1. Consider the constrained NLP:

\[
\begin{align*}
\text{Min} & \quad (A + B) \\
\text{s.t.} & \quad x_1, y_1 \geq R_1, x_1 \leq B - R_1, y_1 \leq A - R_1 \\
& \quad x_2, y_2 \geq R_2, x_2 \leq B - R_2, y_2 \leq A - R_2 \\
& \quad x_3, y_3 \geq R_3, x_3 \leq B - R_3, y_3 \leq A - R_3 \\
& \quad (x_1 - x_2)^2 + (y_1 - y_2)^2 \geq (R_1 + R_2)^2 \\
& \quad (x_1 - x_3)^2 + (y_1 - y_3)^2 \geq (R_1 + R_3)^2 \\
& \quad (x_2 - x_3)^2 + (y_2 - y_3)^2 \geq (R_2 + R_3)^2 \\
& \quad x_1, x_2, x_3, y_1, y_2, y_3, A, B \geq 0
\end{align*}
\]

a) Write the KKT conditions and show that this problem is nonconvex. What are the second order conditions?
b) How does the system of KKT conditions lead to multiple solutions?

2. Consider the NLP:

\[
\begin{align*}
\text{Min} & \quad x_2 \\
\text{s.t.} & \quad x_1 - x_2^2 + 1 \leq 0 \\
& \quad -x_1 - x_2^2 + 1 \leq 0 \\
& \quad x_2 \geq 0
\end{align*}
\]

a) Convert this problem to bound constrained and derive the nonlinear equations for the Newton-based barrier method and solve them for different values of $\mu$.
b) Show the trajectory of $x(\mu)$ and $f(x(\mu))$ as $\mu \to 0$. 
3. Using the coordinate basis, apply range and null space decomposition to solve for the KKT system:
\[
\begin{bmatrix}
W & A \\
A^T & 0
\end{bmatrix}
\begin{bmatrix}
d \\
\lambda
\end{bmatrix}
= -\begin{bmatrix}
\nabla f \\
c
\end{bmatrix}
\quad \text{with} \quad W = \begin{bmatrix}
-1 & 0 & 2 \\
0 & 5 & 3 \\
2 & 3 & 10
\end{bmatrix}, \quad A = \begin{bmatrix}
3 & 5 \\
2 & 1 \\
2 & 2
\end{bmatrix}, \quad \nabla f = \begin{bmatrix}
1 \\
3 \\
5
\end{bmatrix}, \quad c = \begin{bmatrix}
2
\end{bmatrix}
\]

4. Consider the reactor optimal control problem below. Assume that temperature is not a function of time.

Max \( c_2(1.0) \)

s.t. \( \frac{dc_1}{dt} = -k_1(T) c_1^2 \)
\( \frac{dc_2}{dt} = k_1(T) c_1^2 - k_2(T) c_2 \)
\( c_1(0) = 1, c_2(0) = 0 \)

where \( k_1 = 4000 \exp(-2500/T) \), \( k_2 = 62000 \exp(-5000/T) \) and \( T \) lies between \( 298 \) and \( 398 \) K.

a) Develop the direct sensitivity equations to calculate \( \frac{dc_2}{dT} \)

b) Develop the adjoint sensitivity equations to calculate \( \frac{dc_2}{dT} \)

c) Solve both sets of sensitivity equations and compare the results.

5. Consider the reactor optimal control problem below. Assume that temperature is a function of time.

Max \( c_2(1.0) \)

s.t. \( \frac{dc_1}{dt} = -k_1(T) c_1^2 \)
\( \frac{dc_2}{dt} = k_1(T) c_1^2 - k_2(T) c_2 \)
\( c_1(0) = 1, c_2(0) = 0 \)

where \( k_1 = 4000 \exp(-2500/T) \), \( k_2 = 62000 \exp(-5000/T) \) and \( T \) lies between \( 298 \) and \( 398 \) K.

a) Formulate the Euler-Lagrange conditions for this problem. Solve these equations using a boundary value solver such as COLDAE.

b) Formulate this problem as an NLP using orthogonal collocation on finite elements. Solve the problem in GAMS for different numbers of collocation points and finite elements.