Exercises Mixed-Integer Optimization.

1. Formulate linear constraints in terms of binary variables for the following case:

   If A is true and B is true then C is true or D is true. (inclusive OR)

2. It is proposed to model the condition,

   if select item 1 and not item 2, then select item 3 and item 4

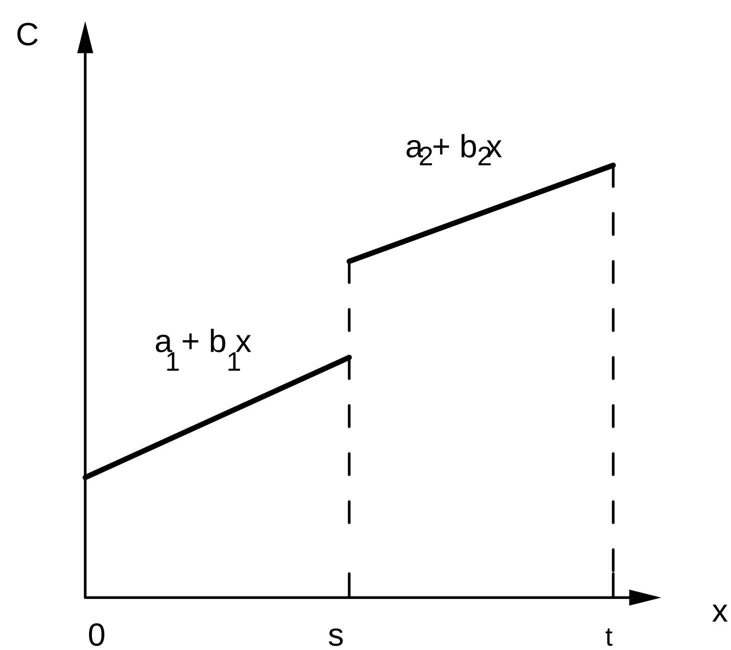
   with the inequality:

   $y_3 + y_4 \geq 2(y_1 - y_2)$

where y_i are binary variables that represent the selection of the corresponding items.

Using propositional logic, derive the inequality (ies) that model the above condition. If you arrive at a different model, determine whether it is better or not, and in what sense than the inequality above.

3. Consider the cost function shown in the graph below.
   (a) Formulate the cost function C as a disjunction.
   (b) Develop the mixed-integer constraints applying the convex hull to the disjunction.



4. For the Generalized Disjunctive Program given below,

a) Reformulate it as an MINLP using the convex hull formulation for the disjunction
b) Reformulate it as a big-M MINLP (M=50)
c) Solve both reformulations and compare their relaxations.

$$\min Z = c + (x_1 - 3)^2 + (x_2 - 2)^2$$

$$st$$

$$\begin{bmatrix} Y_1 \\ x_1^2 + x_2^2 \le 1 \\ c = 2 \end{bmatrix} \vee \begin{bmatrix} Y_2 \\ (x_1 - 4)^2 + (x_2 - 1)^2 \le 1 \\ c = 1 \end{bmatrix} \vee \begin{bmatrix} Y_3 \\ (x_1 - 2)^2 + (x_2 - 4)^2 \le 1 \\ c = 3 \end{bmatrix}$$

$$0 \le x_1 \le 8, 0 \le x_2 \le 8, Y_j = true,\ false,\ j = 1,2,3$$

5. Given the bilinear NLP below, find the global optimal solution using the McCormick convex envelopes and a spatial branch and bound. To obtain good initial lower and upper bounds solve LP's for the bounds of the 4 continuous variables.

$$\min f = x_1 - x_2 - y_1 - x_1y_1 + x_1y_2 + x_2y_1 - x_2y_2$$

$$\begin{aligned} \text{st.}\quad & x_1 + 4x_2 \le 8 \\ & 4x_1 + x_2 \le 12 \\ & 3x_1 + 4x_2 \le 12 \\ & 2y_1 + y_2 \le 8 \\ & y_1 + 2y_2 \le 8 \\ & y_1 + y_2 \le 5 \\ & 0 \le x_1, x_2, y_1, y_2 \le 10 \end{aligned}$$

Optional:
Verify your answer with the webinterface of the software package BARON in GAMS. (Use OPTION NLP=BARON;)

6. A company is considering to produce a chemical C which can be manufactured with either process II or process III, both of which use as raw material chemical B. B can be purchased from another company or else manufactured with process I which uses A as a raw material. Given the specifications below, formulate an MILP model and solve it with GAMS to decide:
   a) Which process to build (II and III are exclusive)?
   b) How to obtain chemical B?
   c) How much should be produced of product C?
   The objective is to maximize profit.

   Consider the two following cases:

   1. Maximum demand of C is 10 tons/hr with a selling price of $1800/ton.

   2. Maximum demand of C is 15 tons/hr; the selling price for the first 10 ton/hr is $1800/ton, and $1500/ton for the excess.

<u>Data:</u>

Investment and Operating Costs

	Fixed ($/hr)	Variable($/ton raw mat)
Process I	1000	250
Process II	1500	400
Process III	2000	550

<u>Prices</u>: A: $500/ton
B: $950/ton

<u>Conversions</u>:		
	Process I	90% of A to B
	Process II	82% of B to C
	Process III	95% of B to C

Maximum supply of A: 16 tons/hr

<u>NOTE</u>: You may want to scale your cost coefficients (e.g. divide them by 100).

7. It is proposed to manufacture a chemical C with a process I that uses raw material B. B can either be purchased or manufactured with either of two processes, II or III, which use chemical A as a raw material. In order to decide the optimal selection of processes and levels of production that maximize profit formulate the MINLP problem and solve with the augmented penalty/outer-approximation/equality-relaxation algorithm in DICOPT++.

Data:

Conversion:	Process I	$C = 0.9B$	
	Process II	$B = \ln(1 + A)$	Maximum capacity: 5 ton prod/hr
	Process III	$B = 1.2 \ln(1 + A)$	
		(A, B, C, in ton/hr)	

Prices:	A $ 1,800/ton
	B $ 7,000/ton
	C $13,000/ton (maximum demand: 1 ton/hr)

Investment cost

	Fixed (10^3\$/hr)	Variable (10^3\$/ton product)
Process I	3.5	2
Process II	1	1
Process III	1.5	1.2

Note: Minimize negative of profit.

7. Seven jobs (tasks) have to be scheduled on two machines. There are no setup times between different tasks. Processing times are known.

Tasks: 1, 2 ..7
Machines: A, B

Processing Times P_{ij}:

	1	2	3	4	5	6	7
A	2	3	4	3	4	2	5
B	4	4	3	2	4	3	2

Develop an MILP model to minimize the makespan.

8. Seven jobs (tasks) have to be scheduled on two machines, as before, but there are also release and due times, R_i, D_i, that have to be satisfied.

Develop a generic MILP formulation for this problem