(IX) EXERCISES ON ADVANCED PROCESS DYNAMICS AND CONTROL

by

OSCAR D. CRISALLE

Department of Chemical Engineering
University of Florida
Gainesville FL 32611 USA
crisalle@che.ufl.edu

GATOR Engineering
UNIVERSITY OF FLORIDA

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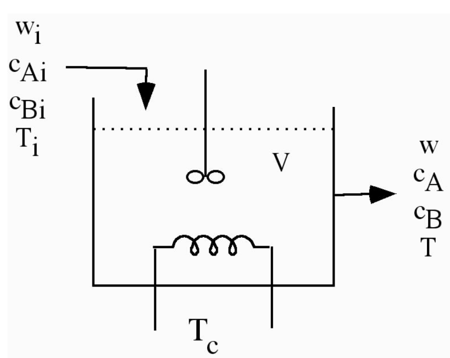
Exercise 1

- (**Problem adapted from problem 4.13 of the textbook by Seborg, Edgar, and Mellichamp**)
  - Derive a nonlinear dynamic model for the reacting system shown in the figure below



w, w_i	mass flow rates
c_A, c_{Ai}, c_B, c_{Bi}	molar concentrations
T, T_i, T_c	temperature
V	reactor volume

- Irreversible chemical reaction
  - $2A \rightarrow 2B$.
  - Exothermic with heat of reaction $(-\Delta H_{rxn})$.
  - Rate of reaction of species A is $r_A = -k\,C$, $k = k_o \exp(-E/RT)$

- Cooling coil
  - Heat transfer area A_c
  - Liquid coolant stays at temperature T_c

- The inlet and outlet mass flow rates may vary with time, depending on upstream and downstream process conditions

Exercise 2

- Consider the matrix $\boldsymbol{A} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$
  - (1) Derive an expression for $det(s\boldsymbol{I} - \boldsymbol{A})$
  - (2) Find the eigenvalues and state their **algebraic** multiplicity
  - (3) Find the eigenvalues and state their **algebraic** multiplicity
  - (4) Find a set of **linearly dependent** eigenvectors
  - (5) Find a set of **linearly independent** eigenvectors
  - (6) State the **geometric** multiplicity of the eigenvalues
  - (7) Calculate the matrix exponential function $e^{\boldsymbol{A}t}$

Exercise 3

- Consider the LTI system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -2 & -20 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

with initial conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

- Calculate the response of the system to the constant-forcing input

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}$$

Exercise 4

- Consider the LTI system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -2 & -20 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

with initial conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

- Is the system controllable?
- Is the homogeneous state-space stable with respect to the initial conditions?
- Is the zero-input system BIBO stable?

Exercise 5

- Consider the LTI system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -2 & -20 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

  with initial conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

- Design a state-feedback control law

$$\boldsymbol{u}(t) = -\boldsymbol{K}\ \boldsymbol{x}(t)$$

  that places the eigenvalues of the closed-loop system at the locations

  { -2, -4 }

- *Hint* 1: Note that the system is controllable when the input matrix is $\boldsymbol{B} = \begin{bmatrix} 2 & 0 \\ -2 & -20 \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 \end{bmatrix}$, and when the input matrix is $\boldsymbol{B} = \boldsymbol{b}_1$. On the other hand, the system is not controllable with respect to the input matrix $\boldsymbol{B} = \boldsymbol{b}_2$

- *Hint* 2. Use Ackermann's formula for input $u_1(t)$ to find a gain $\boldsymbol{k}^{\mathrm{T}} = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$, and then define $\boldsymbol{K} = \begin{bmatrix} k_1 & k_2 \\ 0 & 0 \end{bmatrix}$

- *Question*: why would the approach of *Hint* 2 fail if one were to use input $u_2(t)$ instead of $u_1(t)$?

Exercise 6

- Consider the LTI system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -2 & -20 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

with initial conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

- Design a state-feedback tracking controller

$$\boldsymbol{u}(t) = -\boldsymbol{K}\ \boldsymbol{x}(t) - \boldsymbol{K}_r\ \boldsymbol{r}(t)$$

that ensures zero offset for all set points $\boldsymbol{r}(t)$ that have a constant final value.

