Problems to Try on Model Predictive Control

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1. Suppose the process has three inputs $u_1, \cdots, u_3$ and three outputs $y_1, \cdots, y_3$. y_1 and y_2 has the setpoints of 0.1 and 0.9 respectively. In addition, y_3 needs to be kept below 0.5 (a soft constraint). Finally, it is desirable to minimize the value of u_3 after other requirements are met (a secondary optimization requirement). All three inputs must operate in the range of ± 1. Write down a reasonable *quadratic* objective to use for MPC. Use the prediction horizon of p and the control horizon of m.

2. Derive the form of Ω^u and Ω^d in the note (that appear in the step response model based prediction equation)

3. Consider the following SISO system.

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} 0.9 & 0.2 \\ 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} d(k) \\ y(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \end{aligned} \tag{1}$$

   (a) Calculate the impulse response and step response coefficients for u and d. What is the reasonable truncation point for this system?

   (b) Write down the step response model that corresponds to the above state space system.

   (c) Write down the prediction equation for the above system with $p = 3$ and $m = 2$.

   (d) Derive the unconstrained control law for the above system with $\Lambda^y = 1$ and $\lambda^u = 0.1$.

   (e) How would you derive the unconstrained MPC law for $p = \infty$ and $m = 2$.

   (f) How would the prediction equation change if the state space model is used directly?

4. Consider the following ARX system model:

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2) + \varepsilon(k) \tag{2}$$

   (a) Derive the expression for the one-step-ahead predictor and the prediction error.

   (b) Suppose you are given experimentally obtained time series data $y(1), \cdots, y(12)$ and $u(1), \cdots, u(12)$. Derive the formula for obtaining the parameters a_1, a_2, b_1, b_2 that minimize the prediction error for the given data.