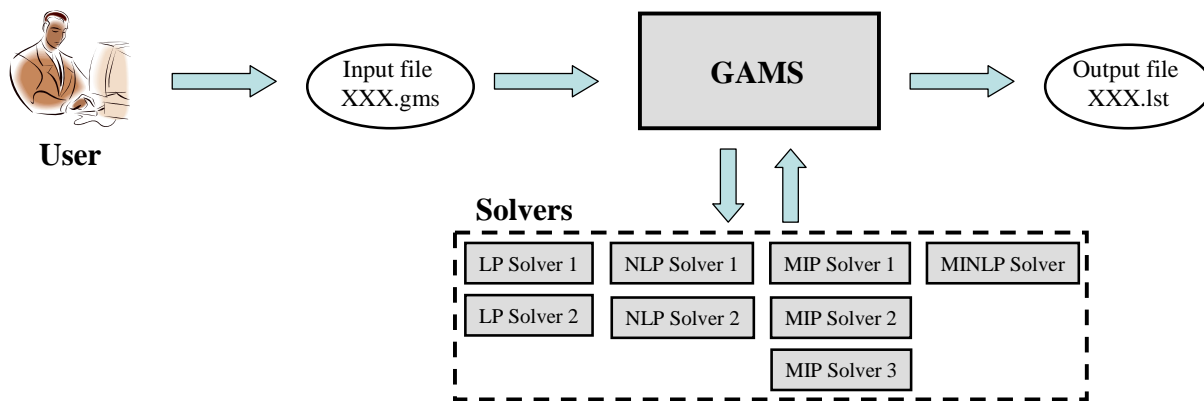


**Pan American Advanced Studies Institute Program:  
Process Modeling and Optimization for Energy and Sustainability**

**GAMS Workshop #1**

GAMS is a modeling system for optimization that provides an interface with a variety of different solvers (algorithms). The user supplies a model (i.e. parameters, variables and equations/constraints) using a modeling language in an input file (XYZ.gms). The model (i.e. the input file) is compiled by GAMS and “passed” to the appropriate solver. The solution found by the solver is communicated back to GAMS, and reported via an output file (XYZ.lst). A simple diagram of the information flow is given bellow.



We will use GAMS to illustrate some basic optimization principles and solve some simple optimization problems.

**1 Starting a New Project:**

1. File → Project → New Project
2. Specify a name for the project and click "Open"

**2 Starting a new .gms (model) file:**

1. File → New
2. Upon saving file will be given .gms suffix

You can now begin editing the .gms file you created

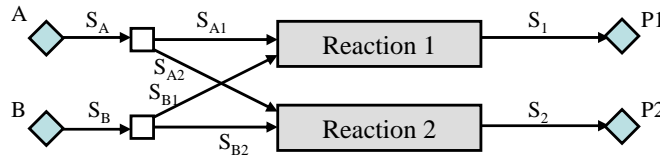
## A Simple Example

Our process is used for the production of final products P1 and P2 from raw materials A and B according to the irreversible reactions (conversion 100%):



If the maximum availability of raw materials A and B is 10 kg/hr, the price of both chemicals is \$100/kg, and the demand for products P1 and P2 is 12 and 13 kg/hr, respectively, what is the optimal production mix?

The problem can be represented as the network in Figure 2 (the labels refer to the streams, not species):



**Figure 2:** Network representation of Example 1

In class we showed that if  $F_{P1}$  and  $F_{P2}$  are the amounts of products P1 and P2, respectively, the optimization problem (P1) can be stated as follows:

$$\begin{aligned} \max Z &= 100F_{P1} + 100F_{P2} \\ \text{st. } 2F_{P1} + F_{P2} &\leq 30 \\ F_{P1} + 2F_{P2} &\leq 30 \\ 0 \leq F_{P1} &\leq 12, \quad 0 \leq F_{P2} \leq 13 \end{aligned} \quad (\text{P1})$$

Model (P1) has two continuous variables ( $F_{P1}$  and  $F_{P2}$ ), two linear inequality constraints and a linear objective function. It is, therefore, a Linear Programming (LP) model. To solve (P1) in GAMS we edit the following 25-line file:

```

$ TITLE Example1                                1
$ OFFSYMXREF                                    2
$ OFFSYMLIST                                    3
                                                    4
POSITIVE VARIABLES FP1, FP2;                    5
VARIABLE Z;                                     6
                                                    7
EQUATIONS CONSTR1, CONSTR2, OBJECTIVE;         8
                                                    9
CONSTR1..      2*FP1 + FP2 =L= 30;              10
CONSTR2..      FP1 + 2*FP2 =L= 30;              11
OBJECTIVE..    Z =E= 100*FP1 + 100*FP2;        12
                                                    13
FP1.UP = 12;                                     14
FP2.UP = 13;                                     15
                                                    16
MODEL P1 /ALL/;                                  17
                                                    18
OPTION LIMROW = 0;                               19
OPTION LIMCOL = 0;                               20
OPTION SOLPRINT = off;                           22
                                                    23
SOLVE P1 USING LP MAXIMIZING Z;                 24
DISPLAY FP1.L, FP2.L;                           25

```

## Notes:

1. The \$ sign in column 1 is a control directive, the first for specifying the title, the other two for suppressing some details in the output. In general, you will always include these keywords (see GAMS User's Guide).
2. The keyword VARIABLE(S) is used to list our variables FP1 and FP2. Note that Z, the objective function value *must* also be included. The keyword POSITIVE VARIABLES is used to specify the non-negativity of FP1 and FP2. The objective function values *should not* be included here as in general it might take positive or negative values. Finally, note that the semicolon ; must be included to specify the end of the lists.
3. Next, the keyword EQUATIONS is for listing the names of the constraints and objective function. The names are arbitrary and here we have selected the names CONSTR1 and CONSTR2 for the constraints and OBJECTIVE for the objective function.
4. The actual equations are then defined by first listing the name followed by two periods. Note that the following syntax must be used for the equality and inequality signs:
  - =E= for *equal* (=)
  - =L= for *less or qual* ( $\leq$ )
  - =G= for *greater or qual* ( $\geq$ )
5. The basic arithmetic operators are:
  - +, - *addition, subtraction*
  - \*, / *multiplication, division*
  - \*\* *exponent*
6. In the input file we can specify the lower and upper bounds as well as initial values. This is done by adding a subfield to the variables. The format is a period followed by a character, and they are as follows:
  - .LO lower bound
  - .UP upper bound
  - .L level value, meaning "actual" value.Note that we do not need to specify lower bounds of zero for FP1 and FP2, because we used the keyword POSITIVE VARIABLES before. Also, it is not a requirement to always specify initial values for the variables. If we do not, GAMS will set them to the lower bounds.
7. The keyword MODEL is used to name our model and to specify which equations should be used. In this case we name our model as P1 and specify that *all* equations be used.
8. Next, the OPTION statements are used to suppress output for debugging. The User's Guide gives a detailed explanation of this keyword. In most cases you want to use both the OPTION LIMROW = 0 and OPTION LIMCOL = 0 to avoid long output files.
9. Finally, we invoke the optimization algorithm with the SOLVE statement. The format is as follows:

SOLVE (model name) USING (solver type) MINIMIZING or MAXIMIZING (objective variable)

The main solver types available in GAMS are the following:

- LP: linear programming
- NLP: nonlinear programming
- MIP: mixed-integer linear programming
- RMIP: relaxed MILP where the integer variables are treated as continuous.
- MINLP: mixed-integer nonlinear programming

If we run the GAMS file (*File* → *Run* or *press F9* or *button with red arrow*) we get the output file Example1.lst, shown in the next page (note that blank lines and repeated output have been omitted).

C o m p i l a t i o n

```
4
5 POSITIVE VARIABLES FP1, FP2;
6 VARIABLE Z;
7
8 EQUATIONS CONSTR1, CONSTR2, OBJECTIVE;
9
10 CONSTR1..      2*FP1 + FP2 =L= 30;
11 CONSTR2..      FP1 + 2*FP2 =L= 30;
12 OBJECTIVE..    Z =E= 100*FP1 + 100*FP2;
13
14 FP1.UP = 12
15 FP2.UP = 13
16
17 MODEL P1 /ALL/;
18
19 OPTION LIMROW = 0;
20 OPTION LIMCOL = 0;
21 OPTION SOLPRINT = off;
22
23 SOLVE P1 USING LP MAXIMIZING Z;
24
25 DISPLAY FP1.L, FP2.L;
```

COMPILATION TIME = 0.000 SECONDS 2.2 Mb WIN213-138 Feb 03, 2004

Model Statistics SOLVE P1 Using LP From line 21

MODEL STATISTICS

BLOCKS OF EQUATIONS	3	SINGLE EQUATIONS	3
BLOCKS OF VARIABLES	3	SINGLE VARIABLES	3
NON ZERO ELEMENTS	7		



GENERATION TIME = 0.010 SECONDS 2.9 Mb WIN213-138 Feb 03, 2004

EXECUTION TIME = 0.010 SECONDS 2.9 Mb WIN213-138 Feb 03, 2004

Solution Report SOLVE P1 Using LP From line 21

S O L V E S U M M A R Y

MODEL	P1	OBJECTIVE	Z
TYPE	LP	DIRECTION	MAXIMIZE
SOLVER	Cplex	FROM LINE	23



```
**** SOLVER STATUS 1 NORMAL COMPLETION
**** MODEL STATUS 1 OPTIMAL
**** OBJECTIVE VALUE 2000.0000
```



RESOURCE USAGE, LIMIT 0.000 1000.000  
ITERATION COUNT, LIMIT 2 10000

GAMS/Cplex Jan 19, 2004 WIN.CP.CL 21.3 025.027.041.VIS For Cplex 9.0  
Cplex 9.0.0, GAMS Link 25

Optimal solution found.  
Objective : 2000.000000

```
**** REPORT SUMMARY : 0 NONOPT
                      0 INFEASIBLE
                      0 UNBOUNDED
```

E x e c u t i o n

```
---- 23 VARIABLE FP1.L = 10.000
      VARIABLE FP2.L = 10.000
```



EXECUTION TIME = 0.000 SECONDS 2.4 Mb WIN213-138 Feb 03, 2004

## **GAMS Output:**

1. The first part of the output is identical to the input file.
2. The statistics on the problem size are reported next (e.g. 3 variables: FP1, FP2 and Z; 3 equations: 2 constraints and objective).
3. The name (P1) and type (LP) of the model, the solver (CPLEX) that was used to solve the problem, the variable for the optimization (Z), the direction of optimization (MAXIMIZE) and the line of the corresponding SOLVE statement (23) are given next.
4. The solve summary indicates that the optimum has been found with an objective function value  $Z = 2,000$ .
5. Finally, due to the DISPLAY command in line 25, the values of variables FP1 and FP2 are reported
6. A detailed explanation of GAMS output can be found in the User's Guide:

*Help* → *GAMS Users Guide*.

You can also see: *Help* → *docs* → *gams* → *Tutorial.pdf*

*Help* → *docs* → *gams* → *gamside.pdf*

### Practice Problem 1

A company has two alkylate plants,  $A_1$  and  $A_2$ , from which a given product is distributed to customers  $C_1$ ,  $C_2$ , and  $C_3$ . The transportation costs are given as follows:

Refinery	$A_1$	$A_1$	$A_1$	$A_2$	$A_2$	$A_2$
Customer	$C_1$	$C_2$	$C_3$	$C_1$	$C_2$	$C_3$
Cost, \$/ton	25	60	75	20	50	85

The maximum refinery production rates and minimum customer demand rates are as follows:

Customer or refinery	$A_1$	$A_2$	$C_1$	$C_2$	$C_3$
Rate, tons/day	1.6	0.8	0.9	0.7	0.3

The cost of production for  $A_1$  is \$30/ton for production levels less than 0.5 ton/day; for production levels greater than 0.5 ton/day, the production cost is \$40/ton.  $A_2$ 's production cost is uniform at \$35/ton. Find the optimum distribution policy to minimize the company's total costs.

### Practice Problem 2

A fertilizer company mixes three different types of chemicals in their Premium blend which sells for \$1.00 per pound. Their less expensive blend (Regular) sells for \$0.80 per pound and contains only two types of chemicals. The mixing formula for both blends is shown below.

Blends:	Chemicals		
	Type A	Type B	Type C
Premium	10%	5%	5%
Regular	0%	15%	5%
Amount available	400 lb	300 lb	500 lb

The remainder of each blend is inert filler material which comes from a nearby pit mine at a cost of \$0.10/lb. An unlimited supply of filler is available. The cost per pound of type A, type B and type C chemicals is \$0.30, \$0.25, and \$0.21, respectively. Determine the optimal amount of each blend that should be prepared; leftover chemicals can be used again. Solve using liner programming.

### Practice Problem 3

A farmer with 100 acres wishes to optimize his crop of tomatoes, cabbage, and lettuce. He can sell tomatoes at 20c a pound, lettuce at 18c a head, and cabbage at 25c a head. The average yield per acre is 1500 pounds of tomatoes, 2500 heads of lettuce, and 1500 heads of cabbage. Labor required per acre during the growing season is five man-days for tomatoes and six man-days for both lettuce and cabbage. Within the cooperative, 400 man-days of labor are available and each laborer who is a member of the cooperative is paid \$30 per man-day. Fertilizer costs 15c per pound; fertilizer requirements are: 125 pounds per acre of tomatoes and 50 pounds per acre of lettuce and cabbage. Based on this information, we desire to find the best way to use land and manpower to maximize profits.

1. Set up this problem as a linear programming problem.
2. Solve using GAMS.
3. Which constraint is not active; why is not all of the acreage being used?
4. Would higher prices for cabbage or lettuce
  - change the optimum?
  - leave it unchanged?
  - do either, depending upon the effect of the price changes?

Discuss your conclusions regarding the effect of these price changes.