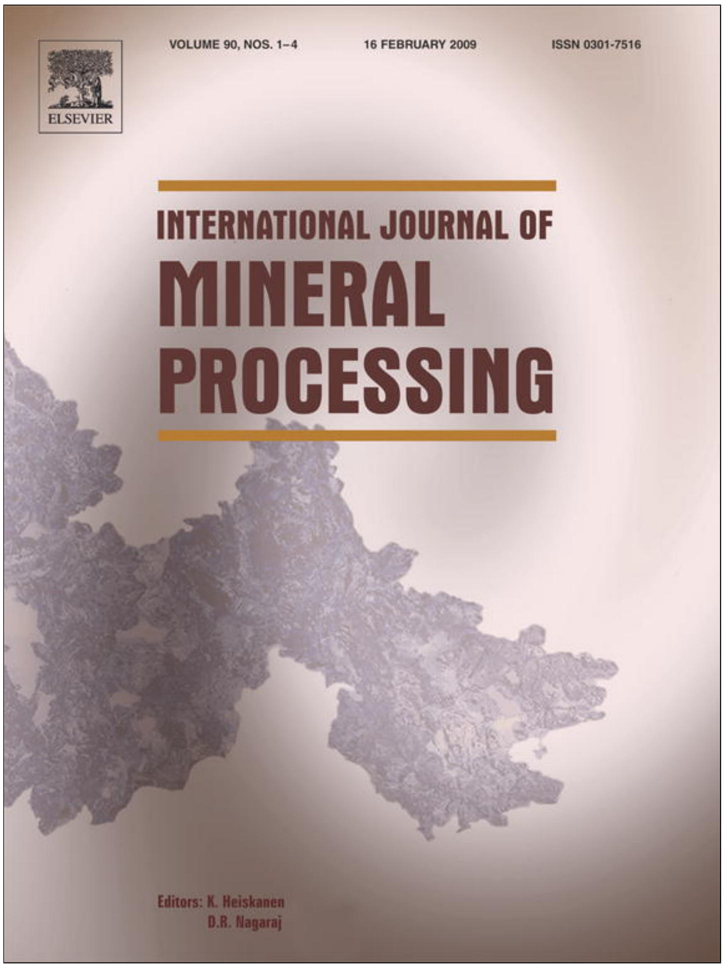
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State of the art in the conceptual design of flotation circuits

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ABSTRACT

The present study summarizes reports on optimal flotation circuits design over the period from 1989 to present. A review is made on the resolution characteristics of each study, leading to a classification of the approaches into four different groups, in which conclusions are based on either (A) mathematical solutions without binary variable, (B) mathematical solutions with binary variable, (C) heuristic solutions, or (D) genetic algorithms. These groups aim at a common scheme of resolution, varying in both flotation and milling models employed, characteristics of their superstructures, and functions which each study proposes to optimize, as well as the effects of the application of each strategy on the form of the solution of the circuit (selection of stages, selection of equipment, and dimensions of equipment).

Within these either implicit or explicit multi-objective optimization problems there are challenges strongly related to the obtaining of better flotation and milling/classification models, as well as the treatment of uncertainty related to important parameters (kinetics, composition, external economic factors) and changes in previously established configurations using methods in which the designer's criteria influence the application of models and restrictions on the problem. Also, the incorporation of environmental issues needs more attention.

Contents

1. Introduction

Flotation is a useful method to separate valuable minerals based on differences in surface properties of the particles from milled mineral mixtures. Various types of equipment exist which promote particles-bubbles encounter which contribute to controlling the balance between high recovery of the desired metal, and a high grade value of the metal in the product outflow. Past experience has shown single-step separation is inefficient, and the inclusion of a greater number of complementary and supportive steps are required which must be supplemented by auxiliary operations. Taking into account the large volume of material to be treated and its associated costs, choices related to the configuration of the separation system are critical.

Assuming the existence of prior reducing stages for mineral mix particle size (crushing and milling), the optimum flotation circuit problem can be summarized as the conversion of a quantity of valuable mineral from a mining operation into a concentrate product with the maximum utility and a minimum impact on the environment.

The first way a designer has to solve a synthesis problem (in any process) is trial-and-error methods. If the trial and error method is

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Nomenclature

GA	Genetic algorithms
b	Fraction of the progeny from the parents of a class which appears in another class (7)
c	Fraction of unliberated mineral from Eq. (5.a)
C_A	Mineral concentration
C_B	Bubble concentration
Co	General costs
D	Coefficient of dispersion in the flotation column
Dg	Discount by grade
E	Energy required per unit of material to provoke a change in the granulometric distribution
F	Feed capacity of a cell
F80	80% of the passing size in milling feed μm
Fp	Paid metal fraction
G	Grade of the concentrate flow stream
$g(x)$	Particle size function describing rupture properties variation in the material
H	Annual hours of functioning of a flotation plant
h	Height of the collection zone
I	Total number of cells
In	Income from concentrate sales
IPSQP	Infeasible Path Succesive Quadratic Programming
J'	Total number of mirror granulometry classes
K'	Total number of mirror mineralogy classes
kin	Kinetic characterization of the mineral
K_1^*, K_1^{**}	Left and right boundaries of the region of the feeder in size class 1 in Eq. (7).
kv	Economical value for valuable species in Eq. (9).b.
L	Number of banks
LP	Linear programming
MILP	Mixed integer linear programming
MINLP	Mixed integer non-linear programming
mJG	Modified jumping genes
mx	Characterization of mineral by species
NLS	Number of not-linked streams
P	Profit
p	Total fraction of the particle population in milled feed.
P80	80% of the passing size in milling product μm
Pm	Metal price
PR	Price for any flow stream in the circuit.
R	Recovery of a valuable species from a concentrate
R_∞	Final recovery of the process
R_{att}	Recovery of attached particles in the froth zone
Rfc	Cost of refining of the smelter
R_W	Water recovery
s	Interfacial bubble area
S	Selection function for particle in milling, Eq. (7)
sz	Mineral characterization by size
T	Total profit per unit mass
t	Instantaneous residence time
Trc	Charge for smelting treatment
u	Speed of the pulp in the recovery zone.
v	Speed of particle sedimentation
w	Mass fraction respect to the total mass of a flow stream
W	Mass flow
WI	Bond's law work index kWh/ton
x	Instantaneous particle size
Y	Flow of particles in suspension due to impeller action
ym	Separation factor

Superscripts

N	Number of mechanical cells in a bank
n	Exponent of Eq. (5)

Subscripts

C	Collection zone in the column
conc	Denotes concentrate stream flow
D	Refers to the discharge from a mill.
F	Froth zone in the column
f	Denotes fast flotation
feed	Denotes feed flow stream
g	Relative to grade G
I	A cell within any bank
J	Element of a granulometric class
J'	Element of a mirror granulometric class
J''	Element of an unliberated granulometric class
k	Element of a mineralogical class
k'	Element of a mirror granulometric class
k''	Element of an unliberated mineralogical class
L	Any bank in the circuit
M	Referent to feed into a mill
mv	Denotes a valuable mineral in a stream flow in any circuit
o	Denotes an outflow stream
r	Relative to the recovery R
s	Denotes slow flotation
W	Water

Greek

α	Conversion fraction or liberation fraction in the mill
β	Fraction of stream division in objective (7.a)
ε	Weight factor of an objective function
Γ	Maximum bubble charge in a flotation column
γ	Constant of proportionality in Eq. (5)
θ	Delay in flotation time
τ	Residence time of mineral in each cell
ϖ	Constant of speed of particles transfer in the flotation columns
Ψ	Non-dimensional exponential constant in Eq. (5.a)
ω	First order flotation kinetic constant
Ω	Particle entrainment function
ω'	Second order flotation kinetic constant
Φ_1, Φ_2	Fitting parameters for Eq. (3.d)
Φ_3	Slow flotation particles fraction entering to processing unit
α_1, β_1	Cost constants from Eqs. (9.b) and (10.a)
α_2, β_2	Fixed cost constants and variable cost, respectively in (10.b)

accepted, there are many ways to arrange a flotation circuit, although a number of them may prove to be incorrect, ineffective and highly expensive, which is showed when the retrofit of an existing process is attempting. That is, (1) circuits can be found which are apparently different, but producing similar metallurgical and/or economic results in which optimization is not truly obtained, or (2) added circuits which remediate those defects in existing circuits, but which introduce their own operational conflicts. In fact it is common to see that flotation circuits changes over time solving some problems, but introducing new problems.

Presently, computer-assisted methods for flotation circuit design have reached an important development level thanks to the advance of the optimization algorithms, the computers speed, and to the new developed strategies/methodology. Those methods have been refined and they are now able to solve problems of realistic size in an acceptable time. They can include aspects like multiple feeds, selection of flotation and grinding stages, economic and technical objectives, and the mineral characteristics, as mineralogy and grade. However, the obtained results and the difficulty of the optimization problem depend on the suppositions and restrictions included by their designers, as the type of superstructure, the flotation and grinding models, as well as the allowed options.

For this reason it is important to know the various approaches presented for establishing a new circuit or modifying one which already exists, based on metallurgical and economic considerations in possession of the designer. There are at least two prior reviews of studies concerning optimal design of a flotation circuit including that of Mehrotra (1988) and Yingling (1993a). Although the studies treated in these two reviews have many points in common, Yingling (1993a) divided them into two groups, with group I representing studies which proposed structural parameters for describing each eventual circuit to be optimized. This group used first-order models of flotation with techniques of linear programming (LP), where the residence time in the cells, and their number, along with the interconnections among stages were decision variables. Group II consisted of models which linearized the problem, resolving the function with direct search methods and indirect use of flotation models.

The present study reviews reports on flotation circuit design which have appeared from the time of the last review mentioned above up to the present, from the perspective of the models for continuous flotation circuits and regrinding stages, characteristics of superstructures or alternative representations, and nature of objective functions and its constraints employed, together with the influence of incorporation of new optimization methods. It is important to point out that for flotation circuit design we means flotation circuit synthesis or conceptual design of flotation circuit.

2. Current algorithm for the design of flotation circuits

At present, the search for the optimal circuit can be represented by Fig. 1, as explained in the following paragraphs. Flotation circuits design can be divided into several sub-problems whose treatment composes the generation of mathematical models describing phenomena such as both flotation and milling. Since mineralogical composition is a particular characteristic for each mining operation, the data needed for modeling these phenomena must be obtained experimentally from laboratory or pilot plant test. However, it is difficult to make direct application of the experimental results to the industrial scale, and data are required from the plant in order to correct or add parameters to be included in the mathematical models applicable to a particular problem. Also external data, as metal price and cost, are needed to process design.

Based on the problem definition, there are a great number of configuration solutions for a single case, but an abstraction of the problem may be conducive to: a) generation of a superstructure which includes a set of possible solutions, and b) representation of the mineral. The generation of this superstructure and the representation of the mineral are key factors in the solution of the problem. For example the superstructure must avoid multiple equivalent configurations, and mineral representation must facilitate the application of models and other constraints as will be discussed below.

Later in the problem definition, restrictions must be applied to satisfy a certain objective function. The objective function is determinant in defining as "optimum" the circuit proposed as a solution. The design strategies are closely related to a mathematical expression which is both representative of the real problem, but is also simple enough to provide rapid responses which are consistent with the metallurgical precepts involved in the construction of circuits.

These three elements (mathematical models, superstructures, and objective functions) need to be integrated in a manner which can be resolved by different optimization techniques. It is precisely, in these optimization techniques, the different models and restrictions where each study establishes differences in the mode of solution and result of optimization. For example, if nonlinear models are included, nonlinear optimization method will be needed, or if the superstructure represent parallel alternative options, then disjunctive optimization can be a selection.

Once synthesized the flotation circuit is possible to obtain and analyze the nearest alternatives to the optimal one. Then the most appropriate conceptual design can be selected. The post design activities correspond to the simulation (experimentally and/or numerically) of the circuit selected with the objective of adjusting in a better way the design and to proceed to the final process design.

Historically, flotation circuits design was resolved by using models of individual or groups of flotation cells and steady state materials balances, together with restrictions on both quality and quantity of the concentrates and final tailings, describing the circuit as both a collection of streams and process stages in a superstructure. These models are used to optimize technical functions (e.g whose aim is to obtain the greatest recovery constrained to a minimum grade in final concentrate), or economic functions (lowest cost circuit) (Mehrotra, 1988; Yingling 1993a).

In recent years (1989–2006) contributions from authors have appeared based on optimization methods which employed gradient search of objective function. For this, in some cases novel superstructures are included to represent connections between the stages, and together with more elaborate models and more intensive numerical computation methods seek to recreate plant conditions with greater realism, treating the mineral as if made up of subclasses having different capabilities for flotation. In general, the dimensions and optimal numbers of flotation cells, the selection of equipment, mass balances among others, convert the optimal design into mixed integer non-linear problem (MINLP). Nevertheless, various strategies allow simplification of the problem to MILP, NLP or LP systems. For the first time, milling models are used to evaluate the milling steps participation in the flotation circuit.

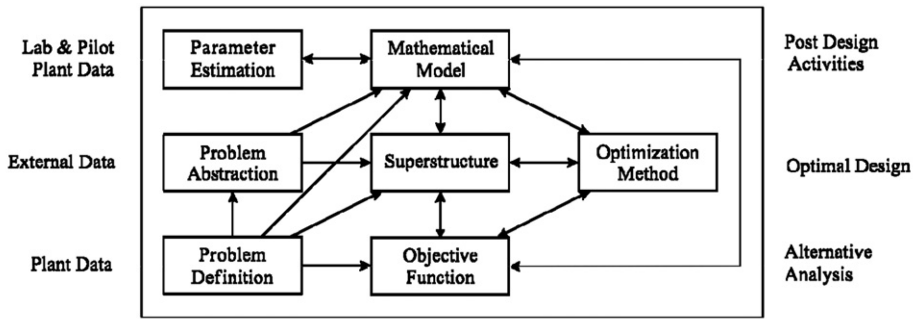

Fig. 1. Design algorithm for optimal flotation circuits.

Yingling (1990, 1993b) introduces an objective function more consistent with the quality-quantity-profit relation and an interpretation of flotation circuit based on Markov chains, assuming that there exists a given probability that each particle will move from one point to another within a circuit (termed "state" in these studies) before leaving the circuit in the outflow. In this way, the circuit can be manipulated to direct the more valuable particles into the concentrated product, reducing the gangue following an objective function as will be analyzed below. The supposition of recovery in probabilistic terms used by Yingling is equivalent to the usual first order model previously used in other works.

Hulbert (1995, 2001) proposed a problem reformulation, conceiving a flotation circuit as a group of countercurrent reactors, *i.e.*; the enrichment direction of the concentrate flows in a stream is opposed to that of the tailing, where it is possible to transform the recovery problem into expressions of solids concentration between the treated pulp and the concentrate through "enrichment functions", obtaining these from steady state macroscopic mass balances. This allows the application of McCabe–Thiele techniques (used for finding the optimal number of distillation steps, or solvent extraction steps) to design flotation circuits. The "enrichment functions" are linear relations between the solid concentration of precious mineral in the froth and the quantity of this mineral arriving in the feed stream. These functions are treated analytically, taking account the volumes cells distribution in the different stages for a total fixed volume.

The studies of Schena et al. (1996, 1997) included various non-linearities which were resolved by dividing the non-linear problem into two subproblems, where the first one delivered solutions to the second subproblem in the NLP problem proposed. By automating both the superstructure construction and the resolution of the problem, the authors developed a system of rules consisting of a prohibition against joining two stream flows from a flotation stage in a flotation step immediately following, leading to non-linear expressions, and requiring a solution strategy reformulation. Schena et al. (1996, 1997) already demonstrate the need for inclusion of a regrinding circuit in order to investigate its effect on the configuration and value of the objective function which is technical-economic.

The papers cited above (Yingling, 1990, 1993b, Hulbert, 1995, 2001; Schena et al., 1996, 1997) form Group A or *mathematical techniques-based methods without integer variables.* The Group A papers do not include logical decision criteria within the mathematical procedures, which may be important when it is desirable to include previous experiences, as for example, the choice of flotation or milling equipment where there may exist an order of succession in the circuit based on industrial experience or designer's criteria.

The studies of Cisternas et al. (2004, 2006) include disjunctive design criteria and logic expressions written in order to apply mixed integer linear programming (MILP) techniques, incorporating for the first time the choice and existence of both flotation columns and mills in the configuration, and providing aid in the determination of streams using binary variables. Later, these authors (Méndez et al., 2007) studied the effect of different grinding circuits using disjunctive equations to represent the alternatives. The studies from Group A and those mentioned immediately above are similar with respect to the models used, the nature of the objective functions, and certain structural parameters presented, as well as the technique of optimization which follows the gradient of the objective function. Then the contributions mentioned qualify the papers of Cisternas et al. (2004, 2006) as belonging to Group B, or *mathematical techniques-based methods with integer variables*.

Some of the obtained configurations may be considered to be too innovative or contradictory compared to empirical rules for circuit design (Dey et al., 1989). Therefore, instead of employing methods which aim at discovering a configuration which in some way optimizes the objective function, strategies can be employed based on the rules of design which lead to solutions acceptable within the criteria of each designer. The studies of Chan and Prince (1989) and Prince and Connolly (1996) apply a series of heuristics which help in obtaining circuits which attempt to improve the technical–economic behavior of the system. This is based on principal rules of determination of concentrates and tailings, and joining flow streams having similar mineralogical and granulometric characteristics, being solved by infeasible path successive quadratic programming (IPSQP) of reduced gradients methods. This method can be used in the retrofit of circuits with a relatively simple algorithm, where the type and order of the heuristics used is more relevant than the complexity of the models, forming another group of studies (Group C studies, or those based on *heuristic methods*). An important aspect is that the circuit is gradually evolved from an initial node (not a superstructure) as long as the proposed rules are followed. Nevertheless the rules order succession in the method does not permit exploring all pathways which could produce better results, and the solution does not necessarily produce an optimal circuit. The goal of the method is the obtaining of feasible circuits, and its use permits exploring alternate designs as shown in Connolly and Prince (2000) where it is applied to the retrofit of an existing circuit.

A circuit can also evolve from others as described by Guria et al. (2005a,b, 2006). In these studies, genetic algorithms (GA) allow the evolution of an initial population of circuit solutions by means of numerical operations which simulate the probabilities of reproduction, crossing, or mutation. These algorithms work with the value of an objective function in any iteration, selecting the solutions which improve it in each generation. The optimization can be mono-objective or multi-objective in the sense of incorporating various small objective functions which can take different directions of optimization. Although the concept is simple, a good portion of these studies is dedicated to demonstrating the use of an interesting improvement, termed modified jumping genes (mJG), and comparing this method with other genetic algorithms, or even with traditional numerical methods. The mJG method reduces the time response of the traditional GA method, and exhibits a greater versatility when the problem includes binary terms of the 0–1 type in flow streams. Although the results of optimization using GA are better than those obtained with mathematical programming there is no reliable demonstration that the optimal solution is a global solution. Thus, the nature of these studies places them in Group D, or *genetic algorithms-based evolutionary methods group.*

The papers in each group present interesting aspects respect to model formulation, problem abstraction, and optimization methods utilized, as summarized in Table 1. The following text explores the most relevant aspects of these studies in terms of the use of models, superstructures, and objective functions, and their influence on the circuits designed.

3. Flotation models

In this section the models used to represent the flotation equipment is revised. As we will see the supposition that the flotation process corresponds to a first order reaction is broadly used. Although this model is useful, its empirical nature impedes to include variables as pulp conditioning and aeration. Also, the limitation of the first order model impedes to identify the limiting conditions (for example the maximum recovery and grade) and to apply techniques as the attainable region.

If a group of solid particles transported in a pulp collide with bubbles within certain defined volume, the valuable (hydrophobic) mineral will adhere to upward bubbles, becoming separated from the gangue. This phenomenon can be considered as a simple mechanism where flotation is a pseudoreaction between the solid particles (A) and the bubbles (B) (Chander and Polat, 1994; Polat and Chander, 2000; Hernáinz and Calero, 2001) where $A+B\rightarrow AB$. Following this simple mechanism, the concentration of mineral C_A decreases over

Table 1
Summary of the characteristics of the studies reviewed

	Group A	Group B	Group C	Group D
Feed characterization	✓	✓	✓	✓
Models of mechanical cells	✓	✓	✓	✓
Models of columns		✓		
Models of milling and/or classification	✓	✓	✓	
Nature of the Objective Function	Technical–economic	Technical–economic	Technical economic	Technical Economic (separately)
Selection of mechanical cell/column		✓		
Selection of milling		✓		
Existence of Milling	✓	✓	✓	
Mathematical strategies	Markov chains Markov, mathematical analysis, MINLP	MILP, disjunctive programming	IPSQP, reduced gradients	Modified Genetic Algorithms
Studies	Yingling (1990, 1993b), Hulbert (1995, 2001), Schena et al. (1996, 1997)	Cisternas et al. (2004, 2006).	Chan and Prince (1989), Prince and Connolly (1996)	Guria et al. (2005a,b, 2006)

time in relation to the bubble concentration C_B and of a kinetic constant ω', following Eq. (1).

$$-\frac{dC_A}{dt} = \omega' C_B C_A \quad (1)$$

When C_B is constant, it develops into a single kinetic flotation constant ω, such that it can be considered a first order kinetic, as shown by Eq. (2). That is, $\omega = \omega'\ C_B$

$$-\frac{dC_A}{dt} = \omega C_A \quad (2)$$

The solution to Eq. (2) leads to a series of models listed in Table 2 which are often used in the simulation of batch flotation, where the mineral recovery R is analogous to the conversion term in chemical reactions. Eq. (2.a) in Table 2 considers the environment in a cell as if it were formed of a single phase, indicating that at the initial time ($t=0$) there is no recovery while over prolonged periods ($t \to \infty$) all the mineral has been carried out on the concentrate stream, although this does not occur in practice. This has promoted modifications of this approach by using a series of terms to include imperfections in batch flotation process (Sripriya et al., 2003), such as final recovery R_∞, a delay in the flotation time θ, or distribution functions for rate constant ω, as shown by Eqs. (2.b), (2.c), (2.d) and (2.e) from Table 2. In any case, an advantage of these models is the relative facility for obtaining the parameters and a good fit of the experimental behavior; therefore many authors accept the supposition of first-order kinetics. Also, Eq. (2.a) represents the response of a batch reactor or plug flow (Levenspiel, 1999). On the other hand, a flotation cell contains more than one phase, which can allow postulation of different mechanisms for mass transfer for flotation as an analogy for a chemical reaction producing an order different than 1 for the phenomenon, especially under extreme operational conditions. (Ek, 1992; Fichera and Chudacek, 1992; Schena et al., 1996). In any case, the differences between the equations are minimal, although they may exist at different degrees. One of these is found in Eq. (2.e) within Table 2, where a kinetic characterization is presented between material showing fast flotation (ω_f) and that with slow flotation (ω_s). In Reuter and Van Deventer (1990) the fitting parameters Φ_1 and Φ_2 must be obtained for Eq. (2.e), such that $(1-\Phi_1)$ is the non-floated fraction, $(\Phi_1-\Phi_2)$ is the slowly floated fraction and $\Phi_1(1-\Phi_2)$ are species with fast flotation.

Also, these equations presuppose that the mineral acts as a group of particles having a single size and composition, ignoring a more correct representation using both granulometrical and mineralogical distributions (Loveday and Brouckaert, 1995). On the other hand, the kinetic constant ω remains unalterable throughout the process, a factor which assumes constant conditions for the conditioning of the pulp, aeration, and addition of reagents to the system, even when it is possible to consider a first-order law where ω changes with the mineral concentration in pulp (Reuter and van Deventer, 1992).

Flotation models for cells and banks of cells described in earlier design studies, and in studies of the A to D groups are summarized in Table 3, and represent applications for continuous operations. In these equations the fed material has a mineralogical characterization, and in some cases granulometrical and kinetic properties characterization (Schena et al., 1997). For example, if it is assumed each flotation cell in a bank behaves as a completely mixed reactor in a steady state, it is possible to deduce Eqs. (3.c) and (3.d) in Table 3 assuming equals residence time (τ) for each cell using appropriate equations given in Levenspiel (1999). Further information about deductions of these equations may be found in Loveday and Brouckaert, (1995).

The expressions vary only when some term is added, such as those indicates a limit of recovery lower than 100% (R_∞), or when, instead of using a recovery term, is used the separation factor $\mathrm{ym}=(1+\omega\tau)^N-1$ in Eq. (3.b) (Mehrotra and Kapur, 1974; Reuter et al., 1988) or when it includes the effect of N cells in series per bank in Eqs. (3.c), (3.e), from Table 3 (Dey et al., 1989; Schena et al., 1997). In fact, Eq. (3.a) is a special case of Eq. (3.c) when $N=1$. In Eq. (3.b), ym is a concentrate/tailing ratio for the whole bank; it makes possible a bank be described as a first order system using an equation similar to Eq. (3a), Thus, the separation factor ym is not analogous to term $\omega\tau$. Eq. (3.b) both simplifies calculations and places the problem into a more realistic scenario. Eq. (3.e) is also presenting overall flotation effect for both slow and fast flotation species, in analogous way than Eq. (2.e) in Table 2. In Eq. (3.e), the authors need to establish differences among the minerals which are submitted to milling, a step which produces redistribution between fast-flotation and slow-flotation particles. They calculated the fitting parameter Φ_3 which represents the fraction

Table 2
Basic expressions for first-order recovery in batch flotation

Key	Recovery expression	References
2.a	$R = 1-\exp(-\omega t)$	Fichera and Chudacek (1992)
2.b	$R = R_\infty(1-\exp(-\omega t))$	Sripriya et al., (2003)
2.c	$R = R_\infty(1-\exp(-\omega(t+\theta)))$	Dowling et al. (1985)
2.d	$R = R_\infty\left(1-\frac{1-\exp(-\omega t)}{\omega t}\right)$	Gorain et al., (2000)
2.e	$R = \Phi_1\left[1-\left[(1-\Phi_2)\exp(-\omega_f t)+\Phi_2\exp(-\omega_s t)\right]\right]$	Reuter and Van Deventer (1990)

Table 3
Expressions for recovery in continuous mechanical cells used in circuit design

Key	Recovery expression	References
3.a	$R = \frac{\omega\tau}{1+\omega\tau}$	Mehrotra and Kapur (1974), Guria et al. (2005a,b, 2006)
3.b	$R = \frac{\mathrm{ym}}{1+\mathrm{ym}}$	Reuter et al. (1988), Cisternas et al. (2004)
3.c	$R = 1-\frac{1}{(1+\omega\tau)^N}$	Cisternas et al. (2006)
3.d	$R = \left[1-\frac{1}{(1+\omega\tau)^N}\right]R_\infty$	Schena et al. (1996)
3.e	$R = \Phi_3\left[1-\frac{1}{(1+\omega_s\tau)^N}\right]+(1-\Phi_3)\left[1-\frac{1}{(1+\omega_f\tau)^N}\right]$	Schena et al. (1997)

of the slow particles which enter the processing unit. It can be shown that the expressions for recoveries from banks of cells arise from a steady state material balance which provides a geometric progression; for the case in which each cell i has a recovery R_i shown in Eq. (3). This equation is valid for any expression of R_i.

$$R = 1-(1-R_i)^N \tag{3}$$

In Table 3, the parameters ω and τ exert a major influence on circuit design. The value of both parameters depend on the stage in which the group of minerals is found (Mehrotra and Kapur, 1974; Green, 1984, Wills, 1997); factors such as hydrodynamic conditions, pulp conditioning, reagent dose, mineralogical distribution, particle size and froth characteristics are hindered by ω. Although parameter ω facilitates both the analysis (the closer this parameter value of the valuable metal respect to the gangue, the more difficult becomes the separation) and the incorporation of the design algorithms, its estimation is difficult because it depends on several variables, as previously indicated. On the other hand, τ is related to the volume required in each cell for treatment of a mass flow of a given density, dictating the dimensions of the cell.

In spite of the existence of many models for flotation columns (Tuteja et al., 1994), the only paper that used some of them is that of Cisternas et al. (2006), due to his discussion of equipment selection. The difference between the column approach and that for mechanical cells is that the recovery in the collection zone is given by a non-ideal plug flow reactor behavior (Eq. (4)) where an dimensionless group has been added describing the dispersion within the column (D/uh), (Levenspiel, 1999). Overall recovery in column cells depends on both collection zone recovery (R_C) and froth zone recovery (R_F) and it can be obtained form a steady state material balance in collection and froth zones (Finch and Dobby, 1990). This type of model is broadly accepted in modeling and fitting of data for flotation columns (Finch and Dobby, 1990; O'Connor et al., 1994), although in Cisternas et al. (2006), recovery in the froth zone R_F is considered constant and the balance between both collection and froth zones gives the total column recovery.

$$R_C = 1-\frac{4a\exp\left(\dfrac{1}{2\mathrm{D/uh}}\right)}{(1+a)^2\exp(a/(2\mathrm{D/uh}))-(1-a)^2\exp(-a/(2\mathrm{D/uh}))} \tag{4}$$

$$a = \sqrt{1+4\omega\tau\frac{\mathrm{D}}{\mathrm{uh}}}$$

In addition to the models presented, which represent those used in problems of circuit design, new models for cells and columns have appeared in recent years as cited in Table 4.

These models maintain first order collision but have yet not been incorporated into design strategies. Eq. (4.a) in Table 4 has been shown to provide a good fit for industrial data and is included in a commercial computer simulation package (Harris et al., 2002). Model (4.a) arise from the result of dividing the volume of aerated pulp into various sections within a single mechanical cell, so that recovery of the mineral is also a function of the recovery of water R_W and of the capture function Ω which depends on the hydrodynamic conditions and where it is assumed that ω is a function of mineral floatability (Gorain et al., 2000). If the aerated pulp content of a mechanical cell is divided into more sections, model (4.b) arises from the material balance between them, where Y is the flow of particles in suspension due to the action of the impeller, while F is the feed capacity. The recovery of the attached particles in the froth zone R_{att} is coupled to the term $\omega\tau$ of the collection zone of the cell. If the effect of these new factors in Eqs. (4.a) and (4.b) is ignored, a return is made to Eq. (3.a). Both Eqs. (4.a) and (4.b) can be included in (3) to obtain the recovery of the bank. More information on froth modelling can be found in Mathe et al. (1998) and Mathe et al. (2000).

With respect to flotation columns, Ityokumbul (1996) disagreed with the collection zone height–recovery relationship given by Eq. (4), and proposed that this equipment be modeled as a mass transfer promoter between phases. In Eq. (4.c) of Table 4, R remains directly associated with the height of the collection zone h, the speed of the pulp in the recovery zone u, the sedimentation particle speed v, the maximum bubble load Γ, the interfacial area of the bubbles s, and the rate transfer particles constant ϖ. Eq. (4.c) is another way to write the plug flow model.

The kinetic expression for the flotation of mineral particles arises from assumptions which impede the direct use of experimental parameters in the industrial environment, since the true mechanisms of the phenomenon are unknown. Although the models are useful in resolutions of the optimal circuit problem, they impede the use of other techniques such as that of the attainable region (Hildebrandt and Glasser, 1990), where it is important to know the true mechanism of a reaction.

Also, in all the studies of the A–D groups, the values of the parameters in the models remain invariable over time, a fact that limits the validity of the results when there are new situations or variations, for example in mineral characteristics, that is, the treatment of uncertainty is omitted in the kinetic parameters.

Of this analysis it is clear that there are challenges related with the flotation models. The introduction of new models available in the literature, which can represent in a better way the flotation phenomena, can introduce new challenges in the optimization problem. Therefore it is necessary to develop models that allow their use in optimization problems, that is to say that these models must capture the phenomena that affect the search of the optimum and at the same time maintain a low mathematical complexity.

4. Models for milling steps

The behavior of a mineral in flotation is affected by its granulometric distribution and its degree of liberation, among other things (Wills, 1997; Gorain et al., 2000). In the first steps of particle size reduction, crushing stages take down material to a maximum size of tens of centimeters, grinding stages reduce maximum particle size to few millimeters. Milling prior to flotation further reduces its granulometry (Jankovic, 2003), with notable increase in the appearance of new mineral surfaces (liberation). This is a strategy used within the flotation circuit to increase the yield of the valuable metal which is present in disseminated form throughout the rocky matrix where its grade value is low. Intermediate milling steps may be installed between flotation steps (Bazin et al., 1994; Schena and Casali, 1994). However, reduction in particle size and the concomitant liberation phenomenon require high inputs of energy because of inherent grinding inefficiency. Also, transport and treatment of slurry adds a water consumption factor not included in optimization reports. Thus the inclusion of milling models in circuit designs complies with the need for representing the changes in the quantities of species liberated and their impact on the configuration of the circuit.

Two forms of modeling the milling process are recognized. One of the more basic forms is to relate the energy required for breaking down the original number of particles following a simple law of the type shown in Eq. (5), where the increase in energy consumed per unit

Table 4
Some recent recovery models for continuous reactors

Key	Recovery expression	References
4.a	$R = \frac{\omega\tau(1-R_W)+\Omega R_W}{(1+\omega\tau)(1-R_W)+\Omega R_W}$	Harris et al., 2002
4.b	$R = \frac{\omega\tau R_{att}(1+Y/F)(1-R_W)+\Omega R_W}{(1+\omega\tau R_{att})(1-R_W)+\Omega R_W}$	Savassi, 2005
4.c	$R = 1-\exp\left(-\frac{hs\varpi\Gamma}{u+v}\right)$	Ityokumbul, 1996

mass dE produces a change in size dx depending on the actual size of the particle, x, raised to a power n, which is a constant, as pointed out by Hukki (1961).

$$dE = -\gamma \frac{dx}{x^n} \tag{5}$$

Different values for n in Hukki's equation make it possible to obtain various equations, such as those of Von Rittinger ($n=2$), Bond ($n=1.5$) and Kick ($n=1$). That proposed by Bond is the most frequently used in calculations of consumption and cost of energy, based on the parameters F80, P80 and where in this case the proportionality constant γ represents 10 times the Bond work index WI, which is the consumption of energy in kWh/ton and represents the inertia of the mineral to be milled (Wills, 1997). F80 is the particle size such that 80% of mill feed weight has a lesser size, i.e., the passing 80% in mill feed (in general, any comminution equipment feed). P80 is the particle size such that 80% of mill product has a lesser size, i.e., the passing 80% in mill product (in general, any comminution equipment product). It has been seen, however, that Bond law does not satisfy the fit for finely sized particles, generating a new approach where the exponent n depends on the particle size *i.e.* $n=f(x)$ such that Eq. (5) simply changes to Eq. (6) (Morrell, 2004). This time, $g(x)$ is the function that describes the variation in the properties of rupture with particle size, where this function must be determined for each material to be milled.

$$dE = -\gamma g(x) \frac{dx}{x^{f(x)}} \tag{6}$$

A second approach for modeling postulates that milling is a process where it is possible to describe the mineral as populations based on size and/or composition, whose relative quantity varies, making it necessary to have parameters for preferential breakage and classification obtained by fitting experimental results in complex expressions. Eq. (7) is the discrete representation of a differential equation which has no analytical solution and arises from batch modeling for milling (King and Schneider, 1998; Fuerstenau et al., 2004; Cho and Austin, 2002; Hoşten, 2005)

$$\frac{dp_{kj}}{dt} = -S_{kj}p_{kj} + \sum_{k=1}^{j-1} \sum_{k'=K_1^*}^{K_1^{**}} S_{k'j'} b_{kjk'j'} p_{k'j'} \tag{7}$$

In Eq. (7) S_{kj} is the selection function for particles of grade class k, and size class j, from here "class kj"; p_{kj} is the fraction of the entire population of particles in class kj which appear in another class $k'j'$. The term $b_{kjk'j'}$ is the fraction of the progeny arising from the breakage of the class kj parent fragments which appear in the other class $k'j'$. K_1^* and K_1^{**} are limits of integration in the j′ size classes (King and Schneider, 1998).

Eq. (7) states that the production of material of class kj over time is a function of the immigration of milled material to this class (positive portion of the right side of this equation) and the emigration of material from this class (negative portion of the equation). It is easily adaptable for considering only the change in particle size.

The balance of population by dispersion developed in Wei and Gay (1999), and Gay (2004a,b) expresses a similar approach, developing an exhaustive statistical study to describe the mineral in classes kj, searching for more complex mathematical representations of milling. The population balance approach is used in the simulation of continuous milling circuits, including adjustments to unequal residence times (Austin, 1999; Austin and Cho, 2002), and considers the parameters obtained as residence time τ, to be constant. Recently a new work has been published with advances using this type of models (Stamboliadis, 2008). These models are an important contribution to represent size changes and liberation in grinding operations, but they are complex to include in synthesis methods using optimization as a part of the flotation circuit synthesis. That complexity is due to the great quantity of necessary classes, and therefore the great quantity of data required and a large number of variables included in the model.

Proposals of population balance and energy are complementary, but there exists the risk of introducing non-linearities which require the use of more complex methods, for example MINLP, if determination of optimal parameters is desired. The treatment can be simplified if the parameters are known, establishing linear mass balances.

Due to the difficulty to include changes in particles size, valuable mineral liberation, and changes in particles surfaces properties simultaneously in design strategies, there are few studies which consider an intermediate milling stage between the flotation stages using the models presented in Table 5; Schena et al. (1996) also considered a milling step, but no expression for this is reported. Studies referred here accept distribution of mineral masses within classes with different ω can change when regrinding stages are included in the optimal flotation circuit configuration. These studies belong to Groups A, B, and C.

Reuter and Van Deventer (1990) states that contact between mineral particles and milling media changes the degree of liberation, even surface chemistry, an effect studied by Martin et al. (1991) and Gonçalves et al. (2003). This causes both a re-ordering of the population, and provokes changes in the flotation constants ω. It was also assumed that no more new kj classes appears which were different from the originals, which speaks for a rigid description of the feed mineral; the specification of a concentration of solids in weight in the pulp is not explicitly included, ignoring the balance of water in the circuit.

Population approach is suggested in Eq. (5.a) of Table 5 where breakage function b depends on energetic requirements for the system. The first equation of (5.a) shows that the mass output flow of the mill for a given class kj (W_{Dkj}) is the contribution of both material which remains in the same class and the quantity coming from the other classes $k'j'$, with a feed flow W_{Mkj}. Its form is suggestive of Eq. (7), but is modeled for steady-state milling. The second expression of (5.a) restricts the milling action, considering that there is a mineral fraction in an unliberated class $k''j''$after the milling, $b_{k''j''}$, which is a function of the specific energy consumption of mill E_{kj} for all the classes, and a function of unliberated mineral fraction, c. These equations are made robust by imposing mass balance restrictions, and introducing the potential for relating the slow flotation fraction with empirical parameters, together with the determination of a size at which the floatability of class kj is maximized, supporting the action (3.e).

In contrast, in (5.b), only the effect of energy consumption by the mill is considered when applying Bond's Law, using the previously explained parameters, whereas in (5.c) only the redistribution of mineral species are employed in the modeling. In (5.c) the discharge flow from mill W_{Dk} depends on the contribution from other classes to class k through a conversion fraction $\alpha_{kk'}$, representing a fraction of

Table 5
Milling models employed in flotation circuits design

Key	Milling equation	References
5.a	$W_{Dkj} = b_{kjkj}\tau_M W_{Mkj} + \sum_{\substack{k'j'=1 \\ k'j' \neq kj}}^{K'J'} b_{kjk'j'}\tau_M W_{Mk'j'}$	Schena et al. (1997)
	$b_{k''j''k''j''} = c + (1-c)\exp\left(-\Psi \frac{\text{Net Power}}{\sum_{kj} W_{Mkj}E_{kj}}\right)$	
5.b	$E = 10WI\left(\frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}}\right)$	Prince and Connolly (1996)
5.c	$W_{Dk} = \sum_{k'} \alpha_{kk'} W_{Mk}$	Cisternas et al. (2006)

liberation in the mill from species k′, $\alpha_{kk'}$ is constant for all stages and classes. In this case, liberation is the main effect which controls the decision to include a mill in the circuit. The concept of using milling for the redistribution of classes appears in the work of Reuter and Van Deventer (1990), where the novelty is they consider that the milling behavior is different if it is carried out on a stream of tailing or on concentrate, where the particle breakage is differentiated between the cases.

None of the group A–D papers mention explicit classification models. In Prince and Connolly (1996) the hydrocyclones divide flows based on certain size fractions. The most commonly utilized models of hydrocyclones in commercial simulation programs are found in the review by Nageswararao et al. (2004). These models require detailed information on the composition and feed to the classifiers, as well as the equipment dimensions, and included highly non-linear functions which were similar in form, but delivered distinct results.

It's important to note equations of Table 5 are linear, or behave as linear functions when the parameters are given. However, Fuerstenau et al. (2004) show that Eqs. (5) and (6) undergo modifications depending on the type of mill used, suggesting that certain non-linearities exist due to the imperfection of the process, *i.e.* the inefficiency of energy use due to collisions between the milling media, which reduce the success on mineral liberation. Besides, there are no studies of ways to determine the quantity of *kj* classes which are sufficient for simulation and design, or even exploring the way to decide between various types for milling-grinding circuits. Also, there is no differentiation in models between different types of milling equipment.

Grinding is an important aspect in the design of flotation circuits. Their use doesn't only affect the mineral liberation, as most of the works that include grinding has considered, but rather it also affects the energy use and water consumption. These last two aspects are subject in the world today and require to be incorporated in the synthesis of flotation circuits. Simple models are also required but that include less parameters and less variables, to be able to integrate those models in the flotation circuit synthesis problem.

5. Superstructures

Circuit conception is strongly influenced by the designer's experience, who may attempt to follow a series of common precepts in metallurgical tradition (Mehrotra, 1988; Dey et al., 1989; Yingling, 1993a); although these precepts may not aim towards providing an optimal result, clear violation of some of them classifies the obtained configuration as unrealistic, doubting the validity of the result (Prince and Connolly, 1996). This led the authors to require inclusion of a superstructure capable of restricting the final form of the optimal circuit. A superstructure is typically represented by a digraph in which arcs represent streams, and the nodes are state, task and/or unit operation of a flotation system. A superstructure allows us to delineate the forms of solutions, and depends on the interconnections between state, task or unit operations and streams proposed by each author. Also, it represents a special restriction within the optimization problem, given that a circuit cannot grow beyond the possibilities offered by the superstructure. We note that all reviewed studies include a superstructure provided as data, except by Schena et al. (1997), where a computational algorithm uses stream flow restrictions to automatically make the superstructure used by optimizer.

Table 6 points out some aspects related to the superstructures construction. Property existence is marked, with the exception of the column labeled (6.6). As can be observed, each circuit receives a single feed flow stream; inclusion of several of these represents an interesting opportunity for simulating, for example, the existence of material coming from different mining operations able to be treated in a single plant. Even though only a single feed flow stream is considered, superstructures exist which permit their splitting, as shown in column (6.1). Column (6.2) shows superstructures which permit division levels in the streams. The presence of split flows in the solution circuits has become a topic under discussion since they are not used in presently existing circuits. The difference should be noted that most studies allow for continuous levels of splitting (Yingling, 1990, 1993b; Guria et al., 2005a,b, 2006) while only one treats discrete levels of splitting (Cisternas et al., 2004). In cases where no levels of division exist, the splitters only direct the flow stream in a single direction toward a following sector of the diagram.

Table 6
Relevant aspects in superstructure construction

Studies	(6.1)	(6.2)	(6.3)	(6.4)	(6.5)	(6.6)	(6.7)
Yingling (1990)		✓				U–E	
Yingling (1993b)	✓	✓				U	✓
Hulbert (1995)				✓		U	
Hulbert (2001)				✓		U	
Schena et al. (1996)						U	
Schena et al. (1997)					✓	U	
Cisternas et al. (2004)		✓			✓	U–T	✓
Cisternas et al. (2006)					✓	U–T–E	✓
Guria et al. (2005a)	✓	✓	✓			U	
Guria et al. (2005b)	✓	✓	✓			U	
Guria et al. (2006)	✓	✓	✓			U	

Legend: (6.1) Feed stream splitting, (6.2) Levels of stream splitting, (6.3) By-production, (6.4) Fixed number of stages, (6.5) Complementary operations, (6.6) Unit Operation (U), Tasks (T) or stages (E), (6.7) Hierarchization.

Column (6.3) shows the circuits design delivering a single product stream, i.e. a single final concentrate and a single final tailing are favored, although some superstructures exist which permit obtaining more than one final product, which is understood to be not re-treated stream of either concentrate or tailing within the same circuit. This type of diagram with by-production occurs in studies of the D group, and can be considered as a valid option for operations where exist more than one valuable species; e.g. operations for the concentration of copper and molybdenum minerals.

Column (6.4) indicates that many of the reports may considering one or a few of the available steps in the superstructure, either because the algorithm is able to include them (groups A, B and C) or because certain stages have been selected for study out from a large superstructure (group D). Nevertheless in the studies of Hulbert (1995, 2001) the circuit remains adjusted to a determinate configuration; the author tries to arrange a better steps dimensioning, a topic treated in the next section on fixed circuits.

A flotation circuit is composed of mutually complementary unit operations, and the studied superstructures reaffirm this idea by considering multiple flotation steps, although not all permit the existence of other complementary operations such as milling. Column (6.5) shows few studies contemplate the inclusion of these steps, discouraged perhaps by the optimization algorithms used. Also, it is note worthy that only the existence of milling-classification is included, omitting other options such as liquid–solid separation between flotation steps (Wills, 1997).

Other relevant characteristics refer to the ways in which superstructures are conceived. For example column (6.6) notifies that superstructures in which the nodes perform as unit operations are common, but also exists the case where the nodes in the superstructure work as state and task.

Column (6.7) also shows that most of the reviewed papers show all the flotation steps on a single level except in the studies of Yingling (1993b) and Cisternas et al. (2004, 2006). Superstructures used in Cisternas et al. (2006) are shown in Fig. 2. Fig. 2A is a hierarchical two-level superstructure where there are two kinds of nodes. In the first level, there are nodes where flotation tasks, i.e., rougher, cleaner and scavenger tasks, are contained in rectangles,

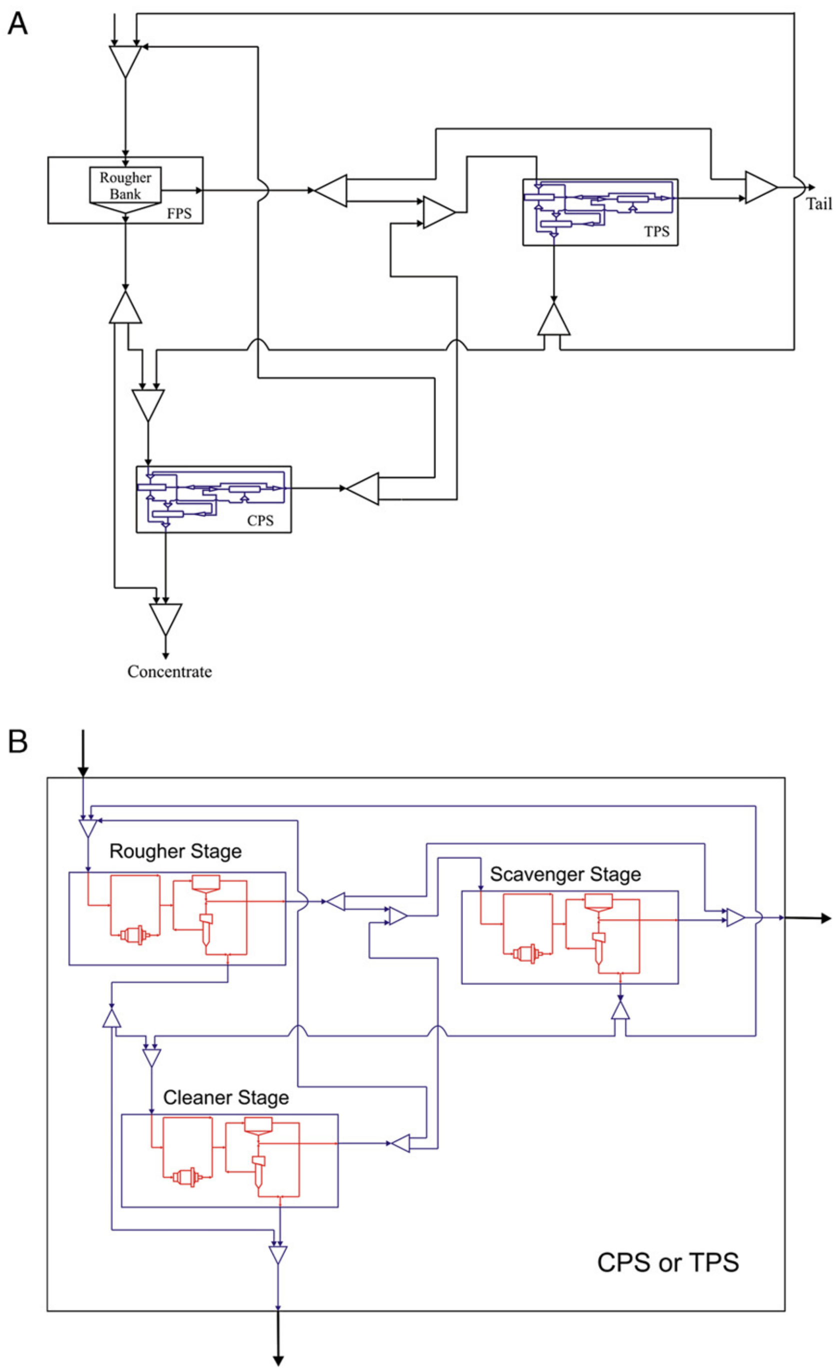

Fig. 2. (A) Two-level superstructure used in Cisternas et al. (2006), (B) Superstructure for equipment selection used in Cisternas et al. (2006).

and also there are nodes performing as streams mixers/splitters in white triangles. In the second level, within each flotation stage the same superstructure is repeated, in such a way it is possible each flotation task be performed by three stages, e.g., cleaner–rougher, cleaner–cleaner and cleaner–scavenger stages in cleaner task. In Cisternas et al. (2006), authors assume only one rougher stage and they give the chance to have three stages for cleaner and scavenger tasks. Besides, Cisternas et al. (2006) introduce a new concept, a third level shown in Fig. 2B, where an equipment selection superstructure is included within each stage in second level of Fig. 2A. In Fig. 2B, there is grinding selection before flotation equipment selection. Although this representation can seem more complex, it allows easily to include circuits until 9 flotation stages with equipment selection, and it avoids to exist alternatives that are equivalent, eliminating in this way possible degenerancies and problems in the optimization using binary variables.

Until now it has been seen that each study group includes superstructures in their own circuit design problem, except for those in Group C. In this group, the circuit analysis begins with a type of seed circuit which evolves by expansion following heuristic methods, without the implication of being constrained by a superstructure. These studies are also based on a single feed stream, which can be treated in cells or columns, as well as including milling treatment.

Thus, superstructures constitute a universe from which the solution can be chosen by limiting the final circuit shapes according to the emphasis and the possibilities proposed by each author for solving the circuit design problem. Together with the models and objective functions, superstructures are introduced in optimization methods, as shown in Fig. 1, which significantly influence the results.

6. Objective function

"Optimal circuit" definition implies a proposed flotation configuration reaches the extreme of some appropriate objective function, considering its constraints, *e.g.*, a configuration may be optimal from the perspective of the best recovery of a valuable species in the concentrate, although not from the perspective of a cost–benefit analysis.

The objective function used in each approach has evolved. A sample of previously used objective functions is presented in Table 7, to provide a perspective for their evolution in the studies of groups A–D. The objective functions (7.a) of Table 7 represent one of the first applications of the scheme presented in Fig. 1. Only a single individual function could be selected per case study in the paper of Mehrotra and Kapur (1974). The notation for recovery R and grade G is based on the superstructure used by the authors, but maintains the traditional concept. It should be recalled that R and G are opposing functions, and the simultaneous optimization of its effects is an interesting challenge which has been reported in several papers from all the groups, as will be mentioned below. Also, maximization of profit P on the sale of the concentrate is shown, where PR_{feed} is the price of the feed having a grade value G_{feed}, and PR_g is the increase in price for each 1% increase in the grade value. Although this function is meaningful, it is inconvenient because PR_{feed}, G_{feed} and PR_g are constant, and then maximization of P in (7.a) is another way of maximizing the quality of the product G as in Guria et al. (2005b). It is important to incorporate technical and economic criteria in obtaining the optimal circuit.

The methods evolved towards a better representation of the problem, with preference for the use of technical functions. Eq. (7.b) represents one of these objectives, attempting to maximize the quantity of valuable species in the concentrate; the importance of each kj class is reflected in weight factors ε_{kj}. Studies which emphasize this kind of objective resolve the problem using LP techniques as shown by Yingling (1993a). This is similar to maximizing R, but can be used when exists a more detailed mineral characterization. Weight factors assignation continues in Eq. (7.c) showing a similarity with the studies of Guria et al. (2005b, 2006) by attempting the simultaneous optimization of R and G. Strength of these works is the attempt for use multi-objective function and obtaining linear functions from originally non-linear expressions. Unfortunately, the results are based on the importance of each ε value assigned by the designer to a particular case, *i.e.* the subjectivity in the problem establishment is basic in obtaining the circuit.

The importance of the economic perspective in circuit design is manifest in the paper by Abu-Ali and Abdel Sabour (2003), although

Table 7
Objective functions used in previous optimization studies

Key	Objective function	References
7.a	$\max R = \dfrac{\sum_{i=1}^{I}\sum_{k=1}^{K} W_{\text{conc},ki}\beta_{i,o}}{\sum_{k=1}^{K} W_{\text{feed},k}}$	Mehrotra and Kapur (1974)
	$\max G = \dfrac{\sum_{i=1}^{I}\sum_{k=1}^{K} W_{\text{conc},ki}\beta_{i,o} w_k}{\sum_{i=1}^{I}\sum_{k=1}^{K} W_{\text{conc},ki}\beta_{i,o}}$	
	$\max P = PR_{\text{feed}} + (G - G_{\text{feed}})PR_g$	
7.b	$\max \sum_{kj=1}^{KJ} \varepsilon_{kj} W_{\text{conc},kj}$	Green (1984), Reuter et al. (1988), Reuter and Van Deventer (1990, 1992)
7.c	$\max \varepsilon_r R + \varepsilon_g G$	Dey et al. (1989)

Table 8
Objective functions used in Groups A–D

Key	Objective function	References
8.a	$\max \sum_{kj=1}^{KJ}\sum_{l=1}^{L} w_{\text{feed},lkj} T_{lkj}$	Yingling (1990, 1993b)
8.b	$\max\ P = \text{In}-\text{Co}$	Schena et al. (1996, 1997), Cisternas et al. (2004, 2006), Chan and Prince (1989), Prince and Connolly (1996)
8.c	$\max\ W_{\text{conc}}$	Hulbert (1995, 2001)
8.d	$\max\ R$	Guria et al. (2005a,b, 2006)
8.e	$\max\ G$	Guria et al. (2005a,b, 2006)
8.f	$\max\ P$	Guria et al. (2005b)
8.g	$\max\ R_{mv}$	Guria et al. (2005b, 2006)
8.h	max solids hold-up	
8.i	max NLS	

they only adopt an economic criterion for determining the number of cells per bank, applying the principle of marginal cost and marginal benefit. Papers in Groups A–D tend to use technical–economic functions for making implicit or explicit multi-objective optimizations as shown in Table 8. In Eq. (8.a), the objective is to maximize the product value, expressed as the sum of total benefits per unit mass accumulated by class *kj* material starting from bank 1 during its trajectory prior to exiting from the circuit T_{lkj}, and where $w_{\text{feed},lkj}$ is the proportion of the total mass of class *kj* which enters bank 1. The concept consists of penalizing an undesired element in the concentrate with a negative benefit while rewarding the presence of a valuable species, with the inverse principle applied to the treatment of tailings. An optimized value of a control policy produces the quantity $w_{\text{feed},lkj}$ to (8.a), as a form of optimizing R, but the grade G remains represented by the benefit T_{lkj}, being a type of multi-objective optimization which is resolved through the use of sub-optimizations. In spite of the benefit factors are similar to the weight factors ε, the idea of monetary representation of the streams makes (8.a) a technical–economic objective, although it only serves as a guide for determining the best interconnection between the banks of cells and the optimum number of cells per bank.

Function (8.b) is more general, and provides more freedom to the authors for using it as a technical–economic function in monetary units. The profit function P is the difference between incomes and the costs Co associated with the process.

In Eq. (8.c) Hulbert (1995, 2001) did not attempt to optimize the interconnections between steps, i.e., the mode in which one cell was linked to another, but rather by improving the condition in which the cells volume was shared within the circuit, optimizing the recovery at a desired grade level, achieved by maximizing the concentrate flow W_{conc}. Since the treatment is analytical, this maximization is equivalent to searching for partial derivative equations for enrichment functions, sequentially resolving the optimization from the final concentrate flow. Stream flows and concentrations are then found which better distribute a total fixed treatment volume, following a treatment similar to the determination of McCabe–Thiele distillation steps.

Finally, Group D employs objective functions from Eq. (8.d) to (8.i). Using modified GA, the authors are able to simultaneously employ many functions in explicit, multi-objective optimizations. For example Guria et al. (2005a), return to the study of Mehrotra and Kapur (1974), maximizing overall recoveries R (8.d) and profits P (8.f) as shown in Eq. (7.a), then a second problem resolving Eqs. (8.d) and (8.e) simultaneously, that is, two antagonistic functions. In the following studies, mono-objective optimizations are resolved with functions such as the concentrate grade G (8.e), or recovery of valuable mineral R_{mv} (8.g). As used superstructure changes, they include problems containing three or up to four objective functions: to the already mentioned objectives are added the simultaneous maximization of solids holdup in cells (8.h), and the numbers of non-interconnected streams NLS (8.i) to obtain simpler circuits. The latter is due to GA presents weakness in binary variables treating. If a rounding-off algorithm is not included, Eq. (8.i) must be added in order to obtain simpler circuits.

Table 9 shows two functions of profits or income. Eq. (9.a) is common in the calculations of benefits from sales of concentrate to a smelter (Bazin and Hodouin, 1997), and is termed *net smelter return function*. This includes the metal grade in the final concentrate G_{conc} and the recovery from the circuit in terms of concentrate flow W_{conc}, together with external restrictions which influence the economics of the process and which can be easily calculated as input data (Schena et al., 1997; Davenport et al., 2002), once again having an implicit multi-objective function.

Flotation operations are sensitive to changes in economic conditions shown in Eq. (9.a) expressed in factors such as the fraction of paid metal Fp, the discount based on grade Dg, and the cost of refining Rfc or treatment charge Trc. These factors can influence the resultant circuit solution. Certainly, the circuit is most sensitive to the variations in metal price, Pm. The factors from (9.a) are considered constants, in spite of their variation during the project period and are only changed in the sensitivity analysis, *i.e.* the uncertainties in G_{conc} (due to variations in mineral grade G_{feed}) or the volatility of Pm, which affects the value of Rfc or Trc, although these are not considered in this class of studies. However, when the feed is classified into *kj* classes, the benefits function allows making comparisons between the relations between grade and recovery, since as the valuable mineral recovery increases, so does the gangue recovery, among other consequences (Cisternas et al., 2004).

A simpler manner of expressing the benefits obtained appears in Eq. (9.b), where $W_{mv,s}$ is the valuable mineral mass flow in a stream *s* in the circuit, and represents the recovery ability of the stages, while $w_{mv,s}$ is the weight-fraction of the valuable mineral on a dry weight basis, and represents the purity of stream where valuable mineral exists. In order to balance the influences of quality and quantity, the authors consider $\beta_1 = 1$ (Chan and Prince, 1989; Prince and Connolly 1996).

Actually, the costs can be classified in a variety of ways, for example those of capital and operation (Schena et al., 1996, 1997; Cisternas et al., 2004), capital only (Chan and Prince, 1989 ; Prince and Connolly, 1996), or even fixed and variable costs (Cisternas et al., 2006).

With regard to capital cost, it is common to use the forms shown in Table 10. Thus it is typical to calculate the capital cost as a power function with respect to the equipment capacity (volume of treated pulp, mass treatment by the equipment, etc.) shown in (10.a), where α_2 and β_2 are cost coefficients. The associated operation costs can be calculated based on, for example, energy consumption for bubbles generation in the cells, milling, or pumping, as well as other operations related to each step of the process. The cost of pumping was originally used to restrict excessive splitting in flow streams within a circuit, but in comparison with the magnitude order of other costs, that of pumping can be considered negligible (Cisternas et al., 2006).

Eq. (10.a) introduces non-linearities into the optimization problem, whose solution depends on number of equipment units (cells in this case) and unit cost. This requires the designer to provide equations in which mass flows of solids and pulp are considered, as well as considerations of residence time as reiterated by Schena et al. (1997). The existence of non-linearities due to this factor can be avoided when this cost function is linearized (e.g. following a Taylor series, expressed as in (10.b)). In this case Cisternas et al. (2006) decided to linearize the function in terms of mass flow treated by the equipment (cells, columns, or mills). Thus the coefficients α_3 and β_3 become those of fixed and variable costs, respectively.

It can be seen objective function has undergone a certain degree of evolution which allows approximation of solutions for circuits which fulfill real requirements. It can also be said the technical–economic concept in problems of implicit multi-objective optimization is the most utilized. Besides the resolution method, the objective function definition depends on the description of the mineral, the models used, and the technical or technical–economic emphasis on the process. Future study is required which allows working with uncertainty in market factors which affect the results and ways to incorporate environmental objective function. Further, for a design that considers current issues, it is necessary to introduce the cost of energy of the grinding systems and the consumption of fresh water in flotation circuits.

Table 9
Benefit functions used in Groups A, B and C

Key	Benefit functions	References
9.a	$[W_{conc}Fp(G-Dg)(Pm-Rfc)-W_{conc}Trc]H$	Schena et al. (1996, 1997), Cisternas et al. (2004, 2006)
9.b	$kv\ W_{mv}\ w_{mv}^{\beta_1}$	Chan and Prince (1989), Prince and Connolly (1996)

Table 10
Equipment costs in the Group A–D studies

Key	Cost functions	References
10.a	α_1 (Equipment capacity)$^{\beta_1}$	Schena et al. (1996, 1997), Chan and Prince (1989), Prince and Connolly, (1996)
10.b	$\alpha_2+\beta_2$ (mass flow of treated material)	Cisternas et al. (2004, 2006)

7. Results in the literature studied

We emphasize, in this portion of the review, that it is not possible the comparison between all the cited studies from their objective function values because their different approaches use different objective function to solve the circuit configuration, and we review only the more important aspects of solutions in terms of 1) what is offered by the methods, 2) which is ultimately decided, and 3) how sensitive are the obtained circuit solutions to the constraints imposed on the problem.

Circuits design is based on the optimization of four common features examined in the reviewed studies, as outlined in Table 11. The most of the optimization algorithms try to calculate or obtain the optimal numbers of cells per bank, N (column 11.1 of Table 11). Yingling (1993b) attempted to resolve this problem by considering the existence of a lead cell. The algorithm was arranged to proceed by including cells to a maximum permitted number, employing a recovery equation for a single cell as in (3.a) and allowing for the existence of different sized cells. In contrast Schena et al. (1996, 1997) decided to resolve the problem by dealing with non-linearity, dividing the problem into sub-problems to be resolved in series, which does not, however, provide an overall optimal solution. In the studies of Hulbert (1995, 2001), the cells number is related to the overall distribution of

Table 11
Main variables considered in the studies

Studies	(11.1)	(11.2)	(11.3)	(11.4)
Yingling (1993b)	✓		✓	
Hulbert (1995)	✓			✓
Hulbert (2001)	✓			✓
Schena et al. (1996)	✓	✓		
Schena et al. (1997)	✓			
Cisternas et al. (2004)	✓	✓	✓	
Cisternas et al. (2006)		✓		
Chan and Prince (1989)	✓	✓		✓
Prince and Connolly (1996)	✓	✓		✓
Guria et al. (2005a)	✓		✓	
Guria et al. (2005b)	✓		✓	
Guria et al. (2006)	✓		✓	

Legend: (1) Number of cells, (2) Number of stages, (3) Levels of division, (4) Improvement of recycling.

volume in the circuit, while in Cisternas et al. (2004) the MILP algorithm allows writing disjunctions to include different options for banks, containing predefined numbers of cells. The evolutionary methods of C and D only use circuits which are constructed cell by cell, since this is inherent in the method (Chan and Prince, 1989; Prince and Connolly, 1996) or even because it is a requirement of the superstructure (Guria et al., 2005a,b, 2006).

Another of the possibilities offered by the design strategies is the selection of flotation stages (column 11.2), that is, where the circuit does not contain a predetermined number of steps, and allowing for the choice of only some of those available. Yingling (1990, 1993b) provides the possibility for having isolated circuits from the rest of the superstructure; even though the banks always have one cell, if the algorithm reveals a zero feed flow, the bank is not considered. Schena et al. (1996, 1997) make flotation stages selection in a similar way, but depending of value for number of cells N, such that if there is zero cell in a bank, then there is no bank selection. Again, the selection of flotation stages in Cisternas et al. (2004, 2006) is related to the disjunctive binary programming employed in order to include a discrete group of possibilities. The heuristic method of Chan and Prince (1989), and Prince and Connolly (1996) gradually review the existence of cells, and once the solution is obtained, groups of cells can be identified which function as banks.

The possibility of flow streams splitting toward various stages have been debated in some studies, although this is not within current industry practice. Nevertheless, this possibility is explored to confirm if the circuits with stream flow divisions are optimal; such papers are shown in column (11.3). The algorithm of Yingling (1993b) is able to accept the division in any proportion, as well as restricting this option by means of the "flow distributor node". The study by Cisternas et al. (2004) suggests division nodes are able to select a discrete group of division levels using disjunctive programming, while in the methods of Group D, it is always possible to divide the flows at any level. The studied examples indicate streams separation does not generally add significant improvement to the circuits.

Recycling improvement (column 11.4 of Table 11) should be understood as the criteria which must be considered for mixing the different flow streams. This occurs only in Hulbert (1995, 2001) and in the Group C studies.

Table 12 shows the characteristics of the streams considered in each works. Also, when the results are obtained, it is important to consider the description of the stream; for example the feed stream, as included in column (12.1). With the exception of Hulbert (1995, 2001), all the reviewed papers provide at least generalized mineralogical characterizations of the feeds (*eg.*; "valuable mineral" or "gangue). Logically, this leads to consideration of different ω constants, although only one study considers different kinetics for the same mineralogical and granulometric class, that is, in terms of "fast flotation kinetics", or slow flotation", the model for which is in (3.f). Few studies consider particle size class, and although none of them specify particle size, some of the cases mention general particle size to be more rigorous in their descriptions (Yingling 1990, 1993b), and others in reference to the needs of the milling model utilized (Schena et al., 1997; Prince and Connolly, 1996).

Table 12
Stream characteristics considered in optimal circuits

Studies	(12.1)	(12.2)	(12.3)	(12.4)
Yingling (1990)	*mx–sz*			
Yingling (1993b)	*mx–sz*		✓	
Schena et al. (1996)	*mx*			
Schena et al. (1997)	*mx–sz–kin*			
Cisternas et al. (2004)	*mx*		✓	
Cisternas et al. (2006)	*mx*			
Chan and Prince (1989)	*mx*			
Prince and Connolly (1996)	*mx–sz*			
Guria et al. (2005a)	*mx*	✓	✓	✓
Guria et al. (2005b)	*mx*	✓	✓	✓
Guria et al. (2006)	*mx*		✓	✓

Legend: (12.1) Mineralogical characterization (12.2) Division of feeds, (12.3) Levels of stream division, (12.4) Intermediate products. Symbology: *mx*: mineralogy, *sz*: size, *kin*: kinetic.

Table 13
Data on equipment considered for circuit solutions

Studies	(13.1)	(13.2)	(13.3)	(13.4)	(13.5)
Yingling (1993b)	✓				
Hulbert (1995)		✓	✓		
Hulbert (2001)			✓		
Schena et al. (1996)	✓	✓			✓
Schena et al. (1997)		✓			✓
Cisternas et al. (2004)	✓				
Cisternas et al. (2006)				✓	✓
Chan and Prince (1989)	✓	✓			
Prince and Connolly (1996)	✓	✓			✓
Guria et al. (2005 a)		✓	✓		
Guria et al. (2005b)		✓	✓		
Guria et al. (2006)		✓			

Legend: (13.1) Number of cells, (13.2) Volume of each cell (13.3) Volume of all fixed cells (13.4) Presence of flotation columns, (13.5) Presence of milling and/or classification.

Although in Table 6 many of the studies suggest splitting the feed stream, only a few does this option appear in the optimal solution. In the study of Guria et al. (2005a,b) this division is effected by the GA utilized; in Guria et al. (2006) the division is obtained using non-genetic algorithms. This does not necessarily mean the division of other streams should be restricted (column 12.3); in fact, stream division can occur in a good number of the studies, although under special conditions. It occurs, for example, in Yingling (1993b), when the algorithm explicitly accepts the division, but in Cisternas et al. (2004) a numerous group of division levels must be delivered, although this is no guarantee for inclusion; in any case, improvements in the objective function like (8.b) are small in comparison with the large computational effort required. Streams splitting are quite common in circuits illustrated by the Group D papers; in some cases this allows for the existence of many intermediate products, that is, either concentrate or tailings streams which are not recycled to another step based on the arrangement of the superstructure.

One of the proposals of the design studies is to obtain the appropriate equipment dimensions required to make up the circuit. In Table 13 some important features in equipment design are shown. All the studies provide for the existence of cells (many times as the only flotation equipment) but in different degrees. For example, the optimal number of cells N per bank (column 13.1) has been highly studied, especially in cases where the recovery depends on it as in Eq. (3), that is, the assumptions of the kinetic model have a great influence on this aspect: as N increases, and knowing that it is impossible to have an infinite number of mixers in series, the solution for recovery approximates to plug flow behavior, theoretically the most recuperative equipment type for flotation circuits (O'Connor et al., 1994; Loveday and Brouckaert, 1995). Furthermore, although it is not clear particle-bubble collisions follow always a first order kinetic in industrial scale (Schena et al., 1997), this assumption, together with the consideration that cells are completely mixed reactors (Schena et al., 1996; Deglon et al., 2000) are useful for the resolution of the problem, because it can be shown that under optimal conditions all the cells in a bank have the same volume for reaching a given conversion (Levenspiel, 1999). In Yingling (1993a,b) N is determined cell by cell, since the existence of a following cell is related to its impact on control policy; which allows considering different volumes in each cell, overcoming the consequence of having mixers in a first-order series. Schena et al. (1996) obtain N on the basis of a discrete group of equipment dimensions, while in Cisternas et al. (2004) the method chooses between a finite set of banks with a given N. The number of cells in Group C is obtained cell by cell based on their inherent design strategy. In practice, there are other aspects such as maintenance, flexibility and operating control points which are also important in order to make a decision about the cell size, number of cells in bank, as well as the number of parallel banks.

The optimum cell unit volume is also studied, as shown in column (13.2), since this is related to the throughput capacity and the residence time τ and also to the cost functions. In all the cited papers there is a calculation of the equipment dimensions, even if an optimal volume is not shown; it is important to note the volume of each cell is not fixed, and can assume mostly continuous values, or discrete values as in Schena et al. (1997). Nonetheless, the sum of these volumes can be fixed in some cases (column 13.4), such that in obtaining N and cells volume, a relationship is obtained which is consonant with the optimal distribution of the equipment.

Although flotation columns are commonly included in many concentration plants (Schena and Casali, 1994), this equipment is not explicitly considered, except by Cisternas et al. (2006) (column 13.4), who obtained flotation columns in cleaner stages, generally supported by a milling unit without size classification, considering their existence by disjunctive programming. In Prince and Connolly (1996), "flotation units" are vaguely mentioned, allowing the possibility for the presence of either columns or cells. This work also allows mills and hydrocyclones separately. In the algorithms of Schena et al. (1996, 1997) there is no option for discriminating between the presence of mills and hydrocyclones, and they are simply included. The difficulty of including milling models to re-create changes in liberation and changes in particle size is due to the complexity of the models available.

Equipment location and functionality within the circuits are important in addition to knowing the inherent equipment properties as shown in Table 14. The numbers given in this Table represent minima and maxima in the configurations, with a single digit implying that there is no selection of stages; here "stages" means cells banks configurations (Groups A and B) or single cells configurations (Groups C and D). All studies in Table 14 have a single primary stage, that is, it assumes the action of taking up the feed stream and separating it into two streams for subsequent treatment. A pair of studies shown it is possible to select a scalper stage before the rougher one which can produce a high-grade concentrate, but its tailing is treated in a bank of rougher cells (column 14.1).

The number of cleaner and scavenger steps (columns 14.2 and 14.3) depend on either superstructure propose to solve the synthesis problem (Groups A,B,D) or the gradual synthesization of the circuit (Group C); it is difficult to obtain a general rule for the average number of stages which emerge from each solution reported, existing a high degree of circuit flexibility. There are cases among the group D studies where no cleaner cells are noted, and in Groups A and D which do not show scavenger stages, leaving the greatest responsibility for recovery to the first stage. In other studies, highly recuperative circuits may exist, i.e., there exist scavenger stages, but finally the concentrate refining was considered most important, i.e. number of cleaner stages is generally greater than number of scavenger stages.

Table 14
Number of flotation and auxiliary stages in circuit solutions

Studies	(14.1)	(14.2)	(14.3)	(14.4)	(14.5)
Yingling (1993b)		1–3			
Hulbert (1995)		3	2		
Hulbert (2001)		3	2		
Schena et al. (1996)	0–1	1–3	1		
Schena et al. (1997)	1	3		2	1
Cisternas et al. (2004)		2–4	0–1		
Cisternas et al. (2006)		2	1	1–2	
Chan and Prince (1989)		4–8	3		
Prince and Connolly (1996)		2–6	2–5	0–2	0–1
Guria et al. (2005a)		1			
Guria et al. (2005b)		0–1	0–3		
Guria et al. (2006)		0–2	1–3		

Number of steps of; (14.1) Scalper, (14.2) Cleaner, (14.3) Scavenger, (14.4) Milling, (14.5) Classification.

Table 15
Economic factors studied in the sensitivity analyses

Studies	(15.1)	(15.2)	(15.3)	(15.4)
Yingling, 1993b			✓	
Schena et al., 1996	✓			✓
Schena et al., 1997	✓	✓		
Cisternas et al., 2004	✓			
Cisternas et al., 2006	✓			
Chan and Prince 1989			✓	

Legend: (15.1) Metal price, Pm, (15.2) Energy cost, (15.3) Capital costs, (15.4) Treatment charges Trc.

In the other hand, the presence of milling and classification stages is important (columns 14.4 and 14.5), which is something which appears not to have been obtained at the time of the Yingling (1993a) review. Number of flotation stages, milling stages and classification stages are shown in separate items in Table 14, to appreciate those works where milling models include classification. Studies of Schena et al. (1996, 1997) require the existence of regrinding with/without classification stages as flotation circuit condition, where the relatively complex models used in the simulation of this portion of the circuit are noteworthy. In studies which select regrinding mills, these are not always accompanied by classification units, that is, hydrocyclones. Also, in the case of Prince and Connolly (1996), a classification step may be present as a single node in the circuit.

The main sensitivity analyses are those which vary the economic conditions of the problem, and those which vary the characteristics of the feed stream. Table 15 shows the studies which analyze the sensitivity of circuits to factors such as the metal price (15.1). This is one of the most-studied factors, and its importance is rooted in the impossibility of the models and optimization strategies for representing changes for volatile price of metal in a broad horizon, concluding that there is a prices range for which the circuit is valid. Thus, by surpassing certain metal price threshold, stages interconnections change for a new optimal flotation circuit; for example, at high metals prices, liberation of the mineral becomes profitable, permitting the occurrence of a mixture of rougher and scavenger concentrates for its liberation in milling units, and subsequent treatment in cleaner stages (Cisternas et al., 2006).

This is certainly not the only possible sensitivity studies. For example, energy cost variations may be analyzed (column 15.2), considering the intensity of its use in concentration operations. Schena et al. (1997) indicate that simultaneous changes in metal price Pm and in energy cost causes modifications in the numbers of cells per bank required at each stage.

Capital costs (column 15.3) can also promote changes in a circuit solution. In Yingling (1993b), the increase in capital cost is a "charge per unit mass of circulating load" which negatively affects the objective profit function, reducing the size of original circuit, creating a group isolated from the rest, and causing a necessary redistribution in the interconnection if the costs are very high; under opposite circumstances, the circuit increases in size. A similar effect was observed in the study by Chan and Prince (1989).

Changes in input conditions of circuit feed stream are also important. For example, Table 16 reviews three important variations. The first of

Table 16
Feed factors studied in the sensitivity analyses

Studies	(16.1)	(16.2)	(16.3)
Cisternas et al., 2004			✓
Cisternas et al., 2006	✓	✓	
Chan and Prince (1989)	✓		✓
Guria et al. (2006)	✓		

Legend: (16.1) Feed stream grade, (16.2) Input mineralogical distribution, (16.3), Change in the kinetic constant ω.

these is related to a decrease in valuable mineral grade (column 16.1), a common occurrence in many extractive mining operations. Guria et al. (2006) studied the influence of the feed grade on circuit configuration for a given recovery level; for high grade feed stream, the circuit tended to recover more, while at lower grades more cleaner steps were required. Chan and Prince (1989) reached the same conclusion, but recommended a decrease in scavenger and cleaner steps number when the grade was low. In Cisternas et al. (2006) cleaning circuits are useful when the grade increased, but also it is found that the sole influence of mineral grade was relative (column 16.2), because optimal circuit configuration may be similar for minerals having different grade values, while for minerals with the same grade value, but with different mineralogical distribution, produce different configurations; the higher or lower quantity of mixed species (that is, a valuable mineral-gangue association) make it more or less attractive to include a mill, respectively, and can change the choice from a column to a mechanical cell. Also, we should notice that none of the reviewed papers include a clear description of the gangue, e.g. the gangue is considered as one species. This aspect is important, since gangue can contain hazardous materials besides the harmless materials.

Another aspect studied is the change in flotation kinetics of the species expressed in the ω (column 16.3), due to changes in the flotation conditions. This can also provoke changes in number of cells and in the cleaning circuit if the recovery is more attractive, that is, if ω_{mv} increases (Cisternas et al., 2004), although Chan and Prince (1989) affirm that this could occur if the separation is more difficult (similar values of ω for gangue and valuable minerals).

The Group D studies explore the sensitivity of the solution of other optimization algorithms, particularly in testing the robustness of the optimum encountered by mJG, which claims to be a global optimum only by seeming to be better than those found using other methods (genetic algorithms, or other types).

Finally, there are many other factors found in the papers cited which may affect the circuit configuration and are not presently analyzed. These studies remain in doubt as to the acceptance of changes in circuit designs where investment in construction and equipment require large amounts of financial input after construction is completed. It is therefore necessary in the near future to have methods which allow management of uncertainty in prices and costs, feed characteristics, and kinetic parameters through the most correct and descriptive models which provide a single, flexible circuit solution.

8. Conclusions

Studies of Groups A–D present resolution schemes for the flotation circuits centered on the use of models, superstructures and objective functions. This implies implicit or explicit multi-objective optimization which is treated conjointly by optimization methods, producing a variety of results. Comparative evaluations of the virtues or deficiencies in the various solution strategies, have not however, led us to a consensus as to which approach(es), specially models and superstructures, represents the best flotation circuit design.

The supposition of first-order kinetic for particle-bubble collision in mineral flotation, which is the basis for modeling efforts, is useful given that under optimal conditions the cells volumes in a bank are identical, simplifying curve fitting and equipment sizing, and allowing the inclusion of models for columns under the same assumption. Nevertheless, a better understanding of flotation mechanism would permit the application of techniques such as the *attainable region* and will allows to include variables not included in the first-order model, which would be useful in identification of possible limits of operation and the identification of more adequate equipment and circuits. On other hand, parameters and grades distribution and determination of a certain optimal number of feed classes has not been carefully studied, and the kinetic values show variability which no method among those studied has been able to resolve.

Inclusion of grinding system is necessary, but the milling models currently used which are oriented towards either energy use or population balance require more detailed study which permits, among other things, the determination of effects such as the liberation, energy use and water consumption. Simple models are required to be able to integrate those models in the flotation circuit synthesis problem.

Superstructures in the presently reviewed papers are a bit more complex than in the older literature, and can be designed for the selection of states, tasks and equipment.

The objective functions are definitely technic–economic, and enter into the decision making process in which the need for a high quality concentrate can be balanced against the requirements for large quantities of concentrate. In some cases functions are added which maximize the number of unconnected streams, thus constructing simpler circuits. The consideration of environmental issues is a challenge not considered and it needs attention. The challenge also remains with respect to the integration of uncertainty, particularly with metals prices, which would allow more reliable economic evaluation over time, and in the feed grades for designing circuits which could be easily adapted to changes in the mineral characteristics. Also, it is necessary to introduce the cost of energy of the grinding systems and the consumption of fresh water in flotation circuits.

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