery location (the destination),
  - a freight to be delivered from one to the other site, and
  - the time required to complete the service at each location.
- All the requests are known in advance.

ILLUSTRATING THE m-PDPTW PROBLEM



MAJOR m-PDPTW PROBLEM FEATURES

- Vehicles depart and return to the central depot (tour constraint).
- Each transportation request must be serviced by a single vehicle. Pickup and delivery locations related to the same request are visited by the same vehicle (pairing constraint).
- Each pickup location has to be visited prior to the associated delivery location (precedence constraint).
- Each vehicle can satisfy one or more customer requests (a "composite" milk run).

MAJOR m-PDPTW PROBLEM FEATURES

- Vehicle capacity must never be exceeded after visiting a pickup node (capacity constraint at pickup nodes).

- A vehicle must transport enough load to meet customer demand when servicing a delivery node (capacity constraint at delivery nodes). Only important for requests involving multiple pickup points.

- The service at each node must be started within the specified time window (time window constraints).

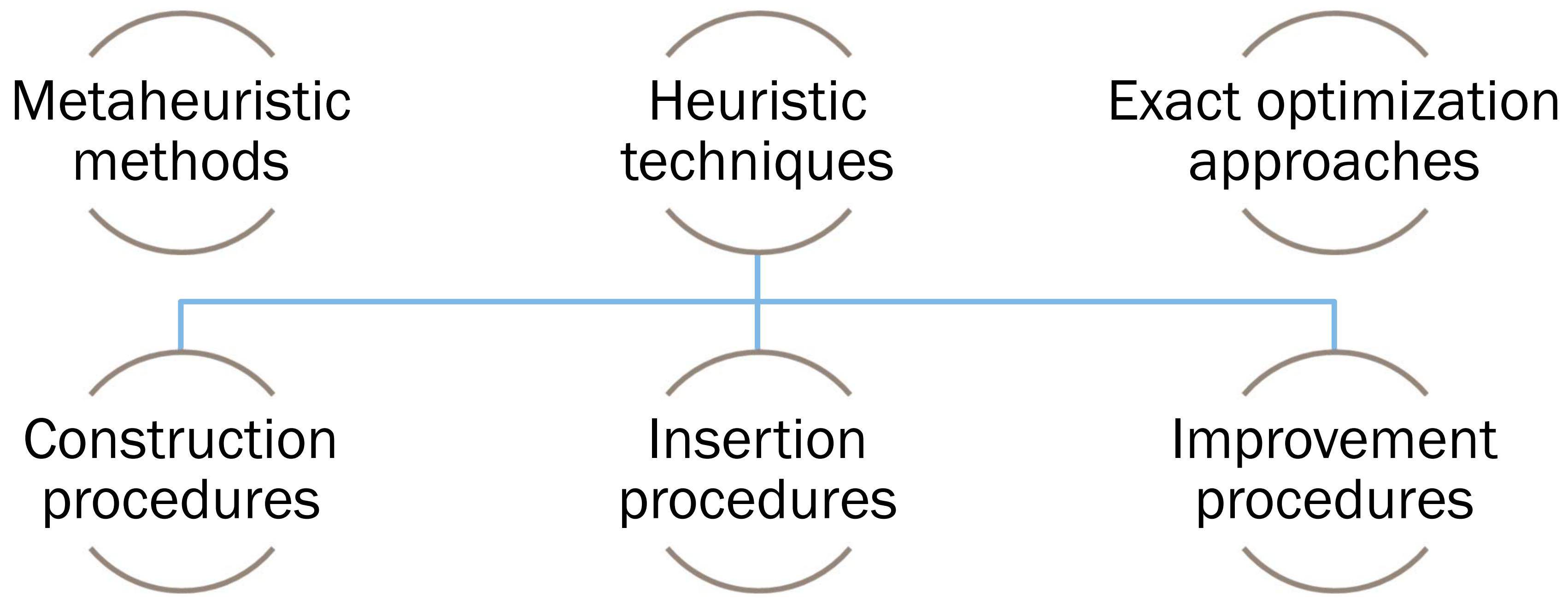
- The total time/distance travelled from the depot to a certain node must be greater than the one required to reach a preceding node on the tour (compatibility between routes and schedules).

MAJOR m-PDPTW PROBLEM FEATURES

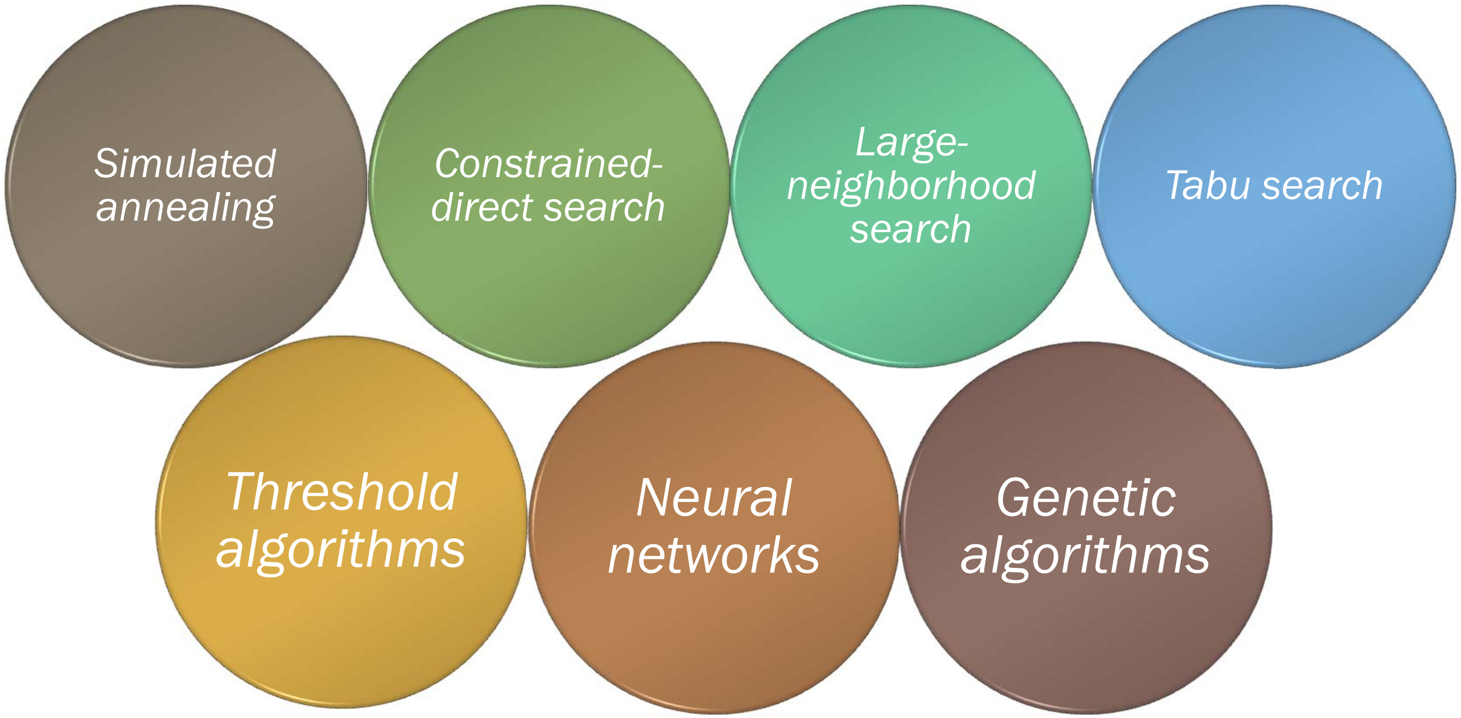
- The alternative problem goals aim to minimize:
  - the total distance travelled
  - the total travel time required to service all customer requests
  - the total customers' inconvenience
  - a weighted combination of total service time and customers' inconvenience.

- Customer inconvenience is usually a linear function of a customer's waiting time (because of late arrivals).

m-PDPTW SOLUTION METHODS



METAHEURISTIC ALGORITHMS FOR m-PDPTW PROBLEMS



EXACT OPTIMIZATION METHODS FOR PDPTW PROBLEMS

1. Dynamic programming
2. Branch-and-price
3. Branch-and-cut

PDPTWT

The PDPTW problem with transshipment

THE PDPTW WITH TRANSSHIPMENT

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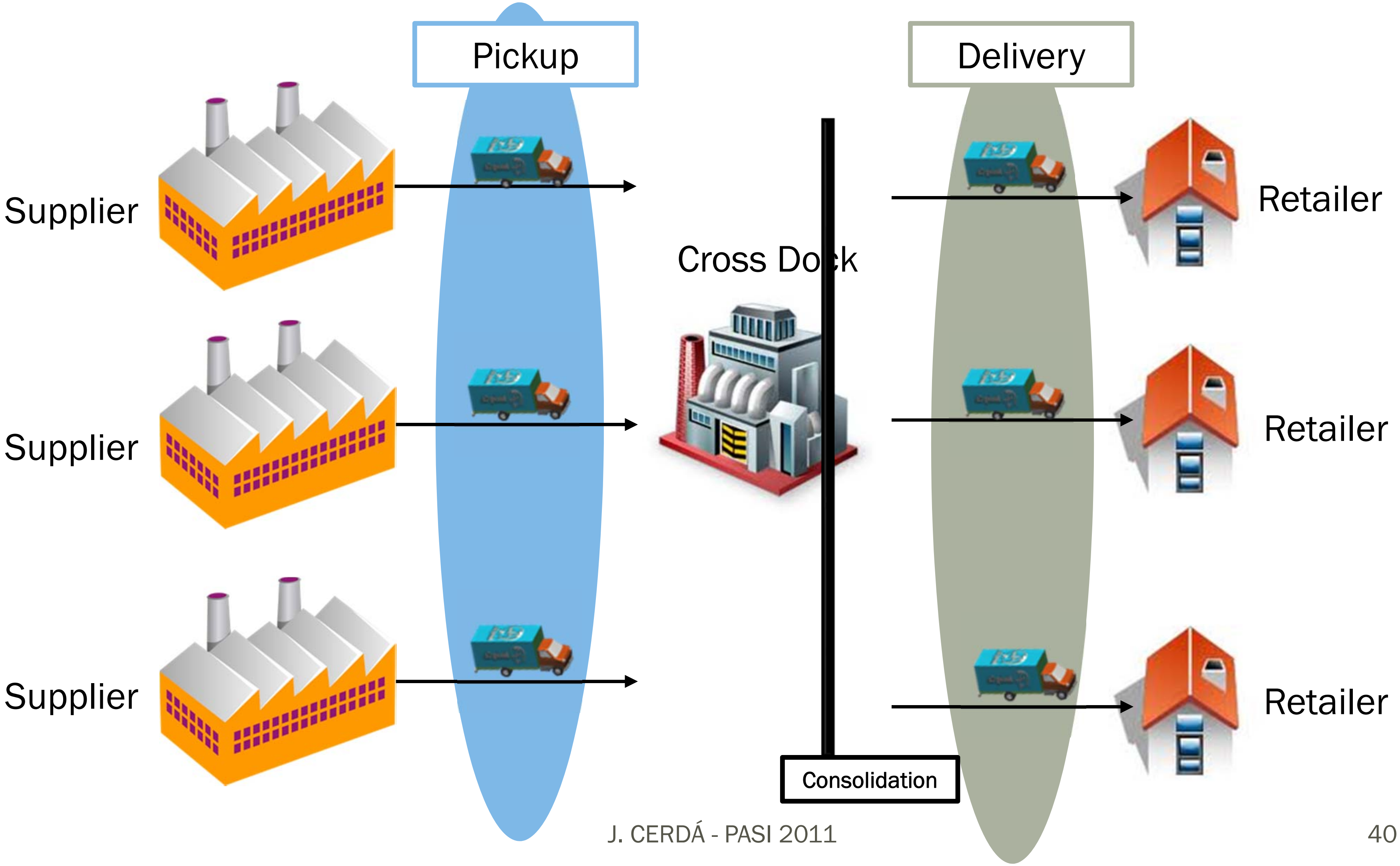
- Loads are transported from suppliers (pickup nodes) to customers (delivery nodes) via a single cross-dock.

- Each request is split into two sub-requests, i.e. a pickup and a delivery sub-request that may be handled by two different vehicles.

- The incorporation of transshipment points may yield solutions with shorter travel distances or fewer vehicles.

- Loads from suppliers are picked up by a fleet of vehicles, consolidated at the cross-dock facility, and immediately delivered to customers by the same set of vehicles, without intermediate storage.

ILLUSTRATING THE PDPTW PROBLEM WITH TRANSSHIPMENT



MAJOR FEATURES OF THE PDPTW WITH TRANSSHIPMENT

- Each node must be visited only once by a single vehicle.
- Each vehicle can pick up loads from more than one supplier and deliver loads to more than one customer. Milk runs are allowed.
- Pickup and delivery routes start and end at the cross-dock facility.
- Loads to pickup/deliver at problem nodes are known data.
- The total amounts unloaded at the receiving dock and loaded into trucks at the shipping dock should be equal. There is no end inventory at the cross-dock facility.

MAJOR FEATURES OF THE PDPTW WITH TRANSSHIPMENT

- Service time windows for the nodes are usually specified.
- The problem goal is to minimize the total transportation cost while satisfying all customer requests.

VRPTW-SCM

The vehicle routing problem with time windows in the supply chain management

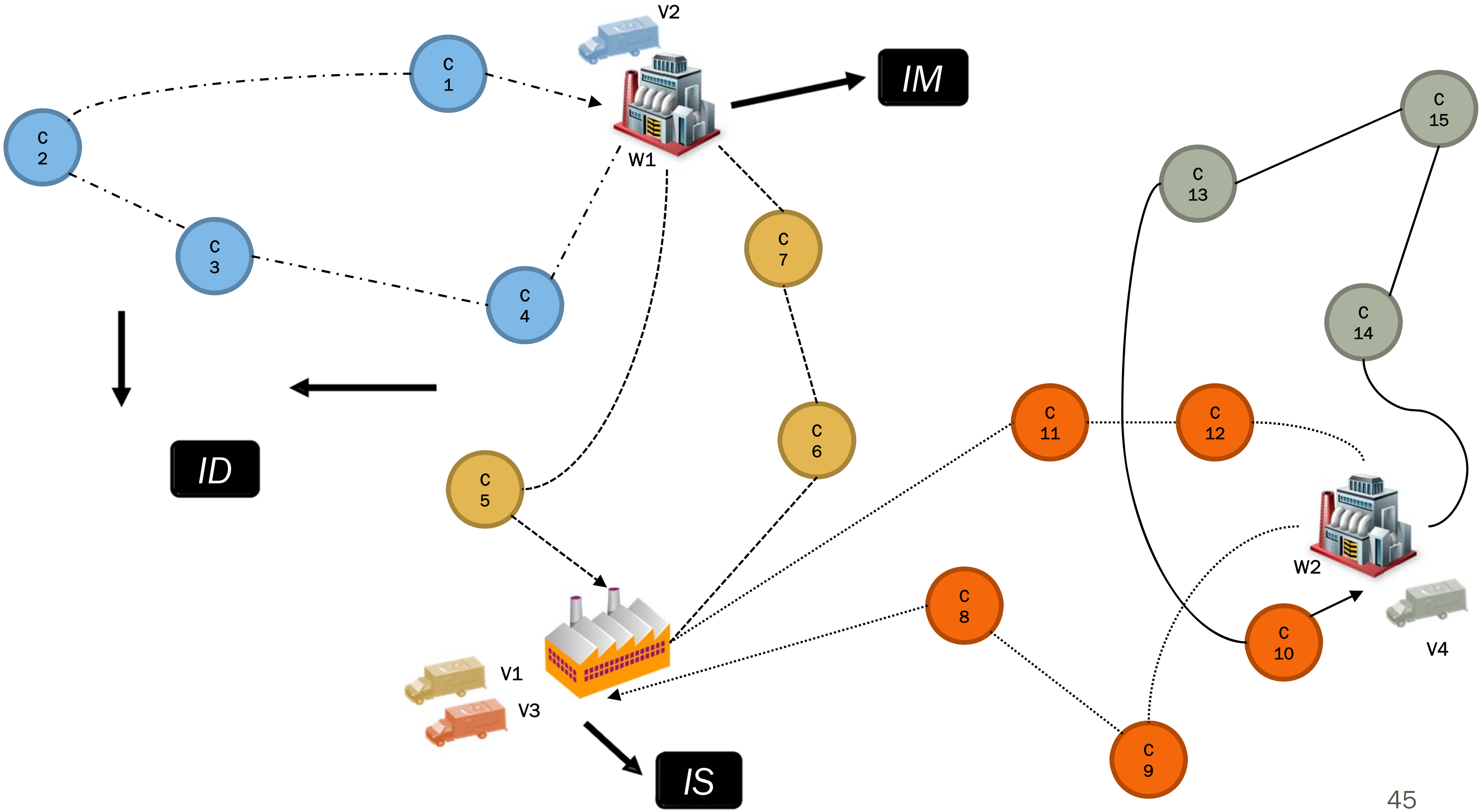
THE VRPTW-SCM PROBLEM

- PROBLEM GOAL: Determine the best operational planning of multi-echelon transportation networks comprising factories, warehouses, and customers (the problem nodes).

- Different types of distribution strategies like direct shipping, shipping via DC or regional warehouses, or a mix of them (hybrid strategies) can be implemented.

- To resemble the logistics activities at multi-site distribution networks, multiple events at every node can occur.

ILLUSTRATING THE VRPTW-SCM PROBLEM



MAJOR ELEMENTS OF THE VRPTW-SCM PROBLEM

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- Types of nodes
  - *"Pure" source nodes (IS),* usually manufacturer storages, where vehicles carry out only pickup operations
  - *Mixed nodes (IM)*, like DCs, where visiting vehicles can accomplish pickup and/or delivery operations.
  - *Destination nodes (ID)*, like consumer zones, where visiting trucks just perform delivery operations

- Number of events. The number of events for a location must be at least equal to the number of vehicles stops for performing pickup/delivery operations at the optimum.

- Global precedence. For each vehicle stop, the model provides all the visits the vehicle has made before.

MAJOR FEATURES OF THE VRPTW-SCM PROBLEM

- Multiple products are distributed from manufacturing plants and warehouses to customers.

- Customer requests may involve several products (multi-commodity distribution) and do not generally have predefined suppliers

- The amounts of products to pick up at source nodes by a vehicle are not predefined but chosen by the model.

- Multiple partial shipments to a customer location are allowed (order splitting)

- Milk runs are performed on both sides, i.e. inbound and outbound sides. Then, several customers can be serviced by the same truck.

MAJOR FEATURES OF THE VRPTW-SCM PROBLEM

- Problem events are the vehicle stops at DCs and customer locations.
- A location can be visited either by multiple trucks or several times by the same vehicle. Then, several events can sequentially occur at any site.
- A set of pre-specified, timely-ordered events for each site are defined. Its number is chosen by the user.
- A vehicle can accomplish pickup and delivery operations during a stop at at DCs.
- The magnitude and composition of the freight transported by a vehicle at any stop must be traced in order to meet:
  - *capacity constraints at pickup locations*
  - *product availability constraints at delivery points*

MAJOR FEATURES OF THE VRPTW-SCM PROBLEM

- Each vehicle must finally return to the assigned depot (tour constraint).
- Customer time windows and the specified maximum service time for each vehicle must be respected.
- Finite inventories at manufacturer storage and distribution centers at the initial time are known.
- In addition to customer demands, specific replenishment orders from DCs are to be fulfilled.
- Cross-docking at DCs is not permitted. Customer demands should be satisfied using the initial inventories.
- At the tactical level, each DC may have been predefined as the exclusive supplier for some nearby customer nodes.

MAJOR FEATURES OF THE VRPTW-SCM PROBLEM

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- The problem goal is to minimize the total transportation cost while satisfying customer requests and meeting the service-level requirements.

- Transportation costs include:
  - fixed expenses incurred by used vehicles,
  - distance-based variable costs, mainly fuel costs
  - time-based variable costs, mainly driver wages.

MODEL VARIABLES IN VRPTW-SCM

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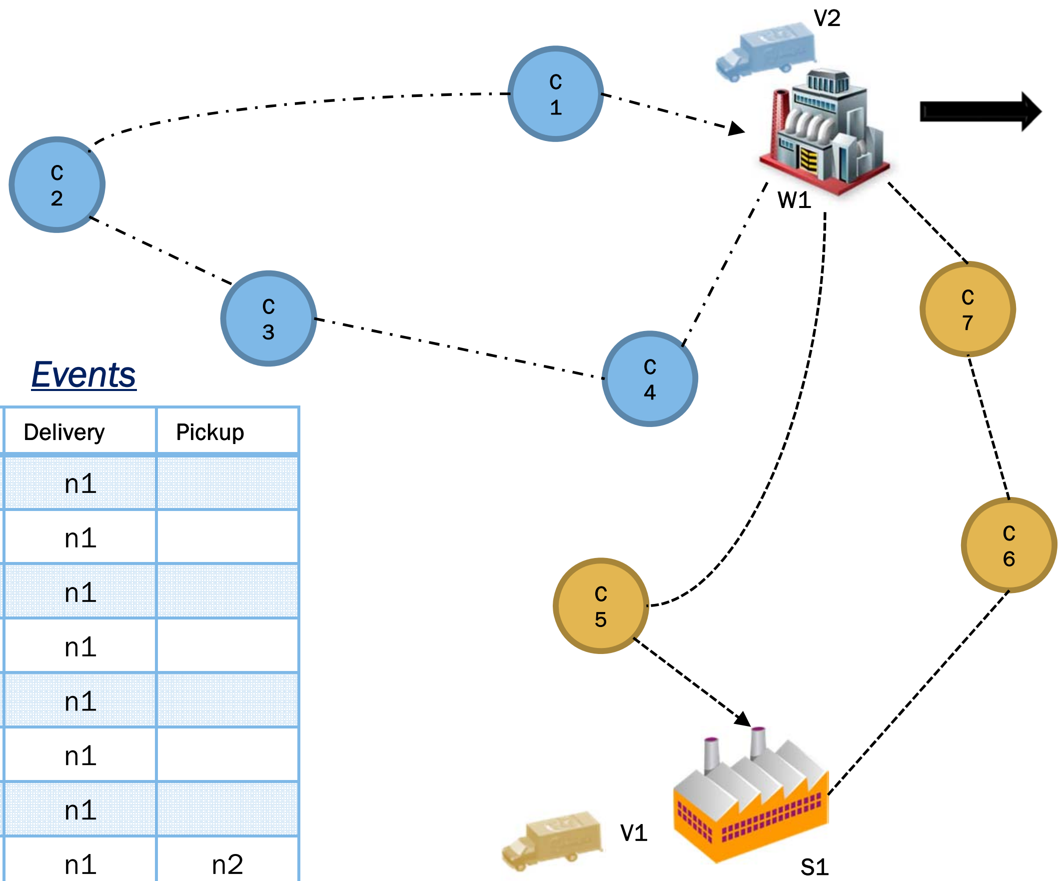
- Binary variables
  - ***Assignment variables*** Y_{niv} : Event *n* at location *i* has been allocated to vehicle *v* whenever Y_{niv} = 1. If so, vehicle *v* stops at node *i* at time event *n*.
  - ***Sequencing variable*** $X_{ni,n'i'}$: Vehicle stop (*n*,*i*) will occur earlier than the vehicle stop (*n'*, *i'*) whenever both nodes (*i*,*i'*) nodes are visited by the same vehicle and $X_{ni,n'i'}$ = 1 .
- Continuous variables
  - ***Cost-variable*** C_{ni}: distance-based transportation cost incurred by the visiting vehicle to move from the base up to node *i* along the assigned route.
  - ***Time-variable*** T_{ni}: total time required by the assigned vehicle to travel from the base up to stop (*n*,*i*).
  - ***Pickup-Load*** $L_{ni,pv}$: amount of product *p* to pick up by vehicle *v* during stop (*n*,*i*)
  - ***Delivery-Load*** $U_{ni,pv}$: amount of product *p* to deliver by vehicle v during stop (*n*,*i*).
  - ***Accumulated-Pickup & Delivery Loads*** $AL_{ni,pv}$ / $AU_{ni,pv}$: accumulated amount of product *p* to pick up/deliver by vehicle *v* along the assigned route from the base up to stop (*n*,*i*),

ILLUSTRATING THE VRPTW-SCM PROBLEM EVENTS



$$Y_{n_1 w_1 v_1} = 1$$

$$Y_{n_2 w_1 v_2} = 1$$

$$Y_{n_1 c_1 v_2} = 1$$

$$X_{n_2 w_1, n_1 c_1} = 1$$

Events

Node	Delivery	Pickup
C1	n1	
C2	n1	
C3	n1	
C4	n1	
C5	n1	
C6	n1	
C7	n1	
W1	n1	n2
S1		n1

MAJOR VRPTW-SCM MODEL CONSTRAINTS

- Route building constraints assigning a particular stop (n,i) to at most a single vehicle, and ordering vehicle stops (n,i) on the same route.
- Product inventory constraints restraining the maximum amount of every product to pick up by visiting vehicles at source nodes accounting for the current product stocks.
- Additional inventory constraints monitoring the amount of every product received at each warehouse over the planning horizon.
- Product demand constraints ensuring that customer requests are satisfied.

MAJOR VRPTW-SCM MODEL CONSTRAINTS

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- Null in-transit inventory constraints requiring that every product unit picked up by a vehicle must be delivered to a customer before the end of the vehicle trip.

- Loading/unloading constraints monitoring the total amount of products transported by each vehicle to prevent from overcapacity or product shortages.

- Time window and maximum service time constraints ensuring that the customer service begins within the specified TW, and each vehicle returns to its base within the allowed working period.

VRPTW-SCM MATHEMATICAL FORMULATION

Vehicle routing constraints

Allocating vehicles to depots

$$\sum_{l \in B_v} W_{lv} \le 1 \quad \forall v \in V$$

Assigning the event n at node i to vehicle v

$$\sum_{v \in V_i} Y_{nv} \le 1 \quad \forall n \in N_i, i \in I$$

Preordering of time events predefined for node i

$$\sum_{v \in V_i} Y_{nv} \ge \sum_{v \in V_i} Y_{n'v} \quad \forall (n, n') \in N_i, i \in I : n < n'$$

Activated vehicle condition

$$\sum_{i \in I_v} \sum_{n \in N_i} Y_{nv} \le M \sum_{l \in B_v} W_{lv} \quad \forall v \in V$$

Travel cost and time from the vehicle depot to the first visited node

$$\begin{cases} C_n \ge \sum_{l \in B_v} dc_v \, d_{li} \, W_{lv} - M_C (1 - Y_{nv}) \\ T_n \ge \sum_{l \in B_v} \left(\frac{d_{li}}{sp_v}\right) W_{lv} - M_T (1 - Y_{nv}) \end{cases} \quad \forall n \in N_i, i \in I, v \in V_i$$

Travel cost and time from the base to vehicle stop (i,n)

$$\begin{cases} C_{n'} \ge C_n + dc_v \, d_{ii'} - M^C (1 - X_{nn'}) - M^C (2 - Y_{nv} - Y_{n'v}) \\ C_n \ge C_{n'} + dc_v \, d_{i'i} - M^C X_{nn'} - M^C (2 - Y_{nv} - Y_{n'v}) \\ T_{n'} \ge T_n + ft_i + \sum_{p \in P_i} vt_{ip} (L_{npv} + U_{npv}) + \frac{d_{ii'}}{sp_v} - M^T (1 - X_{nn'}) - M^T (2 - Y_{nv} - Y_{n'v}) \\ T_n \ge T_{n'} + ft_{i'} + \sum_{p \in P_{i'}} vt_{i'p} (L_{n'pv} + U_{n'pv}) + \frac{d_{i'i}}{sp_v} - M^T X_{nn'} - M^T (2 - Y_{nv} - Y_{n'v}) \end{cases}$$

$$\forall n \in N_i, n' \in N_{i'}, i, i' \in I, v \in V_{ii'} : i < i'$$

Bound on the routing cost and time for the tour assigned to vehicle v

$$\begin{cases} CV_v \ge C_n + \sum_{l \in B_v} dc_v \, d_{il} \, W_{lv} - M_C (1 - Y_{nv}) \\ TV_v \ge T_n + ft_i + \sum_{p \in P_i} vt_{ip} (L_{npv} + U_{npv}) + \sum_{l \in B_v} \frac{d_{il}}{sp_v} W_{lv} - M_T (1 - Y_{nv}) \end{cases}$$

$$\forall n \in N_i, i \in I, v \in V_i$$

Time-window and maximum service time constraints

$$a_i \le T_n \le b_i \qquad n \in N_i, i \in ID$$

$$TV_v \le t_v^{\max} \qquad v \in V$$

VRPTW-SCM MATHEMATICAL FORMULATION

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Vehicle cargo constraints

Product availability constraints

$$\sum_{v\in V_i}\sum_{n\in N_i} L_{npv} \le INV_{ip} \qquad \forall i\in\left(IS_p\cap IM_p\right), p\in P$$

Product demand constraints

$$\sum_{v\in V_i}\sum_{n\in N_i} U_{npv} = DEM_{ip} - BL_{ip} \qquad \forall i\in\left(ID_p\cap IM_p\right), p\in P$$

Null in-transit inventory constraints

$$\sum_{n\in N_i}\sum_{i\in IS\cup IM} L_{npv} = \sum_{n\in N_i}\sum_{i\in IM\cup ID} U_{npv} \qquad , p\in P, v\in V$$

Vehicle loading/unloading operation constraints

$$\begin{Bmatrix} L_{npv} \le INV_{ip}Y_{nv} \\ U_{npv} \le DEM_{ip}Y_{nv} \end{Bmatrix} \qquad \begin{matrix} \forall n\in N_i, i\in\left(IS_p\cap IM_p\right), p\in P, v\in V_i \\ \forall n\in N_i, i\in\left(ID_p\cap IM_p\right), p\in P, v\in V_i \end{matrix}$$

Accumulated amount of product p picked-up by vehicle v up to the stop (i,n)

$$\begin{Bmatrix} AL_{n'pv} \ge AL_{npv} + L_{n'pv} - M_L\left(1-X_{nn'}\right) - M_L\left(2-Y_{nv}-Y_{n'v}\right) \\ AL_{npv} \ge AL_{n'pv} + L_{npv} - M_L X_{nn'} - M_L\left(2-Y_{nv}-Y_{n'v}\right) \end{Bmatrix}$$

$$\forall n\in N_i, n'\in N_{i'}, i,i'\in I, p\in P_{ii'}, v\in V_{ii'} \left(i,n\right)<\left(i',n'\right)$$

Accumulated amount of product p delivered by vehicle v up to the stop (i,n)

$$\begin{Bmatrix} AU_{n'pv} \ge AU_{npv} + U_{n'pv} - M_L\left(1-X_{nn'}\right) - M_L\left(2-Y_{nv}-Y_{n'v}\right) \\ AU_{npv} \ge AU_{n'pv} + U_{npv} - M_L X_{nn'} - M_L\left(2-Y_{nv}-Y_{n'v}\right) \end{Bmatrix}$$

$$\forall n\in N_i, n'\in N_{i'}, i,i'\in I, p\in P_{ii'}, v\in V_{ii'} : \left(i,n\right)<\left(i',n'\right)$$

Maximum volumetric and weight vehicle capacity constraints

$$\begin{Bmatrix} \sum_{p\in P} uv_p\left(AL_{npv} - AU_{npv}\right) \le vq_v Y_{nv} \\ \sum_{p\in P} uw_p\left(AL_{npv} - AU_{npv}\right) \le wq_v Y_{nv} \end{Bmatrix} \qquad \forall n\in N_i, i\in I, p\in P, v\in V_i$$

Lower bounds on the cargo transported by vehicle v after stop (i,n)

$$\begin{Bmatrix} AL_{npv} \ge AU_{npv} \\ AL_{npv} \ge L_{npv} \\ AU_{npv} \ge U_{npv} \end{Bmatrix} \qquad \forall n\in N_i, i\in I, p\in P, v\in V_i$$

Upper bounds on the accumulated amount of product p loaded/unloaded by vehicle v after stop (i,n)

$$\begin{Bmatrix} AL_{npv} \le \sum_{i'\in I}\sum_{n'\in N_{i'}} L_{n'pv} \\ AU_{npv} \le \sum_{i'\in I}\sum_{n'\in N_{i'}} U_{n'pv} \end{Bmatrix} \qquad \forall n\in N_i, i\in I, p\in P, v\in V_i$$

VRPTW-SCM MATHEMATICAL FORMULATION

Alternative Objective Functions

The sum of distance-based travel costs and vehicle fixed costs

$$Min \quad \sum_{v\in V} CV_v + \sum_{v\in V} \sum_{l\in B_v} fc_v W_{lv}$$

The weighted sum of distance-based and time-based travel costs plus vehicle fixed costs

$$Min \quad \sum_{v\in V} CV_v + \sum_{v\in V} \sum_{l\in B_v} fc_v W_{lv} + \sum_{v\in V} utc_v TV_v$$

Fixed and variable transportation costs plus the penalties for unsatisfied demands, late services and overtime journeys

$$Min \quad \sum_{v\in V} CV_v + \sum_{v\in V} \sum_{l\in B_v} fc_v W_{lv} + \sum_{v\in V} utc_v TV_v + \sum_{v\in V} (co_v OVT_v + \sum_{i\in ID_v} \sum_{n\in N_i} cl_i TD_n) + \sum_{p\in P} \sum_{i\in I} c_{B,i} B_{ip}$$

VRPCD-SCM

The VRPCD-SCM problem with cross-docking

MAJOR FEATURES OF THE VRPCD-SCM PROBLEM

- Generalization of the VRP-SCM problem to consider the possibility of cross-docking.
- Intermediate depots may keep finite stocks of fast-moving products (warehousing) and act as cross-dock platforms for slow-moving, high-value items.
- Replenishment orders and cross-docking operations are triggered when the initial stock in a warehouse is insufficient to meet the demand of the assigned customers.
- Inbound and outbound vehicles must stay at receiving/shipping docks of DCs until they complete their delivery/pickup tasks.
- Target product inventories at the end of the planning horizon may be specified.

MAJOR FEATURES OF THE VRPCD-SCM PROBLEM

- Product inventories at cross-dock facilities must be traced over the planning horizon.
- Sequencing of pickup and delivery operations by different vehicles at the same warehouse now become important.
- The problem goal aims to minimize fixed and variable transportation costs.

VRPCD-SCM
MILP MATHEMATICAL MODEL

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Route building constraints

Allocating base nodes to vehicles

$$\sum_{l \in IB_v} \sum_{n \in N_l} Y_{nlv} \leq 1 \qquad v \in V$$

Allocating events at every node to vehicles

$$\sum_{v \in V_i} Y_{niv} \leq 1 \qquad n \in N_i,\, i \in I$$

Pre-ordering events occurring at the same node

$$\sum_{v \in V_i} Y_{niv} \geq \sum_{v \in V_i} Y_{n'iv} \quad (n,n') \in N_i : n' > n,\, i \in I$$

Used vehicle condition

$$\sum_{i \in I_v} \sum_{n \in N_i} Y_{niv} \leq M_v \left(\sum_{l \in IB_v} \sum_{n \in N_l} Y_{nlv} \right) \qquad v \in V$$

Travelling cost constraints

Travelling cost from the base node l to the first serviced node i for vehicle

$$C_{ni} \geq \sum_{l \in IB_v} \sum_{n' \in N_l} c_{li}\, Y_{n'lv} - M_C\,(1 - Y_{niv}) \qquad n \in N_i,\, i \in I,\, v \in V_i$$

Accumulated travelling cost for vehicle v up to the stop (n,i)

$$\begin{Bmatrix} C_{n'i'} \geq C_{ni} + c_{ii'} - M_C\,(1 - X_{ni,n'i'}) - M_C\,(2 - Y_{niv} - Y_{n'i'v}) \\ C_{ni} \geq C_{n'i'} + c_{i'i} - M_C\, X_{ni,n'i'} - M_C\,(2 - Y_{niv} - Y_{n'i'v}) \end{Bmatrix}$$

$$n \in N_i,\, n' \in N_{i'},\, (i,i') \in I,\, v \in V_{ii'} : i < i'$$

- For multiple stops of vehicle *v* at the same node *i* :

$$C_{n'i} \geq C_{ni} - M_C\,(2 - Y_{niv} - Y_{n'iv}) \quad (n,n') \in N_i,\, i \in I,\, v \in V_i : n < n'$$

Overall travelling cost for vehicle *v*

$$CV_v \geq C_{ni} + \sum_{l \in B_v} \sum_{n' \in N_l} c_{li}\, Y_{n'lv} - M_C\,(1 - Y_{niv})$$

$$n \in N_i,\, i \in I,\, v \in V$$

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Travelling time constraints

Travelling cost from the assigned base node $l \in IB_v$ to the first serviced node for vehicle v

$$T_{ni} \geq \sum_{l \in IB_v} \sum_{n' \in N_l} t_{li} \, Y_{n'lv} + ft_l + vt_l \sum_{p \in P_l} L_{nlpv} - M_C \left(1 - Y_{niv}\right)$$

$$n \in N_i, i \in I, v \in V$$

Travelling time for vehicle v from the assigned base node to the stop (n,i)

$$\left\{ \begin{array}{l} T_{n'i'} \geq T_{ni} + ft_i + vt_i \left(\sum_{p \in P_i} L_{ni,pv} + U_{ni,pv} \right) + t_{ii'} - M_C \left(1 - S_{ni,n'i'}\right) - M_C \left(2 - Y_{niv} - Y_{n'i'v}\right) \\ T_{ni} \geq T_{n'i'} + ft_{i'} + vt_{i'} \left(\sum_{p \in P_i} L_{n'i',pv} + U_{n'i',pv} \right) + t_{i'i} - M_C S_{ni,n'i'} - M_C \left(2 - Y_{niv} - Y_{n'i'v}\right) \end{array} \right\}$$

$$n \in N_i, n' \in N_{i'}, (i,i') \in I, v \in V_{ii'} : i < i'$$

- In case of multiple stops of the same vehicle *v* at node *i* at different time events

$$T_{n'i'} \geq T_{ni} + ft_i + vt_i \left(\sum_{p \in P_i} L_{ni,pv} + U_{ni,pv} \right) - M_C \left(2 - Y_{niv} - Y_{n'i'v}\right) \qquad (n,n') \in N_i, i \in I, v \in V_i : n < n'$$

- In case of consecutive stops of two different vehicles *v and v'* at the same warehouse *i*

$$T_{n'i} \geq T_{ni} + ft_i + vt_i \left(\sum_{p \in P_i} L_{ni,pv} + U_{ni,pv} \right) - M_C \left(2 - Y_{niv} - Y_{n'iv'}\right) \qquad (n,n') \in N_i, i \in I, v, v' \in V_i : n < n'$$

VRPCD-SCM
MILP MATHEMATICAL MODEL

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Travelling time constraints

Overall travelling time for vehicle v

$$OT_v \geq T_{ni} + ft_i + vt_i\left(\sum_{p\in P_i} L_{ni,pv} + U_{ni,pv}\right) + \sum_{l\in B_v}\sum_{n'\in N_l} t_{il} Y_{n'lv} - M_C\left(1 - Y_{niv}\right) \qquad n\in N_i, i\in I, v\in V$$

Time window and maximum service-time constraints

$$a_i \leq T_{ni} \leq b_i \qquad n\in N_i, i\in I$$

$$OT_v \leq t_v^{\max} \qquad v\in V$$

Product availability constraints

For pure sources:

$$\sum_{v\in V_i}\sum_{n\in N_i} L_{ni,pv} \leq II_{ip} \qquad i\in IS, p\in P_i$$

For warehouses:

$$\sum_{v\in V_i}\sum_{\substack{n'\in N_i\\ n'\leq n}} L_{n'i,pv} \leq II_{ip} + AINV_{nip} \qquad n\in N_i, i\in IM, p\in P_i$$

Product demand constraints at customer nodes

$$\sum_{v\in V_i}\sum_{n\in N_i} U_{ni,pv} \geq D_{ip} \qquad i\in ID, p\in P_i$$

Overall product balance at each intermediate facility

$$\sum_{v\in V_i}\sum_{n\in N_i} L_{ni,pv} \leq II_{ip} + \sum_{v\in V_i}\sum_{n\in N_i} U_{ni,pv} - FINV_{ip} \qquad p\in P_i, i\in IM$$

VRPCD-SCM
MILP MATHEMATICAL MODEL

Vehicle-related constraints

Overall product balance for every vehicle

$$\sum_{i \in IS \cup IM} \sum_{n \in N_i} L_{ni,pv} = \sum_{i \in (IM \cup ID)} \sum_{n \in N_i} U_{ni,pv} \qquad p \in P, v \in V$$

Relationship between variables $L_{ni,pv}$ / $U_{ni,pv}$ with Y_{niv}

$$L_{ni,pv} \le M_L \, Y_{niv} \qquad n \in N_i, i \in (IS \cup IM), p \in P_i, v \in V_i$$

$$U_{ni,pv} \le M_L \, Y_{niv} \qquad n \in N_i, i \in IM \cup ID, p \in P_i, v \in V_i$$

Accumulated amount of product p picked up by vehicle v up to the stop (n,i)

$$\left\{ \begin{array}{l} AL_{n'i',pv} \ge AL_{ni,pv} + L_{n'i',pv} - M_L \left(1 - S_{ni,n'i'}\right) - M_L \left(2 - Y_{niv} - Y_{n'i'v}\right) \\ AL_{ni,pv} \ge AL_{n'i',pv} + L_{ni,pv} - M_L \, S_{ni,n'i'} - M_L \left(2 - Y_{niv} - Y_{n'i'v}\right) \end{array} \right\} \qquad n \in N_i, n' \in N'_{i'}, (i,i') \in I, p \in P, v \in V_{ii'} : n < n', i \ne i'$$

- In case of multiple stops of the same vehicle v:

$$AL_{n'i,pv} \ge AL_{ni,pv} + L_{n'i,pv} - M_L \left(2 - Y_{niv} - Y_{n'iv}\right) \qquad (n,n') \in N_i, i \in I, v \in V_i : n < n'$$

Accumulated amount of product p delivered by vehicle v up to the stop (n,i)

$$\left\{ \begin{array}{l} UL_{n'i',pv} \ge UL_{ni,pv} + U_{n'i',pv} - M_L \left(1 - S_{ni,n'i'}\right) - M_L \left(2 - Y_{niv} - Y_{n'i'v}\right) \\ UL_{ni,pv} \ge UL_{n'i',pv} + U_{ni,pv} - M_L \, S_{ni,n'i'} - M_L \left(2 - Y_{niv} - Y_{n'i'v}\right) \end{array} \right\} \qquad n \in N_i, n' \in N'_i, (i,i') \in I, p \in P, v \in V_{ii'} : n < n', i \ne i'$$

- In case of multiple stops of the same vehicle v:

$$UL_{n'i,pv} \ge UL_{ni,pv} + U_{n'i,pv} - M_L \left(2 - Y_{niv} - Y_{n'iv}\right)$$

Vehicle capacity constraints

$$\left\{ \begin{array}{l} \sum_{p \in P} uw_p \left(AL_{ni,pv} - AU_{ni,pv}\right) \le qw_v \\ \sum_{p \in P} uv_p \left(AL_{ni,pv} - AU_{ni,pv}\right) \le qv_v \\ AL_{ni,pv} - AU_{ni,pv} \ge 0 \end{array} \right\} \qquad n \in N_i, i \in I_v, v \in V$$

Bounds on variables $AU_{ni,pv}$ and $AL_{ni,pv}$

$$\left\{ \begin{array}{l} L_{ni,pv} \le AL_{ni,pv} \le \sum_{i' \in IS \cup IM} \sum_{n' \in N_{i'}} L_{n'i',pv} \\ U_{ni,pv} \le AU_{ni,pv} \le \sum_{i' \in IS \cup IM} \sum_{n' \in N_{i'}} U_{n'i',pv} \end{array} \right\} \qquad n \in N_i, i \in I_v, v \in V, p \in P$$

VRPCD-SCM MILP MODEL - V

Additional inventory received at cross-docking facilities from other sources

$$AI_{n'ip} \geq AI_{nip} + \sum_{v \in V_i} U_{n'i,pv} \qquad (n,n') \in N_i, i \in IM, p \in P : n < n'$$

$$\sum_{v \in V_i} U_{ni,pv} \leq AI_{nip} \leq \sum_{n' \in N_i} \sum_{v \in V_i} U_{n'i,pv} \qquad n \in N_i, i \in IM, p \in P$$

Alternative Objective Functions

$$Min \left[\sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in IB_v} \sum_{n \in N_l} fc_v \, Y_{nlv} \right]$$

$$Min \left[\sum_{v \in V} OT_v \right]$$

PROBLEM DATA FOR THE VRPCD-SCM PROBLEM

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AVAILABLE INVENTORIES AT SOURCE LOCATIONS

	P1	P2	P3	P4	P5	P6
			Example 4			
Barcelona	800	0	325	0		
Madrid	2500	2500	2500	2500		
Bilbao	200	50	200	100		
Malaga	300	300	300	300		
			Example 5			
Barcelona	800	0	325	0	0	300
Madrid	2600	2600	2600	2600	800	900
Bilbao	200	50	200	100	300	0
Malaga	300	300	300	300	0	0

PROBLEM DATA FOR THE VRPCD-SCM PROBLEM

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PRODUCT DEMANDS AT DESTINATION NODES

	Demands (for all examples)					
	P1	P2	P3	P4	P5	P6
Girona	120		150		50	50
Lerida		75	75			70
Tarragona	50	200		100		
Vic	100		100		150	
Valencia	120	120			50	50
Zaragoza	200		250	150		
Perpignan	150	150			100	50
Andorra	800		200		100	
Valladolid	50	150		200		
S.Sebastián	100	50				
Bilbao(*)	120		120	120		
Teruel	200	100			100	
Soria		200	50	100		150
Santander		150	100	50	200	
Burgos		100	150		100	50
Lugo		100		100		
La Coruña	100		100		100	150
Badajoz		220	430			
Granada	300	250		370		
Murcia		380	200			
Sevilla			200	450		
Cadiz			340			
Córdoba		420		430		
Huesca		50	150			
Castellon	100			50		
Pamplona		100			50	
Zamora	100		100			
Alicante	50			50		
Almeria				100		

PROBLEM DATA FOR THE VRPCD-SCM PROBLEM

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DISTANCE MATRIX

	Madrid	Valencia	Badajoz	Granada	Malaga	Murcia	Sevilla	Cadiz	Cordoba	Alicante	Almeria
Madrid		370	401	434	544	401	538	663	400	422	563
Valencia	370		716	519	648	241	697	808	545	167	461
Badajoz	401	716		438	436	675	217	342	272	697	605
Granada	434	519	438		129	278	256	335	166	354	167
Malaga	544	648	436	129		407	219	265	187	482	219
Murcia	401	241	675	278	407		534	613	444	76	220
Sevilla	538	697	217	256	219	534		125	138	610	423
Cadiz	663	808	342	335	265	613	125		263	689	485
Cordoba	400	545	272	166	187	444	138	263		520	333
Huesca	397	399	783	831	941	612	927	1051	788	564	830
Castellon	417	66	781	585	713	307	749	873	611	232	526
Pamplona	407	502	739	841	951	714	915	1049	798	667	933
Zamora	248	600	361	682	755	649	536	641	591	670	811
Alicante	422	167	697	354	482	76	610	689	520		295

PROBLEM DATA FOR THE VRPCD-SCM PROBLEM

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VEHICLE CAPACITIES

	V1	V2	V3	V4	V5	V6
			Example 1			
Weight capacity (kg)	20000	20000		--	--	--
Volume capacity (m^3)	25	32		--	--	--
			Example 2a			
Weight capacity (kg)	15000	12000	12000	--	--	--
Volume capacity (m^3)	25	18	23	--	--	--
			Example 2b			
Weight capacity (kg)	5000	10000	12000	--	--	--
Volume capacity (m^3)	10	18	23	--	--	--
			Example 3			
Weight capacity (kg)	10000	14000	10000	10000	--	--
Volume capacity (m^3)	18	22	20	20	--	--
			Example 4			
Weight capacity (kg)	12000	14000	10000	8000	20000	15000
Volume capacity (m^3)	20	22	18	18	28	20
			Example 5			
Weight capacity (kg)	12000	18000	12000	10000	18000	12000
Volume capacity (m^3)	20	28	22	15	29	20
Fixed Cost				$ 5000 (V1-V4) ; $ 4000 (V5-V6)		
Variable Cost				3 $/km (V1-V4) ; 2.5 $/km (V5-V6)		
Loading/unloading times						
Fixed						1 h
Variable						250 units/h
Average Speed						70 km/h

TYPE OF RESULTS PROVIDED BY THE MILP MODEL

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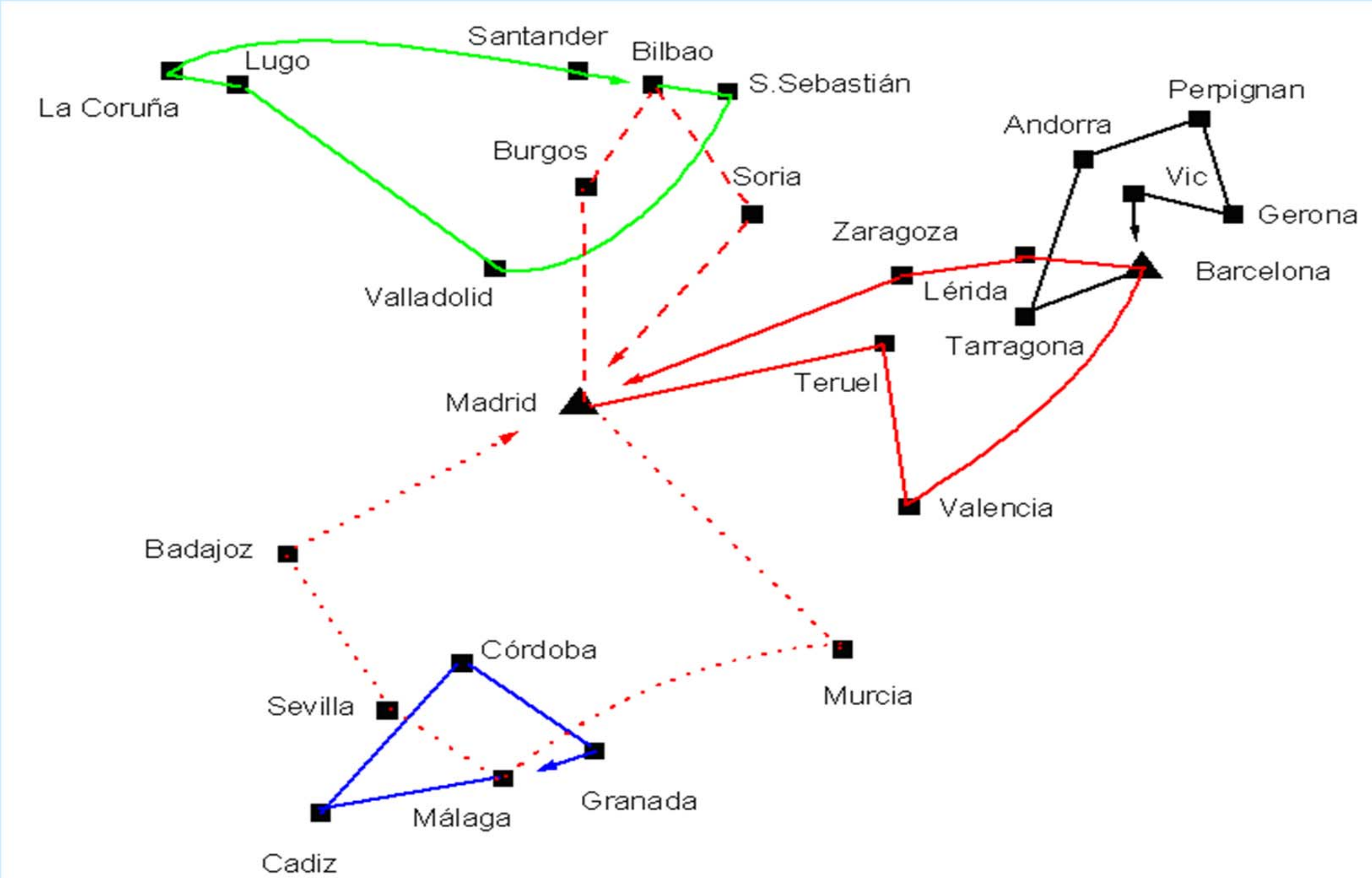
Detailed schedule of vehicle activities

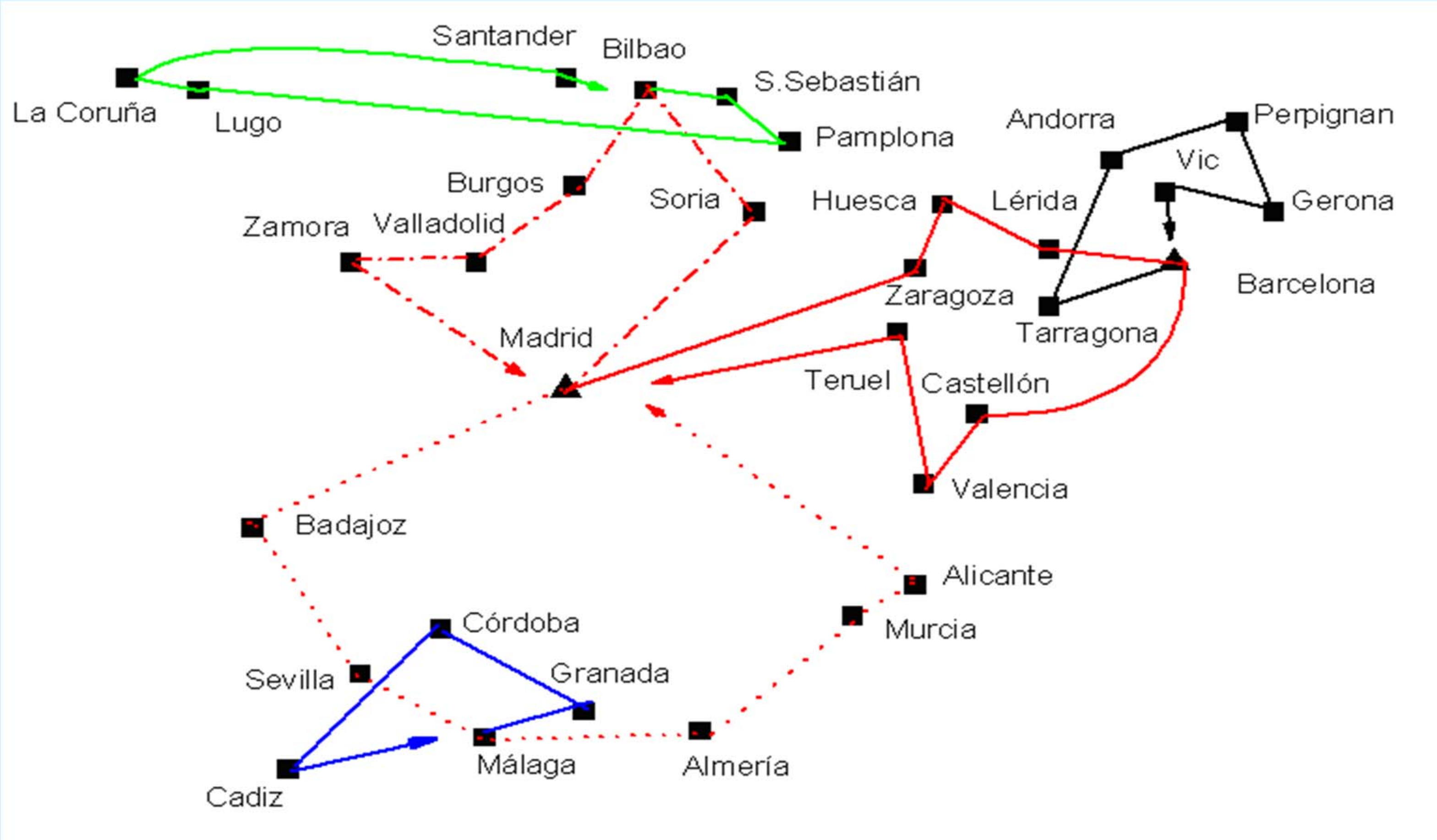
Vehicle	Site	Arrival time	P1	P2	P3	P4	Used Capacity %w	Used Capacity %v
V1	Barcelona	34.6	+1420	+425	+775	+250	59.7	89.9
(——)	Tarragona	48.6	-50	-200		-100		
	Zaragoza	54.4	-200		-250	-150		
	Lerida	59.9		-75	-75			
	Andorra	64.1	-800		-200			
	Perpignan	71.4	-150	-150				
	Gerona	75.0	-120		-150			
	Vic	78.0	-100		-100			
	Barcelona	80.9						

CASE STUDY I
(ONE-FACTORY, THREE WAREHOUSES, 22 CUSTOMERS)



CASE STUDY II

(ONE-FACTORY, THREE-WAREHOUSES, 28 CUSTOMERS)



CONCLUSIONS

- Transportation is a significant link between different stages in a global supply chain.
- Small reductions in transportation expenses could result in substantial total savings over a number of years.
- The use of vehicle routing and scheduling models and techniques can be instrumental in realizing those savings.
- Different types of vehicle routing problems have studied over the years; most of them dealing with single-echelon networks and a single type of operation (pickup or delivery) at every location.
- Since they are NP-hard, solution methods based on metaheuristic techniques are generally applied.
- Recently, new model-based approaches have been developed for the operational planning of multi-echelon distribution networks
- The so-called VRPCD-SCM problem includes many features usually arising in the operation of real-world distribution networks.
- Further work on this area is still under way.

THANKS FOR YOUR ATTENTION

QUESTIONS ?