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**Fruit Industry Supply Chain Model**

This document presents the mathematical equations that represent the supply chain model of the Frutas & Jugos ARG Co. shown in Figure 1.

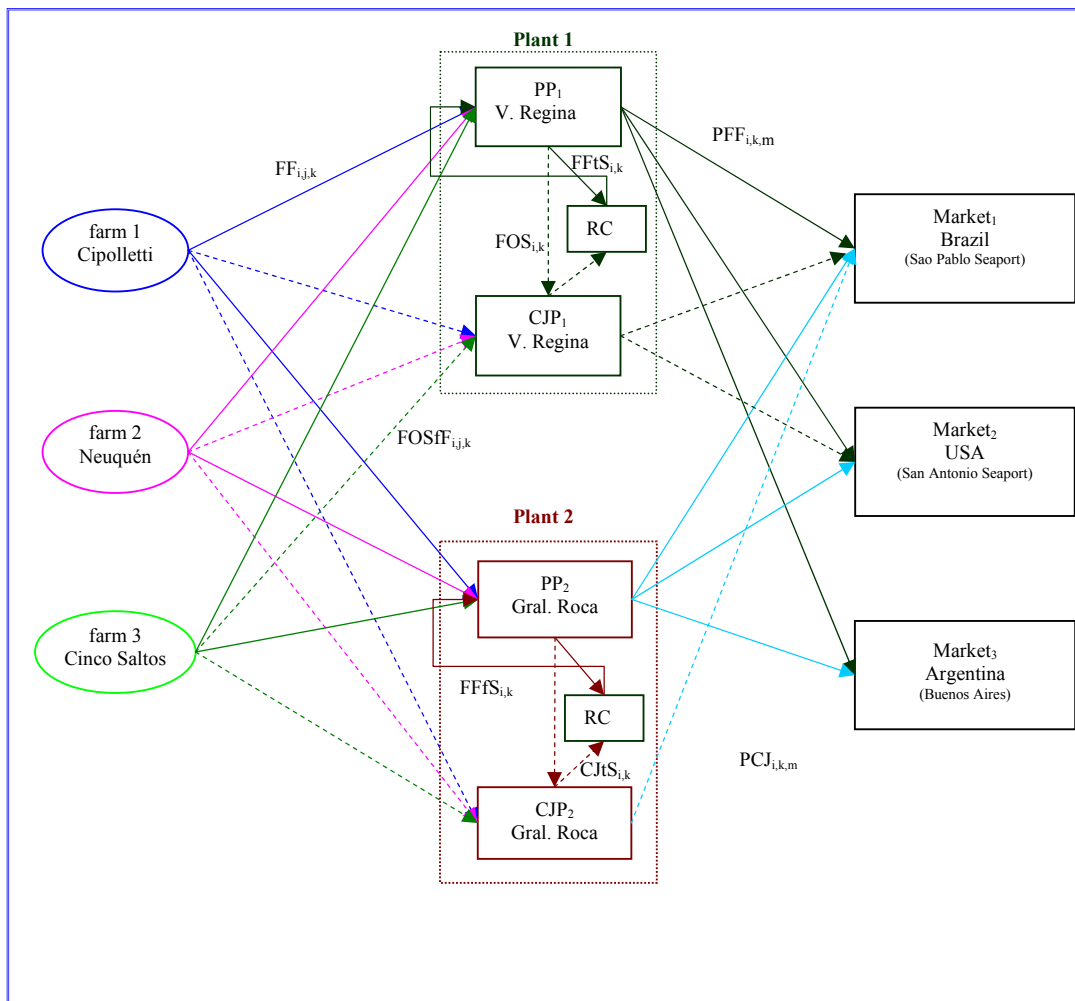


Figure 1 – Frutas & Jugos ARG Supply Chain

This company own two sites in the southwest of Argentina: one at Villa Regina (V.Regina) and the other one at General Roca (Gral.Roca) 25 miles away each other. Each one operates a packaging and concentrated juice plants. They can acquires raw material (apples and pears, streams  $FF_{i,j,k}$ ) from fruit suppliers (farms 1, 2 and 3) to feed packaging plants (PP<sub>1</sub> and PP<sub>2</sub>) or concentrate juice plants (CJP<sub>1</sub> and CJP<sub>2</sub>) The company produces final products consisting in fresh fruit prepared and packed in different ways (PFF<sub>i,k,m</sub>) and concentrate juice of 72 degree Brix (PCJ<sub>i,k,m</sub>).

Many decisions have to be made along the SC to decide where and how much fresh fruit to acquire for processing. At a packaging plant after raw material reception, a decision has to be made whether the fruit is directly sent to cold storage for later processing or to the processing line. The processing stage implies several steps consisting in washing, manual and automatic classification (by size, color, external aspect, etc.), waxing (if required), and packed in different ways depending on customer preferences. Another decision that have to be made at this stage is whether the fruit is keep in cold storage for later selling or processing.

A model has been developed to maximize the gross profit of the company along a one period trough harvest time, January to May. Based on customer demands from three major markets, estimated fruit production, economic information about costs and prices, and yield and availability of processing plants, the model assign plant operation levels, amount and place where raw material should be obtained and final product delivery.

The fruit that do not fulfill quality specifications at the classification stage is transferred to juice plants to produce concentrate juice. Each juice plant can also be fed with fruit from supplier's farms or from third party packaging plants.

Model parameters include cost of each variety of raw materials, selling prices of each product in different markets, fruit production for each farm and fruit variety, distances among farms, processing plants and markets, capacities at packaging and juice plants, demands for each product and market, etc. The description of the parameters and variables are listed at the end of this document.

## MODEL EQUATIONS

### 1. *PP Mass Balance*

1.a. PP Global Balance

$$FF_{i,k} = \frac{FFtS_{i,k} + PFF_{i,k} - FFfS_{i,k}}{\eta_{PP_{i,k}}}$$

1.b. Fresh Fruit send to Stock

$$FFtS_{i,k} = \eta_{PFF_{i,k}} * MaxPFF_{i,k}$$

1.c. PP<sub>k</sub> Yield

$$FOS_{i,k} = (1 - \eta_{PP_{i,k}}) * FF_{i,k}$$

1.d. PP<sub>k</sub> Maximum Production.

$$PFF_{i,k} = \eta_{MaxPFF_{i,k}} * MaxPFF_{i,k}$$

### 2. *CJP Mass Balance*

2.a. CJP Global Balance

$$FOSfF_{i,k} + FOS_{i,k} + FOSfS_{i,k} = \frac{1}{\eta_{CJP_{i,k}}} * PCJ_{i,k} + FOSStS_{i,k}$$

2.b. Fruit Out of Specifications send to Stock

$$FOSStS_{i,k} = \eta_{PCJ_{i,k}} * MaxPCJ_{i,k} / \eta_{CJP_{i,k}}$$

2.c. PCJ<sub>k</sub> Maximum Production

$$PCJ_{i,k} = \eta_{MaxPCJ_{i,k}} * MaxPCJ_{i,k}$$

### 3. *Fruit Supplier's Distribution*

3.a. Fresh Fruit from farm *j* to PP<sub>k</sub>

$$FF_{i,j,k} = \alpha_{PP_{i,j,k}} * FF_{i,k}$$

3.b. Fresh Out of Specification from farm *j* to CJP<sub>k</sub>

$$FOSfF_{i,j,k} = \alpha_{CJP_{i,j,k}} * FOSfF_{i,k}$$

3.c.

$$\sum_j \alpha_{PP_{i,j,k}} = 1 \quad \sum_j \alpha_{CJP_{i,j,k}} = 1$$

### 4. *Demand Distribution*

4.a. Packed Fresh Fruit delivered by plant *k* to market *m*.

$$PFF_{i,k,m} = \beta_{PP_{i,k,m}} * PFF_{i,k}$$

4.b. Packed Concentrate Juice delivered by plant *k* to market *m*.

$$PCJ_{i,k,m} = \beta_{CJP_{i,k,m}} * PCJ_{i,k}$$

4.c.

$$\sum_m \beta_{PP_{i,k,m}} = 1 \quad \sum_m \beta_{CJP_{i,k,m}} = 1$$

4.d. Unsatisfied Demand of Packed Fresh Fruit

$$usDPFF_{i,m} = \sum_k PFF_{i,k,m} - DPFF_{i,m}$$

4.e. Unsatisfied Demand of Packed Concentrate Juice

$$usDPCJ_{i,m} = \sum_k PCJ_{i,k,m} - DPCJ_{i,m}$$

5. Costs

5.a. Total Raw Fruit Cost

$$\begin{aligned} TRFC &= TRFC_{FF} + TRFC_{FOS} = \\ &= \sum_{i,k} (cFF_i * FF_{i,k}) + \sum_{i,k} (cFOS_i * FOSf_{i,k}) \end{aligned}$$

5.b. Total Farms to Plants Fruit Transport Cost

$$\begin{aligned} TFPTTrC &= TFPTTrC_{FF} + TFPTTrC_{FOS} = \\ &= tcFF * \sum_{i,j,k} (dj_{j,k} * FF_{i,j,k}) + tcFF * \sum_{i,j,k} (dj_{j,k} * FOSf_{i,j,k}) \end{aligned}$$

5.c. Total Production Cost

$$\begin{aligned} TPC &= TPC_{PP} + TPC_{CJP} = \\ &= \sum_{i,k} (pcPP_{i,k} * PFF_{i,k}) + \sum_{i,k} (pcCJP_{i,k} * PCJ_{i,k}) \end{aligned}$$

5.d. Total Refrigeration Cost

$$\begin{aligned} TRC &= TRC_{FF} + TRC_{CJ} = \\ &= \sum_{i,k} (rcFF * FFtS_{i,k}) + \sum_{i,k} (rcCJ * FOSSt_{i,k}) \end{aligned}$$

5.e. Total Plants to Market Products Transport Cost

$$\begin{aligned} TPMTTrC &= TPMTTrC_{PFF} + TPMTTrC_{PCJ} = \\ &= tcPFF * \sum_{i,k,m} (dm_{k,m} * PFF_{i,k,m}) + tcPCJ * \sum_{i,k,m} (dm_{k,m} * PCJ_{i,k,m}) \end{aligned}$$

5.f. Unsatisfied Demand PFF Cost

$$usDPFFC = \sum_{i,m} (usDPFF_{i,m} * pPFF_{i,m} * prf_{PFF})$$

5.g. Unsatisfied Demand of Packed Concentrate Juice

$$usDPCJC = \sum_{i,m} (usDPCJ_{i,m} * pPFF_{i,m} * prf_{PCJ})$$

5.h. Total Plant Cost

$$Total Cost = TRFC + TFfSC + TFPTTrC + TPC + TRC + TPMTTrC + usDPFFC + usDPCJC$$

6. *Sales*

$$\begin{aligned} Sales &= Sales_{PP} + Sales_{CJP} = \\ &= \sum_{i,k,m} (pPF_{i,m} * PF_{i,k,m}) + \sum_{i,k,m} (pPCJ_{i,m} * PCJ_{i,k,m}) \end{aligned}$$

7. *Gross Profit*

$$Gross\ Profit = Sales - TotalCost$$

## OPTIMIZATION MODEL

### **Objective Function**

$$ObjF = \text{Max GrossProfit}$$

### **Optimization Variables:**

$\alpha_{CJP_{i,j,k}}$ :	FOS fraction, variety $i$ , supplied from farm $j$ to be processed in $CJP_k$ .
$\alpha_{PP_{i,j,k}}$ :	FF fraction variety $i$ , supplied from farm $j$ to be processed in $PP_k$ .
$\beta_{CJP_{i,k,m}}$ :	PCJ fraction sent from $CJP_k$ to market $m$ .
$\beta_{PP_{i,k,m}}$ :	PFF fraction sent from $PP_k$ to market $m$ .
$\eta_{MaxPCJ_{i,k}}$ :	PCJ variety $i$ , produced by plant $k$ as a fraction of max. capacity.
$\eta_{MaxPFF_{i,k}}$ :	PFF variety $i$ , produced by plant $k$ as a fraction of max. capacity.
$\eta_{PCJ_{i,k}}$ :	Fraction of Packed Concentrate Juice produced to be sent to stock.
$\eta_{PFF_{i,k}}$ :	Fraction of Packed Fresh Fruit produced to be sent to stock.

### **Inequality Constraints**

1.  $PFF_{i,k} \leq MaxPFF_{i,k}$
2.  $PCJ_{i,k} \leq MaxPCJ_{i,k}$
3.  $FFtS_{i,k} \leq MaxPFF_{i,k}$
4.  $FOS_{i,k} \leq MaxPCJ_{i,k} / \eta_{CJP_{i,k}}$
5.  $\sum_k PFF_{i,k,m} \leq DPFF_{i,m}$
6.  $\sum_k PCJ_{i,k,m} \leq DPCJ_{i,m}$
7.  $\sum_k (FF_{i,j,k} + FOS_{i,j,k}) \leq \eta_{F_{i,j}} * MaxFP_{i,j}$
8. All variables  $\geq 0$

## NOMENCLATURE

<i>Parameters</i>	<i>Units</i>	<i>Description</i>
$cFF_i$	\$/kg	cost per unit of <b>Fresh Fruit</b> $i$ .
$cFOS_i$	\$/kg	cost per unit of <b>Fruit Out of Specification</b> $i$ .
$cFFfS_{i,k}$	\$/kg	cost per unit of <b>Fresh Fruit from Stock</b> , variety $i$ , plant $k$ .
$cFOSfS_{i,k}$	\$/kg	cost per unit of <b>Fruit Out of Specification from Stock</b> , variety $i$ , plant $k$ .
$d_{j,k}$	km	<b>distance</b> from farm $j$ to plant $k$ .
$dm_{k,m}$	km	<b>distance</b> from plant $k$ to market $m$ .
$DPCJ_{i,m}$	$10^6$ kg	<b>Demand of Packed Concentrate Juice</b> , variety $i$ , market $m$ .
$DPPF_{i,m}$	$10^6$ kg	<b>Demand of Packed Fresh Fruit</b> ; variety $i$ ; market $m$ .
$MaxFP_{i,j}$	$10^6$ kg	<b>Maximum Farm Production</b> ; variety $i$ , farm $j$ .
$MaxPCJ_{i,k}$	$10^5$ gal	<b>Maximum Packed Concentrate Juice</b> production; variety $i$ ; plant $k$ .
$MaxPPF_{i,k}$	$10^6$ kg	<b>Maximum Packed Fresh Fruit</b> production, variety $i$ ; plant $k$ .
$pcCJP_{i,k}$	\$/gal	<b>production cost of Concentrate Juice Plant</b> ; variety $i$ ; plant $k$ .
$pPCJ_{i,m}$	\$/gal	<b>price per unit of Packed Concentrate Juice</b> ; variety $i$ ; market $m$ .
$pcPP_{i,k}$	\$/kgPPF	<b>production cost of Packaging Plant</b> ; variety $i$ ; plant $k$ .
$pPPF_{i,m}$	\$/kg	<b>price per unit of Packed Fresh Fruit</b> ; variety $i$ ; market $m$ .
$prf_{PCJ}$		<b>price reduction factor</b> due to unsatisfied demand of Packed Concentrate Juice.
$prf_{PPF}$		<b>price reduction factor</b> due to unsatisfied demand of Packed Fruit.
$rcCJ$	\$/ $10^3$ gal	<b>refrigeration cost of Concentrate Juice</b> .
$rcFF$	\$/ $10^3$ kg	<b>refrigeration cost of Fresh Fruit</b> .
$tcFF$	\$/km-kg	<b>transport cost per unit of distance and unit of Fresh Fruit</b> from farms to plants.
$tcPCJ$	\$/km-kg	<b>transport cost per unit of distance and unit of Packed Concentrate Juice</b> from plant to market.
$tcPPF$	\$/km-kg	<b>transport cost per unit of distance and unit of Packed Fruit</b> from plant to market.
$\alpha_{CJP_{i,j,k}}$		<b>Fruit Out of Specification fraction</b> , variety $i$ , supplied from farm $j$ to be processed in $CJP_k$ .
$\alpha_{PP_{i,j,k}}$		<b>Fresh Fruit fraction</b> variety $i$ , supplied from farm $j$ to be processed in $PP_k$ .
$\beta_{CJP_{i,k,m}}$		<b>Packed Concentrate Juice fraction</b> sent from $CJP_k$ to market $m$ .
$\beta_{PP_{i,k,m}}$		<b>Packed Fresh Fruit fraction</b> sent from $PP_k$ to market $m$ .
$\eta_{CJP_{i,k}}$		<b>relationship between concentrate juice volume production and unit of fruit</b> processed by $CJP_k$ .
$\eta_{Fi,j}$		<b>Farm yield production</b> , variety $i$ , farm $j$ .
$\eta_{MaxPCJ_{i,k}}$		<b>Fraction of concentrate juice plant capacity</b> to be produced according to market demands.
$\eta_{MaxPPF_{i,k}}$		<b>Fraction of packaging plant capacity</b> to be produced according to market demands.
$\eta_{PCJ_{i,k}}$		<b>Fraction of Packed Concentrate Juice</b> produced to be sent to stock.
$\eta_{PPF_{i,k}}$		<b>Fraction of Packed Fresh Fruit</b> produced to be sent to stock.
$\eta_{PP_{i,k}}$		<b>Fraction of Fresh Fruit from farm</b> that fulfill PP process specification.
$\eta_{CJP_{i,k}}$		<b>Concentrated Juice Plant production factor</b> .

<b>Variables</b>	<b>Units</b>	<b>Description</b>
FFfS <sub>i,k</sub>	10 <sup>6</sup> kg	<b>Fresh Fruit from Stock (RC)</b> , variety <i>i</i> , to be processed by PP <sub>k</sub> .
FF <sub>i,k</sub>	10 <sup>6</sup> kg	<b>Fresh Fruit</b> variety <i>i</i> to be processed by PP <sub>k</sub> .
FF <sub>i,j,k</sub>	10 <sup>6</sup> kg	<b>Fresh Fruit</b> variety <i>i</i> from farm <i>j</i> to be processed by PP <sub>k</sub> .
FFtS <sub>i,k</sub>	10 <sup>6</sup> kg	<b>Fresh Fruit to Stock (RC)</b> , variety <i>i</i> , plant <i>k</i> .
FOSfF <sub>i,k</sub>	10 <sup>6</sup> kg	<b>Fruit Out of Specification from Farm</b> , variety <i>i</i> , to be processed by CJP <sub>k</sub> .
FOSfF <sub>i,j,k</sub>	10 <sup>6</sup> kg	<b>Fruit Out of Specification</b> , variety <i>i</i> , from farm <i>j</i> to be processed by CJP <sub>k</sub> .
FOS <sub>i,k</sub>	10 <sup>6</sup> kg	<b>Fruit Out of Specification</b> , variety <i>i</i> , from PP <sub>k</sub> to CJP <sub>k</sub> .
usDC	10 <sup>6</sup> kg	<b>unsatisfied Demand Cost</b> .
usDPCJ <sub>i,k,m</sub>	10 <sup>6</sup> kg	<b>unsatisfied Demand of Packed Concentrate Juice</b> , variety <i>i</i> from plant <i>k</i> to market <i>m</i> .
usDPFF <sub>i,k,m</sub>	10 <sup>6</sup> kg	<b>unsatisfied Demand of Packed Fresh Fruit</b> , variety <i>i</i> from plant <i>k</i> to market <i>m</i> .
FOSfS <sub>i,k</sub>	10 <sup>6</sup> kg	<b>Fruit Out of Specification from Stock (RC)</b> , variety <i>i</i> to be processed by CJP <sub>k</sub> .
PCJ <sub>i,k,m</sub>	10 <sup>5</sup> gal	<b>Packed Concentrate Juice</b> , variety <i>i</i> , produced by plant <i>k</i> and delivered to market <i>m</i> .
PCJ <sub>i,k</sub>	10 <sup>5</sup> gal	<b>Packed Concentrate Juice</b> , variety <i>i</i> , produced by plant <i>k</i> .
FOSTS <sub>i,k</sub>	10 <sup>6</sup> kg	<b>Fruit Out of Specification to Stock</b> ; variety <i>i</i> , plant <i>k</i> .
PFF <sub>i,k,m</sub>	10 <sup>6</sup> kg	<b>Packed Fresh Fruit</b> , variety <i>i</i> , produced by plant <i>k</i> and delivered to market <i>m</i> .
PFF <sub>i,k</sub>	10 <sup>6</sup> kg	<b>Packed Fresh Fruit</b> , variety <i>i</i> , produced by plant <i>k</i> .
TFPTrC	\$	<b>Total Farm to Plant Transport Cost</b> .
TFPTrC <sub>FF</sub>	\$	<b>Total Farm to Plant Fresh Fruit Transport Cost</b> .
TFPTrC <sub>FOS</sub>	\$	<b>Total Farm to Plant Fruit Out of Specification Transport Cost</b> .
Total Cost	\$	<b>Final Company operating cost</b> .
TPC	\$	<b>Total Production Cost</b> .
TPC <sub>CJP</sub>	\$	<b>Total Concentrate Juice Plant Production Cost</b> .
TPC <sub>PP</sub>	\$	<b>Total Packaging Plant Production Cost</b> .
TPMTrC	\$	<b>Total Plant to Market Products Transport Cost</b> .
TPMTrC <sub>PCJ</sub>	\$	<b>Total Plant to Market PCJ Transport Cost</b> .
TPMTrC <sub>PFF</sub>	\$	<b>Total Plant to Market PFF Transport Cost</b> .
TRC	\$	<b>Total Refrigeration Cost</b> .
TRC <sub>FF</sub>	\$	<b>Total Fresh Fruit Refrigeration Cost</b> .
TRC <sub>CJ</sub>	\$	<b>Total Concentrate Juice Refrigeration Cost</b> .
TRFC	\$	<b>Total Raw Fruit Cost</b> .
TRFC <sub>FF</sub>	\$	<b>Total Raw Fresh Fruit Cost</b> .
TRFC <sub>FOS</sub>	\$	<b>Total Raw Fruit out of specification Cost</b> .
Sales	\$	<b>Company Incomes</b> .
Sales <sub>pp</sub>	\$	<b>Incomes due to Packaging Plants</b> .
Sales <sub>CJP</sub>	\$	<b>Incomes due to Concentrate Juice Plants</b> .
Gross Profit	\$	<b>Company Profit (Sales - operating costs)</b>