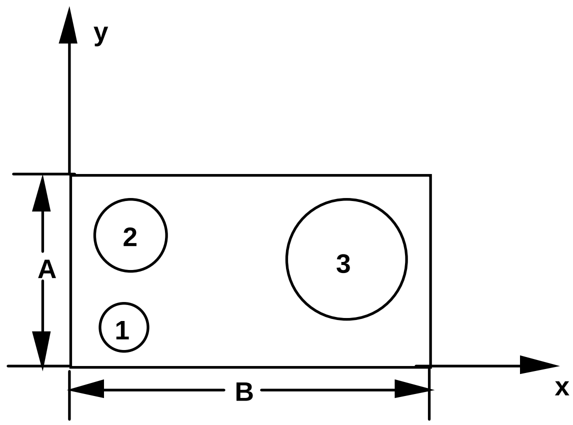
Pan American Advanced Studies Institute
Program on Process Systems Engineering
Nonlinear Programming and Dynamic Optimization
Exercises

1. Consider the constrained NLP:



$$\text{Min} \quad (A + B)$$

$$\begin{cases} x_1, y_1 \geq R_1 & x_1 \leq B - R_1,\ y_1 \leq A - R_1 \\ x_2, y_2 \geq R_2 & x_2 \leq B - R_2,\ y_2 \leq A - R_2 \\ x_3, y_3 \geq R_3 & x_3 \leq B - R_3,\ y_3 \leq A - R_3 \end{cases}$$

$$\begin{cases} (x_1 - x_2)^2 + (y_1 - y_2)^2 \geq (R_1 + R_2)^2 \\ (x_1 - x_3)^2 + (y_1 - y_3)^2 \geq (R_1 + R_3)^2 \\ (x_2 - x_3)^2 + (y_2 - y_3)^2 \geq (R_2 + R_3)^2 \end{cases}$$

$$x_1, x_2, x_3, y_1, y_2, y_3,\ A, B \geq 0$$

a) Write the KKT conditions and show that this problem is nonconvex. What are the second order conditions?
b) How does the system of KKT conditions lead to multiple solutions?

2. Consider the NLP:

$$\begin{aligned} &\text{Min } x_2 \\ \text{s.t.} \quad & x_1 - x_2^2 + 1 \leq 0 \\ & -x_1 - x_2^2 + 1 \leq 0 \\ & x_2 \geq 0 \end{aligned}$$

a) Convert this problem to bound constrained and derive the nonlinear equations for the Newton-based barrier method and solve them for different values of μ.
b) Show the trajectory of $x(\mu)$ and $f(x(\mu))$ as $\mu \rightarrow 0$.

3. Using the coordinate basis, apply range and null space decomposition to solve for the KKT system:

$$\begin{bmatrix} W & A \\ A^T & 0 \end{bmatrix}\begin{bmatrix} d \\ \lambda \end{bmatrix} = -\begin{bmatrix} \nabla f \\ c \end{bmatrix} \quad with\ W = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 5 & 3 \\ 2 & 3 & 10 \end{bmatrix} \quad A = \begin{bmatrix} 3 & 5 \\ 2 & 1 \\ 2 & 2 \end{bmatrix} \quad \nabla f = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \quad c = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

4. Consider the reactor optimal control problem below. Assume that temperature is not a function of time.

Max $c_2(1.0)$

s.t. $dc_1/dt = - k_1(T)\, c_1^2$

$dc_2/dt = k_1(T)\, c_1^2 - k_2(T)\, c_2$

$c_1(0) = 1,\ c_2(0) = 0$

where $k_1 = 4000 \exp(-2500/T)$, $k_2 = 62000 \exp(-5000/T)$ and T lies between 298 and 398 K.

a) Develop the direct sensitivity equations to calculate dc_2/dT

b) Develop the adjoint sensitivity equations to calculate dc_2/dT

c) Solve both sets of sensitivity equations and compare the results.

5. Consider the reactor optimal control problem below. Assume that temperature is a function of time.

Max $c_2(1.0)$

s.t. $dc_1/dt = - k_1(T)\, c_1^2$

$dc_2/dt = k_1(T)\, c_1^2 - k_2(T)\, c_2$

$c_1(0) = 1,\ c_2(0) = 0$

where $k_1 = 4000 \exp(-2500/T)$, $k_2 = 62000 \exp(-5000/T)$ and T lies between 298 and 398 K.

a) Formulate the Euler-Lagrange conditions for this problem. Solve these equations using a boundary value solver such as COLDAE.

b) Formulate this problem as an NLP using orthogonal collocation on finite elements. Solve the problem in GAMS for different numbers of collocation points and finite elements.