PROCESS CONTROL DESIGN
PASI 2005

Pan American Advanced Studies Institute
Program on Process Systems Engineering
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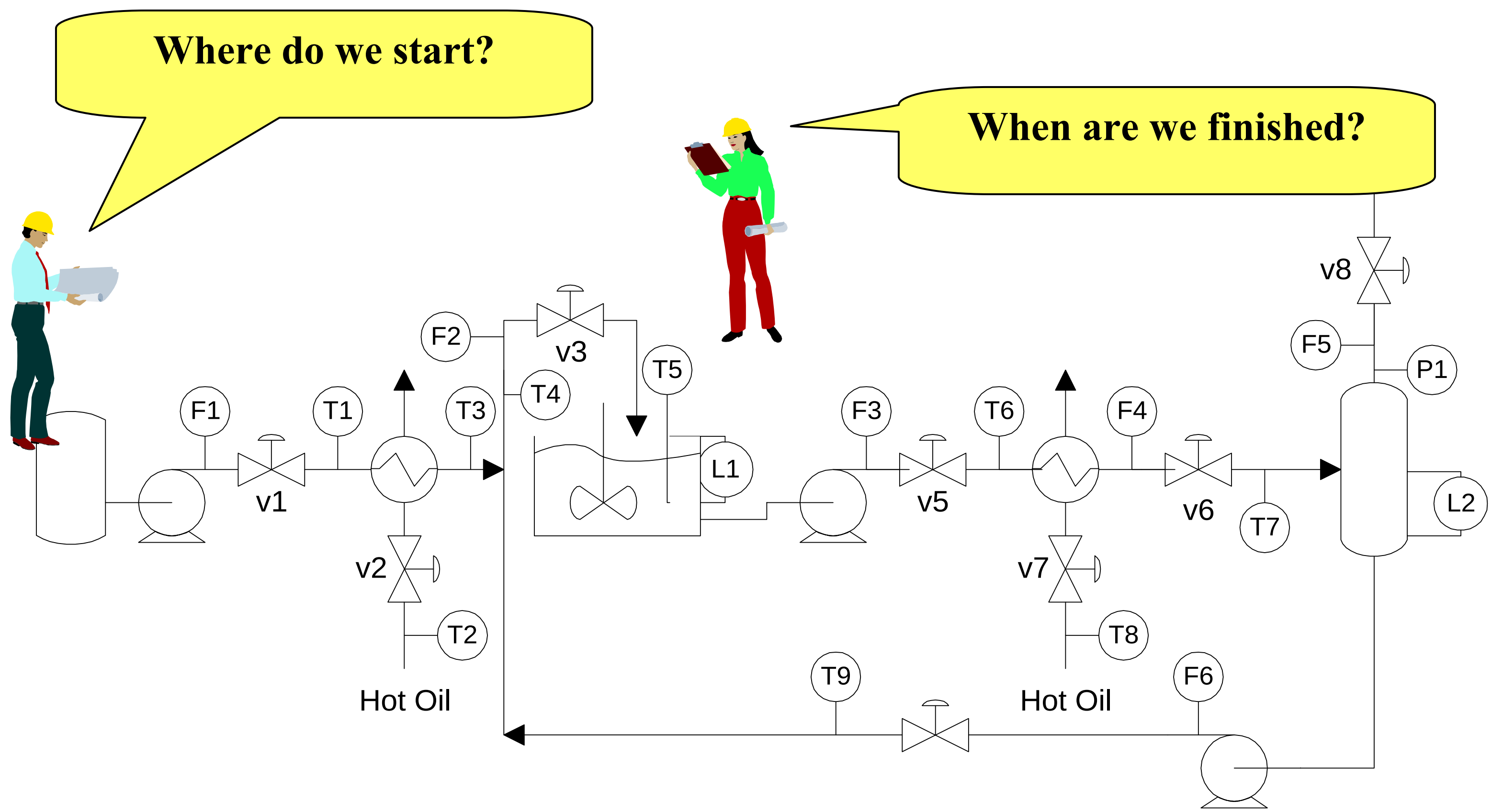
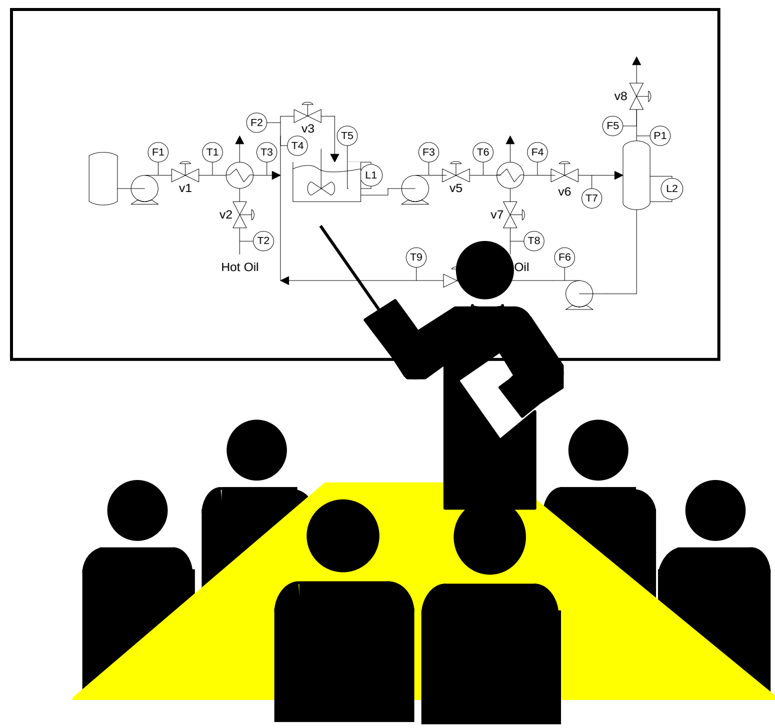
Design is a challenging task. We must use all of our technical and problem-solving skills.



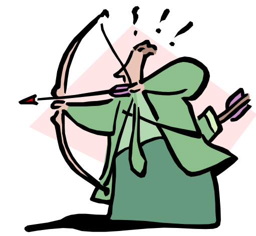
You will be the leaders in the PSE community; practitioners, educators, and researchers.

You need to know

- The **process needs**, independent of PSE technology
- The current best practices, **strengths and gaps**
- **Principles** informing research and practice
- Key breakthroughs in associated technologies



Course Goals



- Be able to define the control objectives for this goal-driven engineering task.
- Evaluate unique features of multivariable dynamic systems resulting from interaction
- Design simple control strategies using process insights and performance metrics
- Design challenging problems using a systematic, optimization method
- Become enthusiastic and investigate further

Course Resources

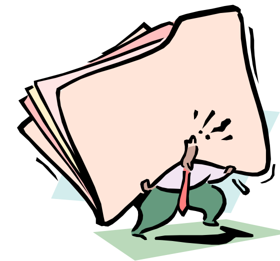
- **Lecture Notes**
- **Annotated Reading List**
- **Solutions to Workshops** **(38 problems, 73 Pages)**
- **WEB sites**

Undergraduate Control: www.pc-education.mcmaster.ca
Graduate Control Design: http://www.chemeng.mcmaster.ca/graduate/CourseOutlines/764Course_WEB_Page/764_Course_WEB_Table.doc

This course will be a success if you study, apply, and improve good control design techniques

Theory & Practice

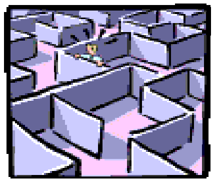
Course Content



- Defining the Control Design Problem & Workshop
- ~~Single-Loop Control Concepts~~ & Workshop
- Multivariable Principles
  - Interaction & Workshop
  - Controllability & Workshop
  - Integrity & Workshop
  - Directionality & Workshop
- Short-cut Design Procedure & Workshop
- Optimization-Based Control Design

Slides provided but not covered because of limited me.

This is a small subset of the topics in the graduate course noted on the previous slide.



Course Emphasis

So many great topics!

A Foss: Key Challenge is "which variables to measure, which inputs to manipulate, and what links should be made between these two sets"

CONTROL STRUCTURE!

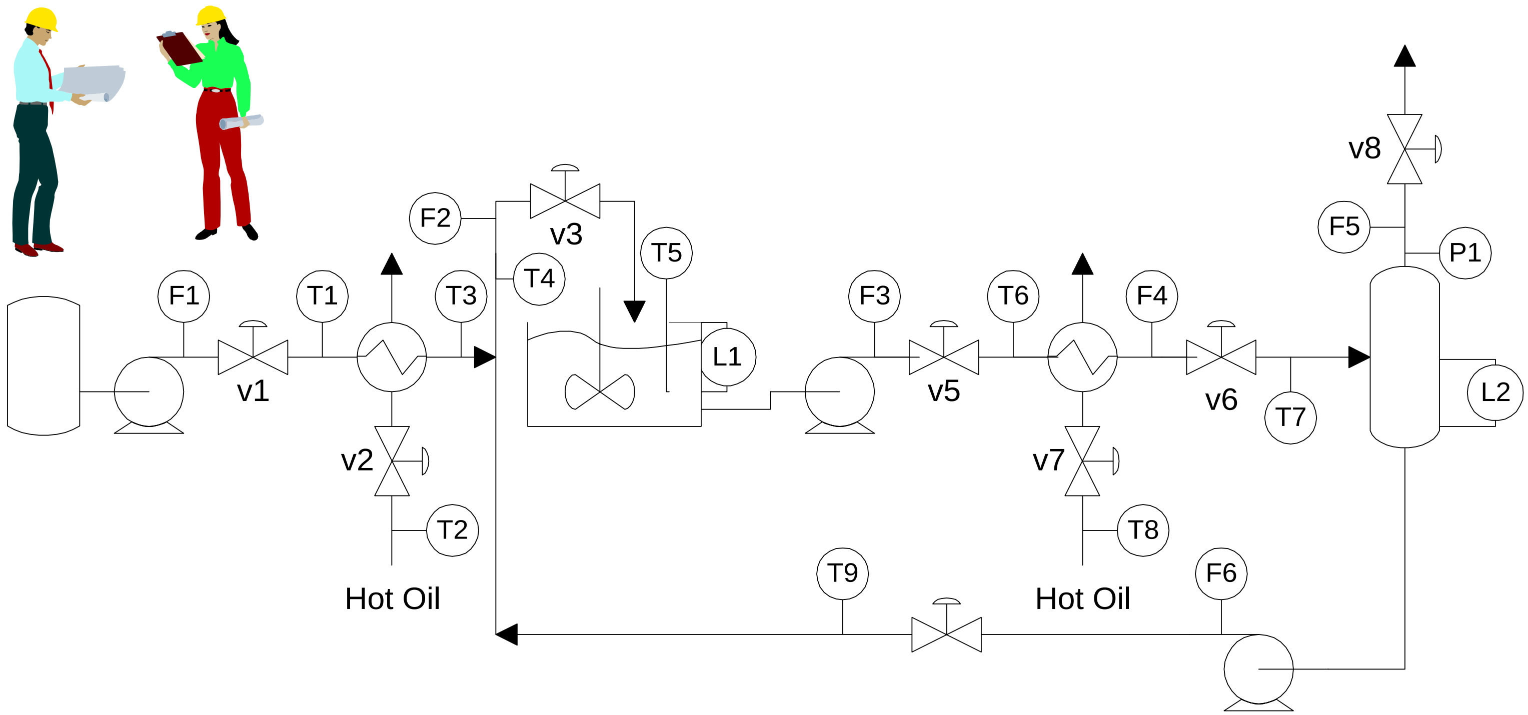
C. Nett: Objective "minimize control system complexity subject to achievement of accuracy specifications"

SIMPLICITY!

PERFORMANCE!

CONTROL DESIGN: DEFINING THE PROBLEM

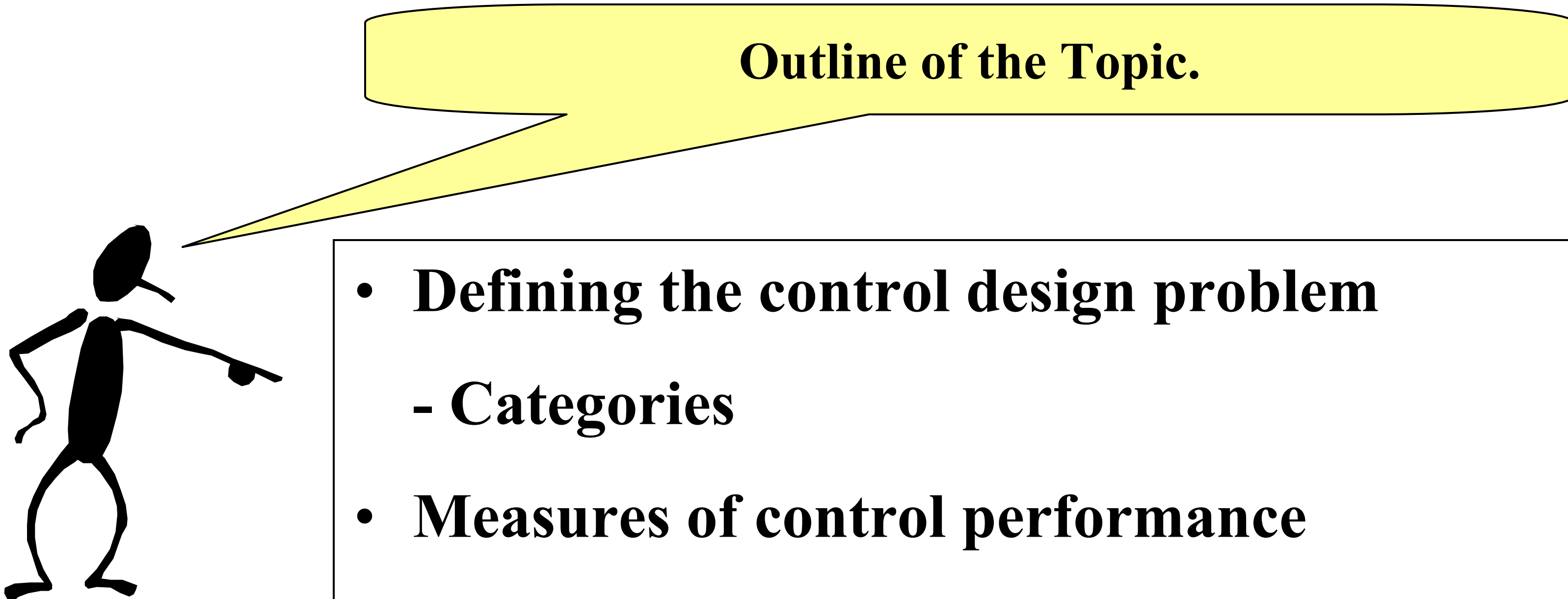
Control Design is a <u>Goal-Driven</u> Problem



The problem is defined based on knowledge of safety, process technology, sales, market demands, legal requirements, etc.

Process systems technology is applied to achieve these goals.

CONTROL DESIGN: DEFINING THE PROBLEM



- Defining the control design problem

  - Categories

- Measures of control performance

  - Controlled variable
  - Manipulated variable

- Benefits of reduced variation

  - Class exercise

- Workshop

CONTROL DESIGN: DEFINING THE PROBLEM

The Control Design Form

The form provides a useful check list for items that should be considered.

It also provides a concise yet complete presentation of the important decisions that must be reviewed by all stake holders.

The objectives and performance descriptions must be stated in process terms, not limited by anticipated control performance.

See Marlin (2000), Chapter 24

CONTROL DESIGN FORM

TITLE: | ORGANIZATION:
PROCESS UNIT: | DESIGNER:
DRAWING | ORIGINAL DATE:
PROCESS FLOW: | REVISION No. DATE:
PIPING AND INSTR.: | PAGE No. OF

CONTROL OBJECTIVES:
1) SAFETY OF PERSONNEL
2) ENVIRONMENTAL PROTECTION
3) EQUIPMENT PROTECTION
4) SMOOTH, EASY OPERATION
5) PRODUCT QUALITY
6) EFFICIENCY AND OPTIMIZATION
7) MONITORING AND DIAGNOSIS

[Define the control performance required by specifying standard deviations, approach to constraint, response to major upset, or other relevant, quantitative performance measure for each objective.]

MEASUREMENTS:

VAR.	SENSOR	RANGE	ACCURACY&DYNAMICS	RELIABILITY	PRINCIPLE	LOCATION	REPRODUCIBILITY

MANIPULATED VARIABLES:

VAR.	AUTO/ MANUAL	LOCAL/ REMOTE	CONTIN. DISCRETE	RANGE	PRECISION	DYNAMICS	RELIABIL.

CONSTRAINTS:

VARIABLE	LIMIT VALUES	MEASURED/ INFERRED	PENALTY FOR VIOLATION

DISTURBANCES:

SOURCE	MAGNITUDE	FREQUENCY	MEASURED?

DYNAMIC RESPONSES: (ALL INPUTS, MANIPULATED AND DISTURBANCE)

INPUT	OUTPUT	GAIN	DYNAMIC MODEL

ADDITIONAL CONSIDERATIONS:

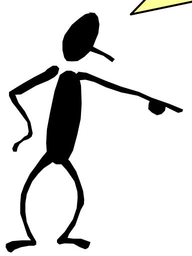
CURRENT OPERATION:

CONTROL DESIGN: (INCLUDE DRAWING)

BENEFITS:

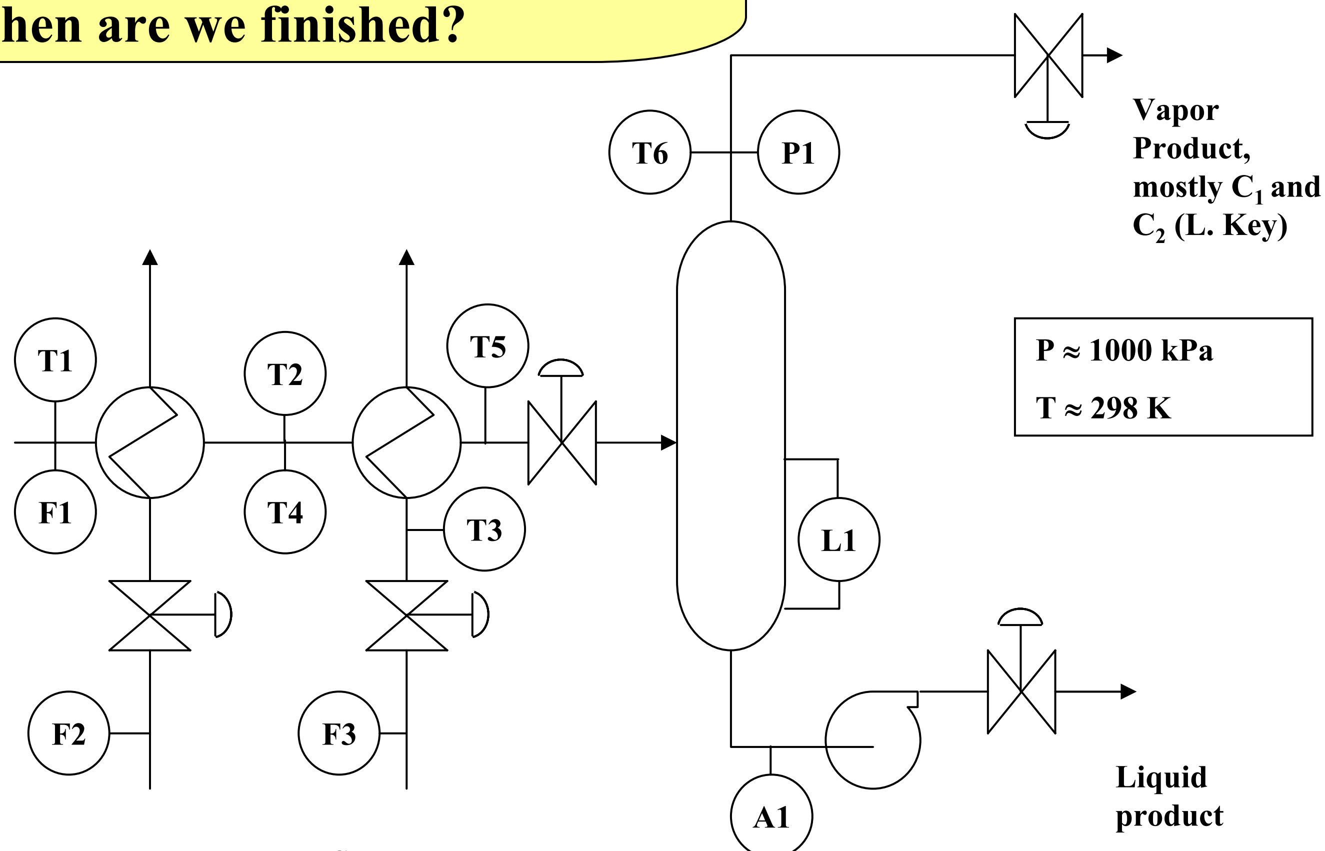
COSTS:

CONTROL DESIGN: DEFINING THE PROBLEM



Let's design controls for this process

- **How do we start?**
- **What are the common steps?**
- **When are we finished?**



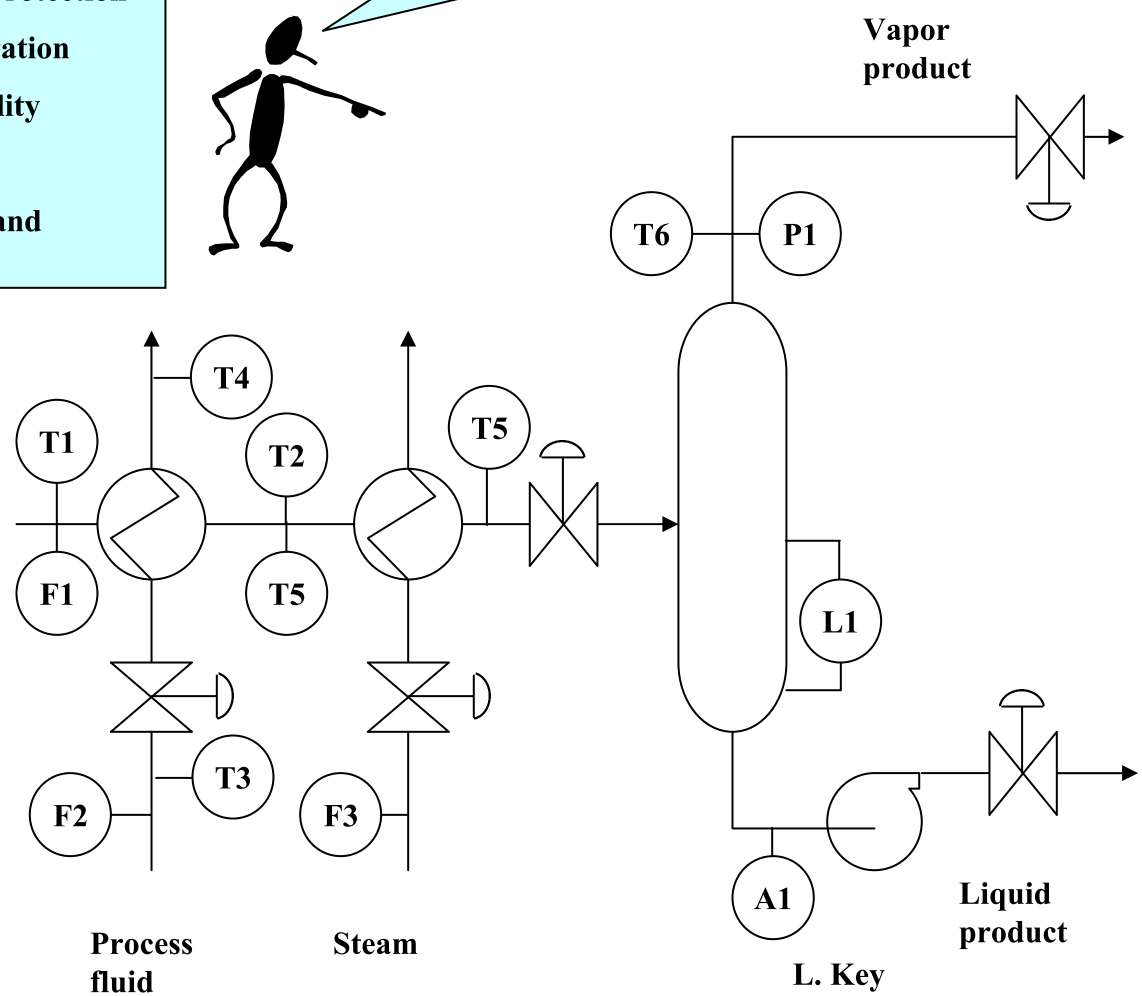
CONTROL DESIGN: DEFINING THE PROBLEM

<u>Control Design Form</u>

- **Objectives** →
- **Measurements**
- **Manipulated variables**
- **Constraints**
- **Disturbances**
- **Dynamic responses**
- **Additional considerations**

1. Safety
2. Environmental protection
3. Equipment protection
4. Smooth operation
5. Product quality
6. Profit
7. Monitoring and diagnosis

<u>WORKSHOP</u>: Complete a Control Design Form



CONTROL DESIGN: DEFINING THE PROBLEM

Entries must be specific and measurable to guide design

CONTROL OBJECTIVES:

1) SAFETY OF PERSONNEL
   a) the maximum pressure of 1200 kPa must not be exceeded under any (conceivable) circumstances

2) ENVIRONMENTAL PROTECTION
   a) material must not be vented to the atmosphere under any circumstances

3) EQUIPMENT PROTECTION
   a) the flow through the pump should always be greater than or equal to a minimum

4) SMOOTH, EASY OPERATION
   a) the feed flow should have small variability

5) PRODUCT QUALITY
   a) the steady-state value of the ethane in the liquid product should maintained at its target of 10 mole% for operating condition changes of +20 to -25% feed flow, 5 mole% changes in the ethane and propane in the feed, and -10 to +50 °C in the feed temperature.
   b) the ethane in the liquid product should not deviate more than ±1 mole % from its set point during transient responses for the following disturbances
      i) the feed temperature experiences a step from 0 to 30 °C
      ii) the feed composition experiences steps of +5 mole% ethane and -5 mole% of propane
      iii) the feed flow set point changes 5% in a step

6) EFFICIENCY AND OPTIMIZATION
   a) the heat transferred should be maximized from the process integration exchanger before using the more expensive steam utility exchanger

7) MONITORING AND DIAGNOSIS
   a) sensors and displays needed to monitor the normal and upset conditions of the unit must be provided to the plant operator
   b) sensors and calculated variables required to monitor the product quality and thermal efficiency of the unit should be provided for longer term monitoring

CONTROL DESIGN: DEFINING THE PROBLEM

Entries must be specific and measurable to guide design

CONSTRAINTS:

VARIABLE	LIMIT VALUES	MEASURED/ INFERRED	HARD/ SOFT	PENALTY FOR VIOLATION
drum pressure	1200 kPa, high	P1, measured	hard	personnel injury
drum level	15%, low	L1, measured	hard	pump damage
Ethane in F5 product	± 1 mole%, (max deviation)	A1, measured & T6, inferred	soft	reduced selectivity in downstream reactor

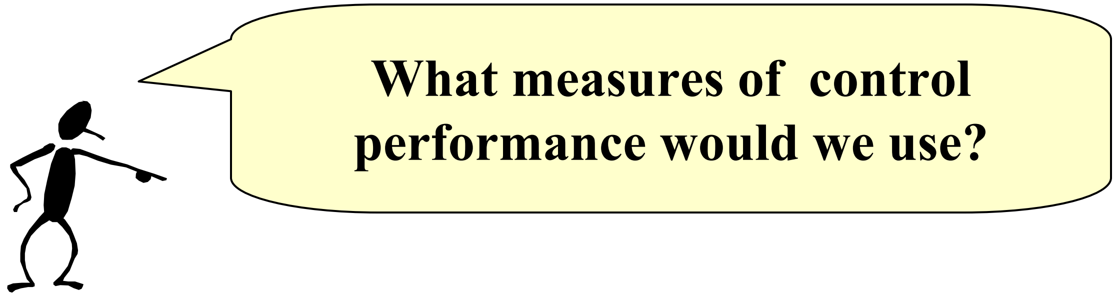
DISTURBANCES:

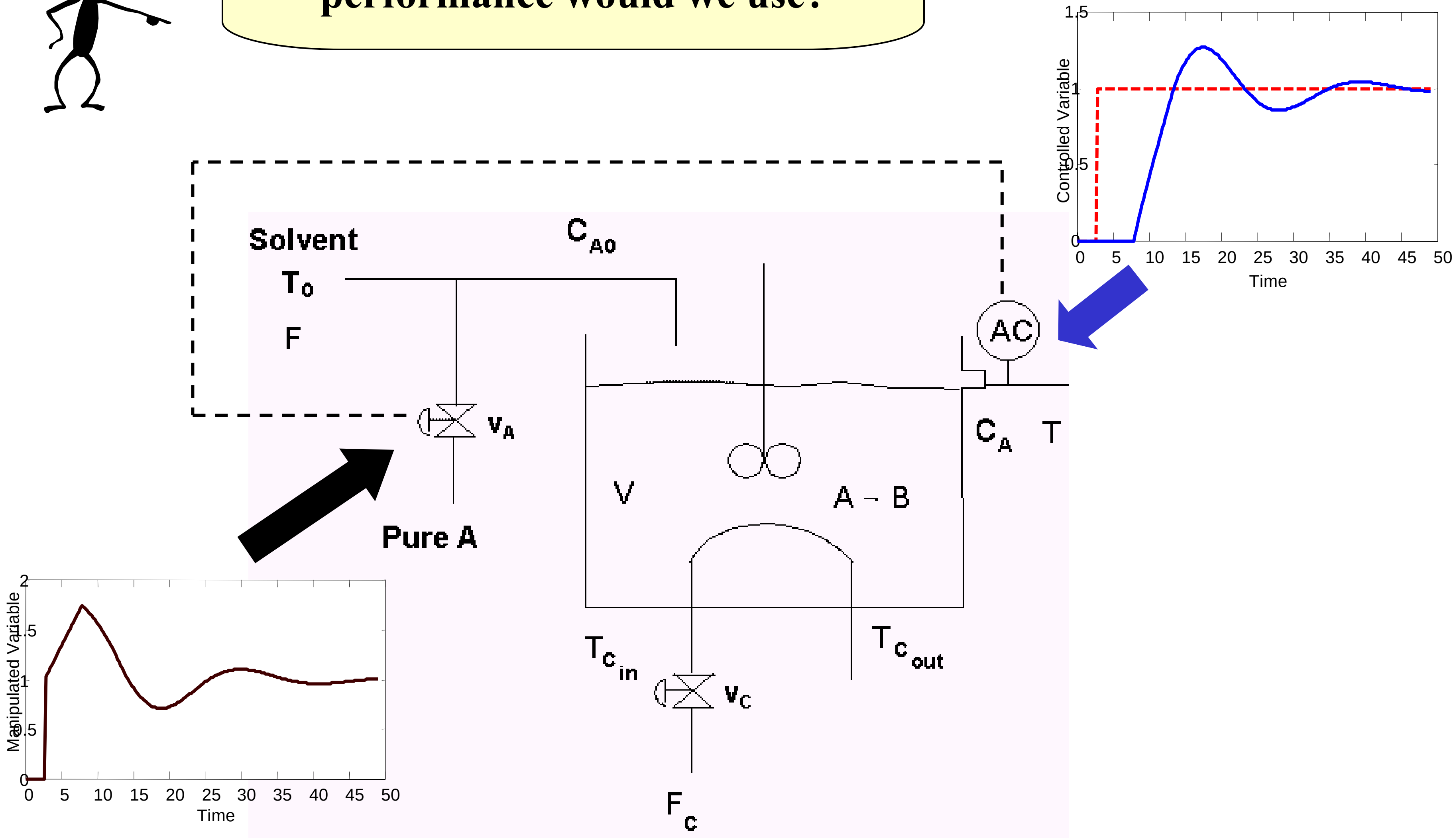
SOURCE	MAGNITUDE	DYNAMICS
feed temperature (T_1)	-10 to 55°C	infrequent step changes of 20°C magnitude
feed rate (F_1)	70 to 180	set point changes of 5% at one time
feed composition	±5 mole% feed ethane	frequent step changes (every 1-3 hr)

For an excellent problem definition, see the Tennessee Eastman design challenge problem.

Downs, J. and E. Vogel (1993) "A Plant-wide Industrial Process Control Problem", *Comp. Chem. Engr.*, 17, 245-255.

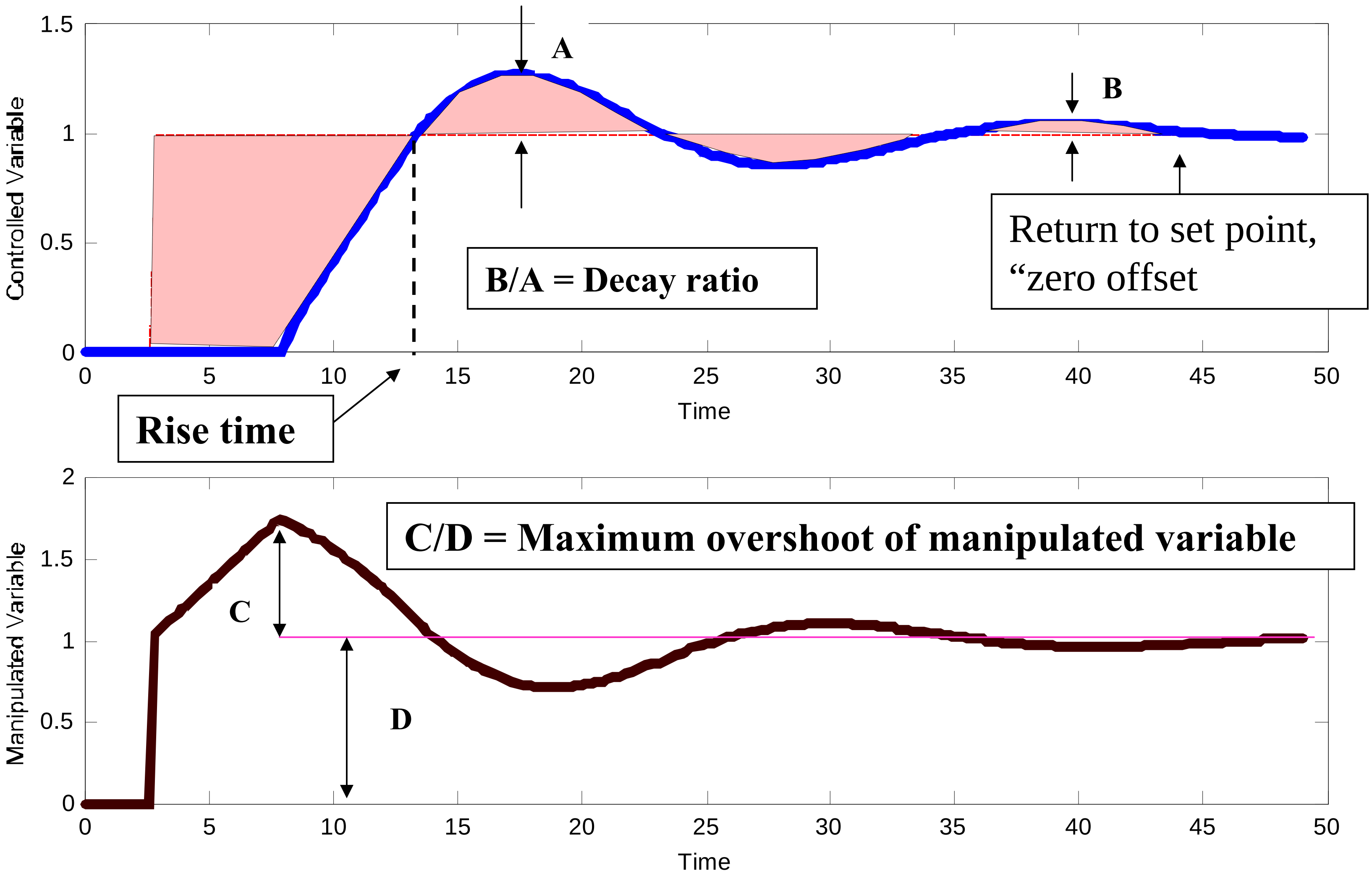
CONTROL DESIGN: DEFINING THE PROBLEM





CONTROL DESIGN: DEFINING THE PROBLEM

$$= IAE = \int |SP(t)-CV(t)|\, dt$$



CONTROL DESIGN: DEFINING THE PROBLEM

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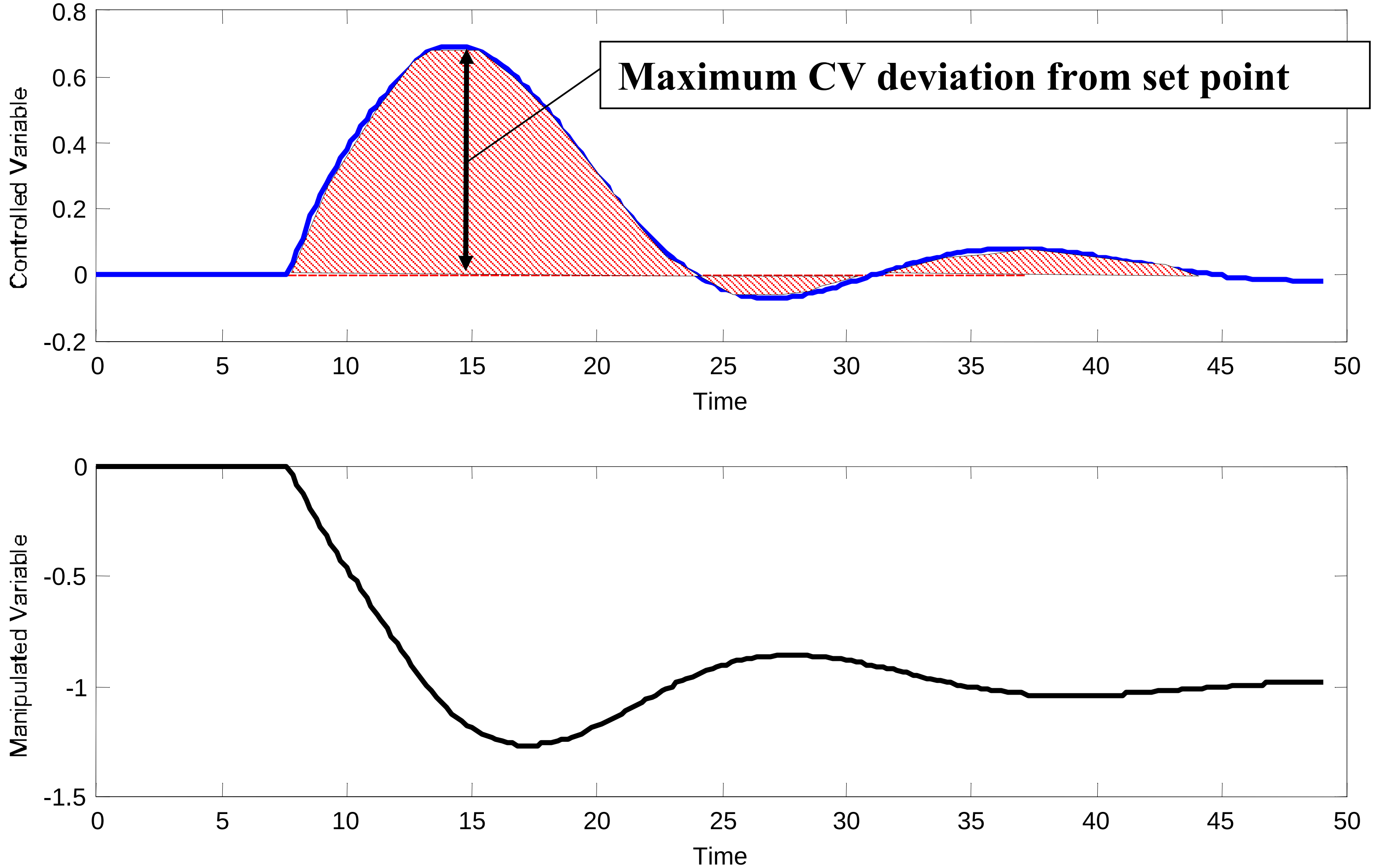
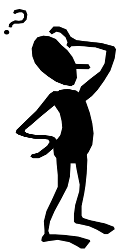
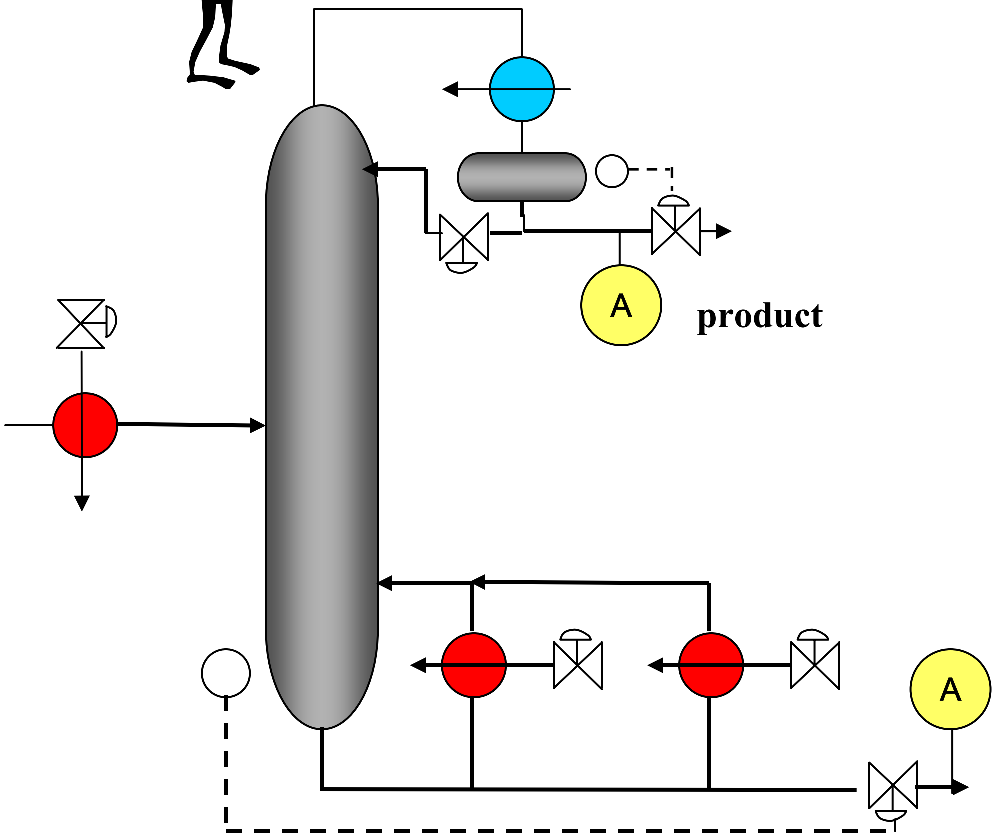


CONTROL DESIGN: DEFINING THE PROBLEM

An important, but often overlooked, factor



All objectives and CVs are not of equal importance!

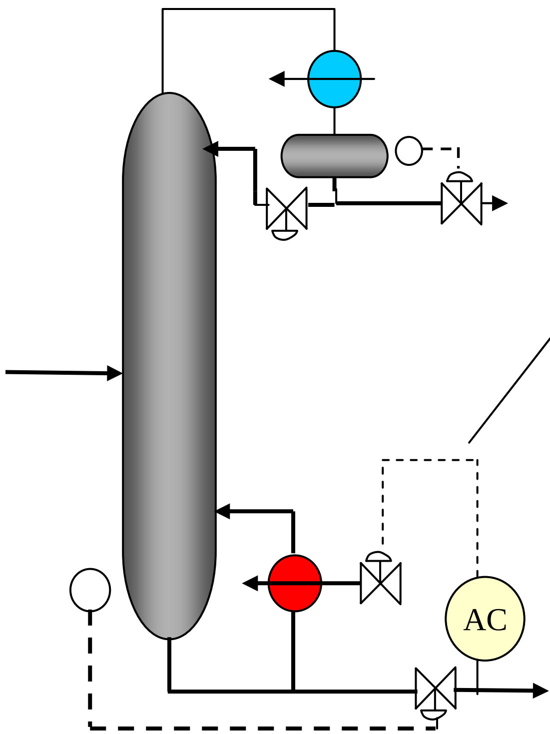


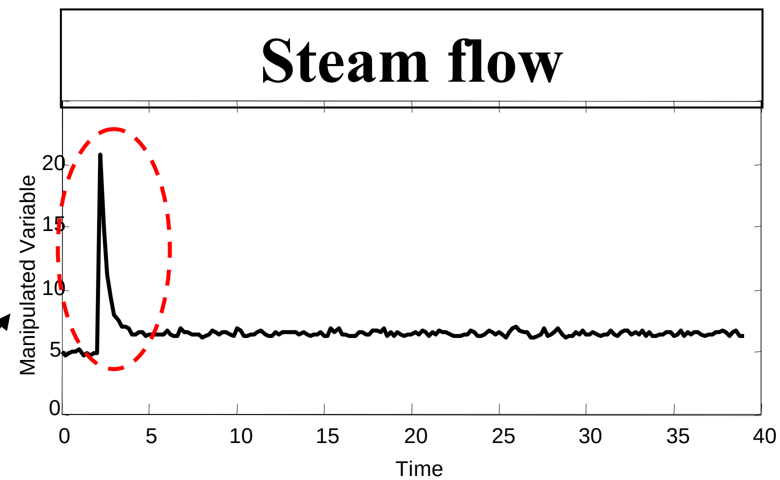
CV Priority Ranking

CONTROL DESIGN: DEFINING THE PROBLEM

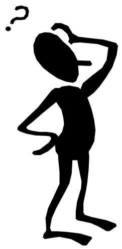
An important, but often overlooked, factor

Our primary goal is to maintain the CV near the set point. Besides not wearing out the valve, why do we have goals for the MV?





CONTROL DESIGN: DEFINING THE PROBLEM



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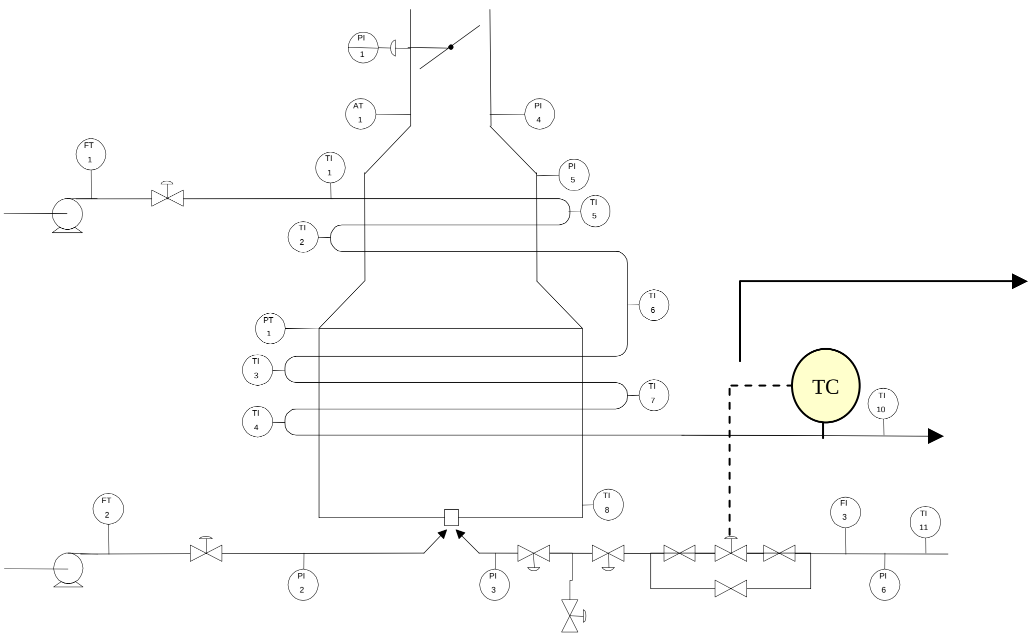
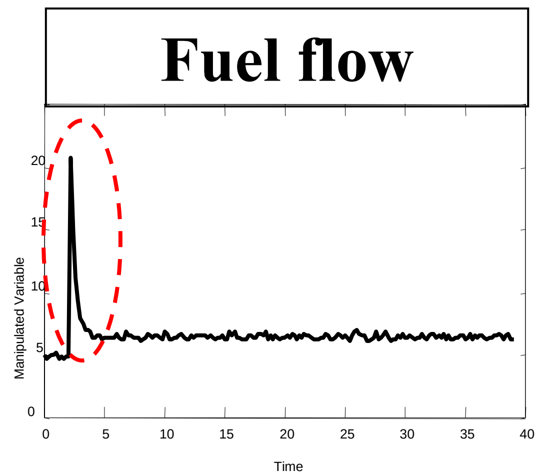


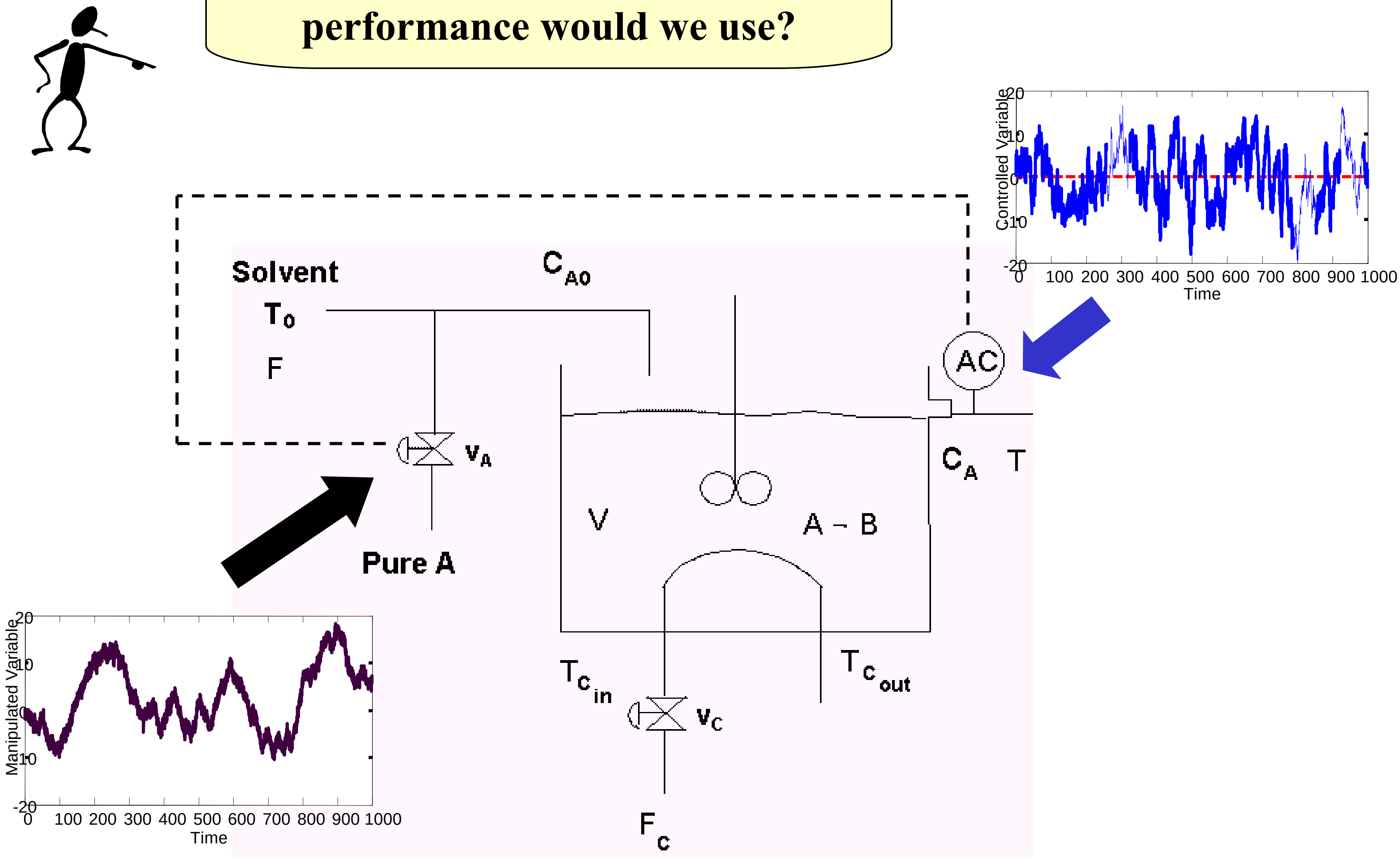
Fuel



For more on Life Extending Control, see Li, Chen, and Marquez (2003)

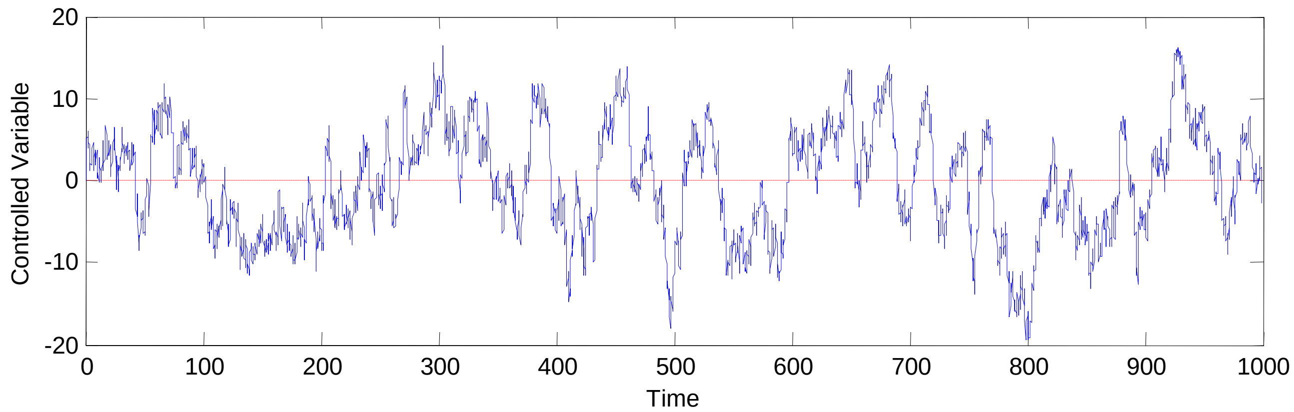
CONTROL DESIGN: DEFINING THE PROBLEM

What measures of control performance would we use?

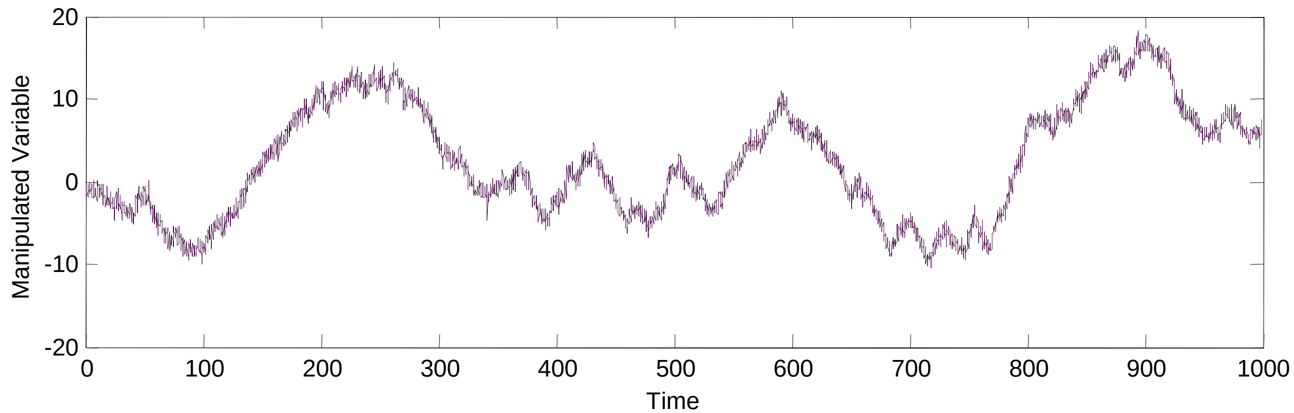


CONTROL DESIGN: DEFINING THE PROBLEM

Often, the process is subject to many large and small disturbances and sensor noise. The performance measure characterizes the variability.



Variance or standard deviation of CV



Variance or standard deviation of MV

CONTROL DESIGN: DEFINING THE PROBLEM

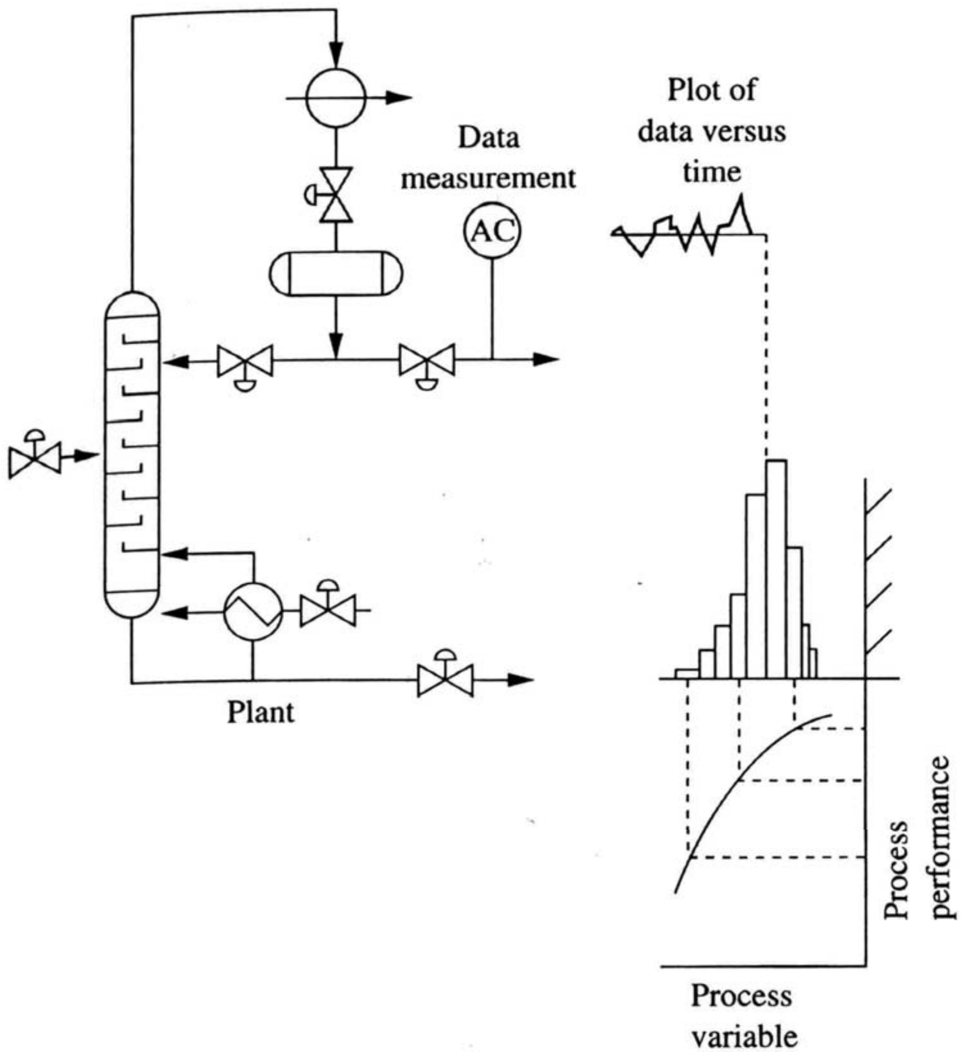


FIGURE 2.10

Process performance = **efficiency, yield, production rate, etc. It measures performance for a control objective.**

Calculate the process performance using the CV distribution, not the average value of the key variable!

Class Exercise: Benefits for reduced variability for chemical reactor

Goal: Maximize conversion of feed ethane but do not exceed 864C

Which operation, A or B, is better and explain why.

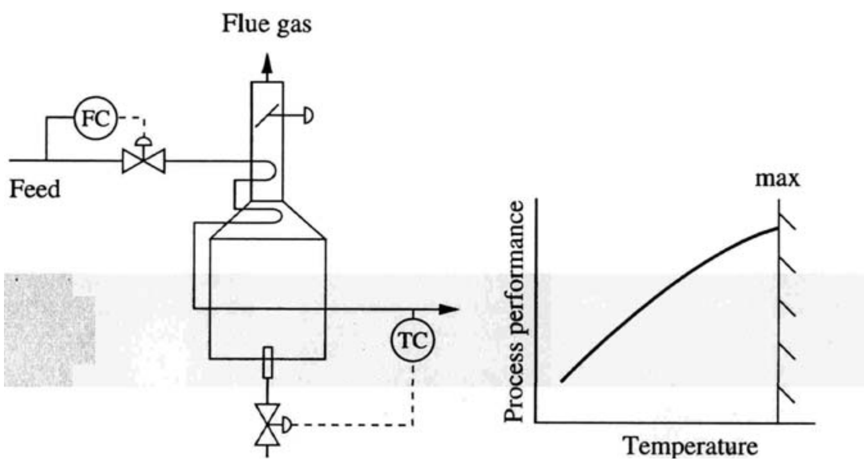


FIGURE 2.5

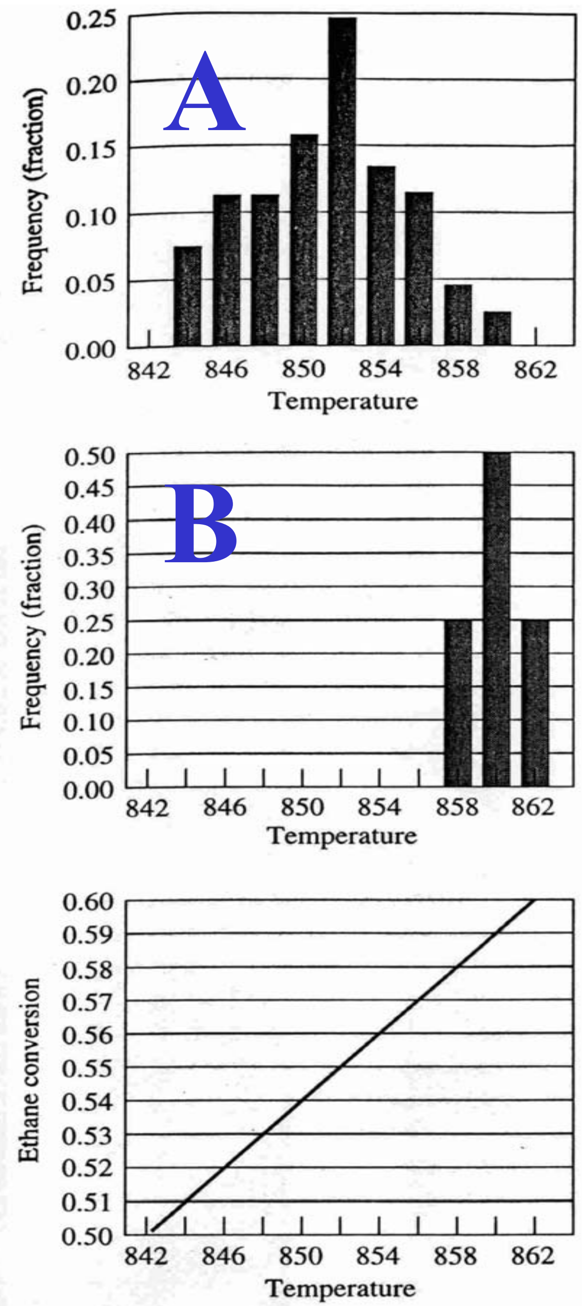


FIGURE 2.11

Class Exercise: Benefits for reduced variability for boiler

Goal: Maximize efficiency and prevent fuel-rich flue gas

Which operation, A or B, is better and explain why.

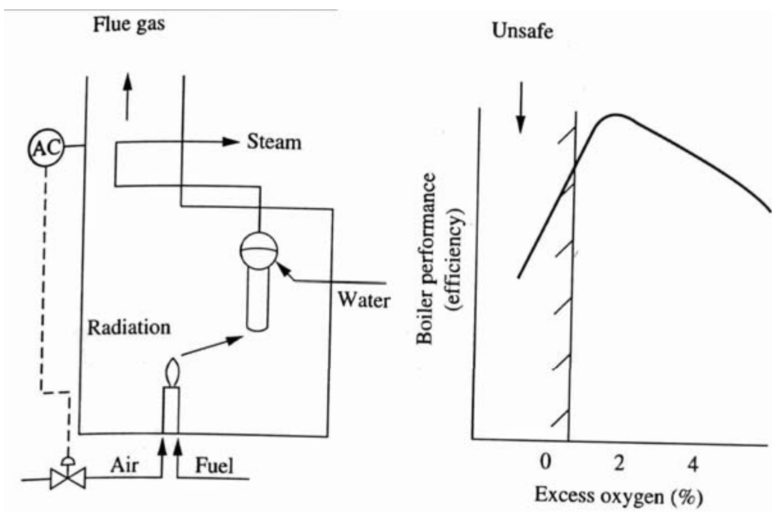


FIGURE 2.6

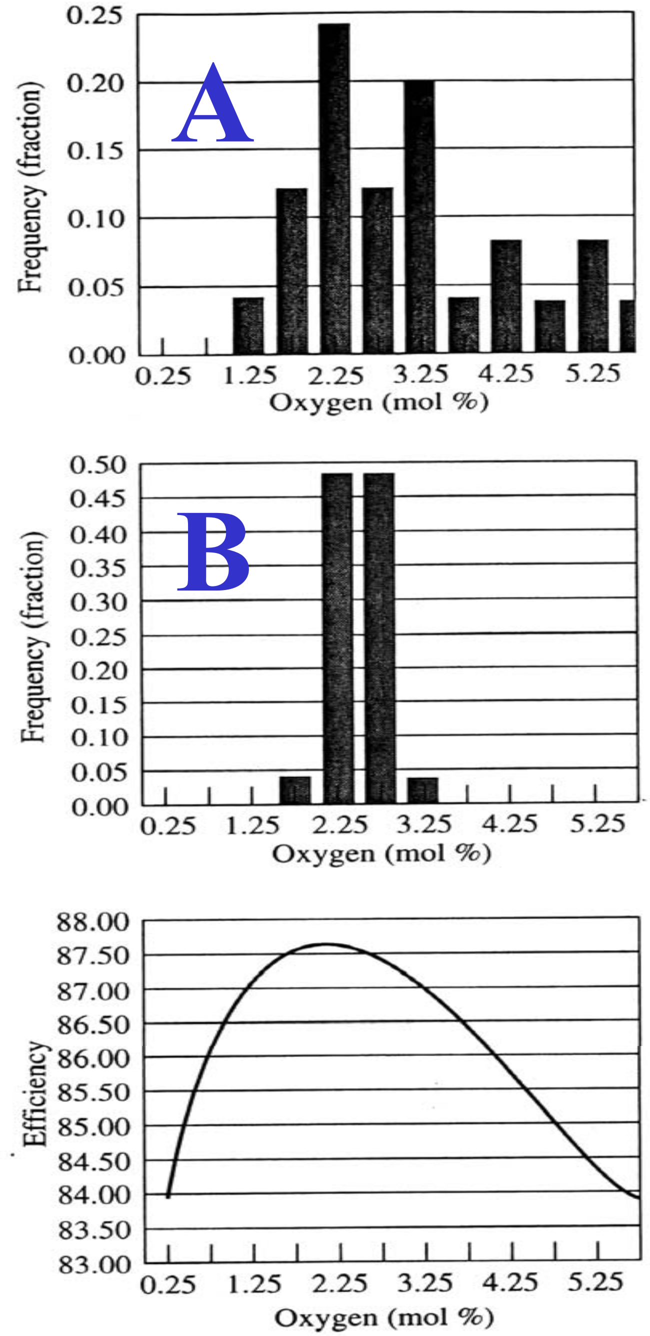


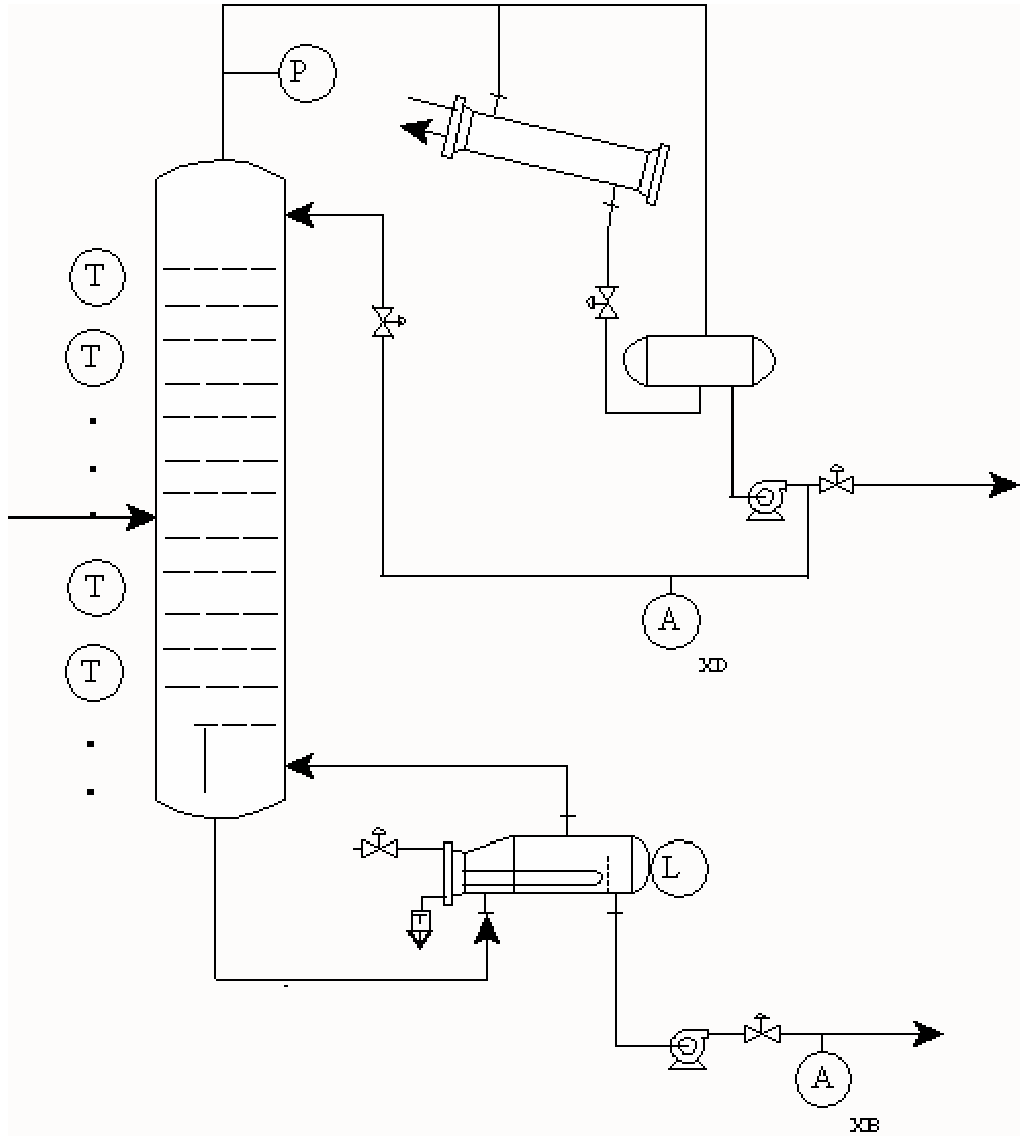
FIGURE 2.12

DEFINING THE PROBLEM: Workshop 1

Complete a control design form for a typical 2-product distillation tower.

Make reasonable assumptions and note questions you would ask to verify your assumptions.

Note that the figure is not complete; you are allowed to make changes to sensors and final elements.

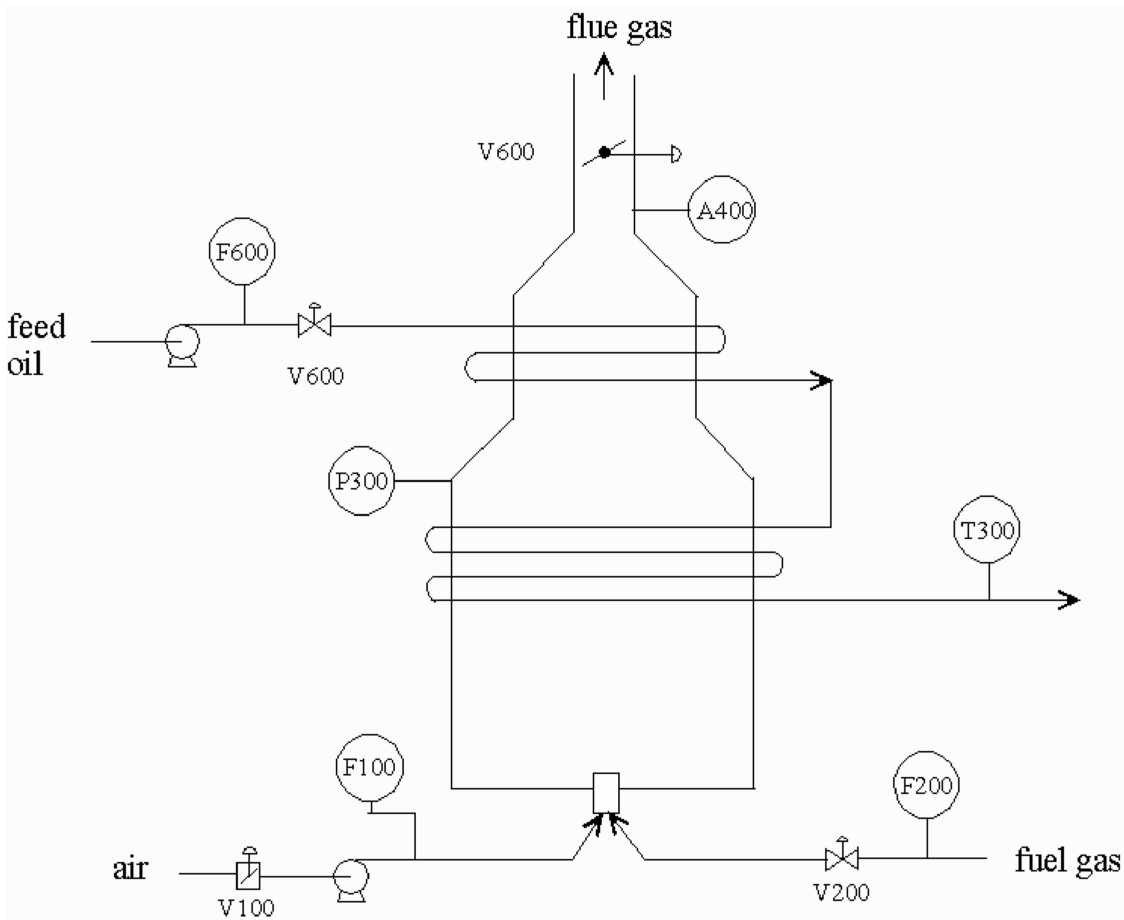


DEFINING THE PROBLEM: Workshop 2

Complete a control design form for a typical fired heater.

Make reasonable assumptions and note questions you would ask to verify your assumptions.

Note that the figure is not complete; you are allowed to make changes to sensors and final elements.



DEFINING THE PROBLEM: Workshop 3

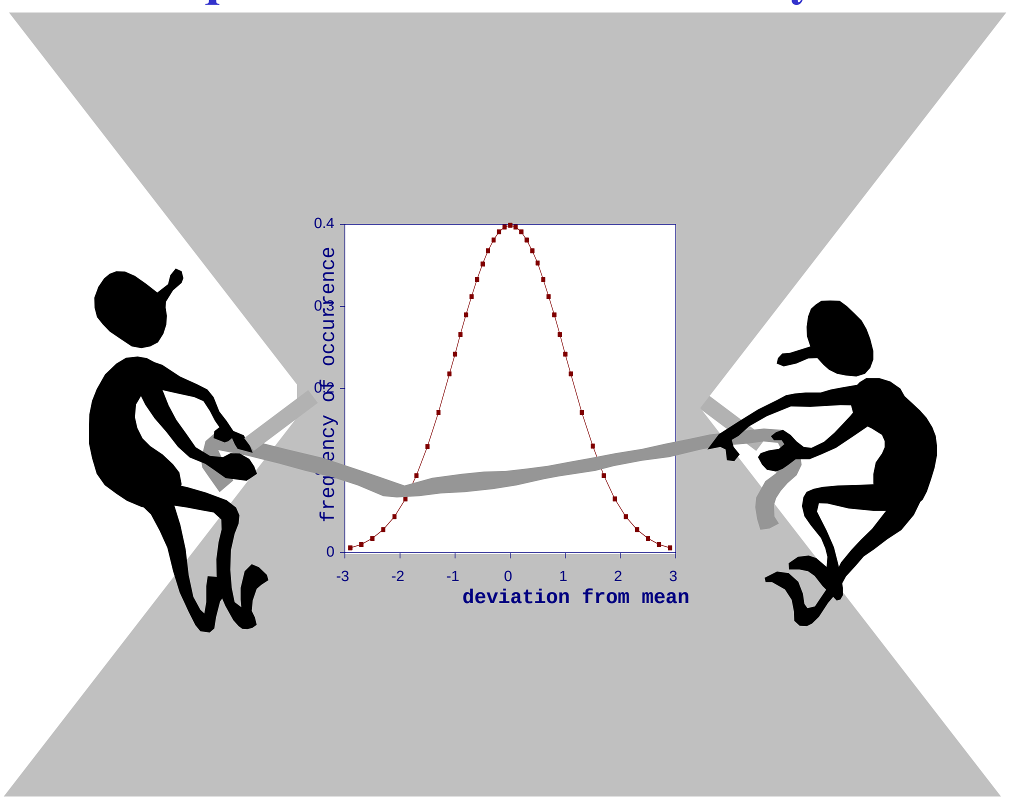
Typically, we have only a steady-state flowsheet (if that) when designing a plant.

- **Discuss the information in the Control Design Form that can be determined at this stage of the design.**
- **Discuss the information in the Control Design Form that is not known at this stage of the design.**

DEFINING THE PROBLEM: Workshop 4

Two process examples show the benefit of reduced variability, the fired heater reactor and the boiler. Discuss the difference between the two examples. Can you think of another example that shows the principle of each?

Squeeze down the variability



DEFINING THE PROBLEM: Workshop 5

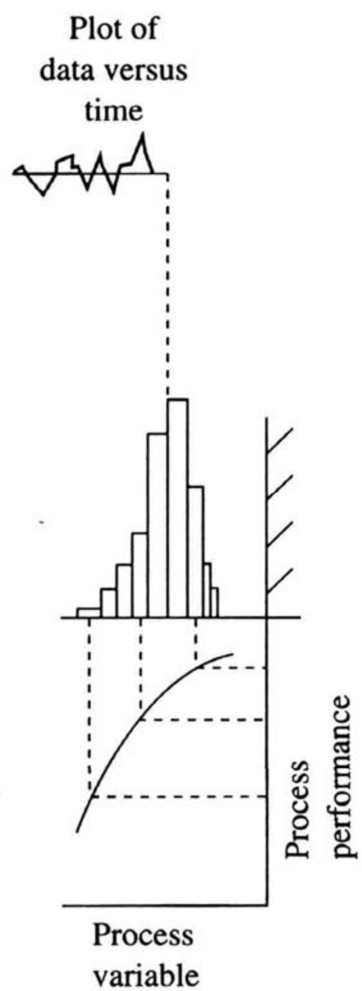


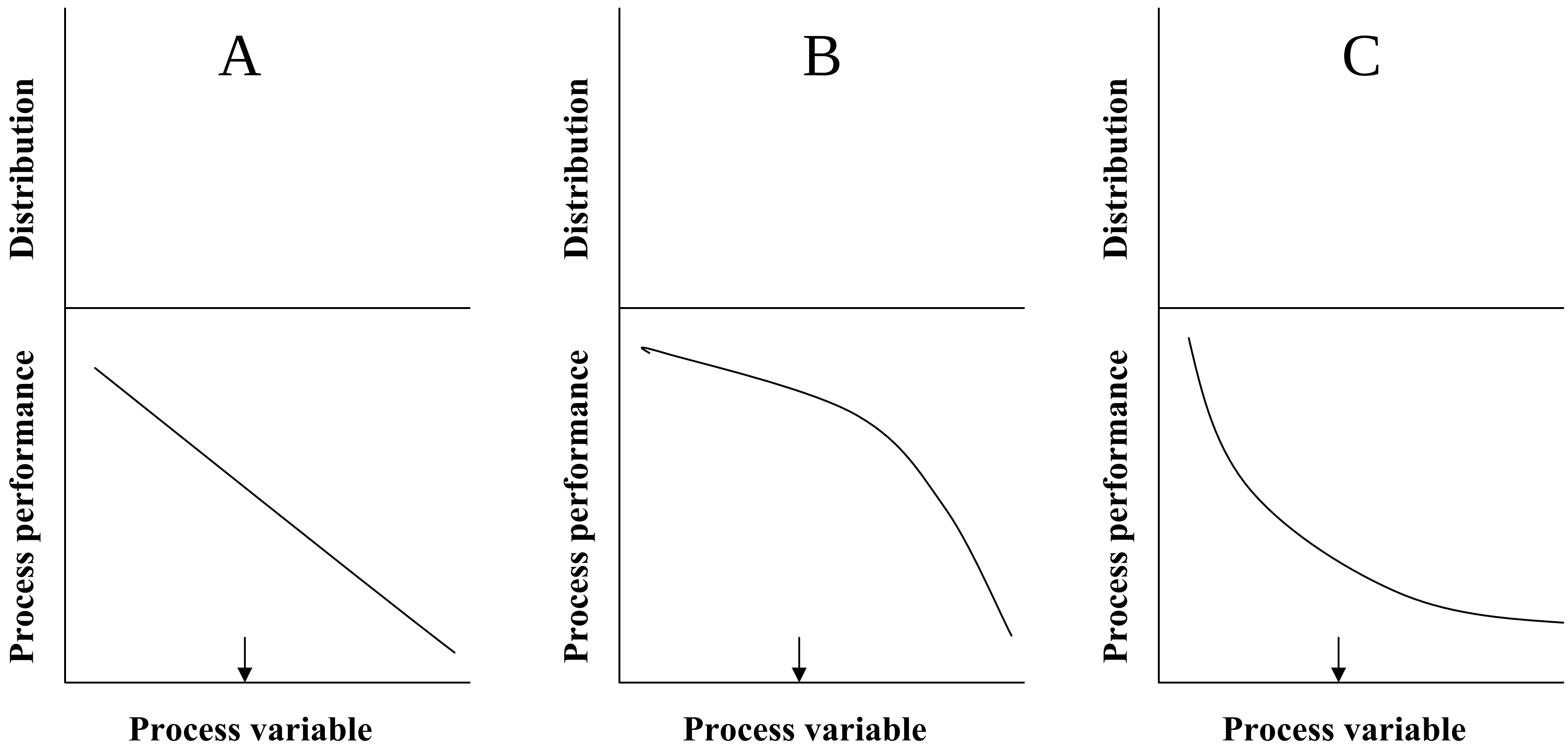
FIGURE 2.10

Discuss an important assumption that is made on the procedure proposed for calculating the average process performance. (Hint: consider dynamics)

How would you evaluate the assumption?

DEFINING THE PROBLEM: Workshop 6

The following performance vs. process variable correlations are provided. All applications require the same average value for the process variable (see arrow). What is the best distribute for each case? (Sketch histogram as your answer.)



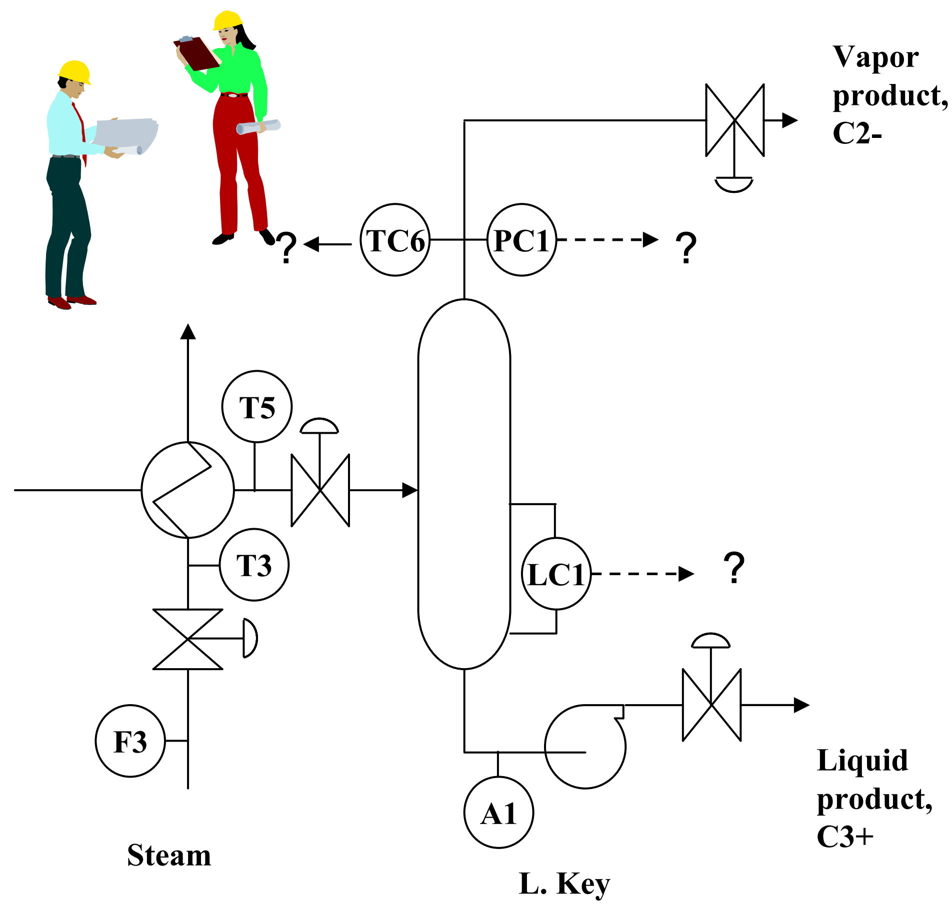
Principles of Single-Loop Feedback Control

Multiloop control contains many single-loop systems.

Conclusion: We need to understand single-loop principles.

Lesson Outline

- **Ten observations on what affects single-loop feedback**
- **Workshop**

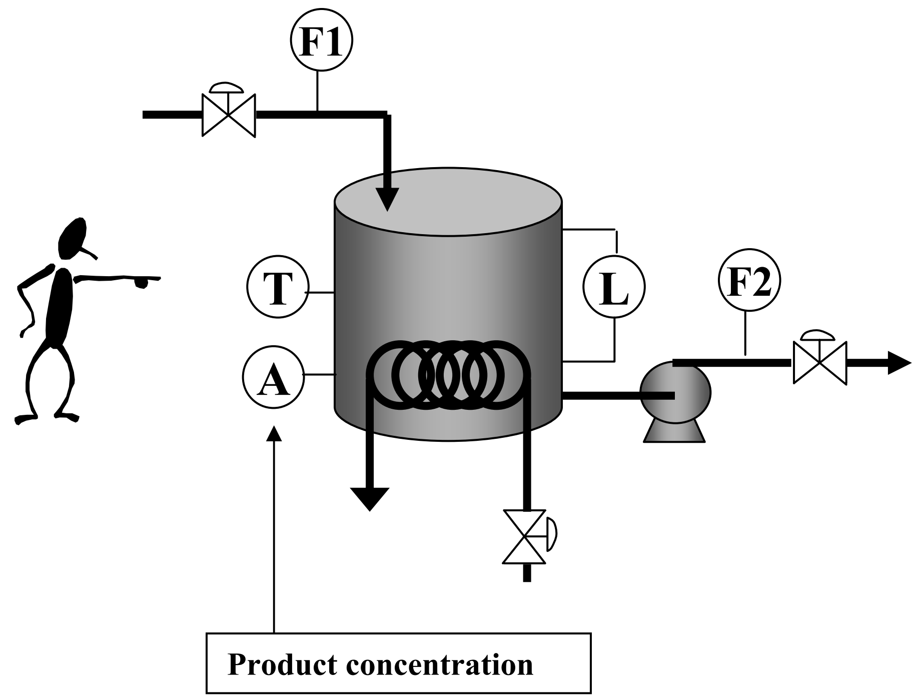


Principles of Single-Loop Feedback Control

Performance Observations #0. **Very obvious, but not so obvious for multivariable systems.**

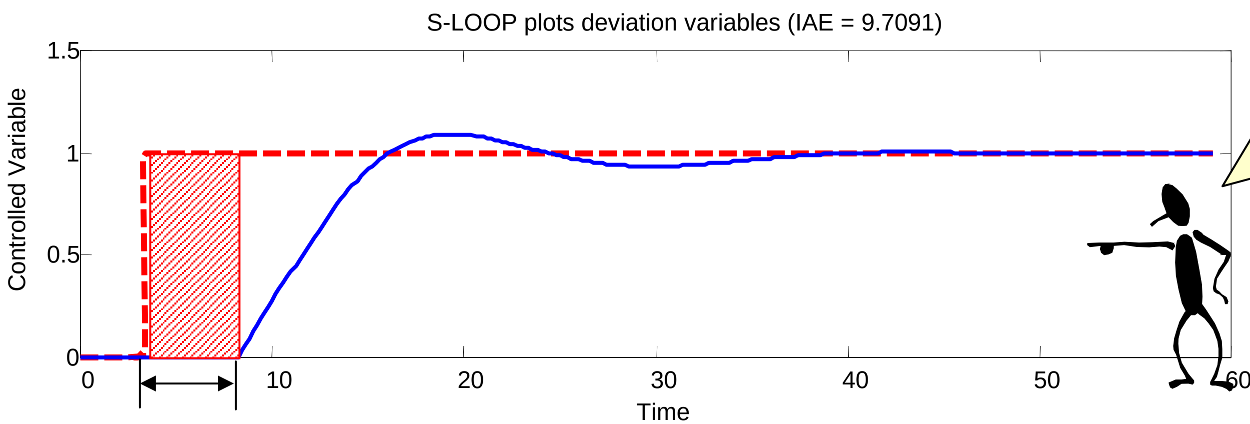
Process gain must not be zero ($K_p \neq 0$). Gain should not be too small (range) or too large (sensitivity).

- Which valves can be used to control each measured variable?
- Would the answer change if many single-loops were implemented at the same time?



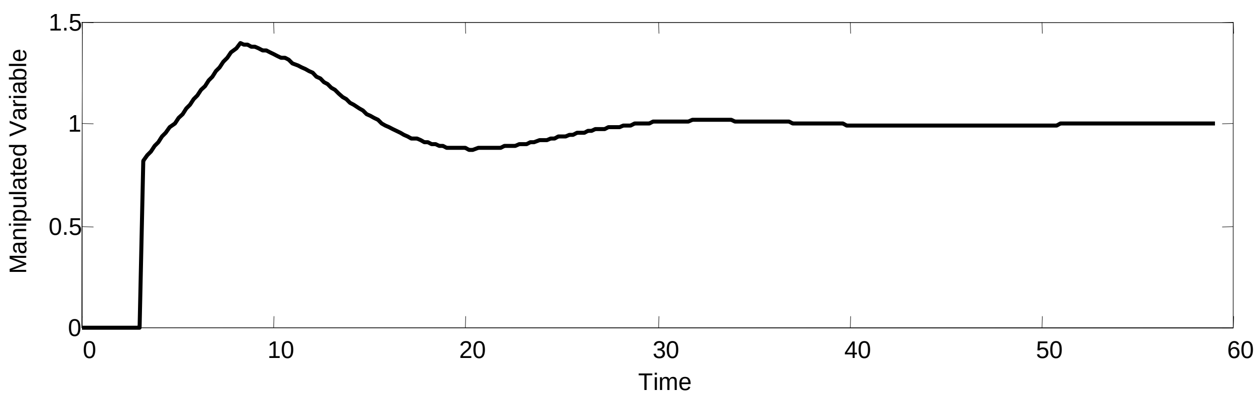
Principles of Single-Loop Feedback Control

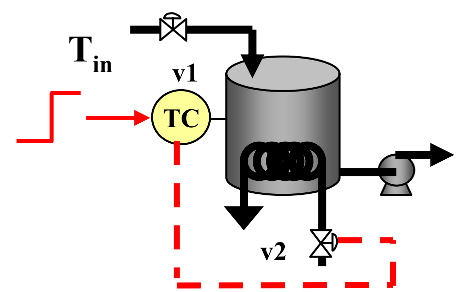
Performance Observation #1. Feedback dead time limits best possible performance



Discuss why the red area defines deviation from set point that cannot be reduced by any feedback.

θ_p, feedback dead time





Principles of Single-Loop Feedback Control

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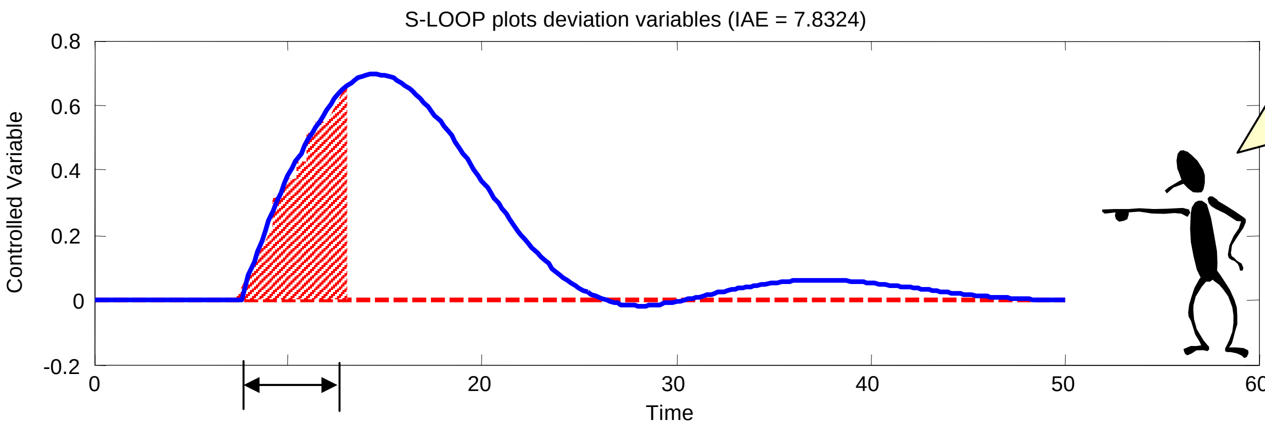

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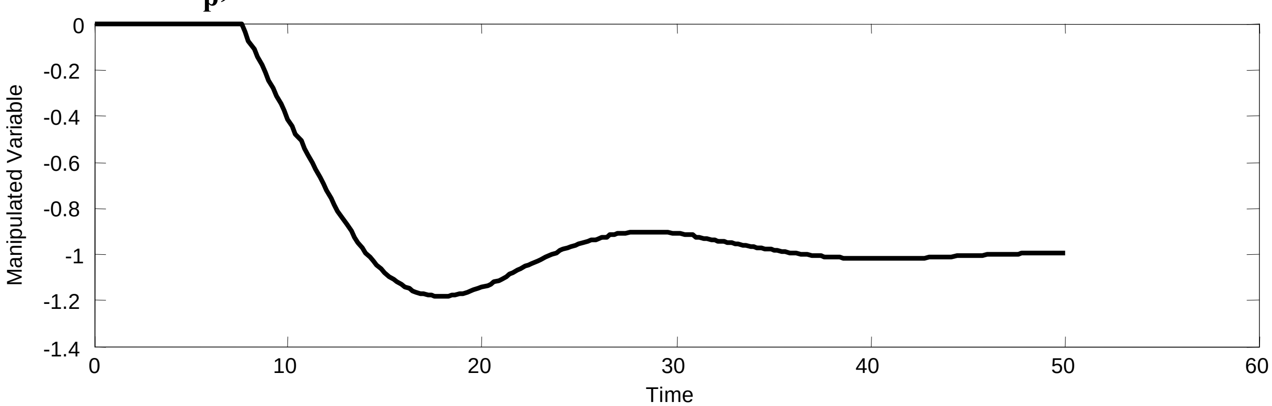
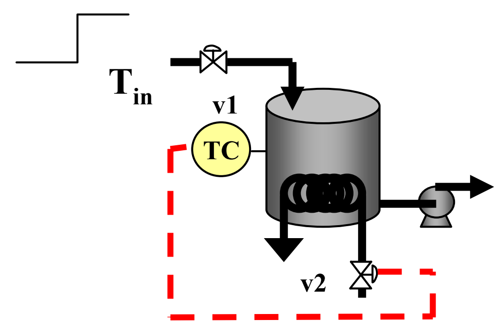
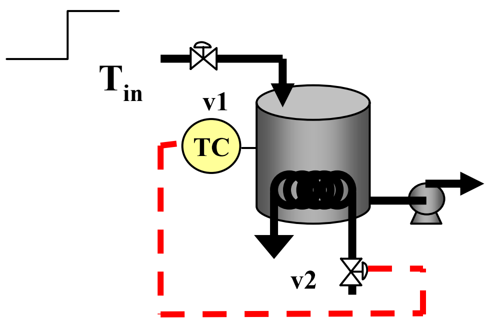
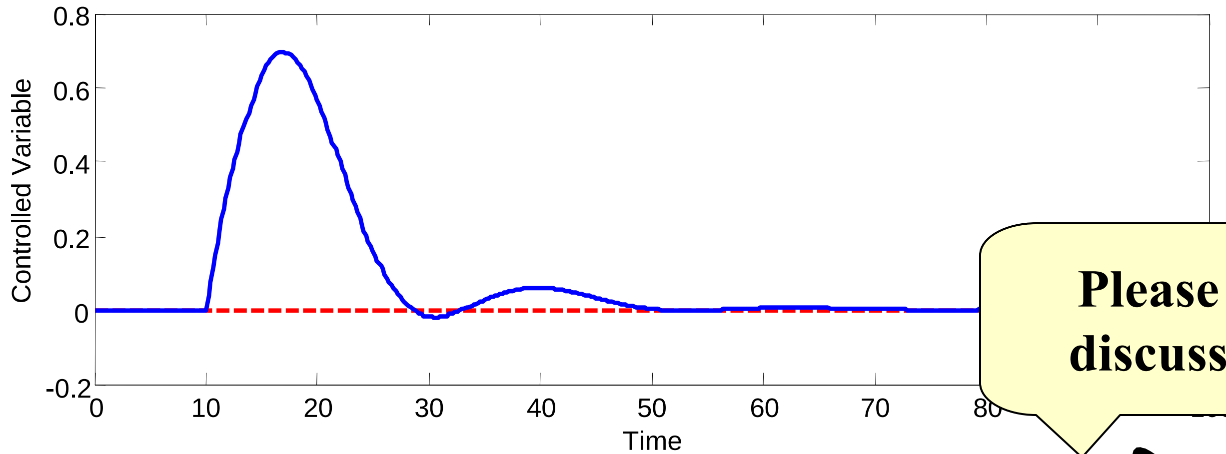


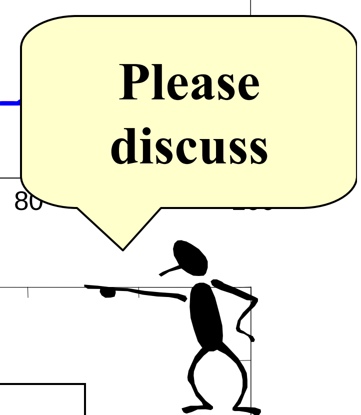


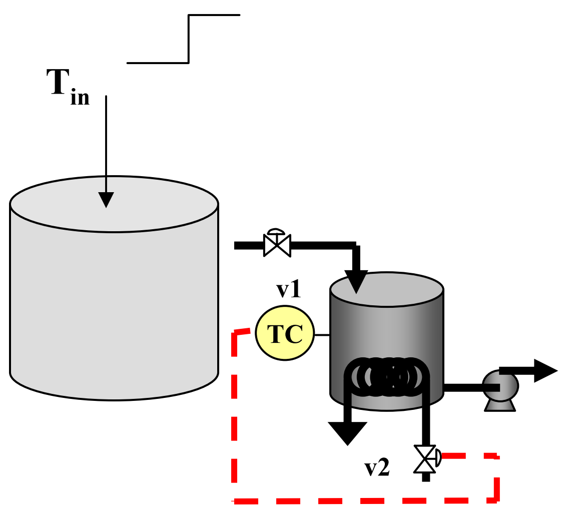
Principles of Single-Loop Feedback Control

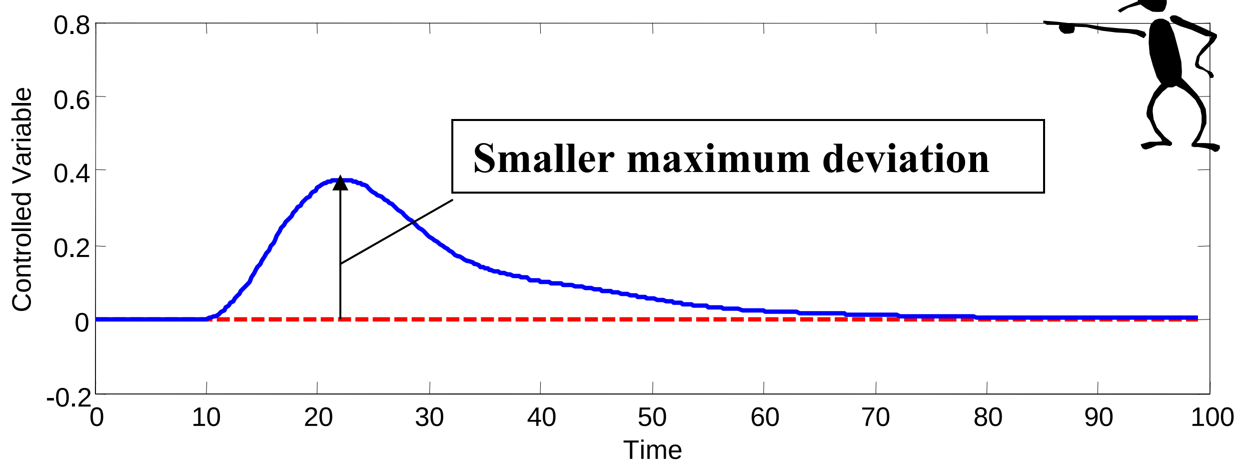
Performance Observation #2. Large disturbance time constants slow disturbances and improve performance.







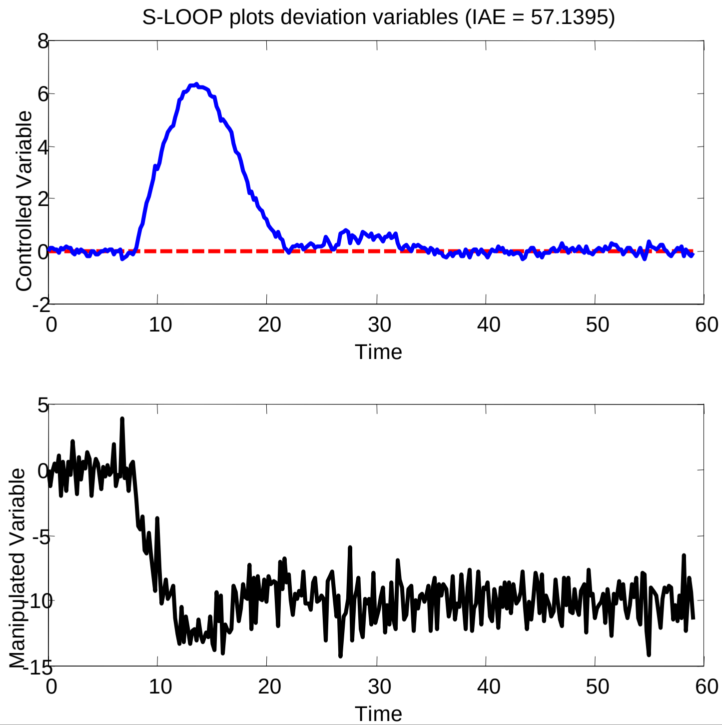




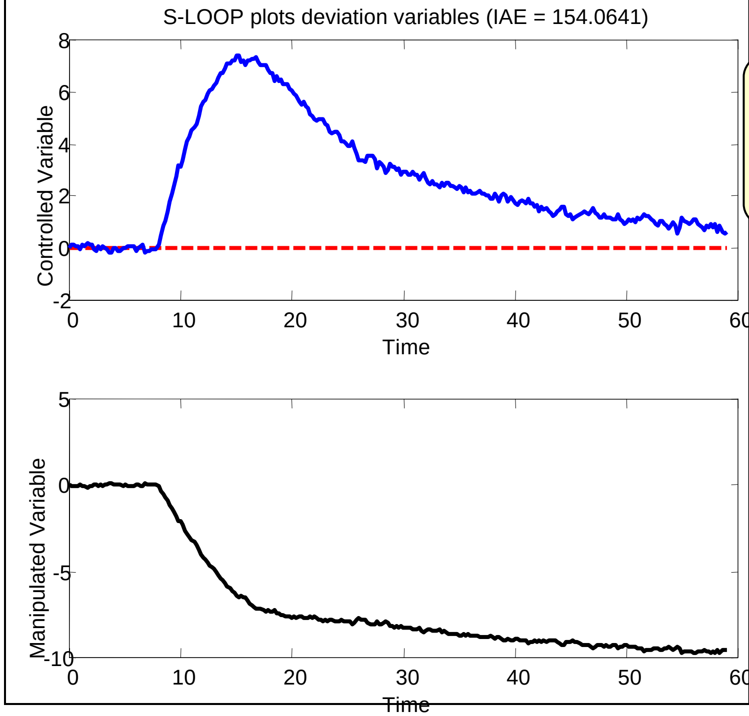
Principles of Single-Loop Feedback Control

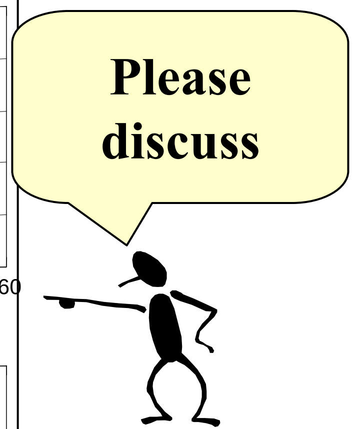
Performance Observation #3. Feedback must change the MV aggressively to improve performance.

Kc = 1.3, TI = 7, Td = 1.5



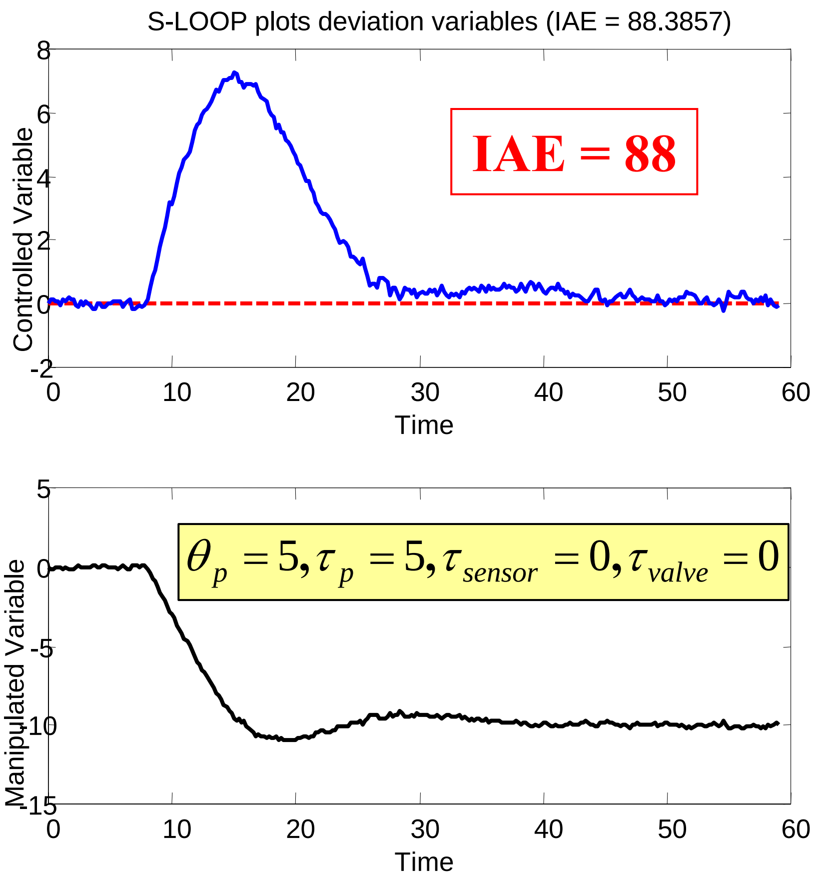
Kc = 0.6, TI = 10, Td = 0

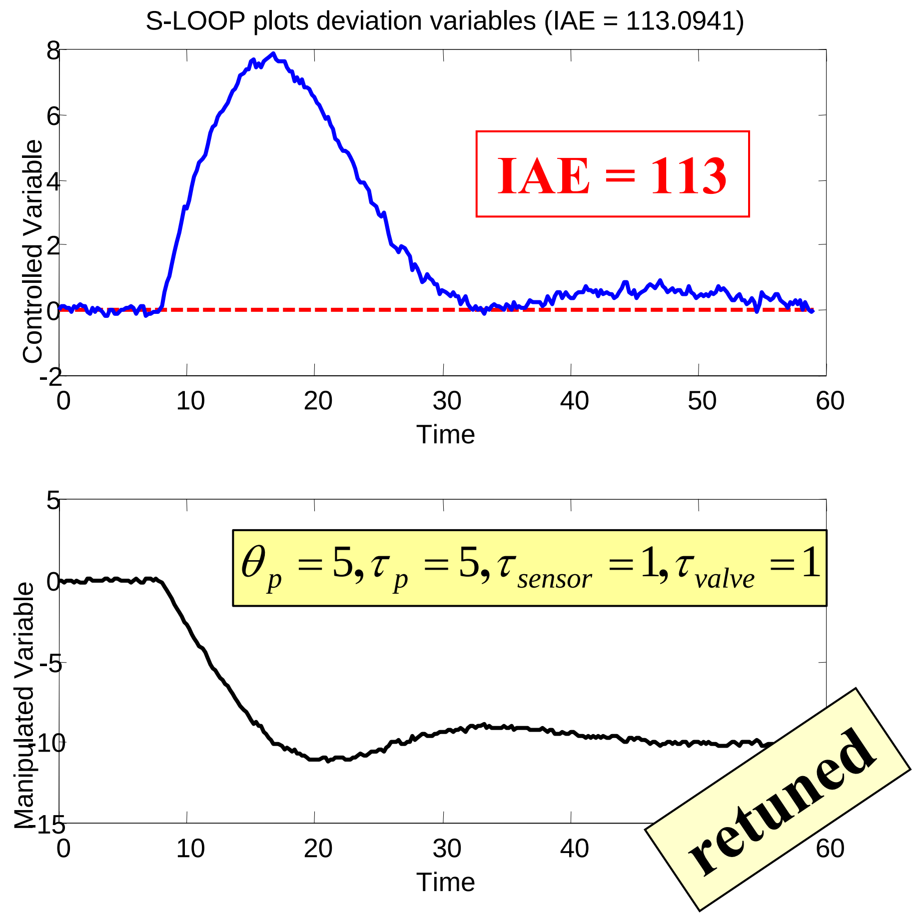




Principles of Single-Loop Feedback Control

Performance Observation #4. Sensor and final element dynamics are in feedback loop, slow responses degrade performance.

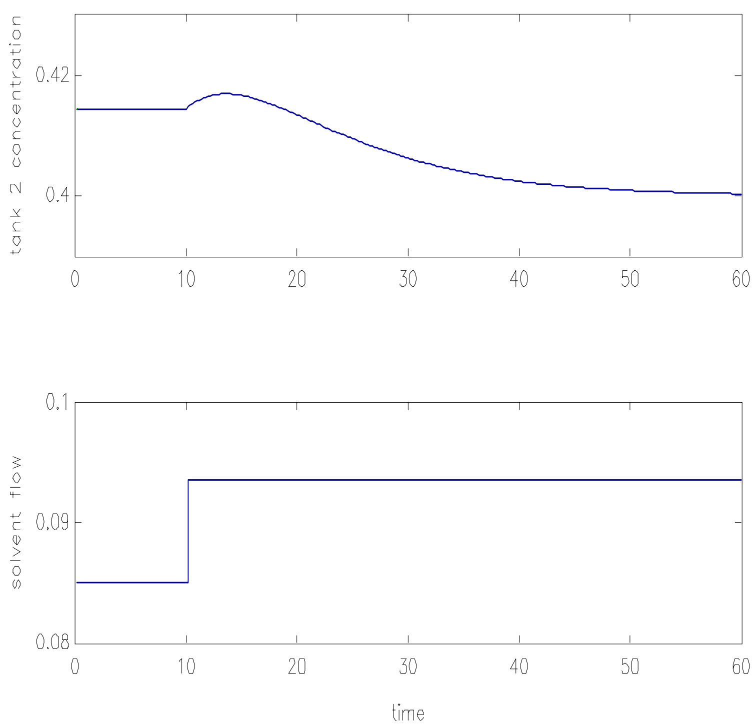




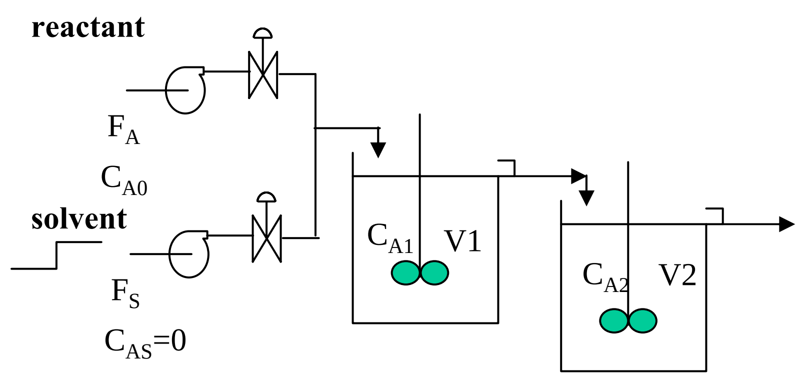
Principles of Single-Loop Feedback Control

Performance Observation #5. Inverse response (RHP zero) degrades feedback control performance.

Process reaction curve for the effect of solvent flow rate on the reactor effluent concentration.



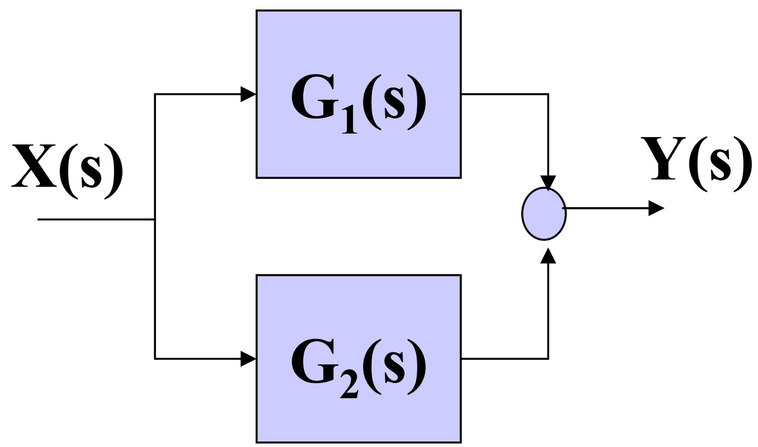
Two, isothermal CSTRs with reaction A → B and $F_S >> F_A$



Principles of Single-Loop Feedback Control

Performance Observation #5. Inverse response degrades feedback control performance.

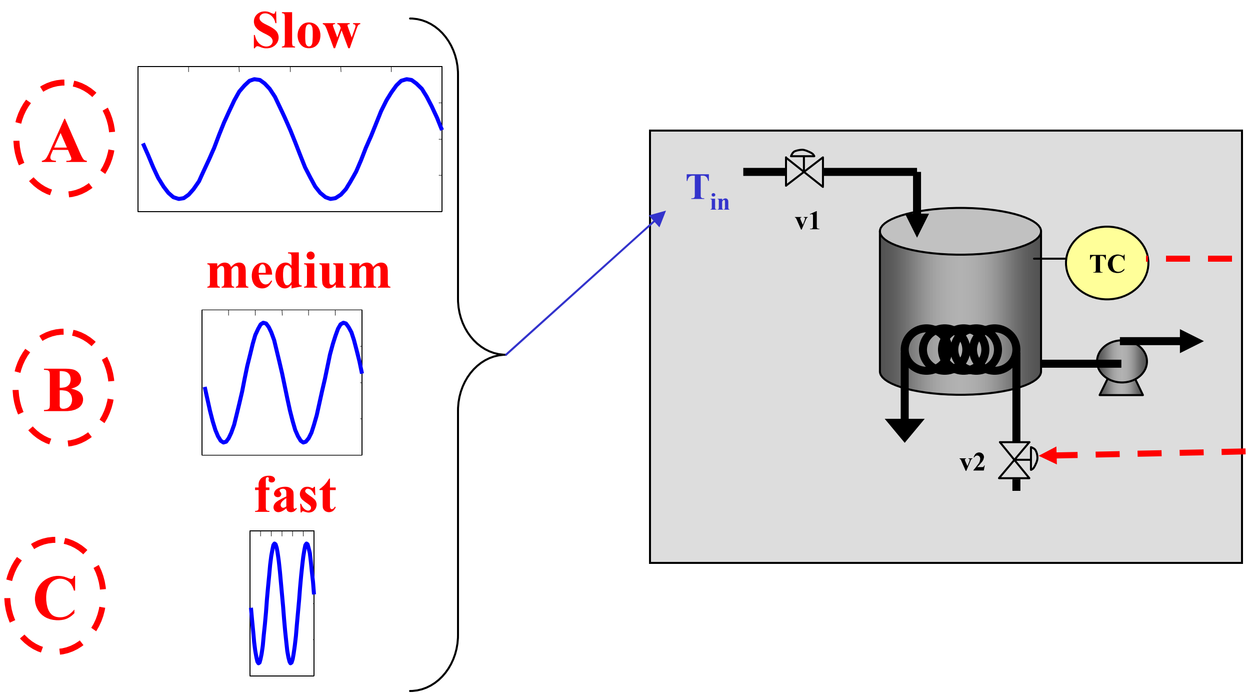
PARALLEL STRUCTURES



Inverse response occurs when parallel paths have different signs for their steady-state gains and the path with the "smaller" magnitude gain is faster.

Principles of Single-Loop Feedback Control

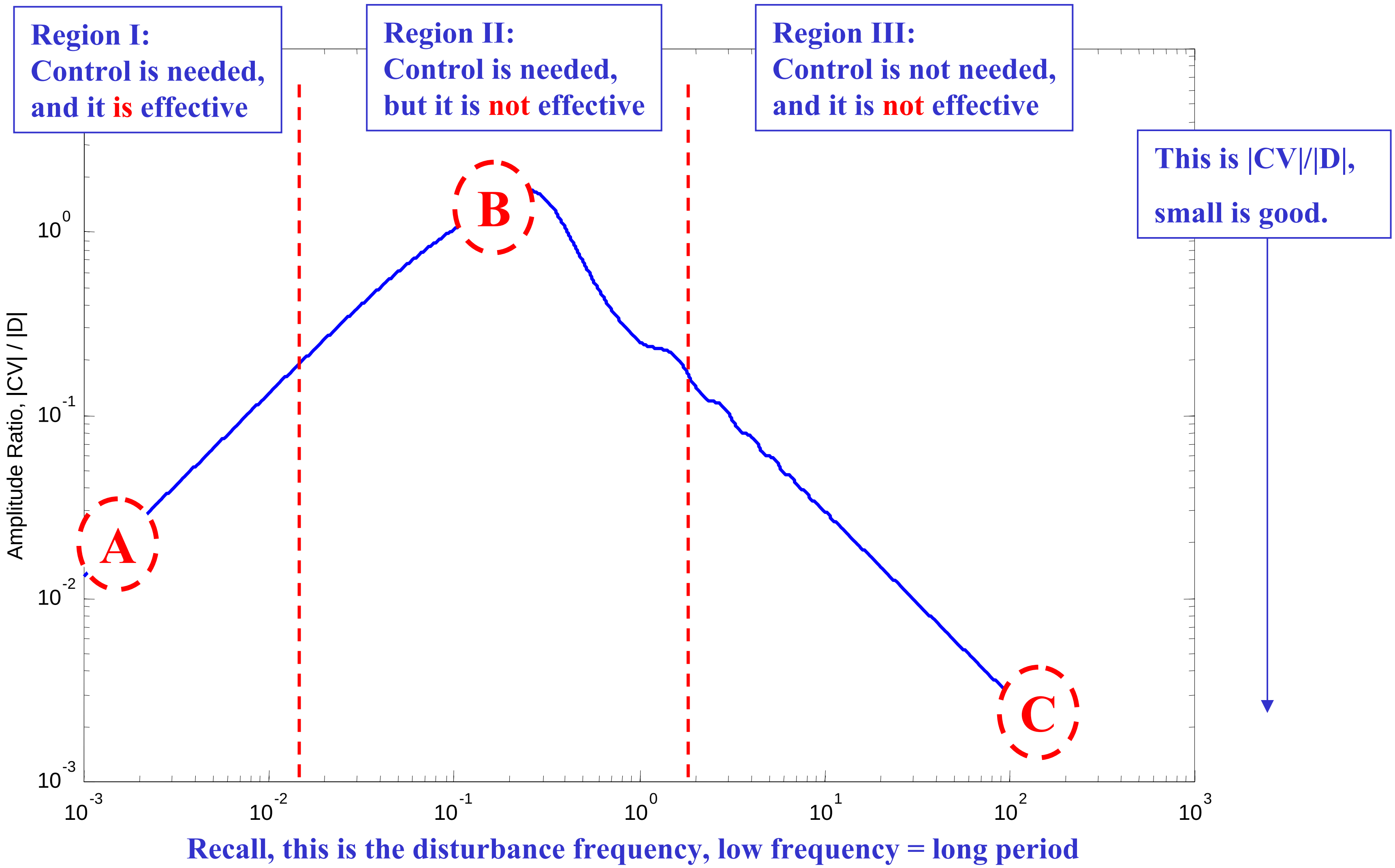
Performance Observation #6. Disturbance frequencies around and higher than the critical frequency cannot be controlled .



Behavior of the tank temperature for three cases?

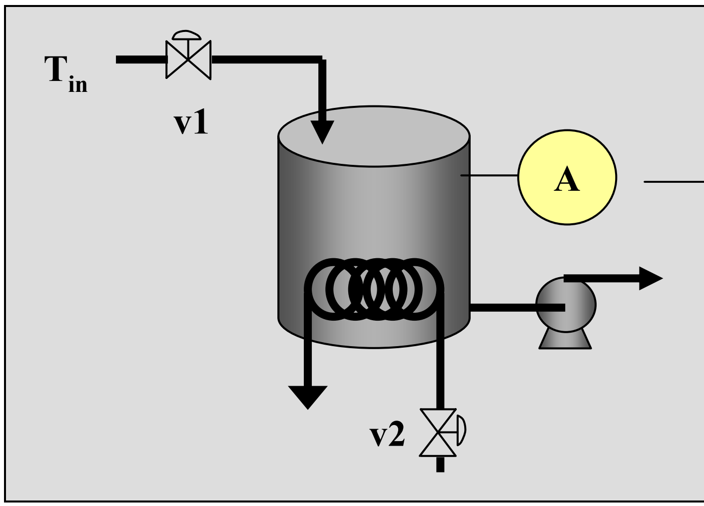
Principles of Single-Loop Feedback Control

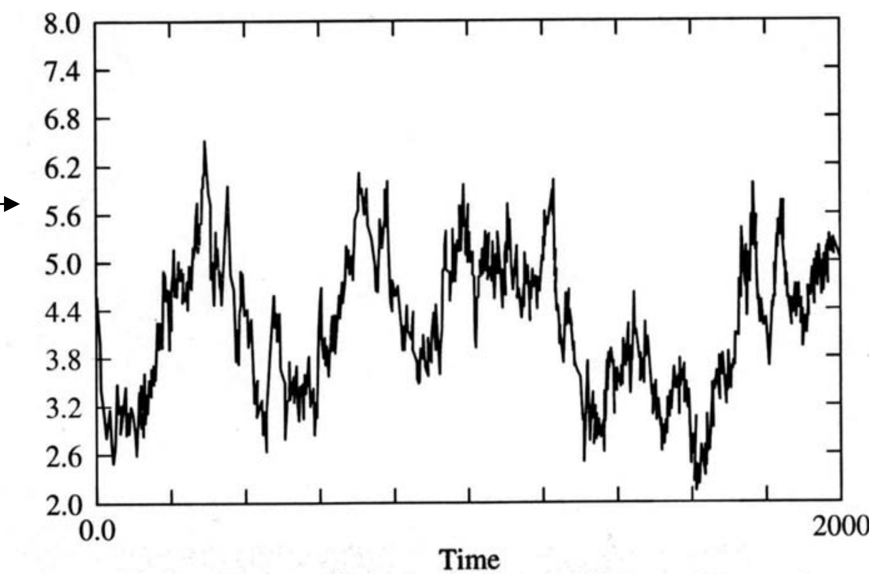
Performance Observation #6: Frequency Response



Performance Observation #6. **Disturbance frequencies around and higher than the critical frequency cannot be controlled .**

Let's apply frequency response concepts to a practical example. Can we reduce this open-loop variation?





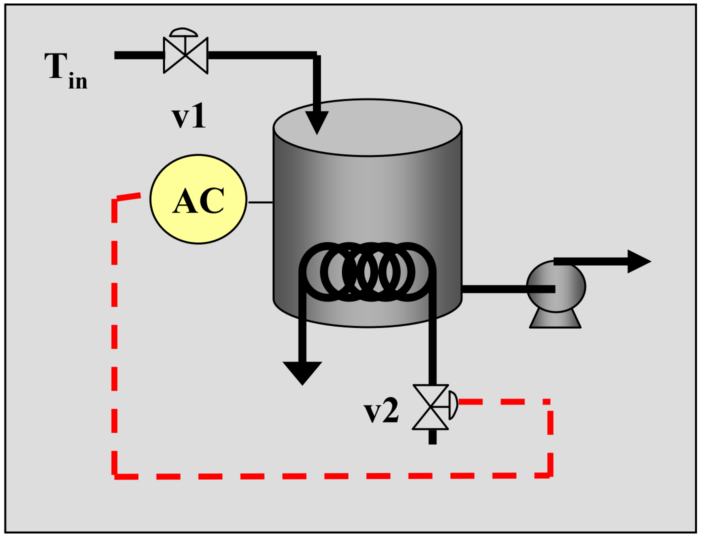
Feedback dynamics are:

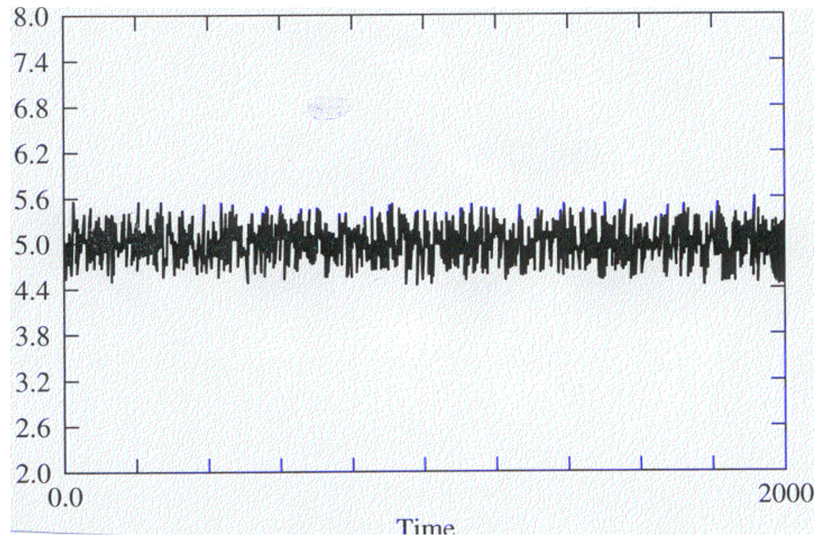
$$\frac{A(s)}{v(s)} = \frac{1.0e^{-2s}}{2s+1}$$

We note that the variation has many frequencies, some much slower than the feedback dynamics.

Performance Observation #6. Disturbance frequencies around and higher than the critical frequency cannot be controlled .

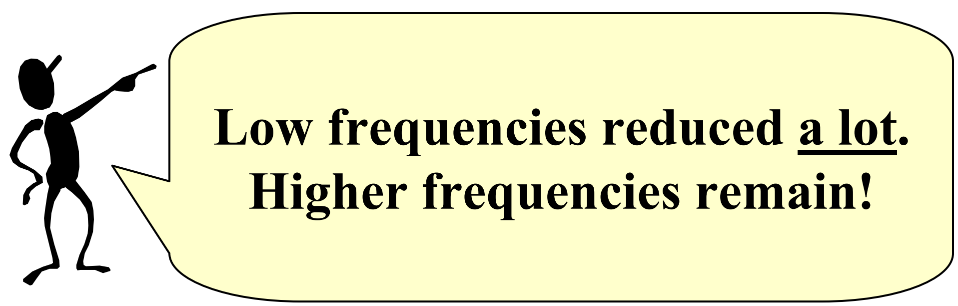
Yes, we can we reduce the variation substantially because of the dominant low frequency of the disturbance effects.





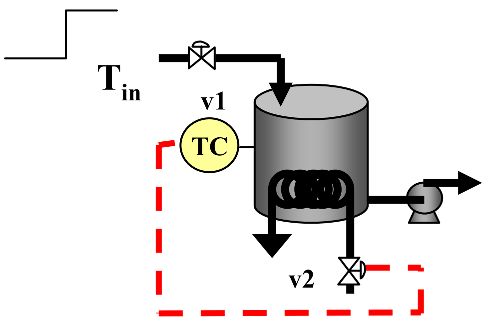
Feedback dynamics are:

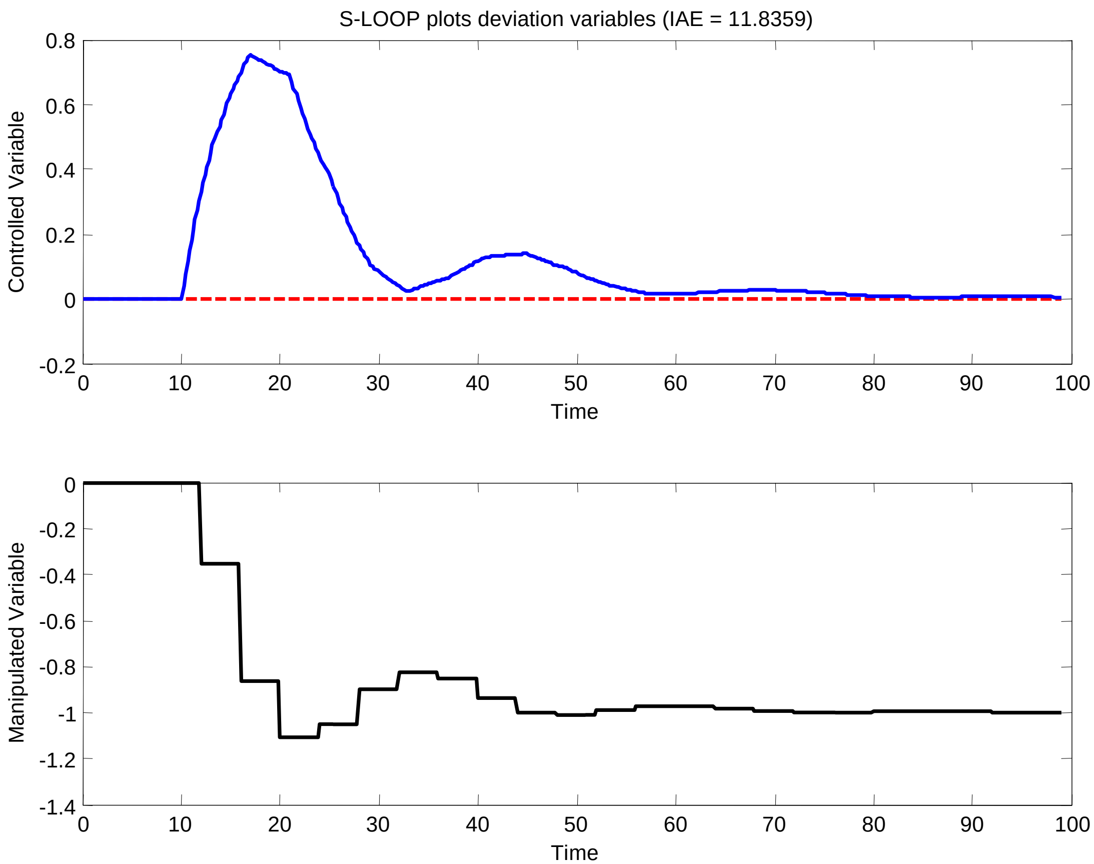
$$\frac{A(s)}{v(s)} = \frac{1.0e^{-2s}}{2s+1}$$



Principles of Single-Loop Feedback Control

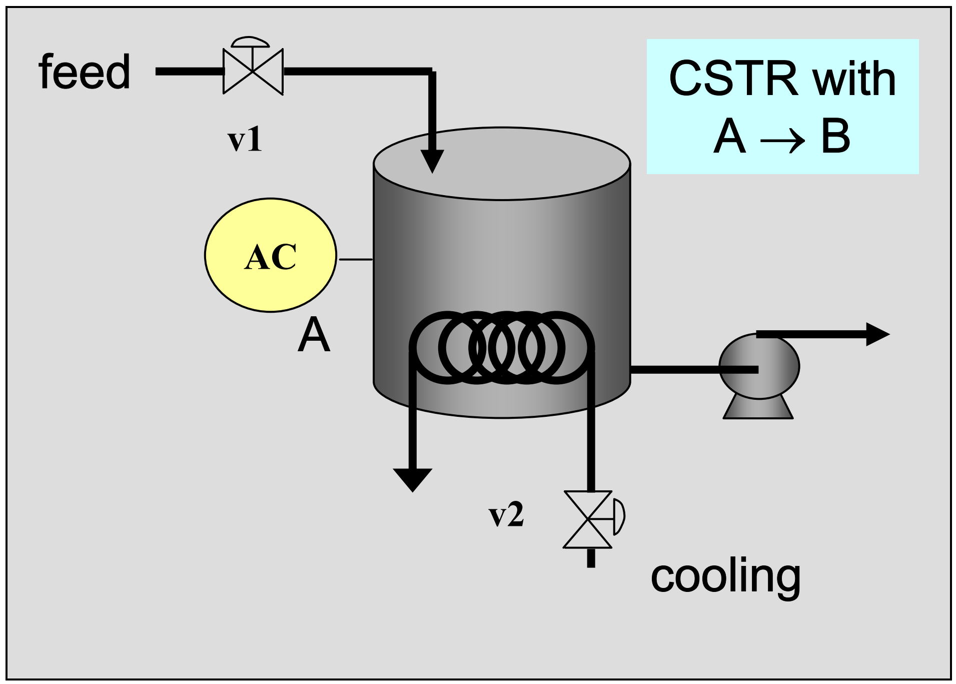
Performance Observation #7. Long controller execution periods degrade feedback control performance.





Principles of Single-Loop Feedback Control

Performance Observation #8. Use process understanding to provide a CV-MV pairing with good steady-state and dynamic behavior.

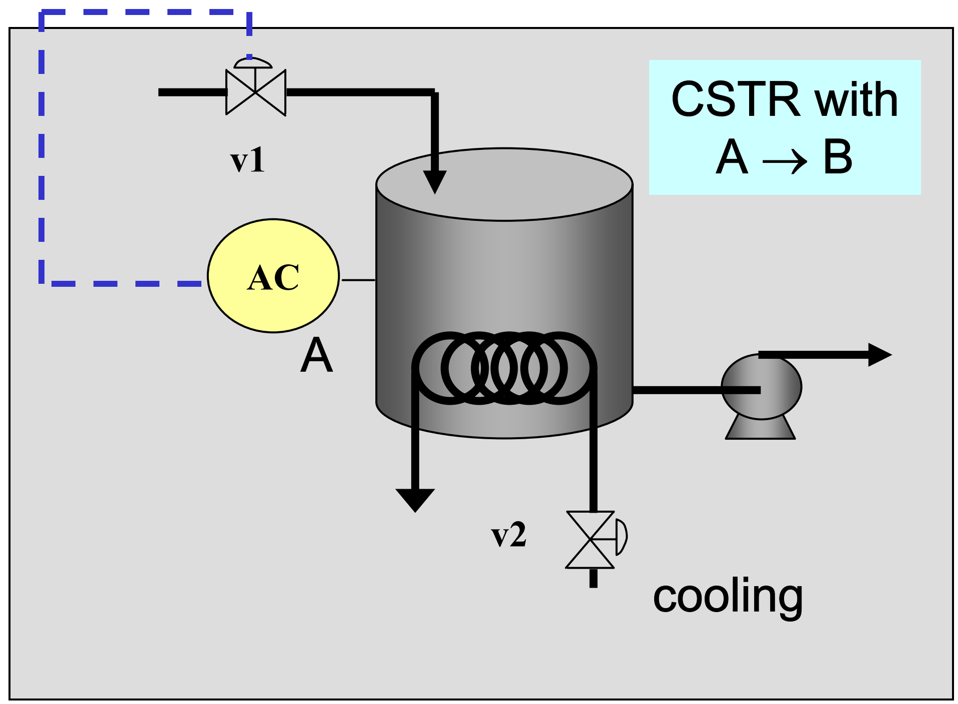


Principles of Single-Loop Feedback Control

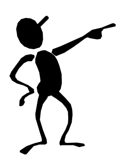
Performance Observation #8. Use process understanding to provide a CV-MV paring with good steady-state and dynamic behavior.

v1 → Component material balance

v2 → Energy balance → Component material balance



v1 gives faster feedback dynamics

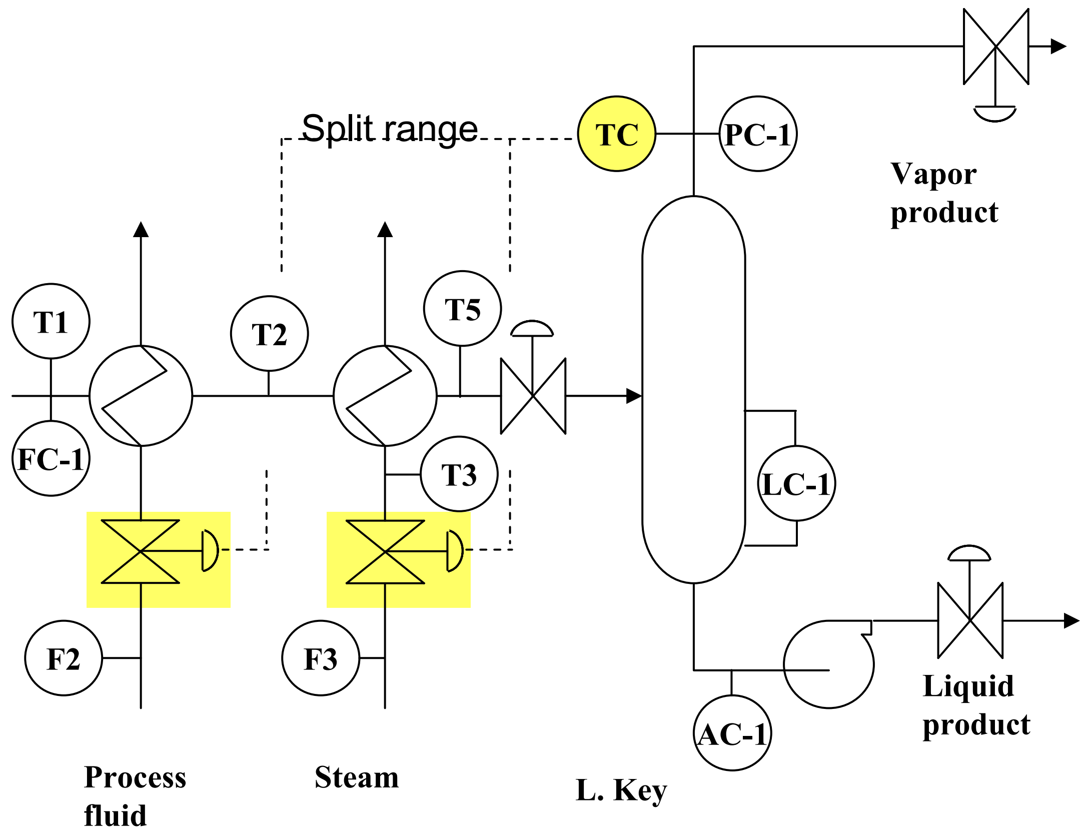


Principles of Single-Loop Feedback Control

Performance Observation #9. Some designs require a loop to adjust two valves to achieve the desired range and precision.

Two heating valves are available for manipulation.

Adjust only one at a time.

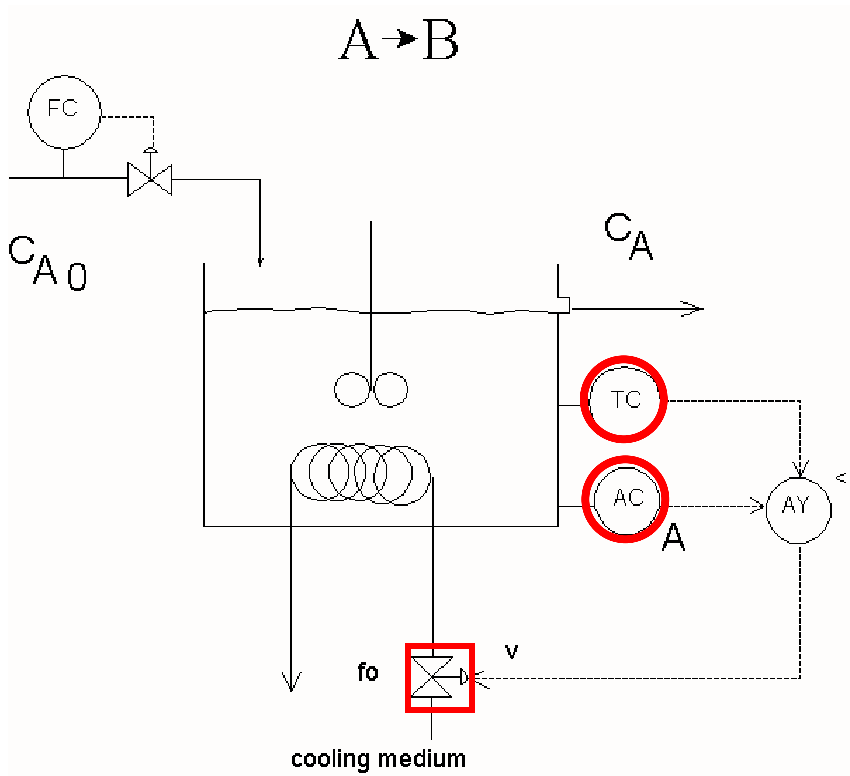


Principles of Single-Loop Feedback Control

Performance Observation #10. Some designs require several loops to adjust the same manipulated variable to ensure that the highest priority objective is achieved.

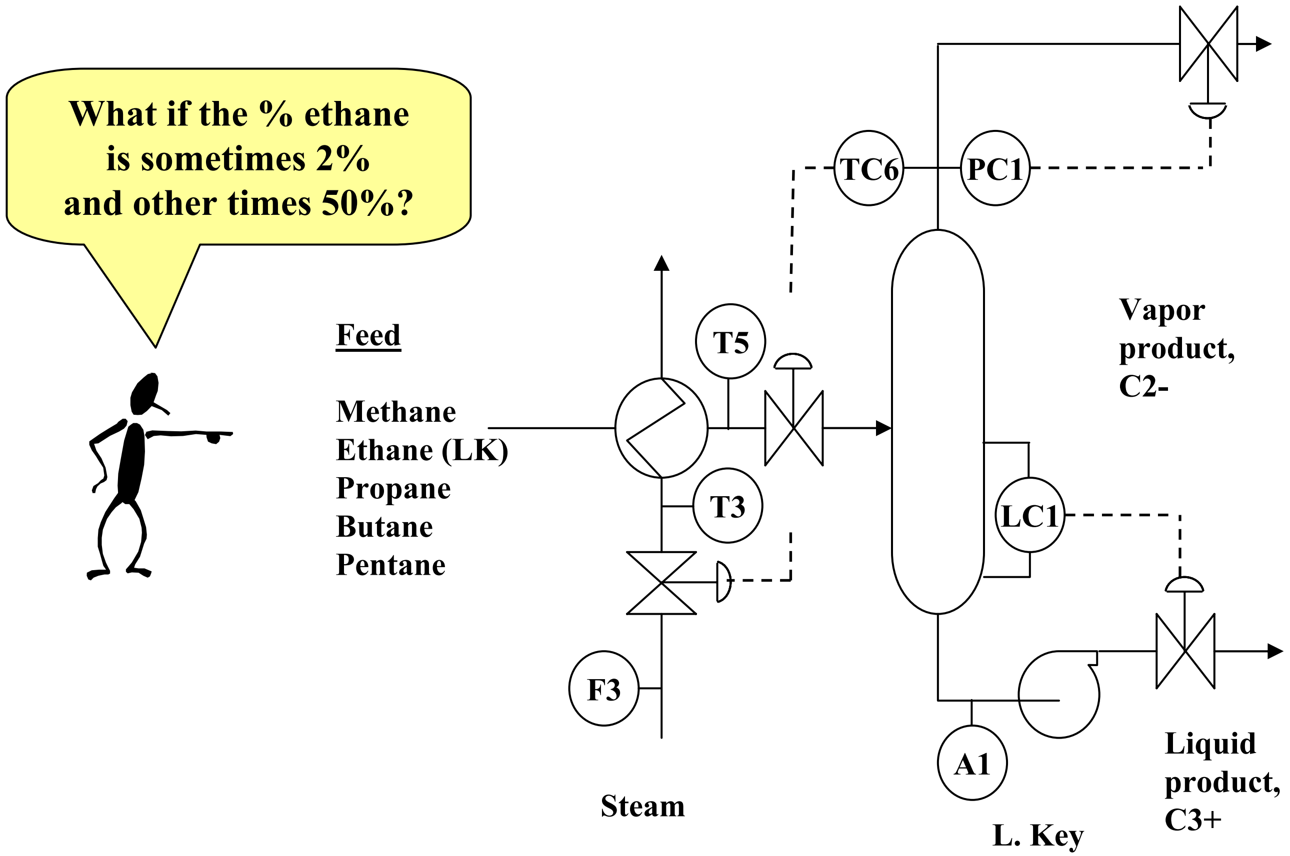
Control the effluent concentration of A but do not exceed a maximum reactor temperature.

Only the cooling medium may be adjusted.



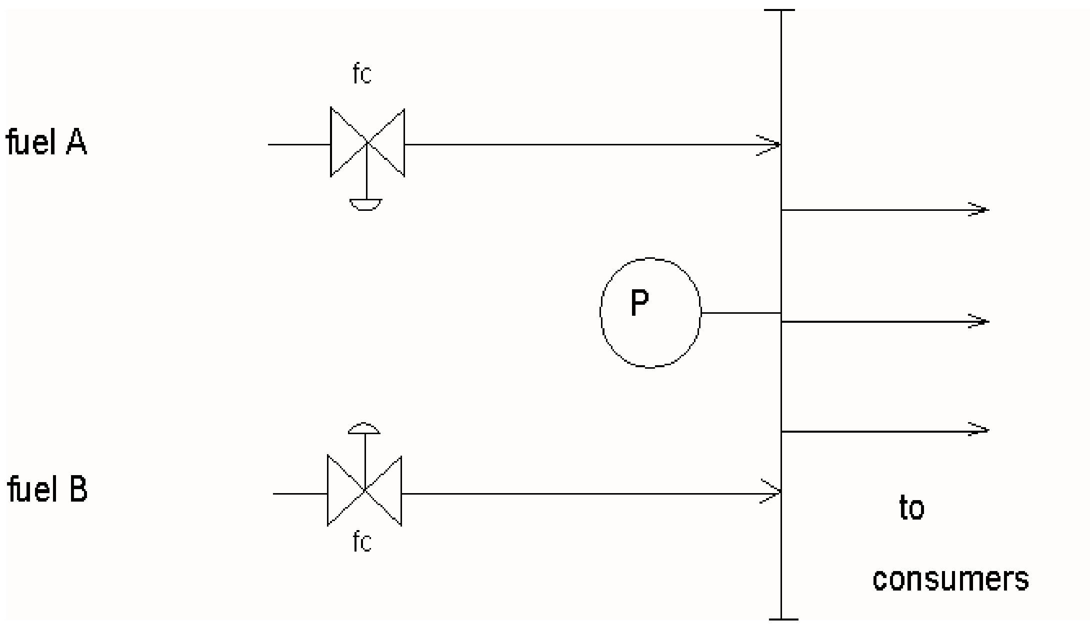
signal select

Single-loop Control, Workshop #1



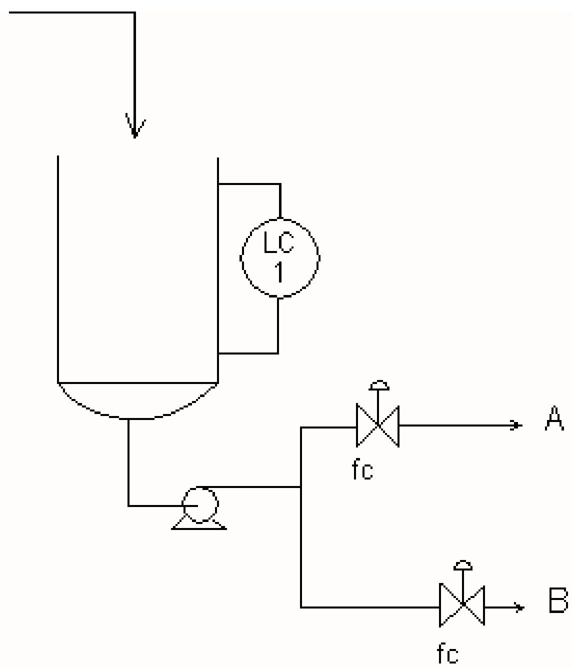
Single-loop Control, Workshop #2

The consumers vary and we must satisfy them by purchasing fuel gas. Therefore, we want to control the pressure in the gas distribution network. Design a control system. By the way, fuel A is less expensive.



Single-loop Control, Workshop #3

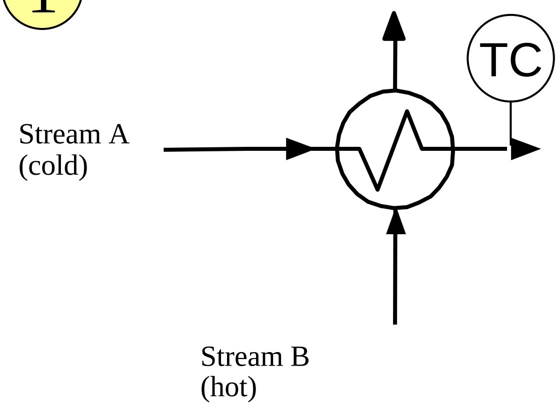
Design a controller that will control the level in the bottom of the distillation tower and send as much flow as possible to Stream A



A = downstream unit processing (profitable)
B = waste (costly, minimize this!)

Single-loop Control, Workshop #4

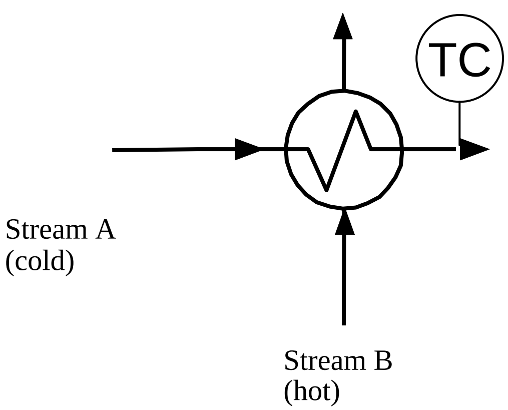
1



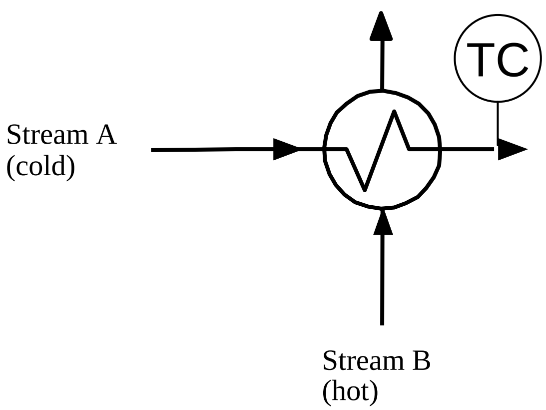
Freedom to adjust flows

	Stream A	Stream B
1.	Constant	Adjustable
2.	Adjustable	Constant
3.	Constant	Constant

2



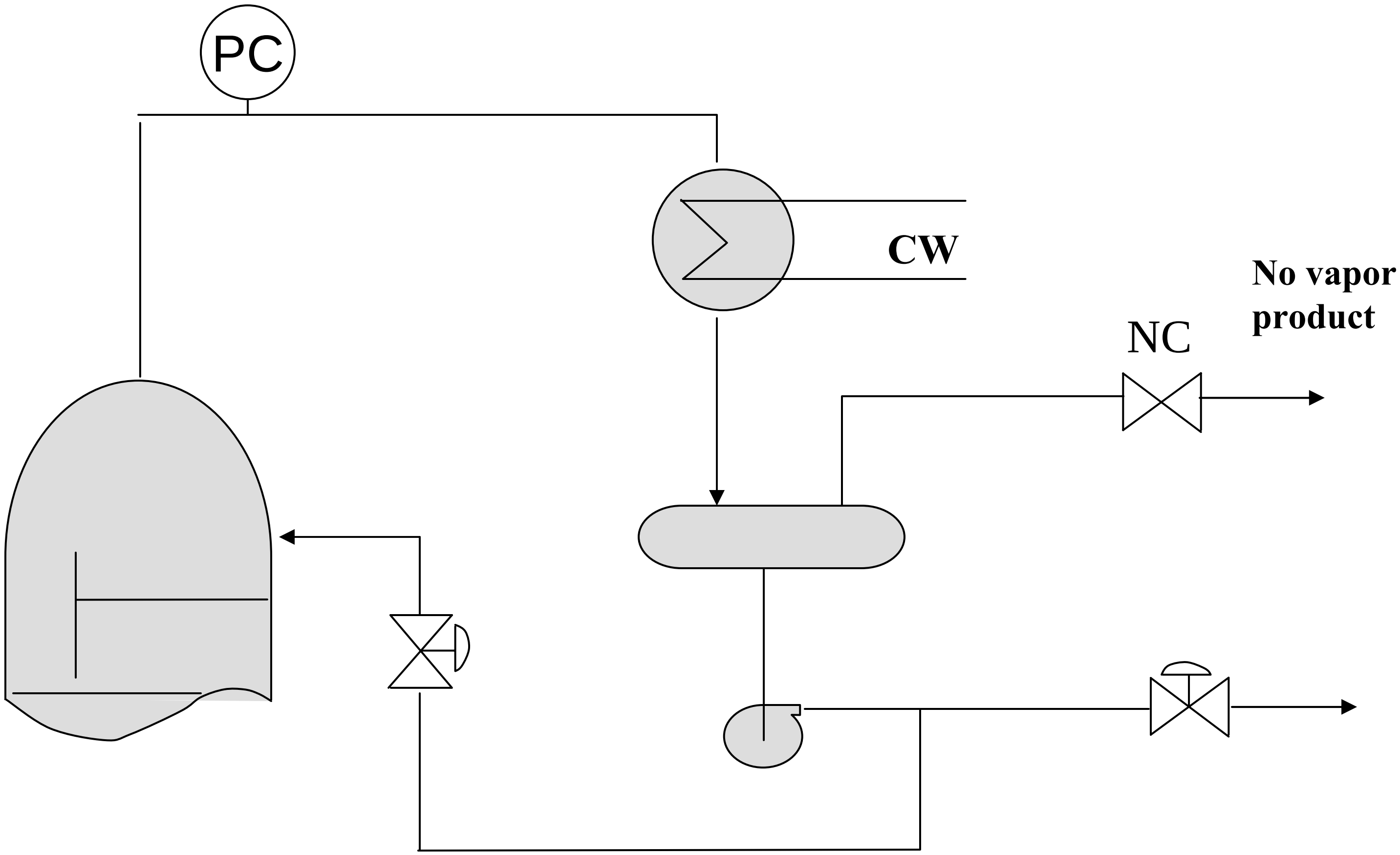
3



You can add valve(s) and piping.

Single-loop Control, Workshop #5

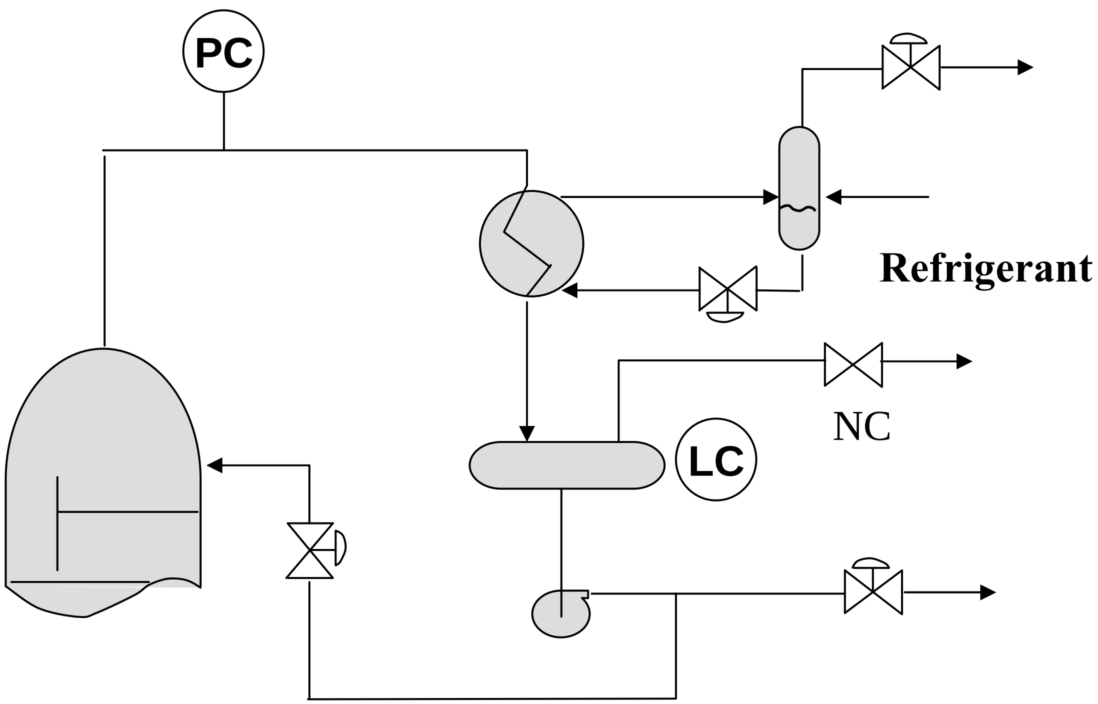
Class exercise: Distillation overhead system. Design a pressure controller. (Think about affecting U, A and ΔT)



You can add valve(s) and piping.

Single-loop Control, Workshop #6

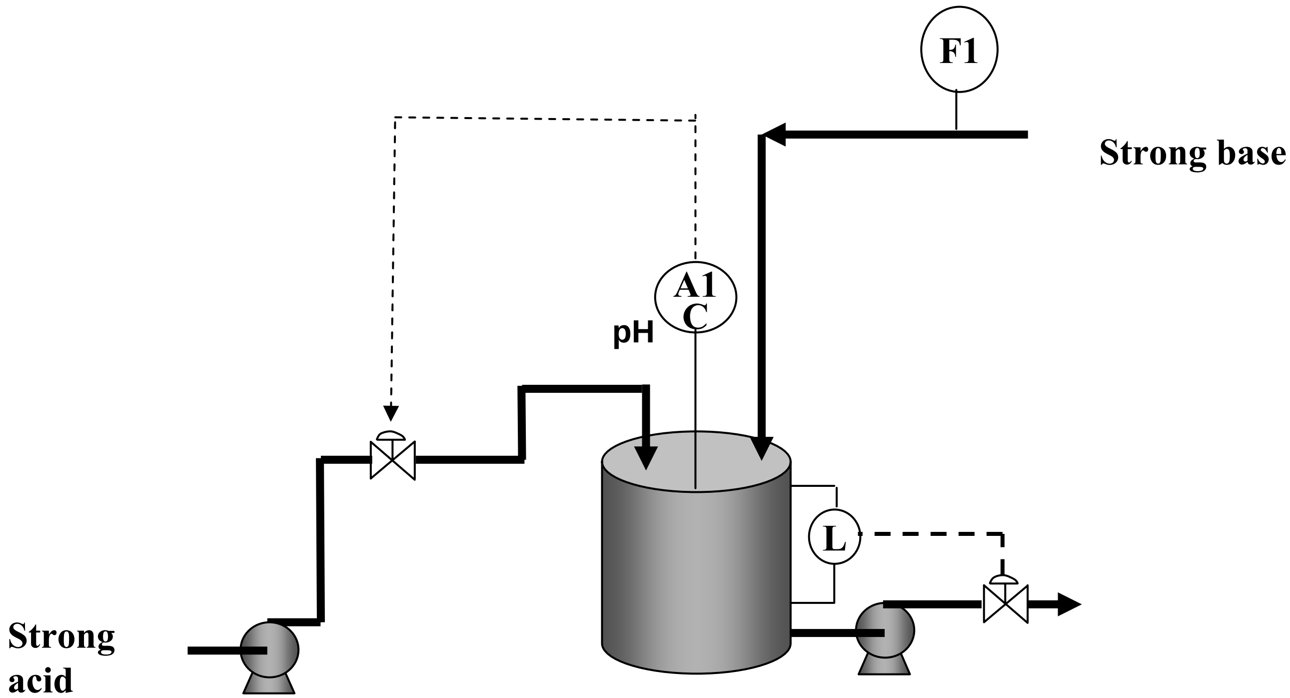
Class exercise: **Distillation overhead system. Design a pressure controller. (Think about affecting U, A and ΔT)**



You can add valve(s) and piping.

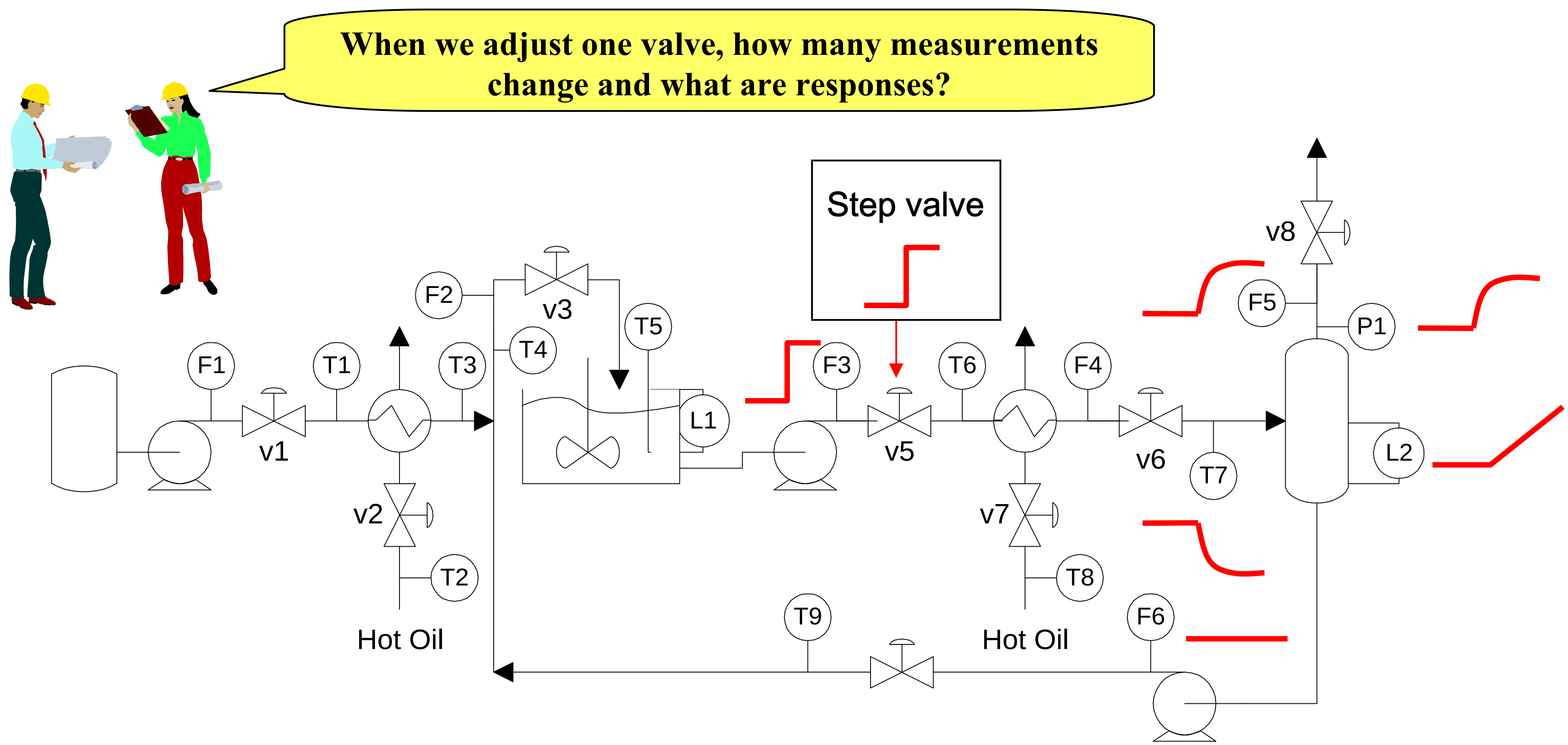
Single-loop Control, Workshop #7

The following control system has a very large gain near pH = 7. For a strong acid/base, performance is likely to be poor. How can we improve the situation?



Principles of Multivariable Dynamic Processes and MultiLoop Control

Basically, multiloop designs are simply many single-loop (PID) controllers. Then, what is new? ** INTERACTION **



Principles of Multivariable Dynamic Processes and MultiLoop Control

** INTERACTION **

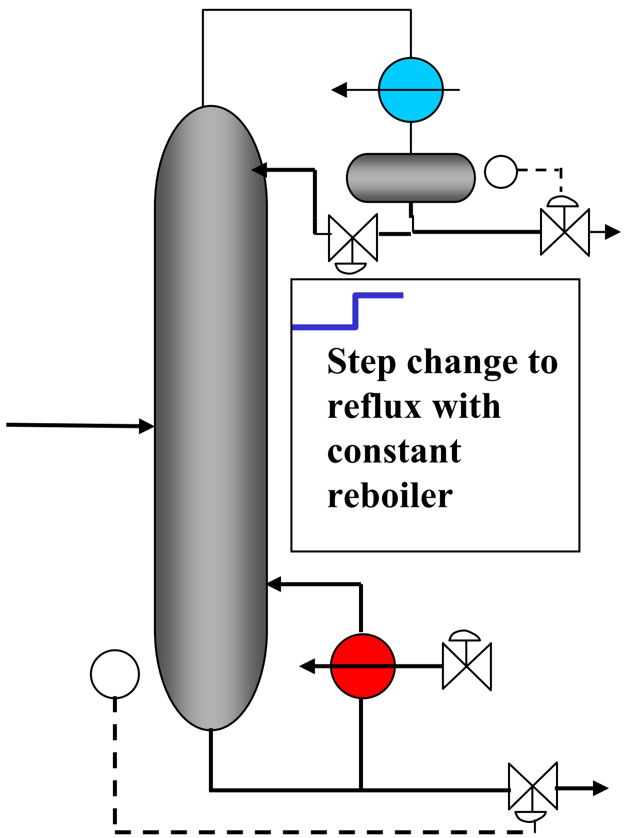
LESSON OUTLINE

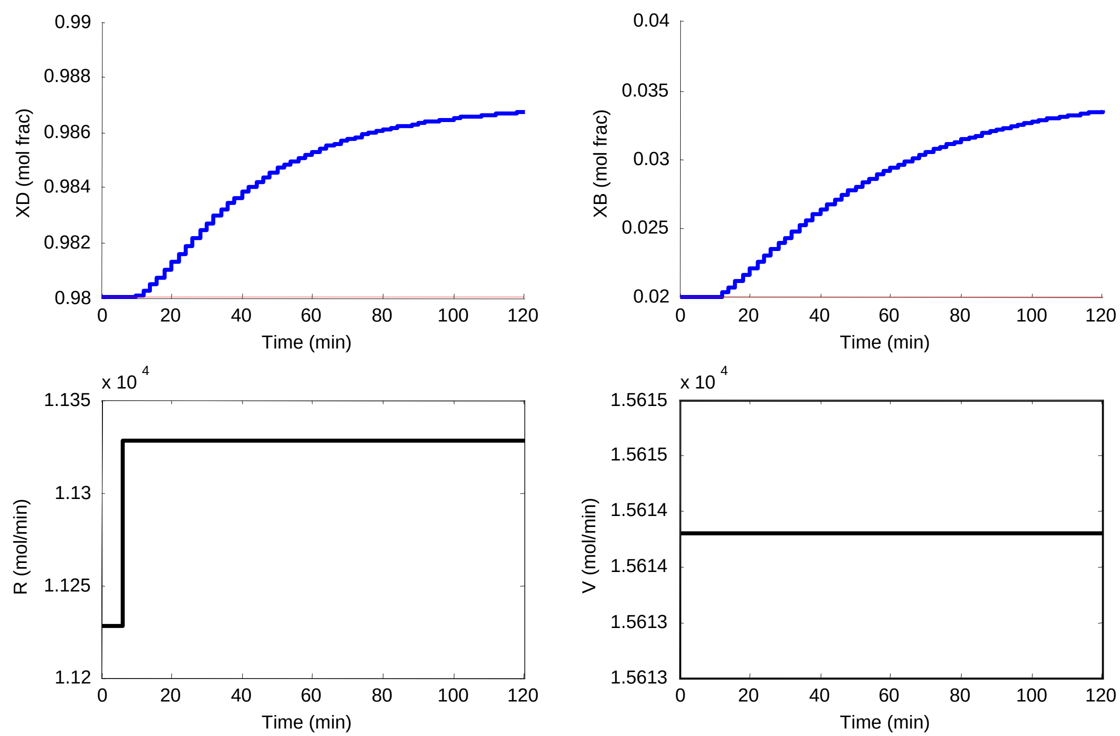
- Interaction – A *brief* definition
- Controllability – Can desired performance be achieved?
- Integrity – What happens to the system when a controller stops functioning?
- Directionality – A key factor in control performance

Class Exercises and Workshops throughout

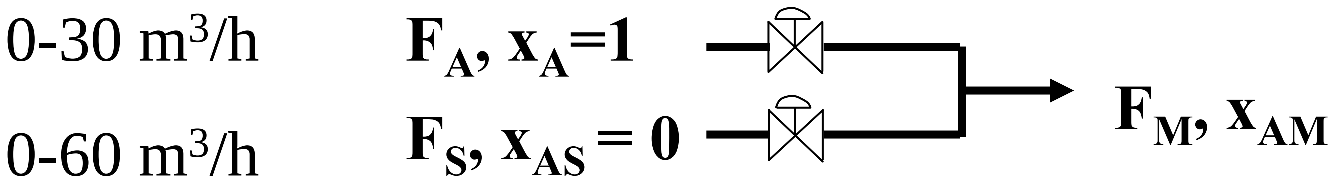
INTERACTION: The difference in multivariable control

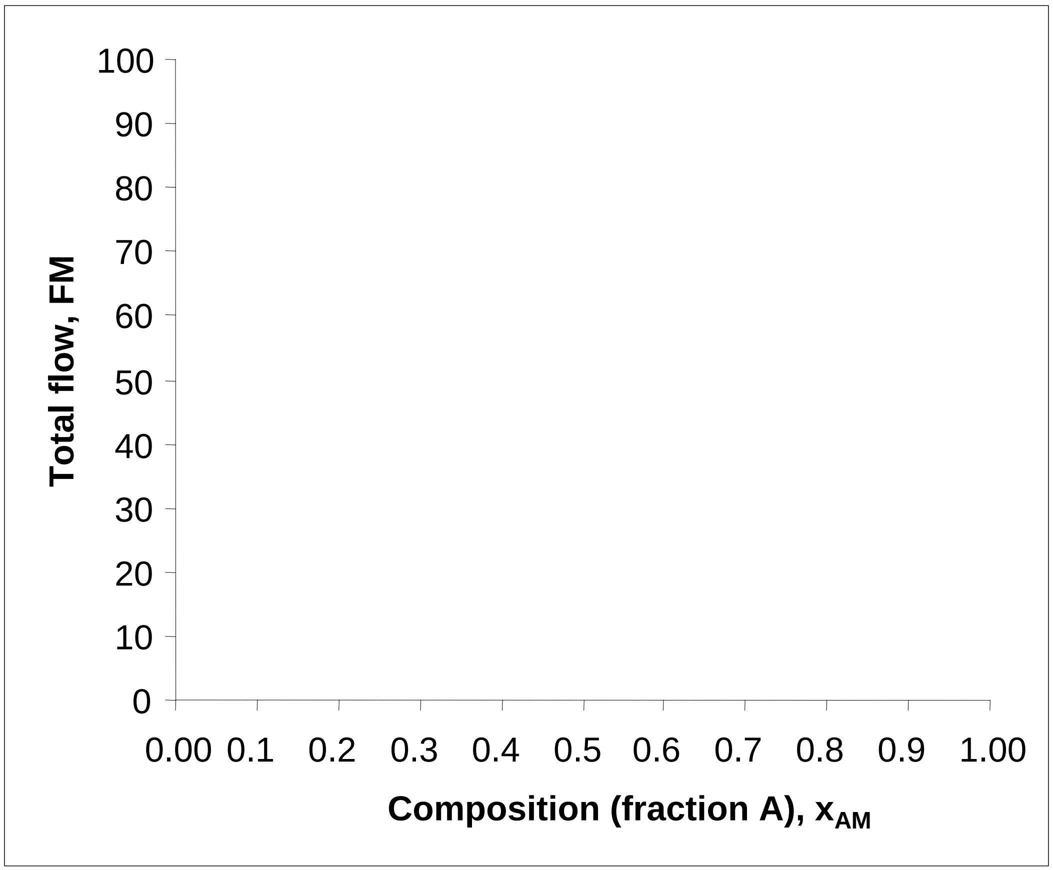
Definition: A multivariable process has interaction when input (manipulated) variables affect more than one output (controlled) variable.





Multivariable Interaction, Workshop #1





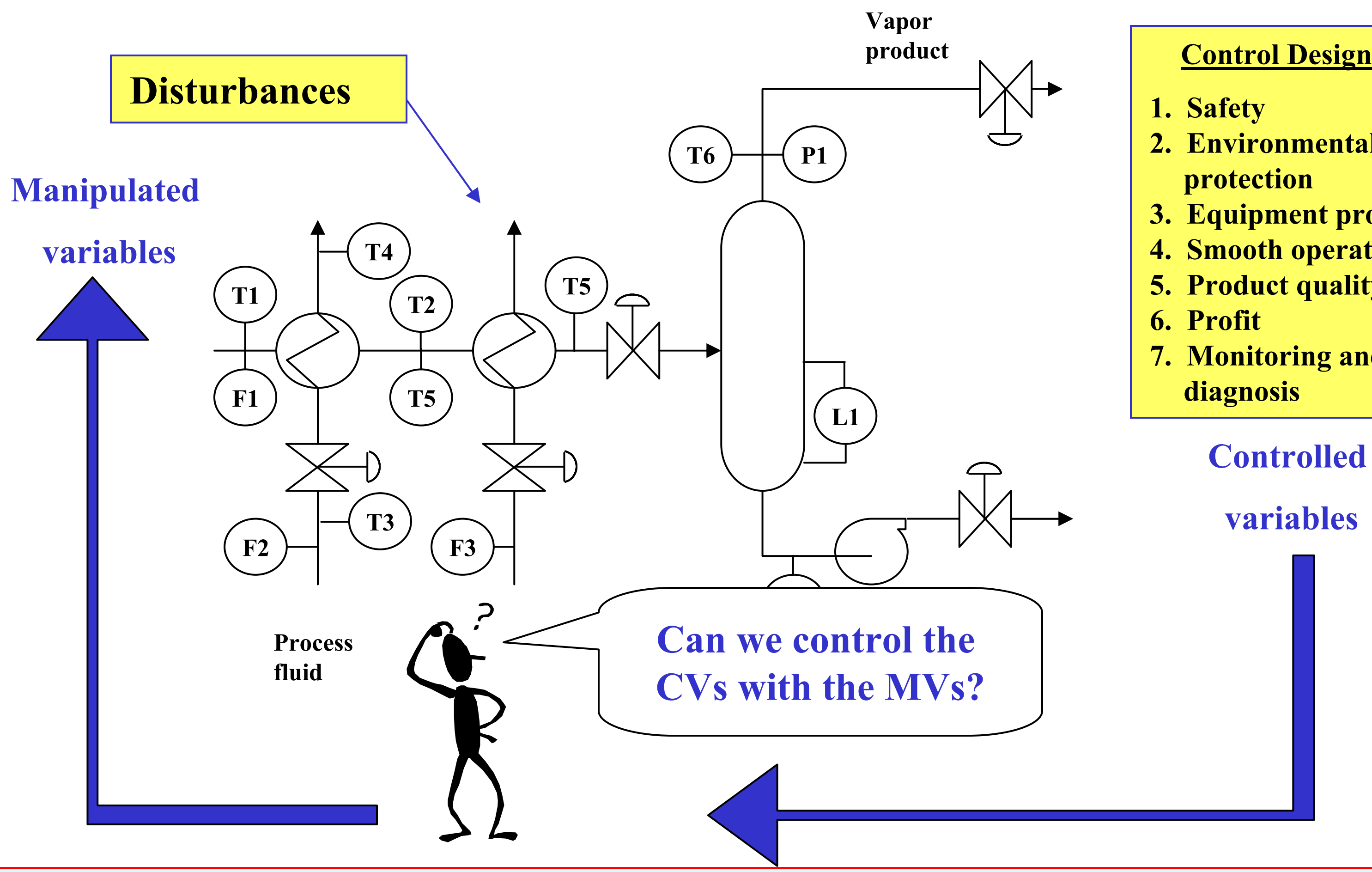
The ranges of the two mixing flows are given in the figure.

Sketch the feasible steady-state operating window in the Figure.

Note:

You may assume no disturbances for this exercise.

Controllability: Is the desired performance achievable?



Control Design Form

1. Safety
2. Environmental protection
3. Equipment protection
4. Smooth operation
5. Product quality
6. Profit
7. Monitoring and diagnosis

If the required performance is not achievable, fix the process; don't design controllers!

Controllability: Is the desired performance achievable?

CONTROLLABILITY - A characteristic of the process that determines whether a specified dynamic behavior can be achieved with a defined set of controlled and manipulated variables and a defined scenario.

Various specifications for dynamic behavior are possible; we will review a few commonly used.

We seek a fundamental property of the process independent of a specific control design or structure.

Controllability: Is the desired performance achievable?

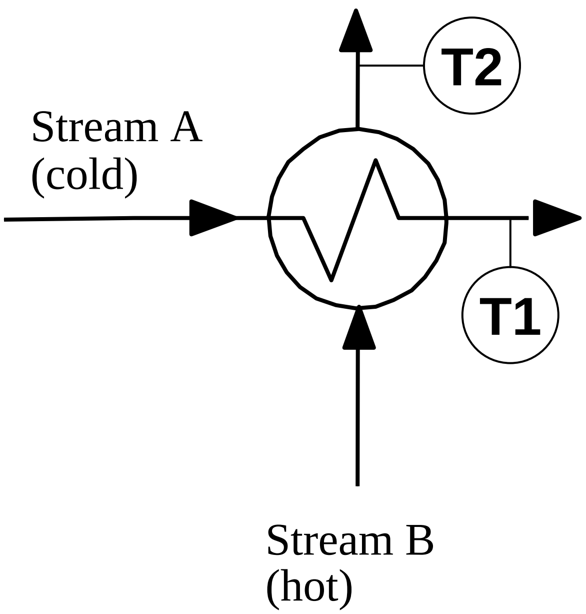
Interaction influences Controllability.

Is this behavior possible?

Goals: Maintain cold effluent T_1 and
Maintain hot effluent at T_2
final steady-states = set points

Freedom to adjust flows

Stream A	Stream B
Adjustable	Adjustable



Controllability: Is the desired performance achievable?

Interaction influences Controllability.

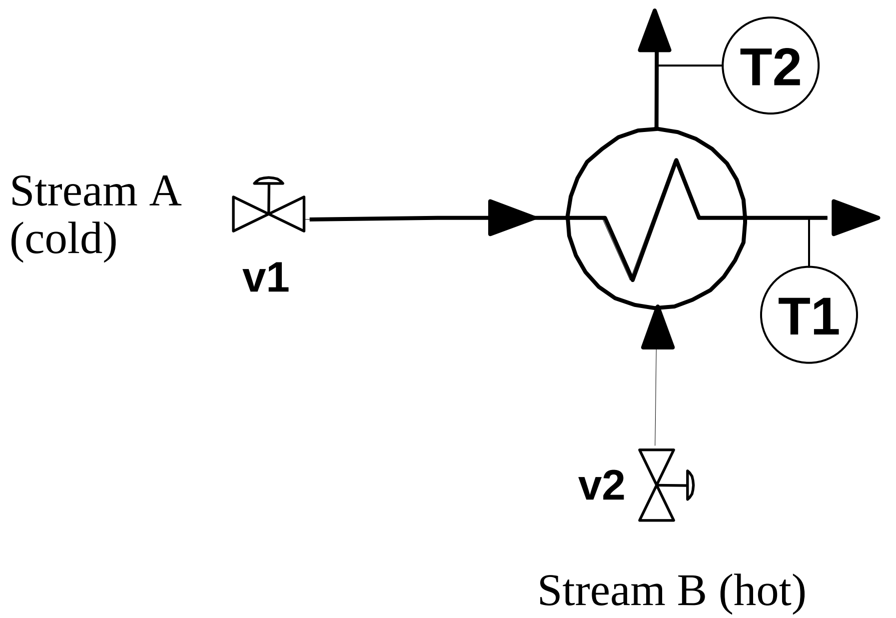
Quick check for each loop

- **Can we control T2 with v2?**

  YES

- **Can we control T1 with v1?**

  YES



Since each individual loop is OK, both loops are OK?

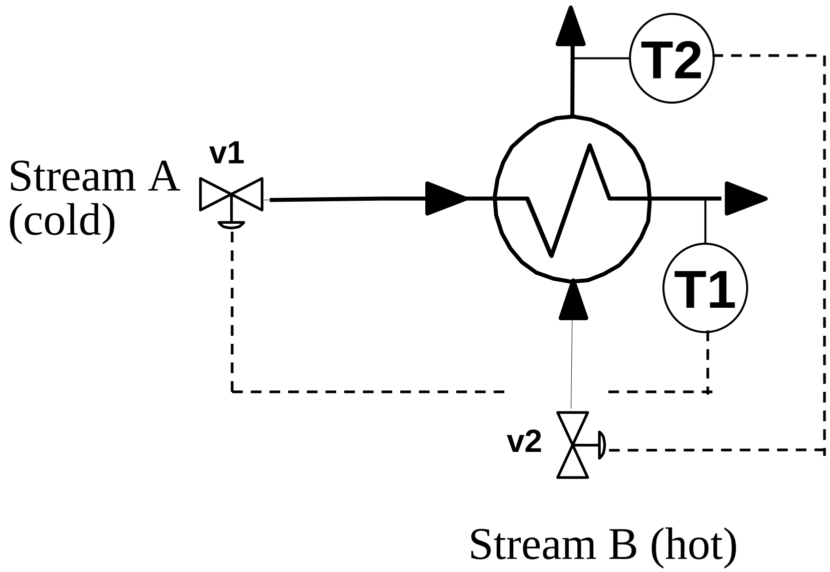
Controllability: Is the desired performance achievable?

Interaction influences Controllability.

Energy balance on each stream

$$Q_{Hot} = F_{Hot} C_{p_H} (T_{Hout} - T_{Hin})$$

$$Q_{Cold} = F_{Cold} C_{p_C} (T_{Cout} - T_{Cin})$$



Controllability: Is the desired performance achievable?

A few typical controllability performance specifications

- **Steady-state: Achieve desired steady-state when disturbances occur**

  For processes that operate at steady-state, but no information about transition.

- **Point-wise state (output): Move to specified initial values of states (or CVs) to final values in finite time**

  Often in textbooks. Perhaps, useful for batch end-point control.

- **Functional: Strictly follow any defined trajectory for CVs**

  Useful for transition control between steady states and for batch processes. However, likely too restrictive (strictly, any).

Controllability: Is the desired performance achievable?

Steady-state Controllable: A mathematical test.

The system will be deemed controllable if the steady-state I/O gain matrix can be inverted, i.e.,

$$\text{Det}\,[\,G(0)\,] \neq 0$$

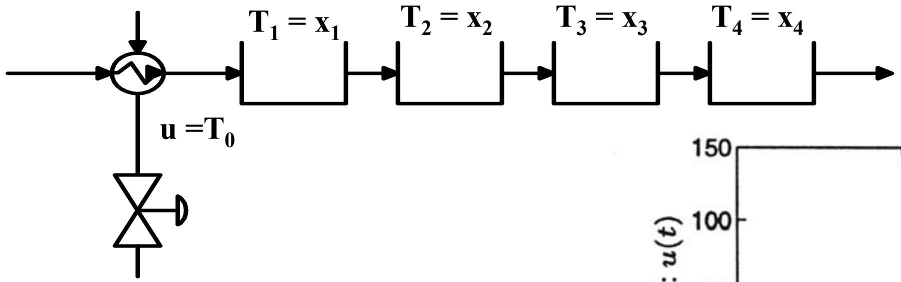
$$[G(0)]^{-1} \text{ exists}$$

This is only applicable to open-loop stable plants. It is a point-wise test that gives no (definitive) information about other conditions.

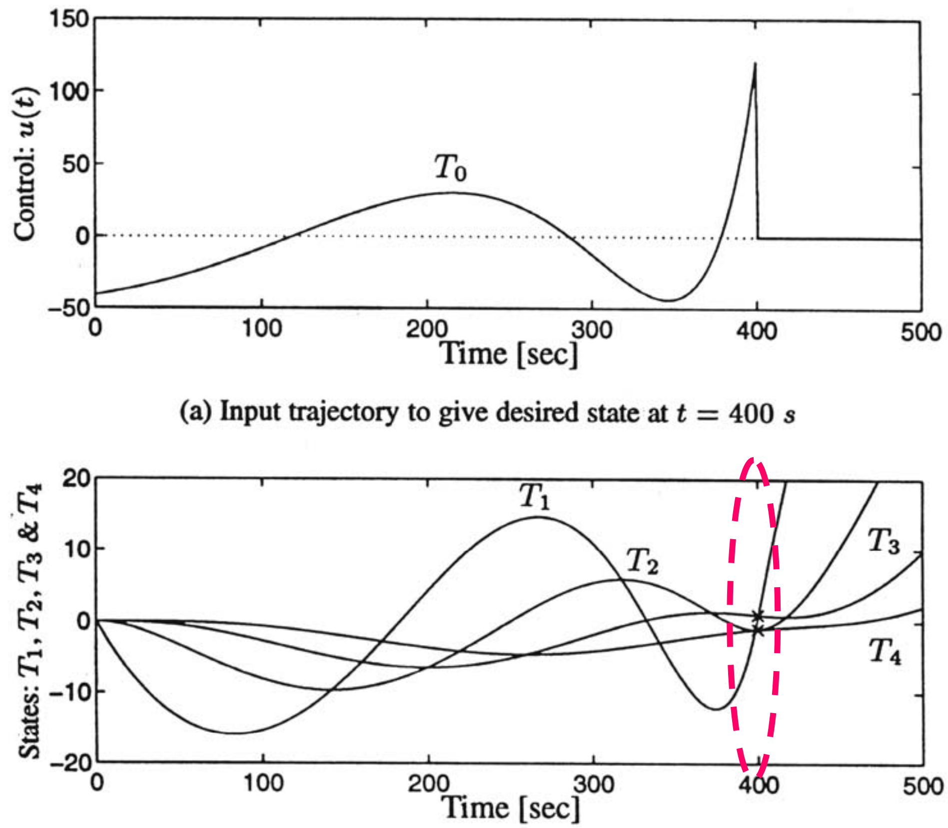
No information about the transient behavior or the changes to MV's to achieve the desired CV's (set points).

Controllability: Is the desired performance achievable?

Pointwise State Controllable: A process example for p.s. controllability.



From Skogestad & Postlethwaite, 1996



(a) Input trajectory to give desired state at $t = 400\ s$

(b) Response of states (tank temperatures)

<u>**One MV and four CVs.**</u>
The control performance can be achieved!

Nothing is specified for MV at any time or for CV between t_0 and t_1 or after the time t_1.

Controllability: Is the desired performance achievable?

Functional Controllable* - A system is controllable if it is possible to adjust the manipulated variables MV(t) so that the system will follow a (smooth) defined path from $CV(t_0)$ to $CV(t_1)$ in a finite time.

A system G(s) is (output) functionally controllable when dimensions of CV and MV are the same (say n)

The rank of $G(j\omega) = n$

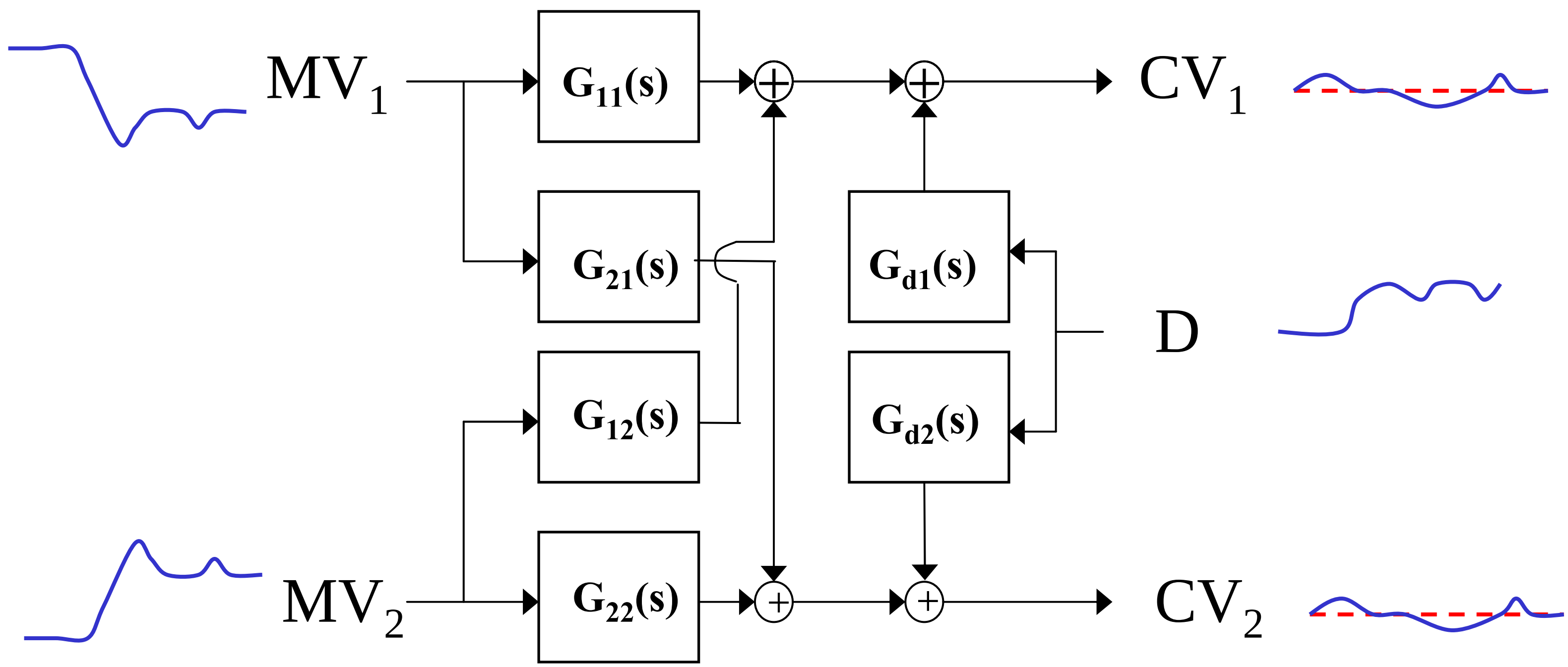
Stated differently, $G^{-1}(j\omega)$ exists for all ω

Stated again, $\sigma_{min}(G(j\omega)) > 0$ [minimum singular value]

Unfortunately, dead times and RHP zeros prevent the controller from implementing the inverse for most process. Also, no specification on the MV's.

Controllability: Is the desired performance achievable?

Class Exercise: Let's define controllability with a test that is computable.



Controllability: Is the desired performance achievable?

Controllability Test: Solve as open-loop optimization problem, which can be an LP or convex QP.

Note that in the formulation, slacks s_{1n} define allowable deviation from desired output, and s_{2n} are violations for excessive deviation.

If all violation slacks (s_{2n}) on the performance specifications are zero, i.e. if f = 0, the system is controllable!

$$\min_{\mathbf{u}} \quad \mathbf{f} = \sum (s_{2n}{}^{+} + s_{2n}{}^{-})$$

$s.t.$

$$x_n = Ax_{n-1} + Bu_n + Dd_n$$

$$y_n = Cx_n$$

$$y_n - s_{1n}{}^{+} - s_{2n}{}^{+} \le (y_{SP})_n$$

$$y_n + s_{1n}{}^{-} + s_{2n}{}^{-} \ge (y_{SP})_n$$

$$(u_{\min})_n \le u_n \le (u_{\max})_n$$

$$(\Delta u_n)_{\min} \le u_n - u_{n-1} \le (\Delta u_n)_{\max}$$

$$0 \le s_{1n}{}^{+} \le (s_{1n}{}^{+})_{\max}$$

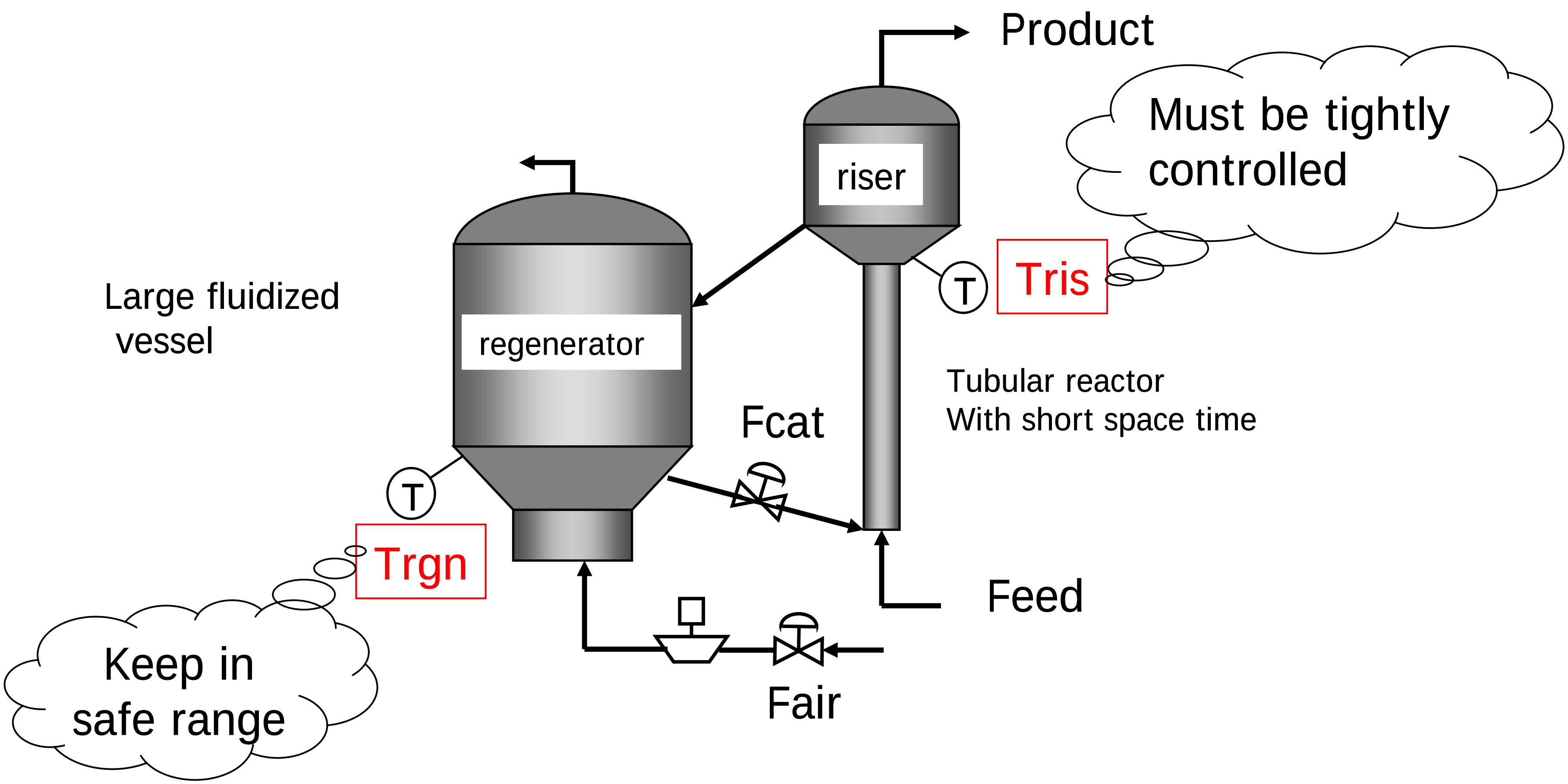
$$0 \le s_{1n}{}^{-} \le (s_{1n}{}^{-})_{\max}$$

$$0 \le s^{-}{}_{2n}, \; 0 \le s^{+}{}_{2n}$$

$$given \quad u_o, x_o, y_0, d_n$$

Controllability: Is the desired performance achievable?

Example of controllability test for 2x2 Fluidized Catalytic Cracking Reactor



Controllability: Is the desired performance achievable?

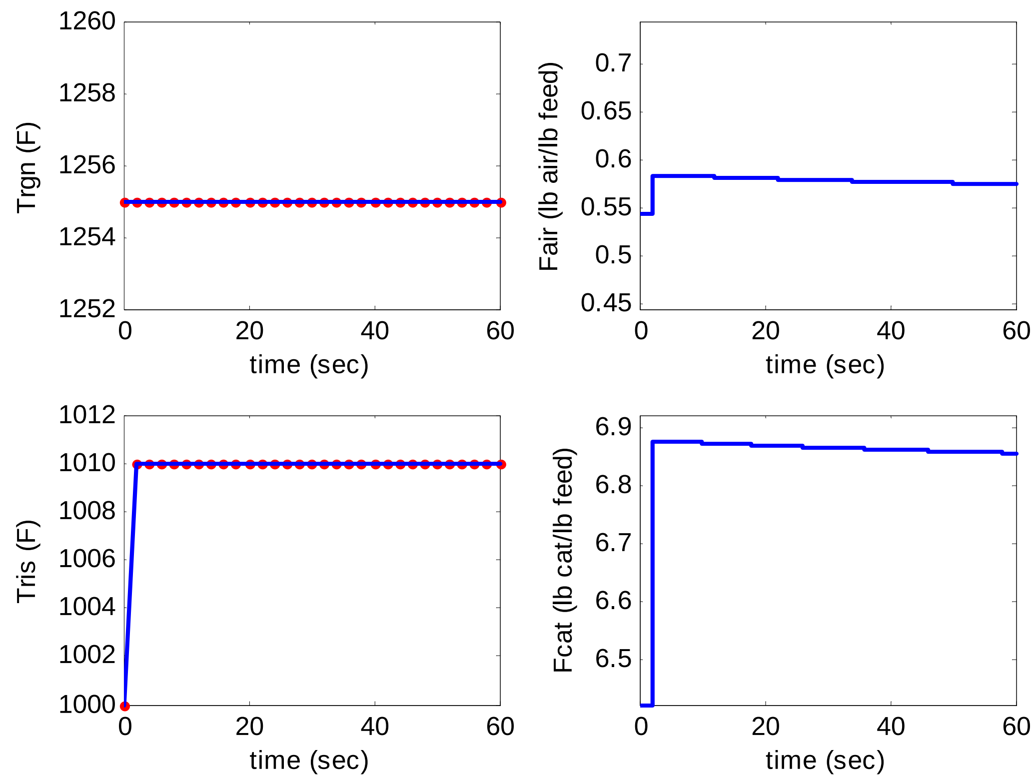
Example of controllability test for 2x2 Fluidized Catalytic Cracking Reactor

FCC Case 1

Tris ΔSP = +10 °F at t=0

With no bounds on the speed of adjustment, the set points can be tracked exactly.

Is the system controllable?



Controllability: Is the desired performance achievable?

Example of controllability test for 2x2 Fluidized Catalytic Cracking Reactor

<u>FCC Case 2</u>

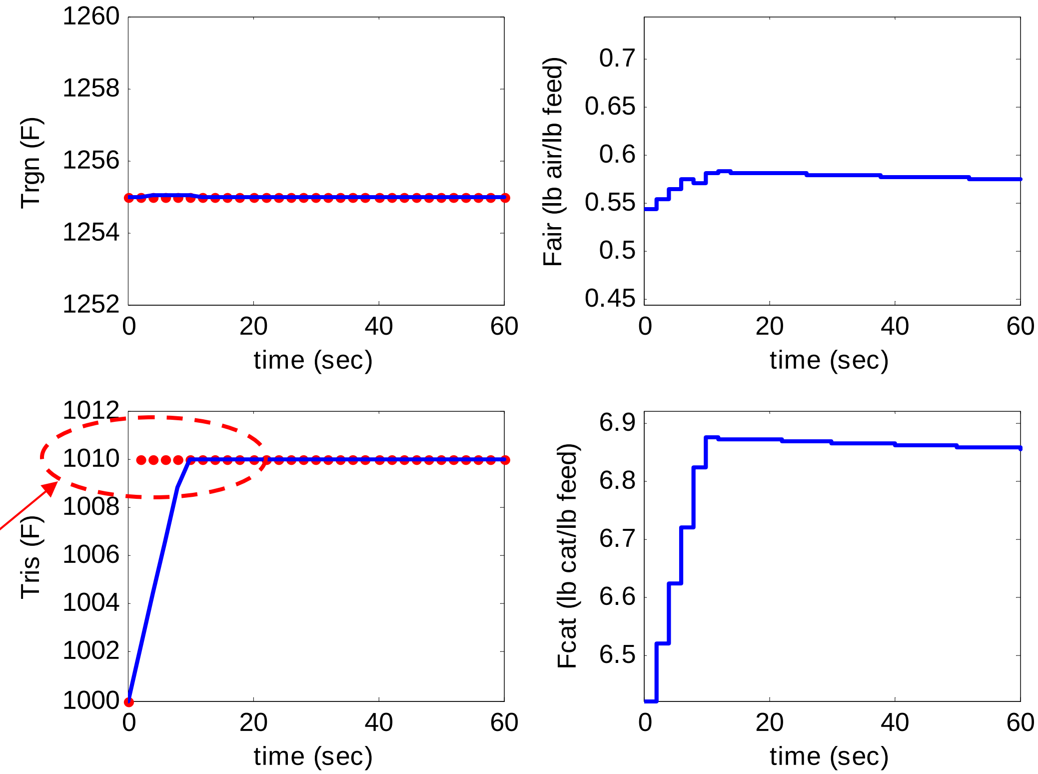
Tris ΔSP = +10 °F at t=0

With bounds on the speed of adjustment,

ΔFair ≤ 0.01 (lb air/lb feed)/2sec

ΔFcat ≤ 0.1 (lb cat/lb feed)/2sec

Is the system controllable?



Controllability: Is the desired performance achievable?

Evaluation of the proposed approach

GOOD ASPECTS

- **Can define relevant time-domain performance specifications**

  - on CVs (or states)
  - on MVs

- **Great flexibility on form of specifications**
- **Easily computed**
- **No limitation on disturbance**
- **Includes most other definitions/tests as special cases**

SHORTCOMINGS

- **Linear Model**
- **No robustness measure**
- **No guarantee that any controller will achieve the performance**
- **Finite horizon**

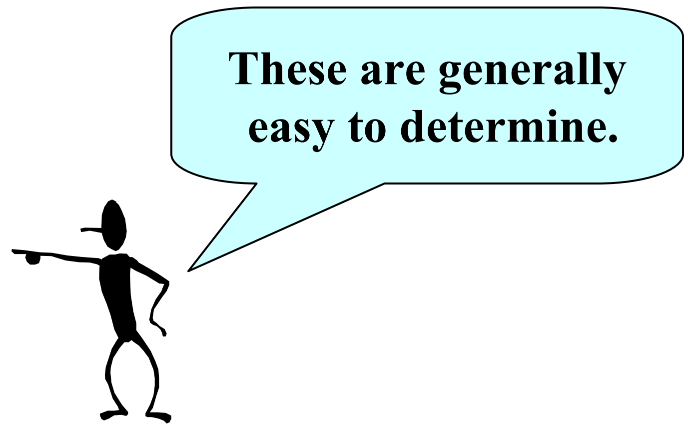
True for all controllability tests (applies to the first three shortcomings)

Controllability: Is the desired performance achievable?

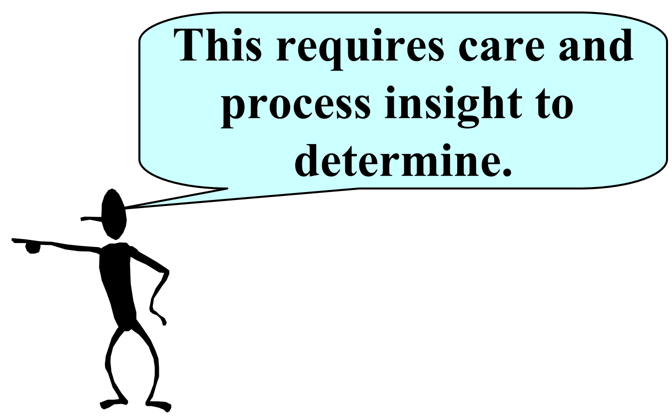
Determining controllability from process sight

Lack of controllability when

1. **One CV cannot be effected by any valve**
2. **One MV has no effect on CVs**



3. **Lack of <u>independent</u> effects.**
   Look for “contractions”

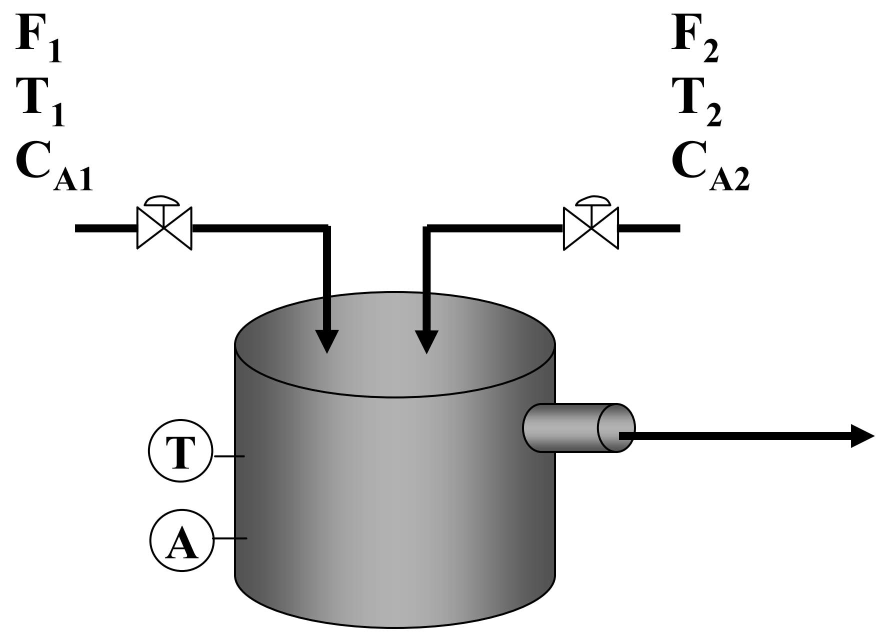


CONTROLLABILITY : Workshop 1

We need to control the mixing tank effluent temperature and concentration.

You have been asked to evaluate the steady-state controllability of the process in the figure.

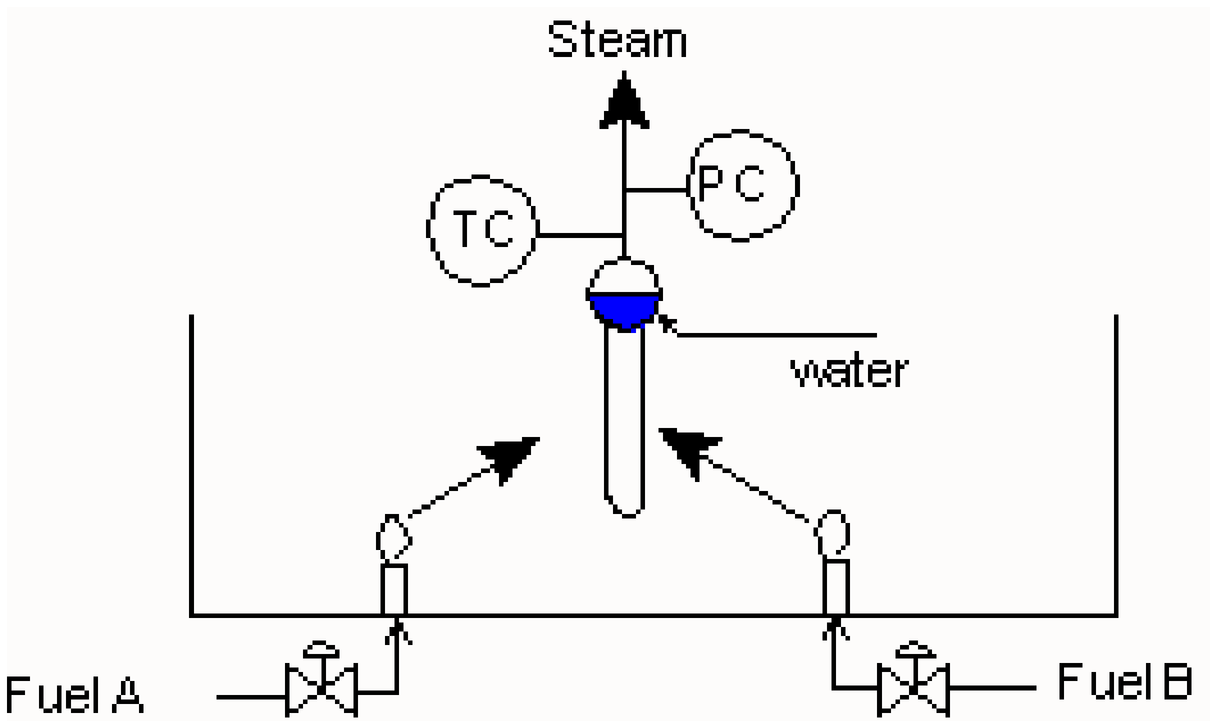
Discuss good and poor aspects and decide whether you would recommend the design.



Controlled variables are the temperature and concentration in the tank effluent.

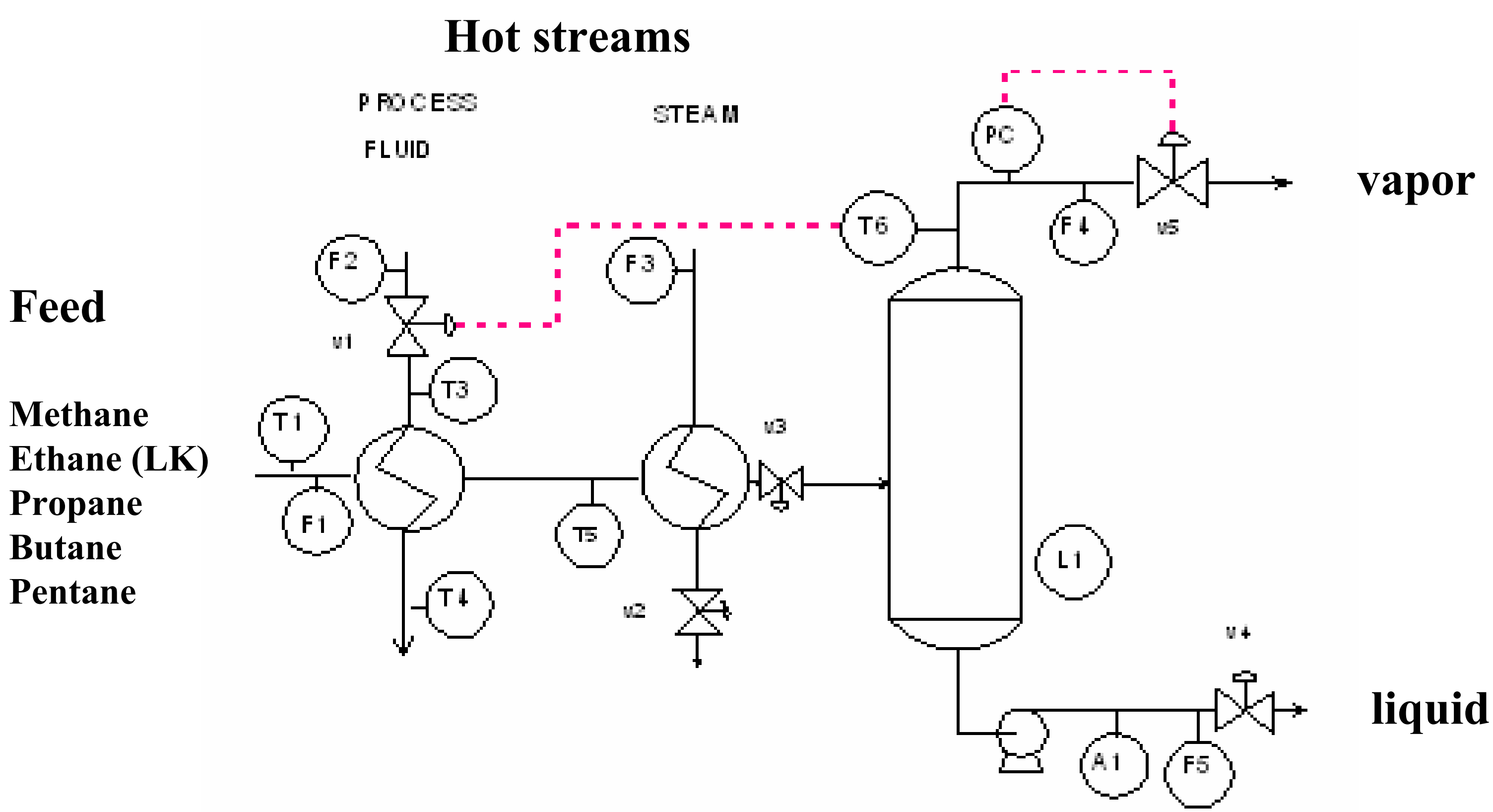
CONTROLLABILITY : Workshop 2

The sketch describes a simplified boiler for the production of steam. The boiler has two fuels that can be manipulated independently. We want to control the steam temperature and pressure. Analyze the controllability of this system and determine the loop pairing.



CONTROLLABILITY : Workshop 3

The sketch describes a simplified flash drum. A design is proposed to control the temperature and pressure of the vapor section. Analyze the controllability of this system and determine if the loop pairing is correct.



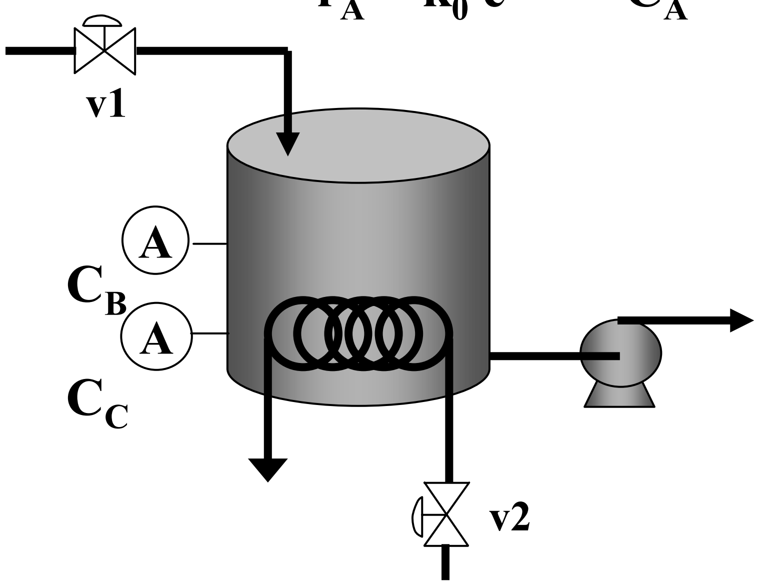
CONTROLLABILITY : Workshop 4

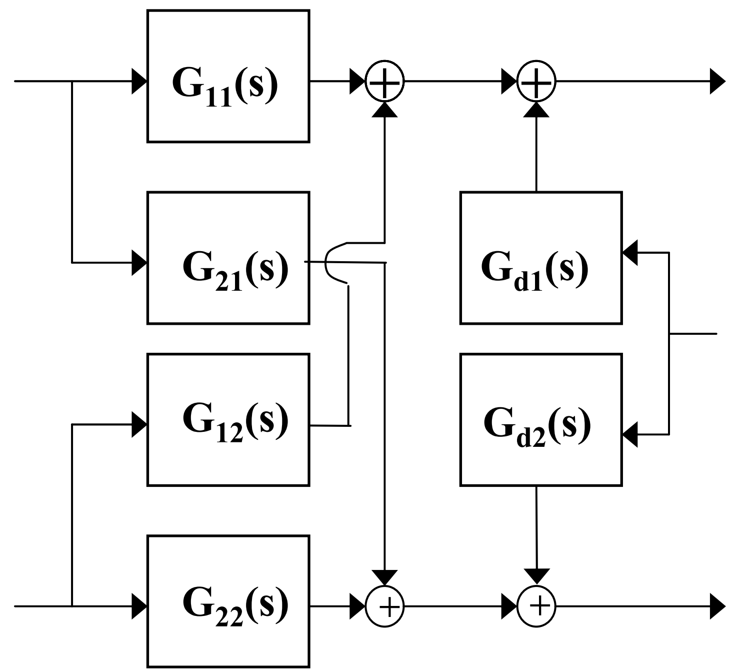
A non-isothermal CSTR

- **Does interaction exist?**
- **Are the CVs (concentrations) independently controllable?**

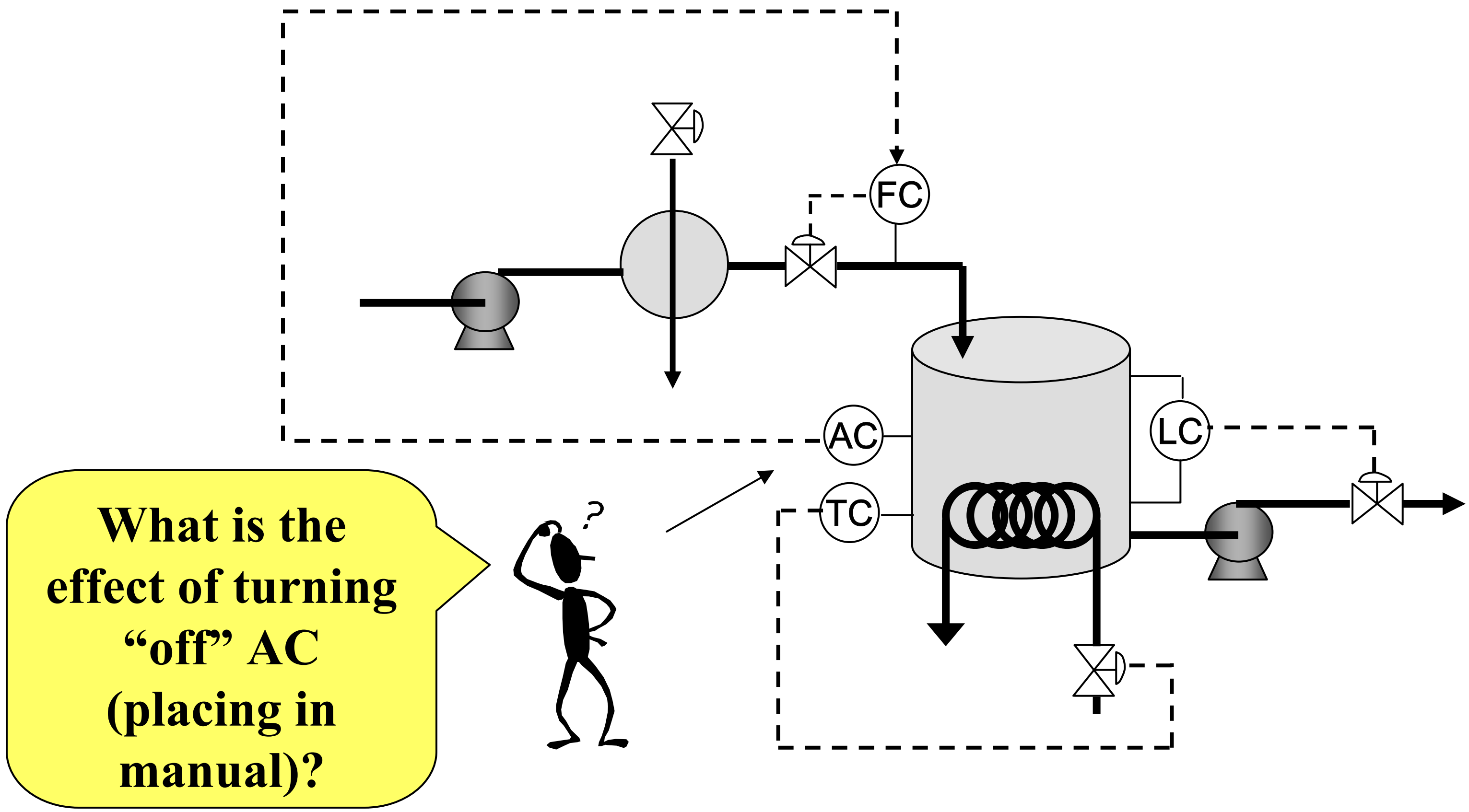
$$A \rightarrow B + 2C$$

$$-r_A = k_0\, e^{-E/RT}\, C_A$$





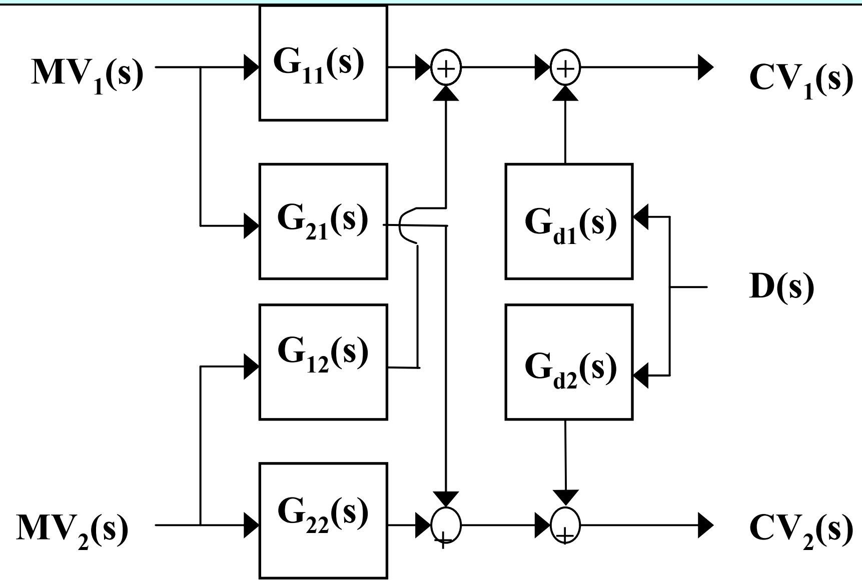
Integrity: Is the design "robust" to changes in controller status?



We will first introduce the Relative Gain; then, we will apply it to the Integrity Question

RELATIVE GAIN: Definition

The relative gain between MV_j and CV_i is λ_{ij}. It is defined in the following equation.



Explain in words.

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

What have we assumed about the other controllers?

RELATIVE GAIN: Properties

1. The RGA can be calculated from open-loop gains (only).

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$$[CV] = [K][MV] \qquad \mathrm{k_{ij}} = \left(\partial CV_i \big/ \partial MV_j\right)_{MV_i}$$

Open-loop

$$[MV] = [K]^{-1}[CV] \qquad \mathrm{kI_{ij}} = \left(\partial MV_i \big/ \partial CV_j\right)_{CV_j}$$

Closed-loop

The relative gain array is the element-by-element product of K with K^{-1}. (❑ = product of ij elements)

$$\Lambda = K \otimes \left(K^{-1}\right)^{\mathrm{T}} \qquad \lambda_{ij} = \left(k_{ij}\right)\left(kI_{ji}\right)$$

RELATIVE GAIN: Properties

2. In some cases, the RGA is very sensitive to small errors in the gains, K_{ij}.

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$$\lambda_{11} = \frac{1}{1 - \dfrac{K_{12}K_{21}}{K_{11}K_{22}}}$$

When this term is nearly equal 1.0

When is this equation very sensitive to errors in the individual gains?

RELATIVE GAIN: Properties

2. In some cases, the RGA is very sensitive to small errors in the gains, K_{ij}.

$$\lambda_{ij} = \frac{\left(\dfrac{\Delta CV_i}{\Delta MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\Delta CV_i}{\Delta MV_j}\right)_{CV_k = \text{constant}}}$$

We must perform a thorough study to ensure that numerical derivatives are sufficiently accurate!

Change in F_D used in finite difference for derivative	λ_{11} for a positive change in F_D	λ_{11} for a negative change in F_D	Average λ_{11} for positive and negative changes in F_D
2%	.796	.301	.548
0.5%	.673	.508	.590
0.2%	.629	.562	.596
0.05%	.605	.588	.597

Results for a distillation tower, from McAvoy, 1983

The δx must be sufficiently small (be careful about roundoff).

RELATIVE GAIN: Properties

2. In some cases, the RGA is very sensitive to small errors in the gains, K_{ij}.

$$\lambda_{ij} = \frac{\left(\dfrac{\Delta CV_i}{\Delta MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\Delta CV_i}{\Delta MV_j}\right)_{CV_k = \text{constant}}}$$

We must perform a thorough study to ensure that numerical derivatives are sufficiently accurate!

Convergence tolerance of equations (some of all errors squared)	λ_{11} for a positive change in F_D	λ_{11} for a negative change in F_D	Average λ_{11} for positive and negative changes in F_D
10^{-4}	-4.605	8.080	-.887
10^{-6}	-.096	1.068	.503
10^{-8}	.556	.615	.586
10^{-10}	.622	.568	.595
10^{-16}	.629	.562	.596

Average gains from +/-

Results for a distillation tower, from McAvoy, 1983

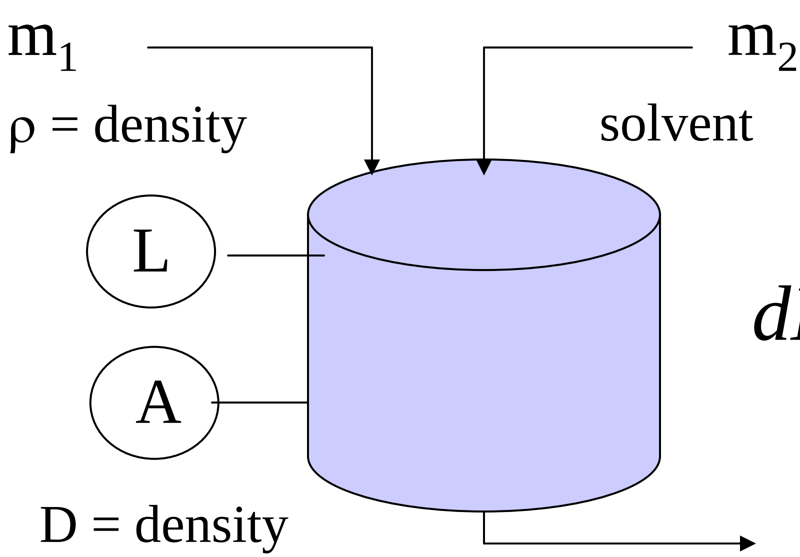
The convergence tolerance must be sufficiently small.

RELATIVE GAIN: Properties

3. We can evaluate the RGA of a system with integrating processes, such as levels.

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

Redefine the output as the derivative of the level; then, calculate as normal.



	m_1	m_2
dL/dt	D/ρ	$(1-D/\rho)$
D	$(1-D/\rho)$	D/ρ

RELATIVE GAIN: Properties

Additional Properties, stated but not proved here

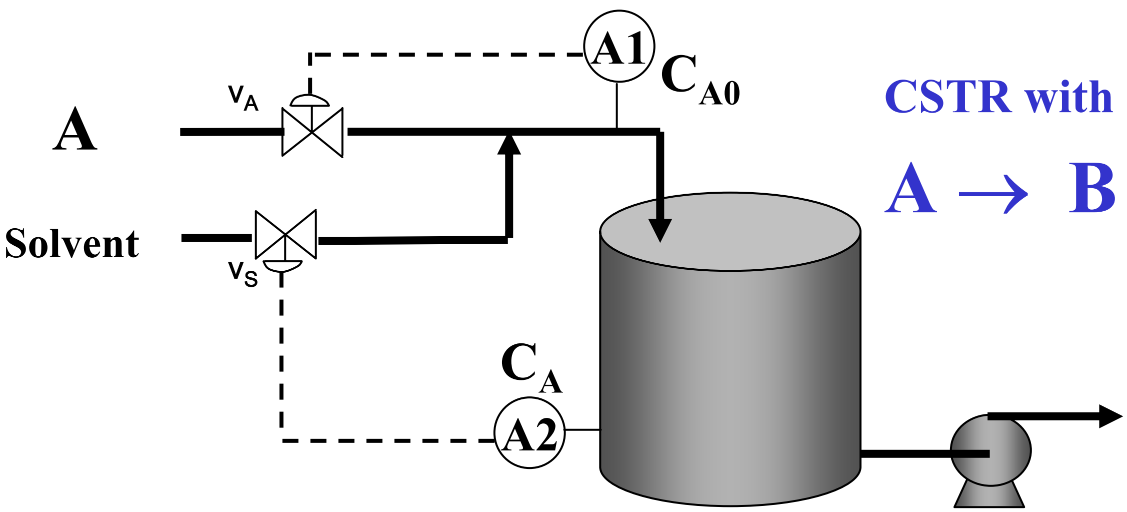
4. **Rows and column of RGA sum to 1.0**
5. **Elements in RGA are independent of variable scalings**
6. **Permutations of variables results in same permutation in RGA**
7. **RGA is independent of a specific I/O pairing – it need be evaluated only once**

RELATIVE GAIN: Interpretations

$$MV_j \rightarrow CV_i$$

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} < 0$ In this case, the steady-state gains have <u>different signs</u> depending on the status (auto/manual) of the <u>other loops</u>.



<u>Discuss interaction</u>

What sign is process gain of A2 loop with A1

(a) in automatic

(b) in manual.

RELATIVE GAIN: Interpretations

$$\mathbf{MV_j \rightarrow CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} < 0$ In this case, the steady-state gains have different signs depending on the status (auto/manual) of other loops!

We can achieve stable multiloop feedback by using the sign of the controller gain that stabilizes the multiloop system.

Discuss what happens when the other interacting loop is placed in manual!

RELATIVE GAIN: Interpretations

$MV_j \rightarrow CV_i$

$$\lambda_{ij} = \frac{\left(\frac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\frac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\frac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\frac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} < 0$ the steady-state gains have different signs

<u>For $\lambda_{ij} < 0$, one of three BAD situations occurs</u>

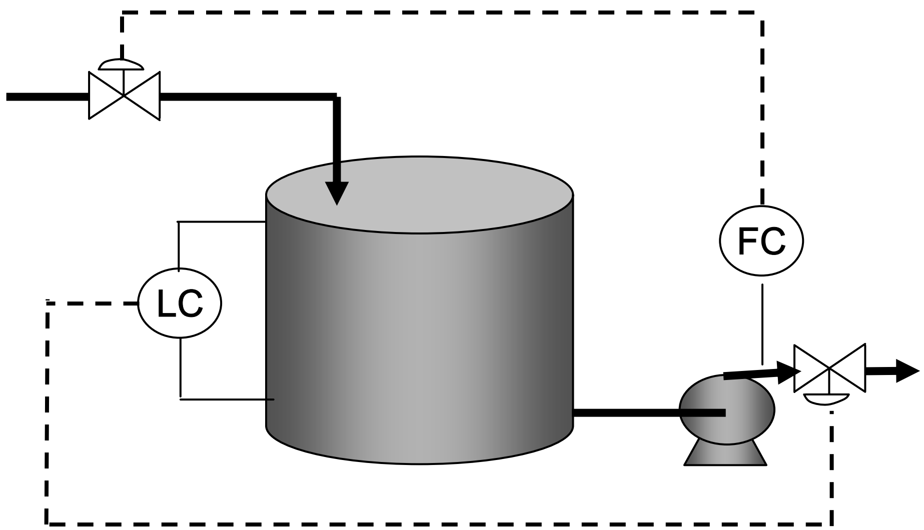
1. **Multiloop is unstable with all in automatic.**
2. **Single-loop ij is unstable when others are in manual.**
3. **Multiloop is unstable when loop ij is manual and other loops are in automatic**

RELATIVE GAIN: Interpretations

$MV_j \rightarrow CV_i$

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} = 0$ In this case, the steady-state gain is zero when all other loops are open, in manual.



Could this control system work?

What would happen if one controller were in manual?

RELATIVE GAIN: Interpretations

$$\mathbf{MV_j \rightarrow CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$0<\lambda_{ij}<1$ In this case, the multiloop (ML) steady-state gain is larger than the single-loop (SL) gain.

What would be the effect on tuning of opening/closing the other loop?

Discuss the case of a 2x2 system paired on $\lambda_{ij} = 0.1$

RELATIVE GAIN: Interpretations

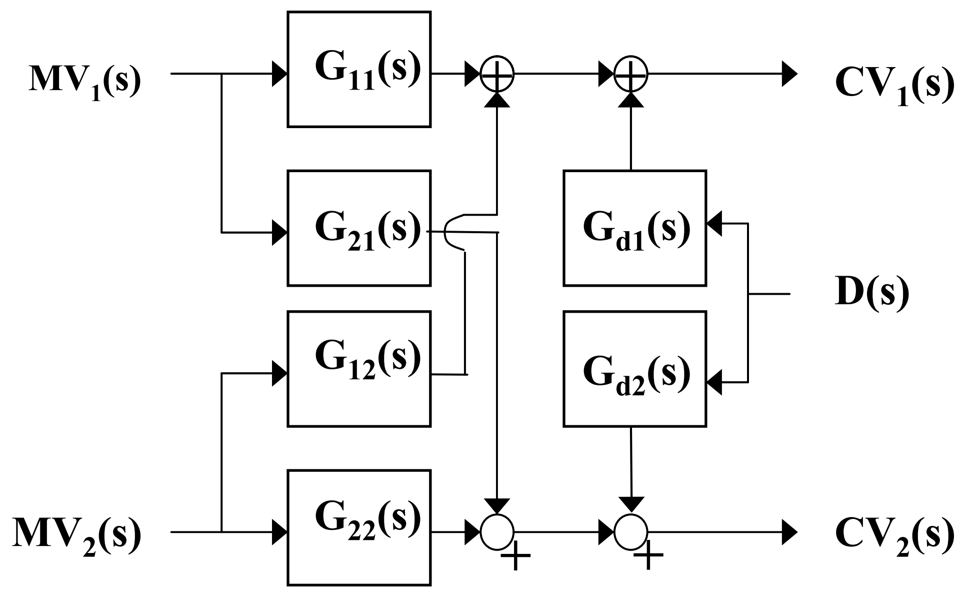
$$MV_j \rightarrow CV_i$$

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} = 1$ In this case, the steady-state gains are identical in both the ML and the SL conditions.

What is generally true when $\lambda_{ij} = 1$?

Does $\lambda_{ij} = 1$ indicate <u>no</u> interaction?



RELATIVE GAIN: Interpretations

$\lambda_{ij} = 1$ In this case, the steady-state gains are identical in both the ML and the SL conditions.

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

Both give an RGA that is diagonal!

Diagonal gain matrix

$$\mathbf{K} = \begin{bmatrix} \mathbf{k}_{11} & & 0 & \\ & \mathbf{k}_{22} & & \\ & & .. & \\ 0 & & & .. \\ & & & & .. \end{bmatrix}$$

Lower diagonal gain matrix

$$\mathbf{K} = \begin{bmatrix} \mathbf{k}_{11} & & 0 & & \\ \mathbf{k}_{21} & \mathbf{k}_{22} & & & \\ .. & & .. & & \\ .. & & & .. & \\ \mathbf{k}_{n1} & .. & .. & .. & .. \end{bmatrix}$$

$$\mathbf{RGA} = \begin{bmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} = \mathbf{I}$$

RELATIVE GAIN: Interpretations

$$MV_j \rightarrow CV_i$$

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$1 < \lambda_{ij}$ In this case, the steady-state multiloop (ML) gain is smaller than the single-loop (SL) gain.

If a ML process has a smaller process gain, why not just increase the associated controller gain by λ_{ij} ?

What would be the effect on tuning of opening/closing the other loop?

Discuss a the case of a 2x2 system paired on $\lambda_{ij} = 10$.

RELATIVE GAIN: Interpretations

$$\mathbf{MV_j \rightarrow CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial CV_i}{\partial MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} = \infty$ **In this case, the gain in the ML situation is zero. We conclude that ML control is not possible.**

Have we seen this result before?

INTEGRITY

Increasingly restrictive ↓

- **Integral Stabilizability**

  - Can be stabilized with feedback using same sign of controller gains as single-loop

- **Integral Controllability**

  - Can be stabilized with feedback using same sign of controller gains as single-loop and all controllers can be detuned "equally"

- **Integral Controllability with Integrity**

  - Can be stabilized with feedback using same sign of controller gains as single-loop and some controllers can be placed off, on "manual" while retaining stability (with retuning)

- **Decentralized Integral Controllability**

  - Can be stabilized with feedback using same sign of controller gains as single-loop and any controller(s) can be detuned any amount

INTEGRITY

INTEGRITY is strongly desired for a control design.

SOME ASSUMPTIONS FOR RESULTS PRESENTED

- **Limited to stable plants; if open-loop unstable plants, extensions to analysis are available**
- **All controllers have "integral modes". They provide zero steady-state offset for asymptotically constant ("step-like") inputs**
- **All "simple loops" ; variable structure (split range and signal select) are not considered unless explicitly noted.**
- **See references (Campo and Morari, Skogestad and Postlethwaite, etc.) for limitations on the transfer functions.**

INTEGRITY

- **Integral Stabilizability**

  - Can be stabilized with feedback using same sign of controller gains as single-loop

  Niederlinski Index

- **Integral Controllability**

  - Can be stabilized with feedback using same sign of controller gains as single-loop and all controllers can be detuned "equally"

  Not very important

- **Integral Controllability with Integrity**

  - Can be stabilized with feedback using same sign of controller gains as single-loop and some controllers can be off ("manual") while retaining stability (with retuning)

  Relative Gain Array*

- **Decentralized Integral Controllability**

  - Can be stabilized with single-loop feedback using same sign of controller gains as any individual controller(s) can be detuned any amount

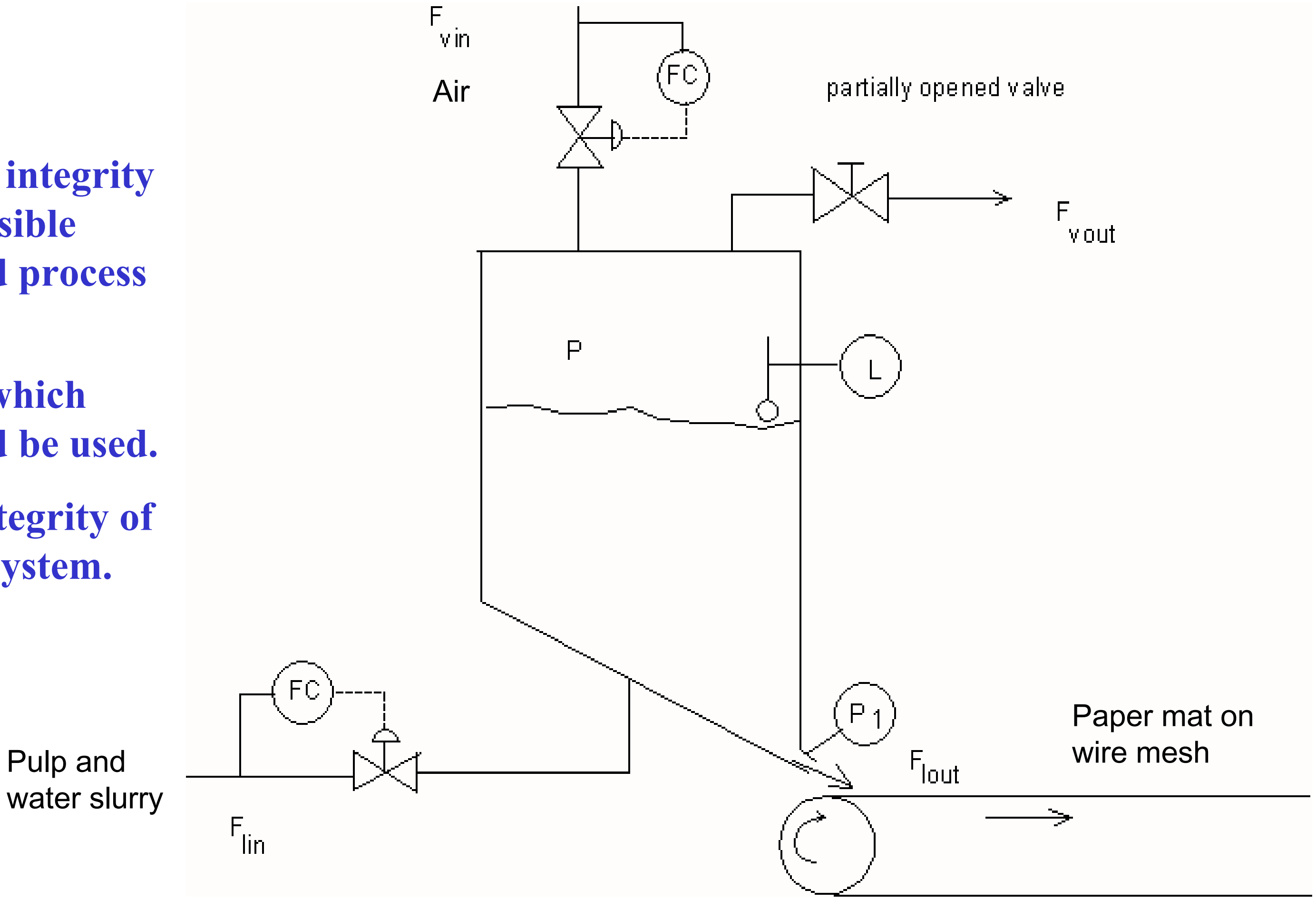
  Important, no short-cut test

*** All possible sub-systems with controller(s) off**

Integrity Workshop 1

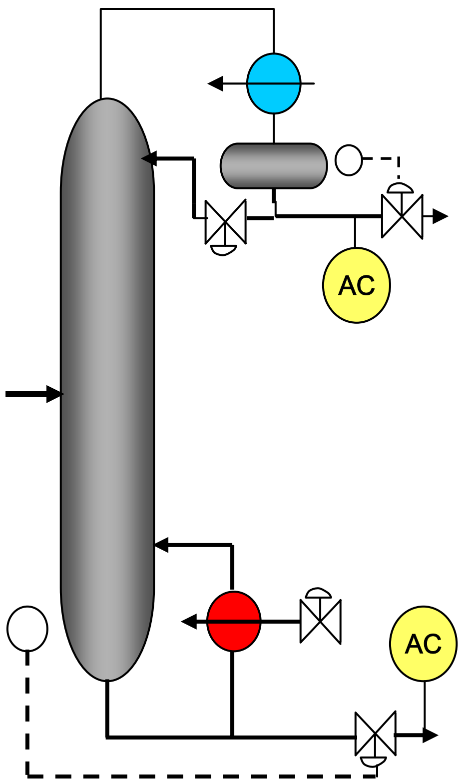
The process in the figure is a simplified head box for a paper making process. The control objectives are to control the pressure at the bottom of the head box (P1) tightly and to control the slurry level (L) within a range. The manipulated variables are the slurry flow rate in (F_{lin}) and the air vent valve opening.

1. **Determine the integrity of the two possible pairings based process insight.**
2. **Recommend which pairing should be used.**
3. **Discuss the integrity of the resulting system.**



Integrity Workshop 2

The following transfer function matrix and RGA are given for a binary distillation tower. Discuss the integrity for the two loop pairings.

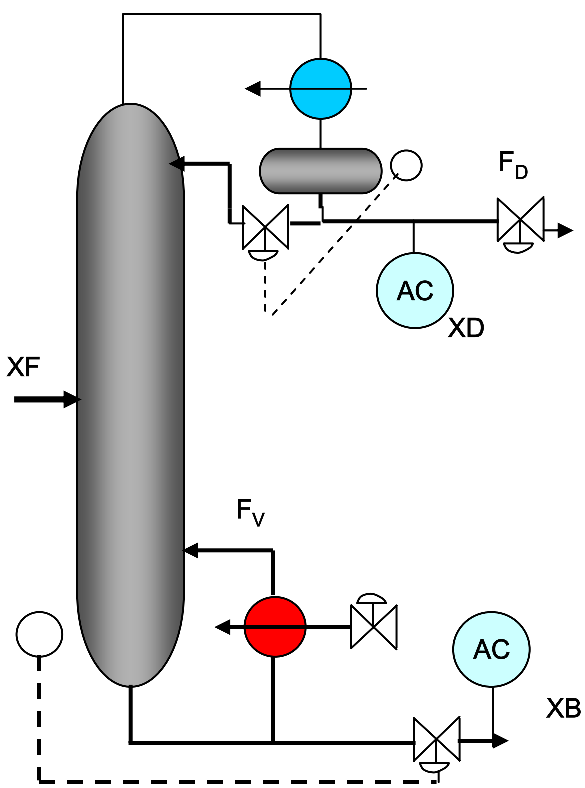


$$\begin{bmatrix} XD(s) \\ XB(s) \end{bmatrix} = \begin{bmatrix} \dfrac{0.0747e^{-3s}}{12s+1} & \dfrac{-0.0667e^{-2s}}{15s+1} \\ \dfrac{0.1173e^{-3.3s}}{11.75s+1} & \dfrac{-0.1253e^{-2s}}{10.2s+1} \end{bmatrix} \begin{bmatrix} F_R(s) \\ F_V(s) \end{bmatrix}$$

	FR	FV
XD	6.1	−5.1
XB	−5.1	6.1

Integrity Workshop 3

The following transfer function matrix and RGA are given for a binary distillation tower. Discuss the integrity for the two loop pairings.



$$\begin{bmatrix} XD(s) \\ XB(s) \end{bmatrix} = \begin{bmatrix} \dfrac{-0.0747e^{-3s}}{12s+1} & \dfrac{0.008e^{-2s}}{5s+1} \\ \dfrac{-0.1173e^{-3.3s}}{11.75s+1} & \dfrac{-0.008e^{-2s}}{3s+1} \end{bmatrix} \begin{bmatrix} F_D(s) \\ F_V(s) \end{bmatrix}$$

	FD	*FV*
XD	0.39	0.61
XB	0.61	0.39

Integrity Workshop 4

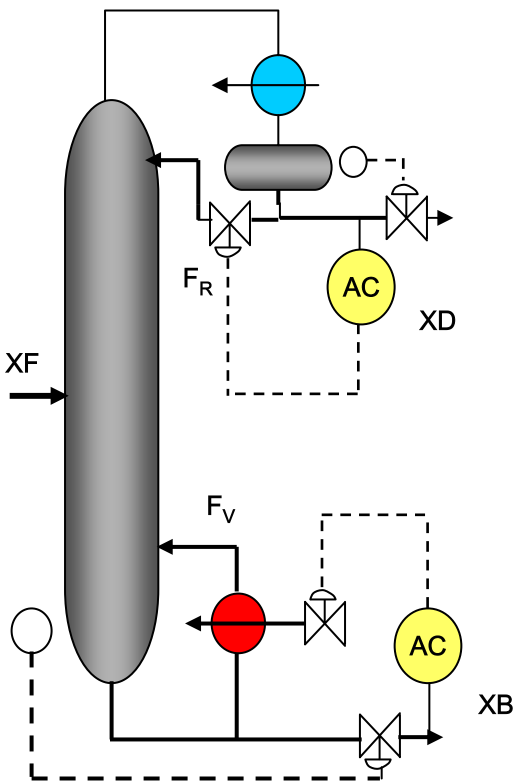
We will consider a hypothetical 4 input, 4 output process.

- How many possible combinations are possible for the square mutliloop system?
- For the system with the RGA below, how many loop pairings have good integrity?

	$mv1$	$mv2$	$mv3$	$mv4$
$CV1$	0	1	0	0
$CV2$	1.83	0	0	$-.83$
$CV3$	$-.83$	0	0	1.83
$CV4$	0	0	1	0

Integrity Workshop 5

The table presents RGA(1,1) for the same 2x2 process with different level controllers (considered "part of the process") and different operation conditions. **What do you conclude about the effects of regulatory level controls and operating conditions on the RGA?**

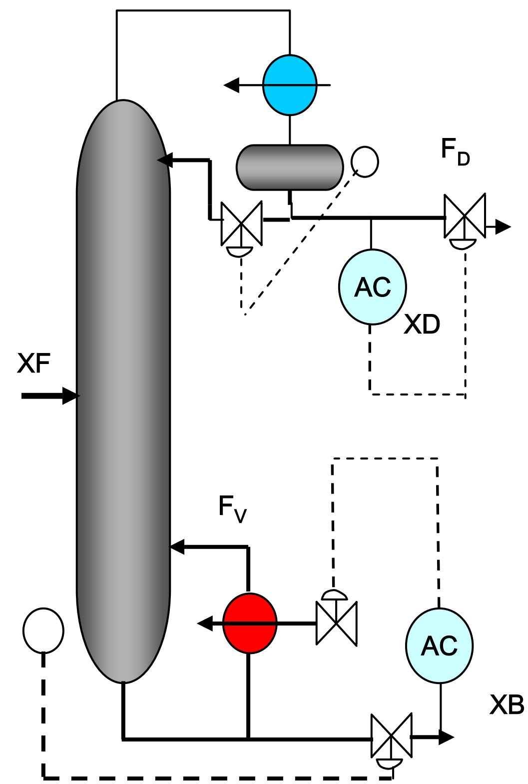


Small distillation column

Rel. vol = 1.2, R = 1.2 R_{min}

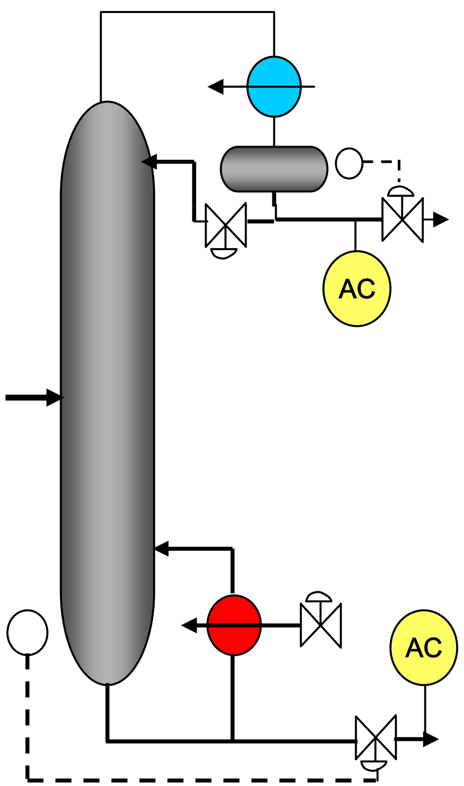
XD, XB	Feed Comp.	RGA	RGA
.998,.02	.25	46.4	.07
.998,.02	.50	45.4	.113
.998,.02	.75	66.5	.233
.98, .02	.25	36.5	.344
.98, .02	.50	30.8	.5
.98, .02	.75	37.8	.65
.98, .002	.25	66.1	.787
.98, .002	.50	46	.887
.98, .002	.75	48.8	.939

From McAvoy, 1983



Directionality: Its effect on Control Performance

Let's gain insight about control performance and learn one short-cut metric

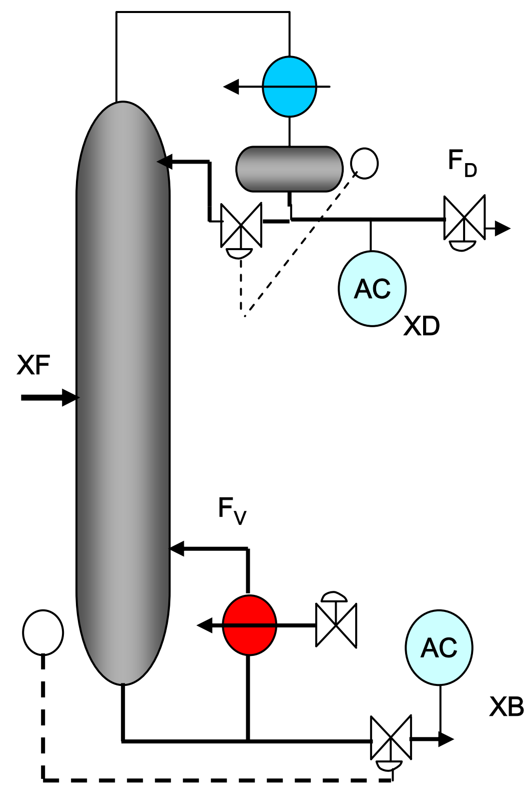


$\lambda(1,1) = 6.1$

Do we need more than

- **Controllability**
- **Integrity (RGA)**

Before we design controls?



$\lambda(1,1) = 0.39$

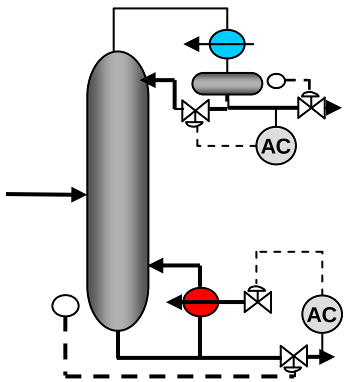
Important Observation: Case 1

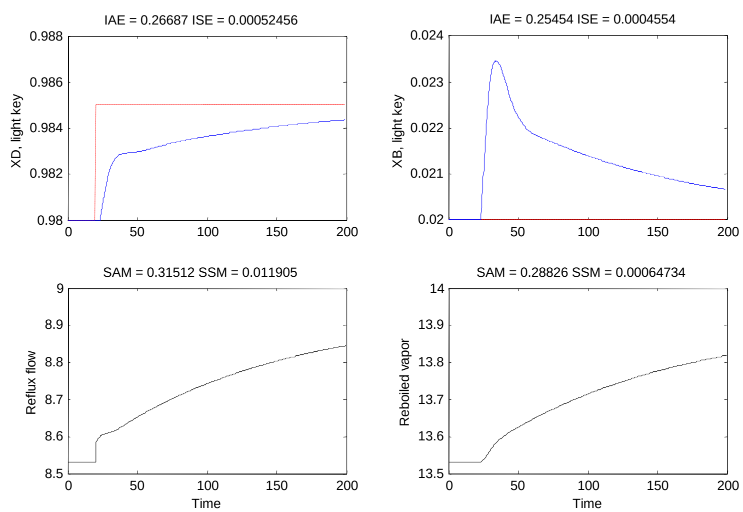
Design A

FR → XD

FRB → XB

RGA = 6.09



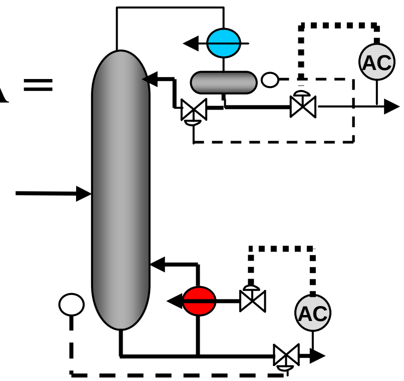


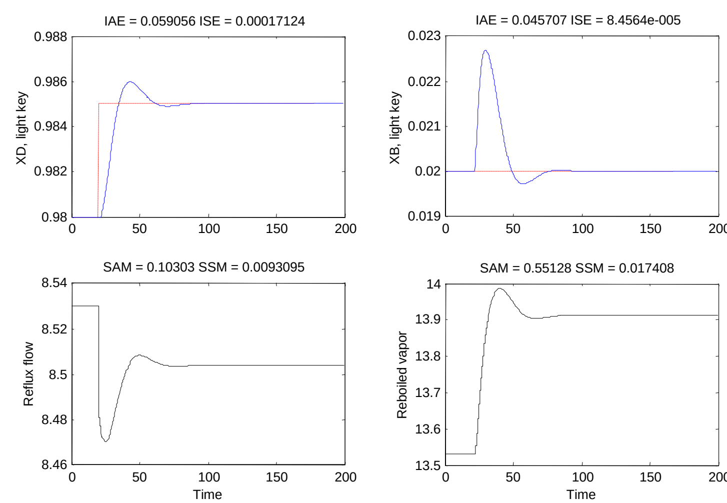
Design B

FD → XD

FRB → XB

RGA = 0.39





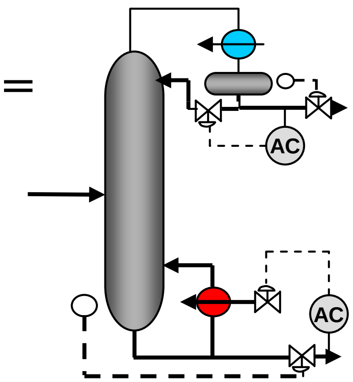
Important Observation: Case 2

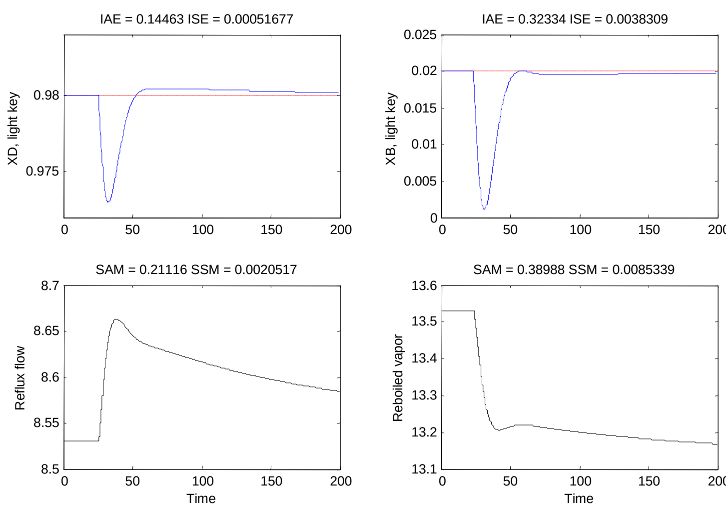
Design A

FR → XD

FRB → XB

RGA = 6.09



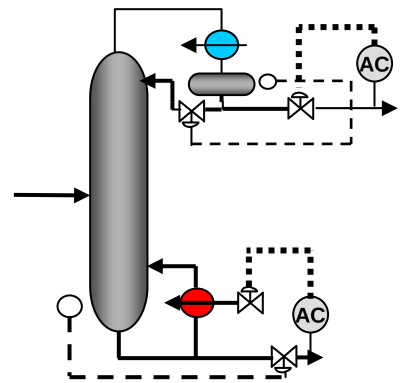


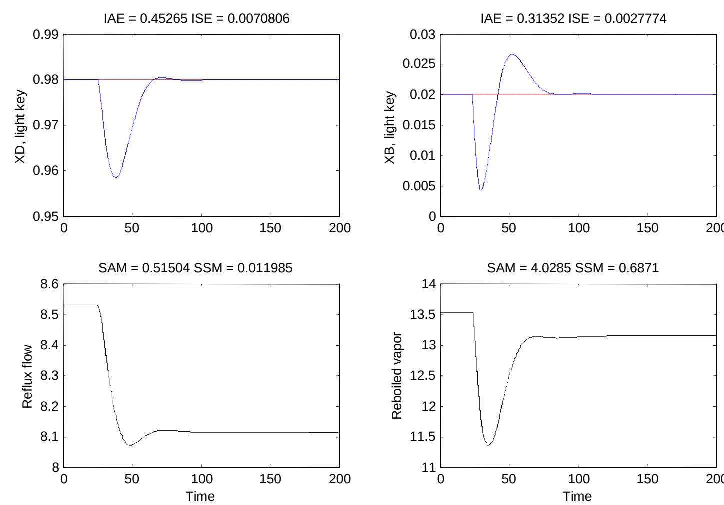
Design B

FD → XD

FRB → XB

RGA = 0.39





Directionality: Its effect on Control Performance

Conclusion from the two observations (and much more evidence)

The best performing loop pairing is <u>not</u> always the pairing with

- **Relative gains elements nearest 1.0**
- **The "least interaction"**

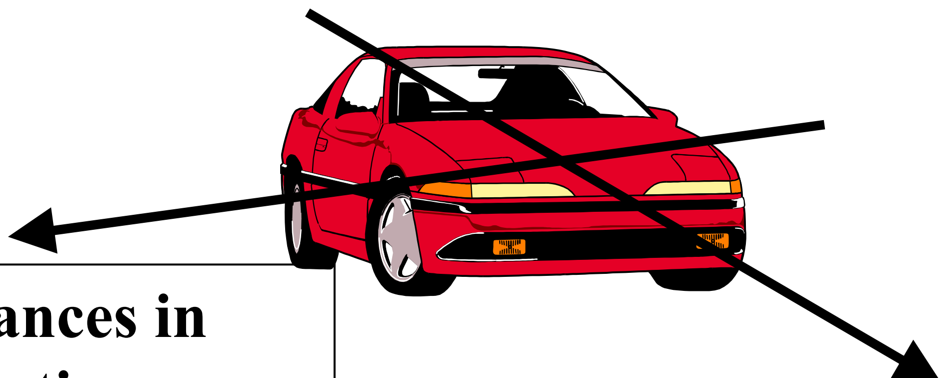
This should not be surprising; we have not established a direct connection between RGA and performance.

So, what is going on?

Directionality: Its effect on Control Performance

Strong directionality is a result of Interaction

- The best multivariable control design depends on the feedback and input!
- A key factor is the relationship between the feedback and disturbance directions.



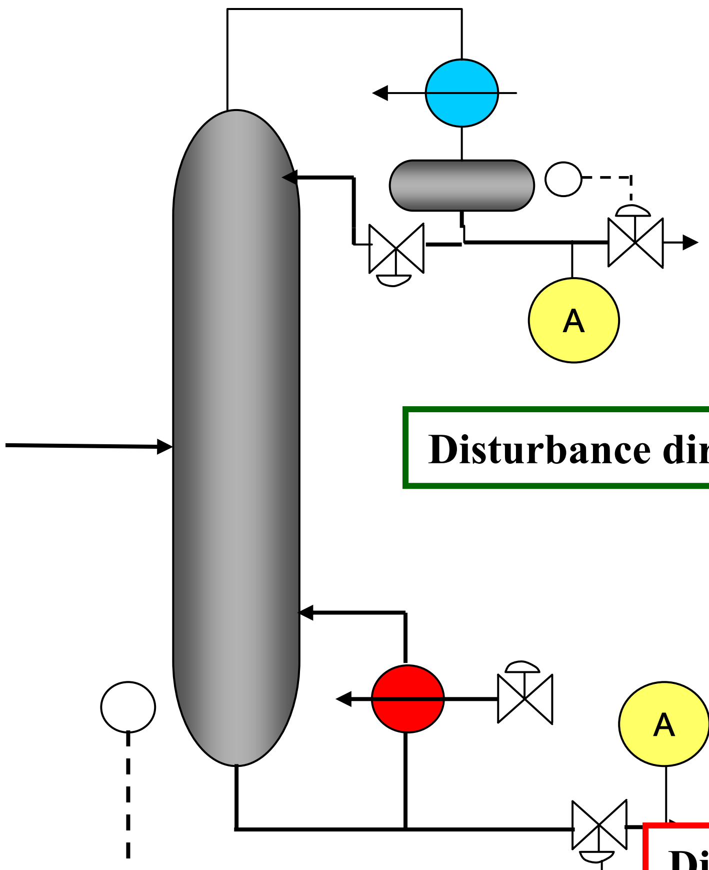
Disturbances in this direction are difficult to correct.

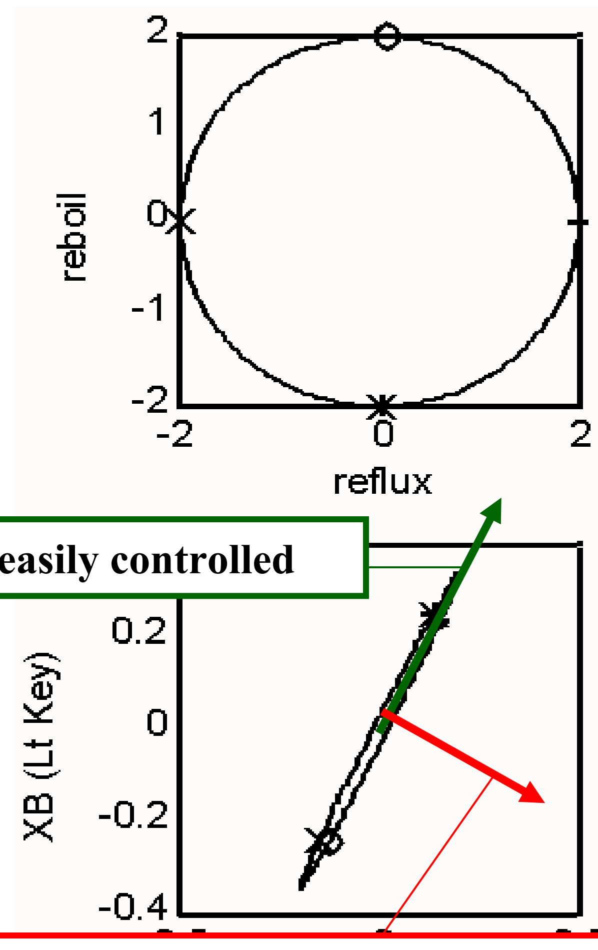
Disturbances in this direction are easily corrected.

Directionality: Its effect on Control Performance

Strong directionality is a result of Interaction

Distillation with "energy balance"





Disturbance direction easily controlled

Disturbance direction not easily controlled

Directionality: Its effect on Control Performance

KEY MATHEMATICAL INSIGHT
For Short-cut Metric

Definition of the Laplace transform and take lim as s → 0

$$E(s) = \int_0^\infty E(t) e^{-st} dt \qquad \lim_{s \to 0} E(s) = \int_0^\infty E(t) dt$$

Measure of control performance,

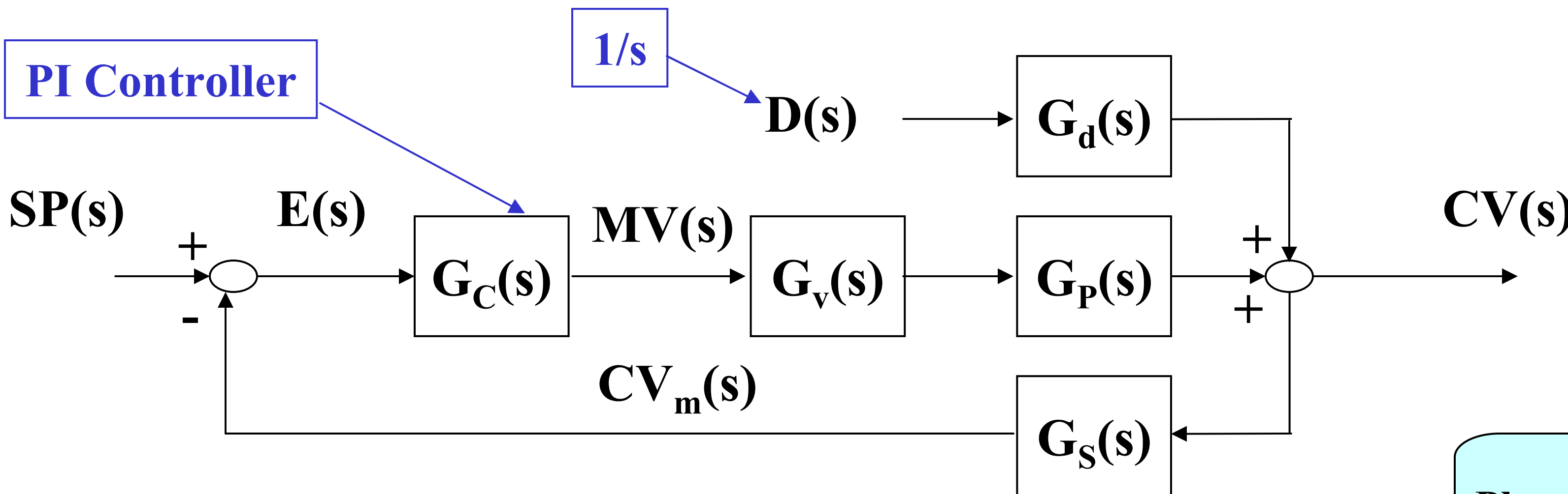
Large value = BAD

By the way, this is an application of the method for evaluating the moment of a dynamic variable.

$$\text{n}^{\text{th}} \text{ moment}: \quad \int_0^\infty t^n Y(t) dt = (-1)^n \left[\frac{d^n}{ds^n} Y(s) \right]_{s=0}$$

Directionality: Its effect on Control Performance

KEY MATHEMATICAL INSIGHT
APPLIED FIRST TO A SINGLE-LOOP SYSTEM



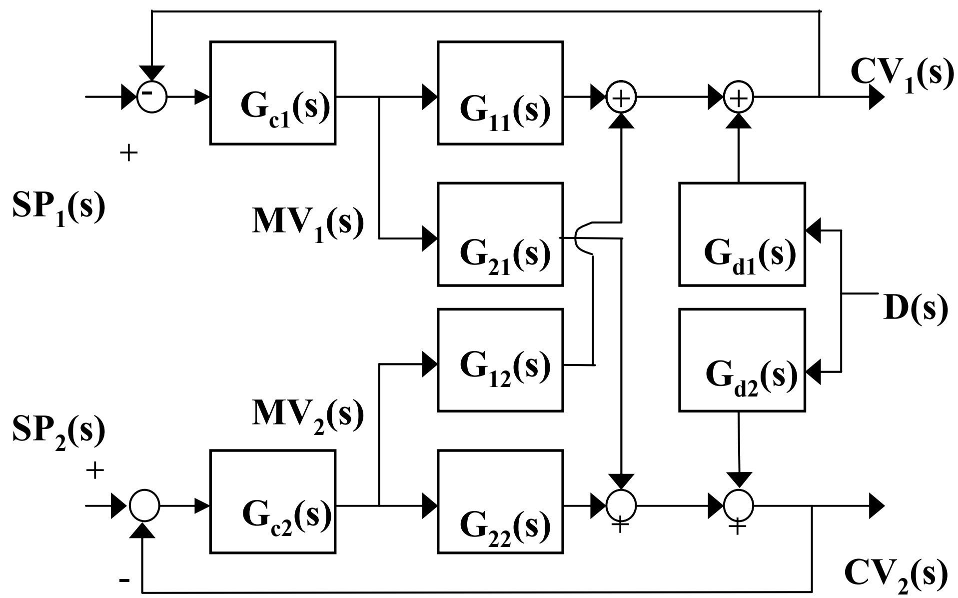
Apply this concept to a single-loop control system

$$E(s) = \frac{G_d(s)}{1 + G_p(s) G_c(s)} D(s)$$

$$\int_0^{\infty} E_{SL}(t)dt = -\frac{K_D T_I}{K_p K_c}$$

Directionality: Its effect on Control Performance

- Apply the approach just introduced to a 2x2 multiloop system
- Identify the key aspects - group terms!



$$\int_0^\infty E_{iML}(t)dt = RDG_{ij} \quad f_{tune} \int_0^\infty E_{iSL}(t)dt$$

Directionality: Its effect on Control Performance

$$\int_0^\infty E_{iML}(t)dt = RDG_{ij} \quad f_{tune} \quad \int_0^\infty E_{iSL}(t)dt$$

Relative Disturbance Gain

- dimensionless
- only s-s gains
- can be +/- and > or < 1.0
- different for each disturbance
- Usually the dominant term for interaction

Tune Factor

Change in tuning for multi-loop

Single-loop performance (dead times, large disturbances, etc. are bad)

Directionality: Its effect on Control Performance

$$\int_0^\infty E_{iML}(t)dt = RDG_{ij} \quad f_{tune} \quad \int_0^\infty E_{iSL}(t)dt$$

Relative disturbance gain

$$\left(\frac{1}{1 - \left({K_{12}K_{21}} \big/ {K_{11}K_{22}} \right)} \right) \left(1 - \frac{K_{d2}K_{12}}{K_{d1}K_{22}} \right)$$

$$\frac{\left({K_c} \big/ {T_I} \right)_{SL}}{\left({K_c} \big/ {T_I} \right)_{ML}}$$

$$\frac{K_D T_I}{K_p K_c}$$

Typical range 0.5 – 2.0.

What unique information is here?

What is this term?

Directionality: Its effect on Control Performance

The dominant factor!

$$\int_0^\infty E_{iML}(t)dt \Big/ \int_0^\infty E_{iSL}(t)dt = RDG_{ij}\, f_{tune}$$

The change in performance due to interaction in multiloop system

RDG include feedback interaction (RGA) and disturbance direction

Directionality: Its effect on Control Performance

$$\int_0^\infty E_{iML}(t)dt = RDG_{ij} \quad f_{tune} \quad \int_0^\infty E_{iSL}(t)dt$$

RDG is easily calculated

2x2 Multiloop

$$\left(\frac{1}{1 - \left(K_{12}K_{21} / K_{11}K_{22} \right)} \right) \left(1 - \frac{K_{d2}K_{12}}{K_{d1}K_{22}} \right)$$

General Square Multiloop

$$\text{Multiloop:} \qquad [\Delta MV]_{ML} = -K_p^{-1} K_d$$

$$\text{Single-loop:} \qquad [\Delta MV]_{SL} = -(K_{P_{diagonal}})^{-1} K_d$$

$$RDG_{ij} = \frac{(\Delta MV_j)_{ML}}{\Delta D} \Big/ \frac{(\Delta MV_j)_{SL}}{\Delta D}$$

Directionality: Its effect on Control Performance

Relative Disturbance Gain (RDG)
Gives effects of Interaction on Performance

$$\int_0^\infty E_{iML}(t)dt \Big/ \int_0^\infty E_{iSL}(t)dt = RDG_{ij}\; f_{tune}$$

Advantages

- Steady-state information
- Includes feedback and disturbance directions
- Direct measure of CV performance

Shortcomings

- Allows +/- cancellation
  - large is bad performance
  - small might be good performance
- Represents only CV performance
- Measures only one aspect of CV behavior

Directionality: Its effect on Control Performance

Some important results. You can prove them yourself.

- **For a single set point change, RDG = RGA**
- **For a disturbance with same effect as an MV, the |RDG| = 0 to 2.0 (depending on the output variable)**
- **For one-way interaction, RDG = 1**
- **Decouple only for unfavorable directionality, i.e., large |RDG|**

Large RGA indicates poor performance for ΔSP

For these common disturbances, interaction is favorable and performance similar to SL!

Performance similar to SL!

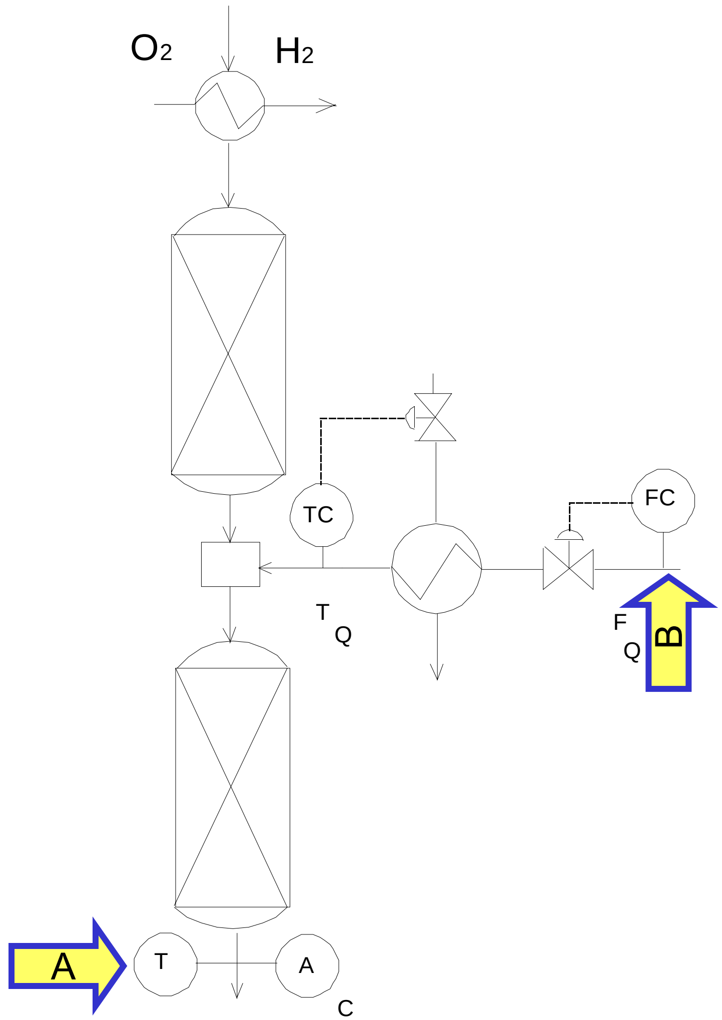
Decoupling can make performance worse!

Directionality: Its effect on Control Performance

Process Example: FOSS packed bed chemical reactor

1. **Calculate the RGA and tuning**
2. **Select pairings and predict performance**
   A. **Temperature set point change (single ΔSP)**
   B. **Quench pressure change (disturbance same as MV)**

$$\begin{bmatrix} T(s) \\ C(s) \end{bmatrix} = \begin{bmatrix} \frac{-2.265e^{-1.326s}}{0.786s+1} & \frac{0.746e^{-2.538s}}{0.092s+1} \\ \frac{1.841e^{-0.445s}}{0.917s+1} & \frac{-0.654e^{-0.786s}}{0.870s+1} \end{bmatrix} \begin{bmatrix} F_Q(s) \\ T_Q(s) \end{bmatrix}$$



Directionality: Its effect on Control Performance

1. Calculate the RGA and tuning

For T - FQ and C - TQ pairing

RGA = 13.7, Detune factor = 2.0

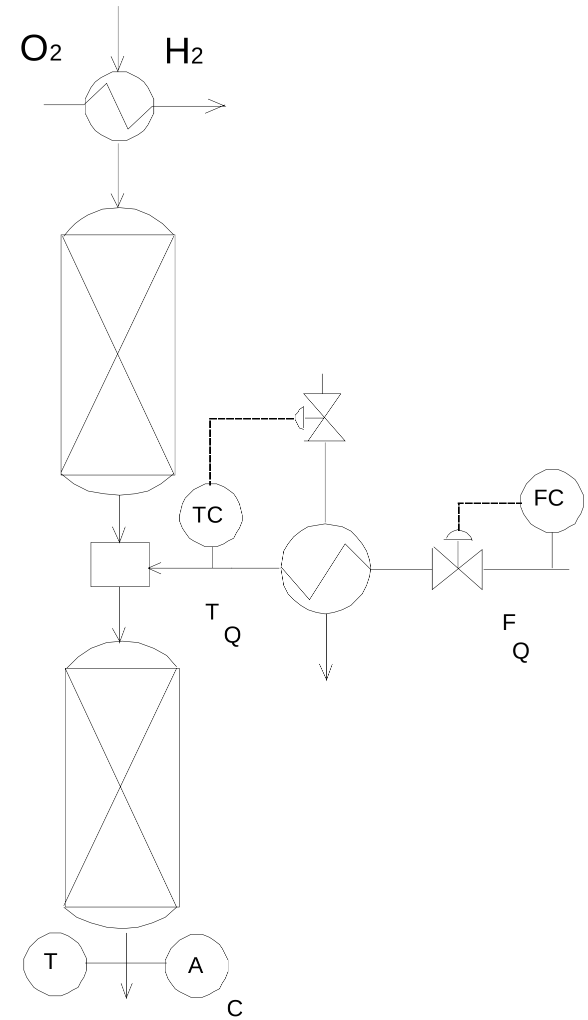
Kc_T = -0.115, TI_T = 1.37

Kc_C = -0.61, TI_C = 1.19

2. Select pairings and predict performance

A. For set point change, RDG = RGA = large!!
Predict poor performance!

B. For disturbance, RDG small (between 0 and 2)!
Predict good performance, near single-loop

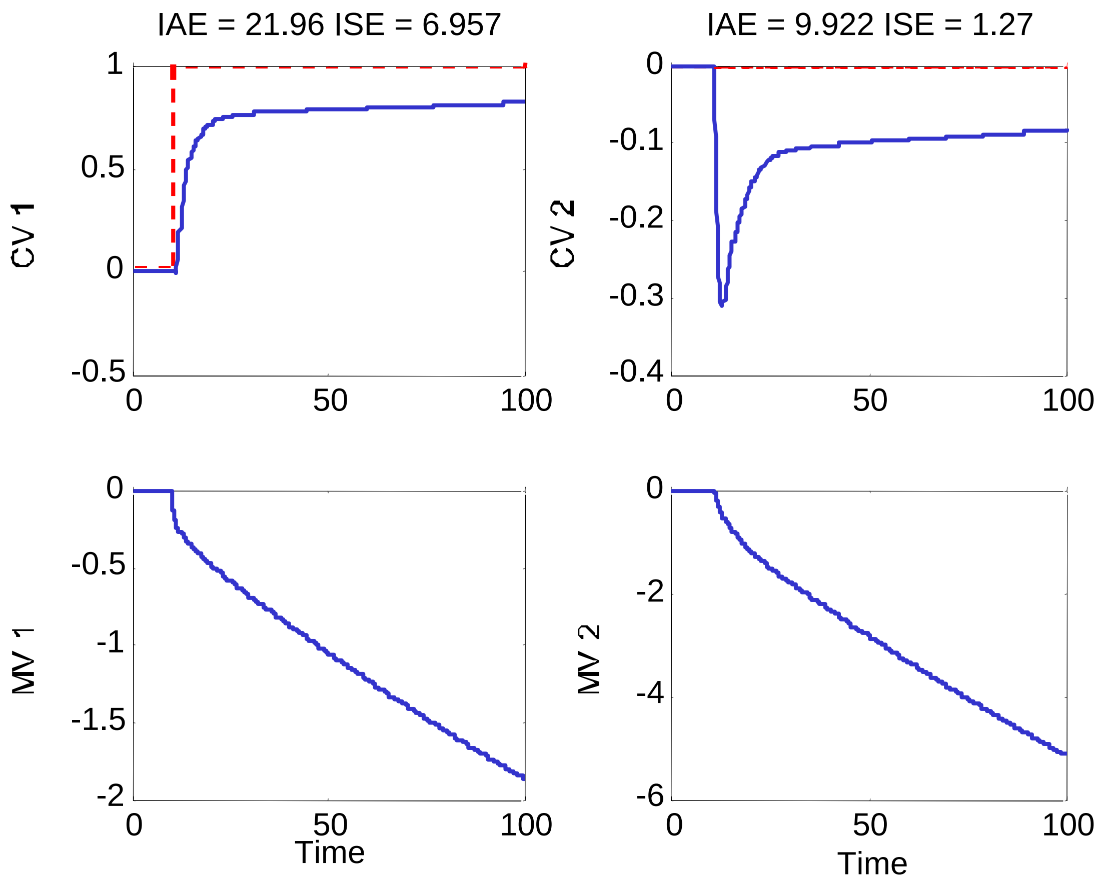


Directionality: Its effect on Control Performance

$$\int E_{ML}\,dt \approx (RGA)\, f_{detune} \int E_{SL}\,dt \qquad \int E_{ML}\,dt \approx (RGA)(Kij / Kii)\, f_{detune} \int E_{SL}\,dt$$

Set point change.

Discuss the performance

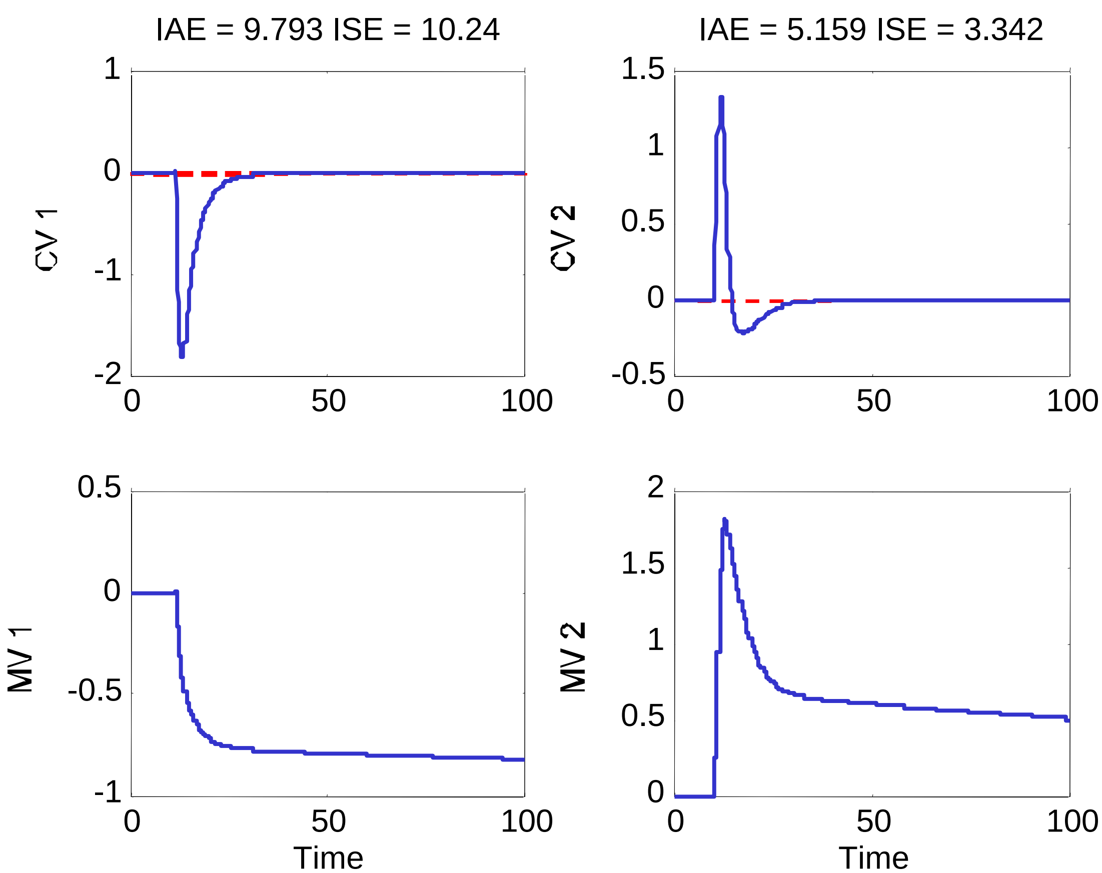


Directionality: Its effect on Control Performance

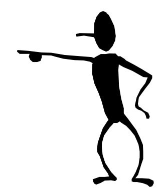
Disturbance in quench pressure, which is through MV dynamics.

Discuss the performance

$$\int E_{ML}\, dt \approx f_{detune} \int E_{SL}\, dt \qquad\qquad \int E_{ML}\, dt = 0$$

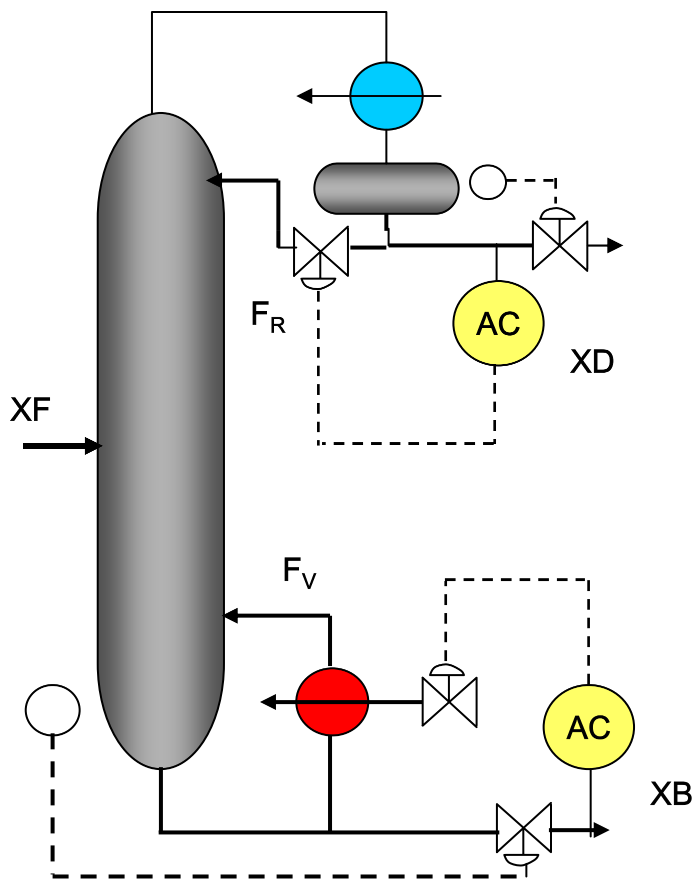


How can this good performance occur with high interaction?



Directionality: Its effect on Control Performance

Process Example: Binary distillation shown in figure.



1. **Calculate the RGA, RDG and tuning**

2. **Predict performance**

   A. XD set point change

   B. Feed composition disturbance

$$\begin{bmatrix} XD(s) \\ XB(s) \end{bmatrix} = \begin{bmatrix} \frac{0.0747e^{-3s}}{12s+1} & \frac{-0.0667e^{-2s}}{15s+1} \\ \frac{0.1173e^{-3.3s}}{11.75s+1} & \frac{-0.1253e^{-2s}}{10.2s+1} \end{bmatrix} \begin{bmatrix} F_R(s) \\ F_V(s) \end{bmatrix} + \begin{bmatrix} \frac{0.70e^{-5s}}{14.4s+1} \\ \frac{1.3e^{-3s}}{12s+1} \end{bmatrix} X_F(s)$$

Directionality: Its effect on Control Performance

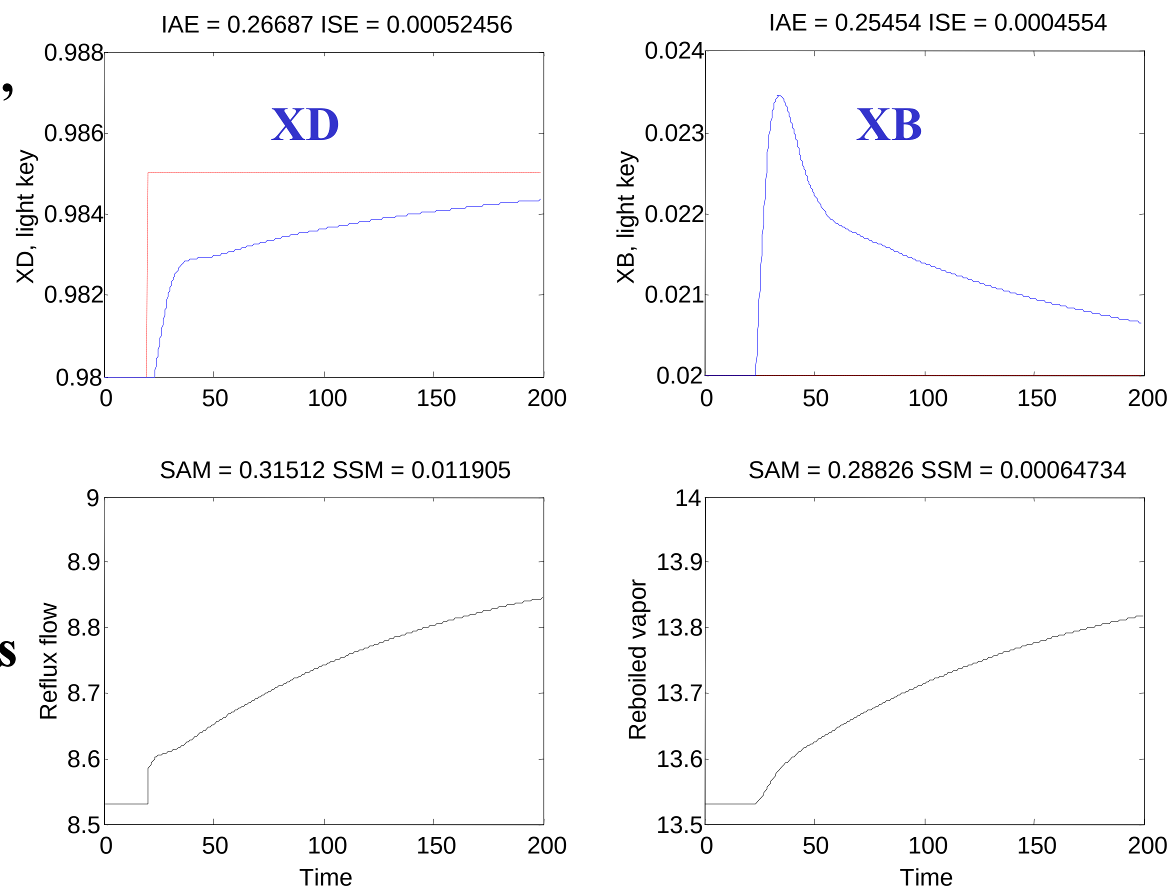
Distillation tower (R,V) with <u>both controllers in automatic</u> for XD set point change

For input = ΔSP_{XD},

RDG = RGA = 6

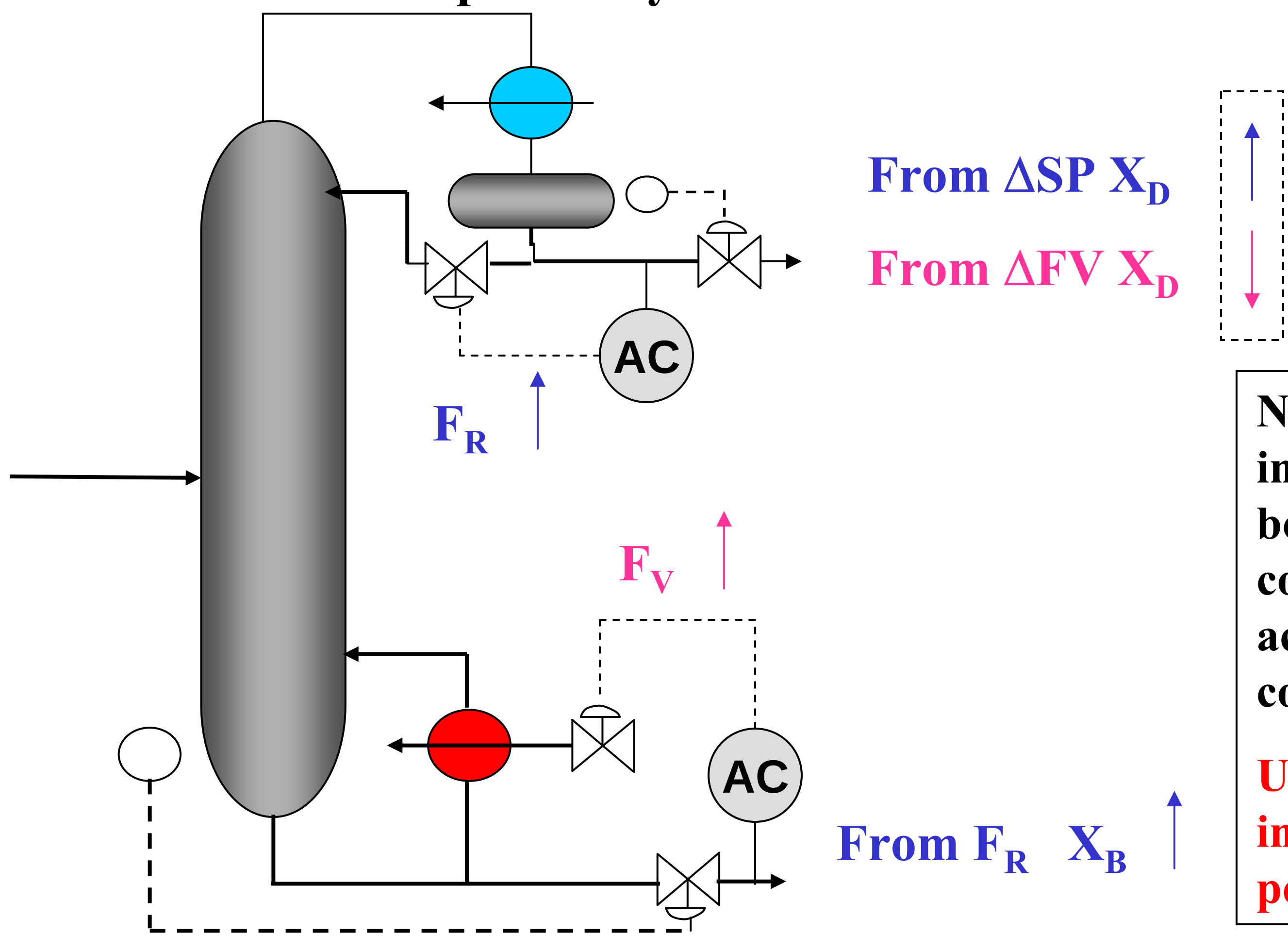
RDG large indicates much worse than single-loop

Transient confirms the short-cut prediction



Directionality: Its effect on Control Performance

Distillation for XD set point change
Let's explain why the interaction is unfavorable



Note that the interaction from the bottom loop tends to counteract the action of the XD controller!

Unfavorable interaction, poor performance

Directionality: Its effect on Control Performance

Distillation tower (R,V) with **both controllers in automatic** for feed composition disturbance

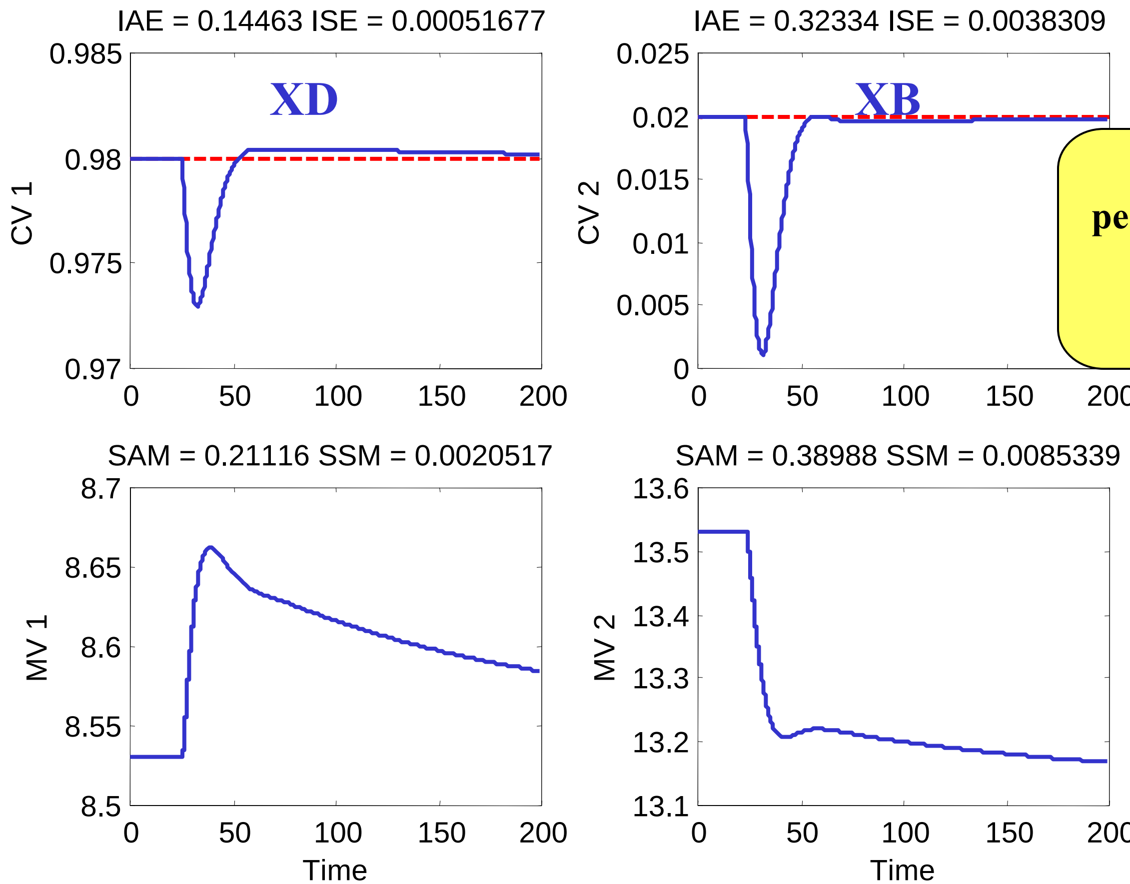
For input = ΔXF,

$|RDG_{XD}| = 0.07$
$|RDG_{XB}| = 0.90$

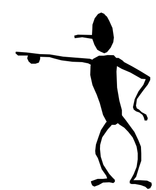
(RGA = 6)

RDG small indicates multiloop can be as good as single-loop

Transient confirms the short-cut prediction



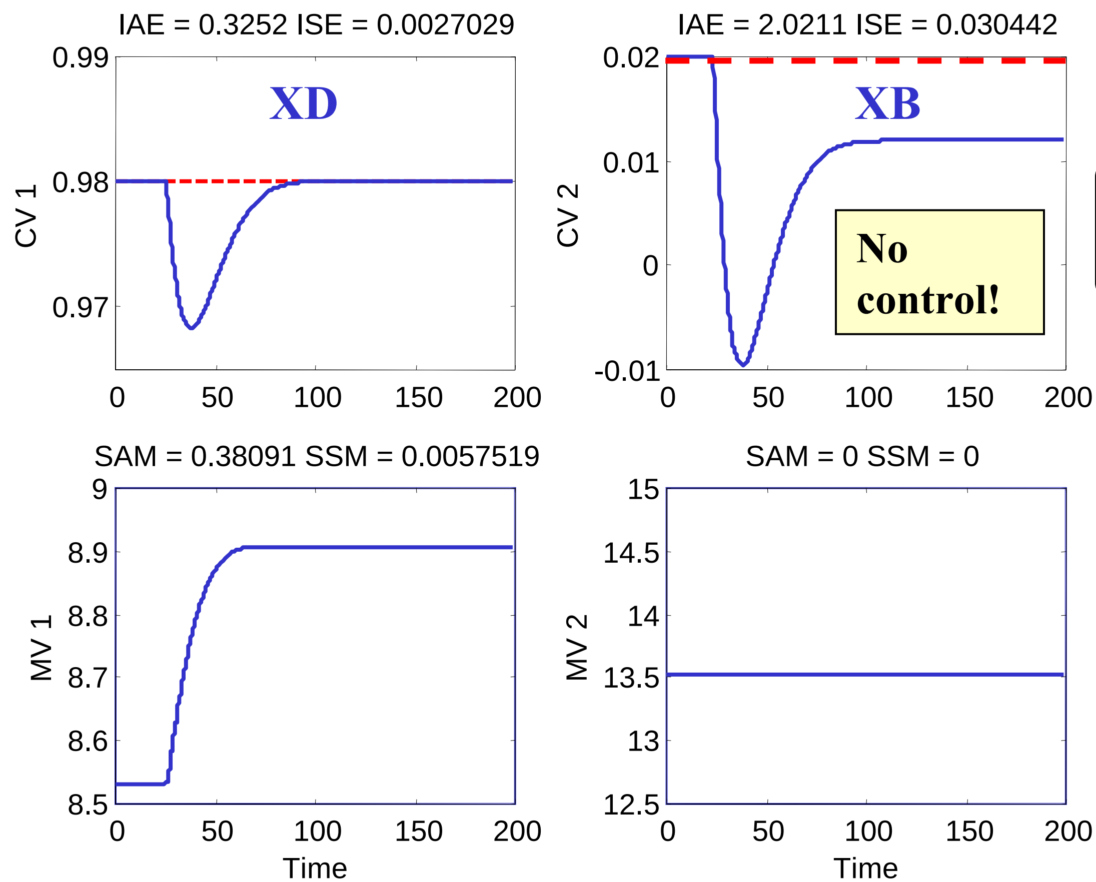
Good performance without MPC!



Directionality: Its effect on Control Performance

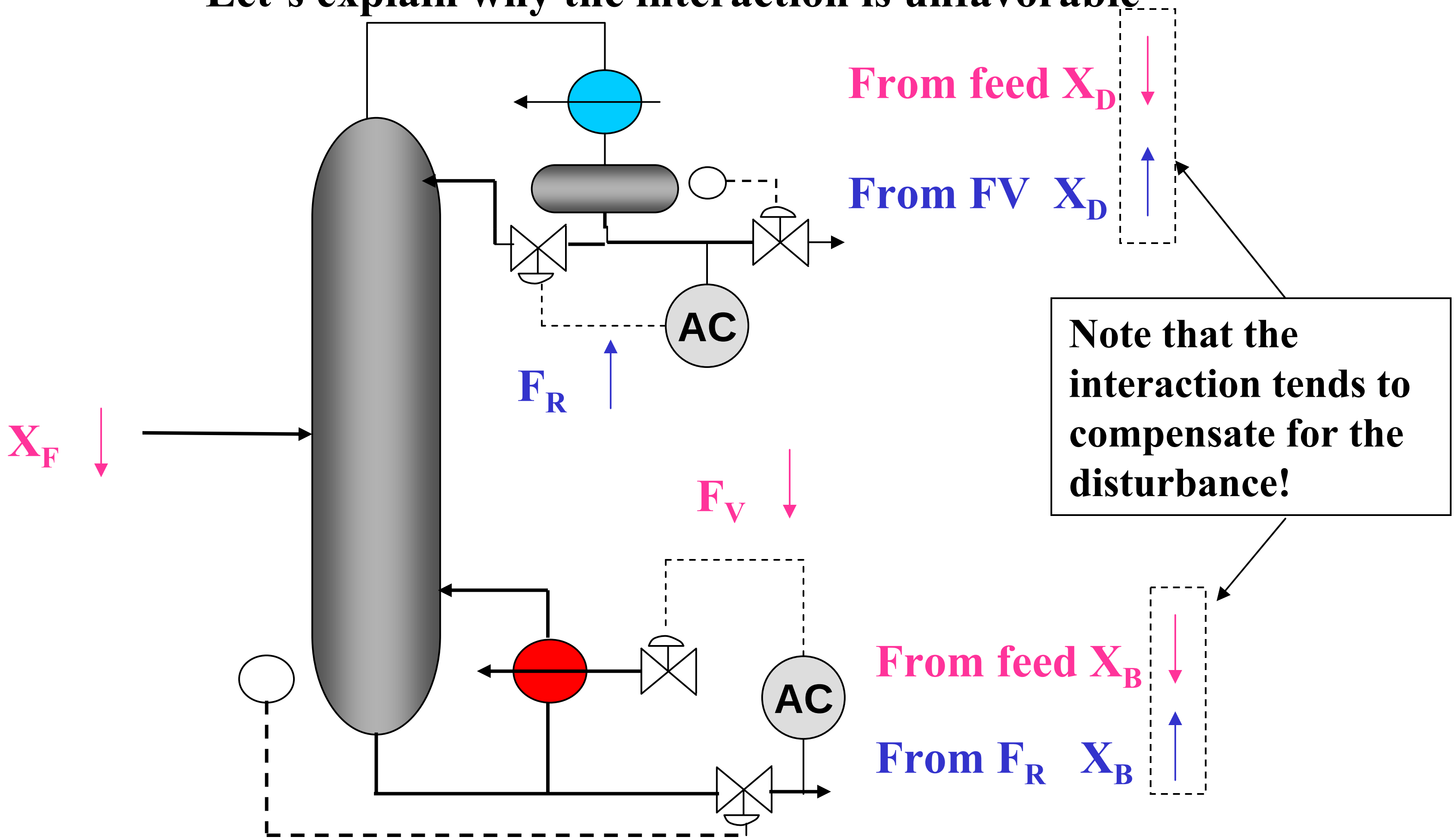
Distillation tower (R,V) with only XD controller in automatic for feed composition disturbance

Favorable interaction results in small XB deviation although XB is not controlled!



Directionality: Its effect on Control Performance

Distillation for XD set point change
Let's explain why the interaction is unfavorable



Directionality: Its effect on Control Performance

We have a short-cut measure that

- **Is dimensionless**
- **Indicates CV performance for each disturbance (relative to single-loop performance)**

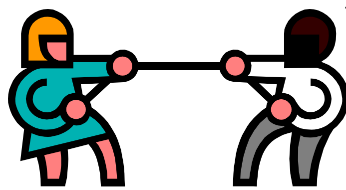
  - But is not definitive! Large is always bad, small might be good.

- **Gives general insights for**
  - **SP changes,**
  - **one-way interaction,**
  - **disturbances with MV model, and**
  - **decoupling**

Directionality: Its effect on Control Performance

We have two short-cut measures, and many more exist. Which do we use?

I need to know integrity, I want RGA!



I need to know performance, I want RDG!

- Use them both (and other relevant short-cut metrics)
- They are especially useful for eliminating candidates
- Do not design based solely on short-cut metrics

Directionality & Performance Workshop 1

Prove the following important results.

A. For a single set point change, RDG = RGA

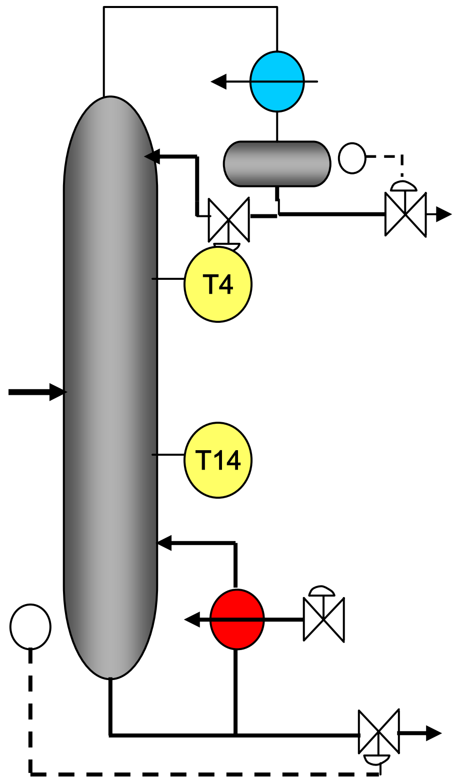
B. For a disturbance with same effect as an MV, the |RDG| = 0 to 2.0 (depending on the output variable)

C. For one-way interaction, RDG = 1

D. Decouple only for unfavorable directionality, i.e., large |RDG|

Directionality & Performance Workshop 2

The following model for a two-product distillation tower was presented by Waller et. al. (1987).



$$\begin{bmatrix} T4(s) \\ T14(s) \end{bmatrix} = \begin{bmatrix} \dfrac{-0.045e^{-0.5s}}{8.1s+1} & \dfrac{0.048e^{-0.5s}}{11s+1} \\ \dfrac{-0.23e^{-1.5s}}{8.1s+1} & \dfrac{0.55e^{-.5s}}{10.2s+1} \end{bmatrix} \begin{bmatrix} F_R(s) \\ F_V(s) \end{bmatrix} + \begin{bmatrix} \dfrac{0.004e^{-s}}{8.5s+1} \\ \dfrac{-0.65e^{-s}}{9.2s+1} \end{bmatrix} X_F(s)$$

Determine the following.

a. Is the system controllable in the steady state?

b. What loop pairings have good integrity?

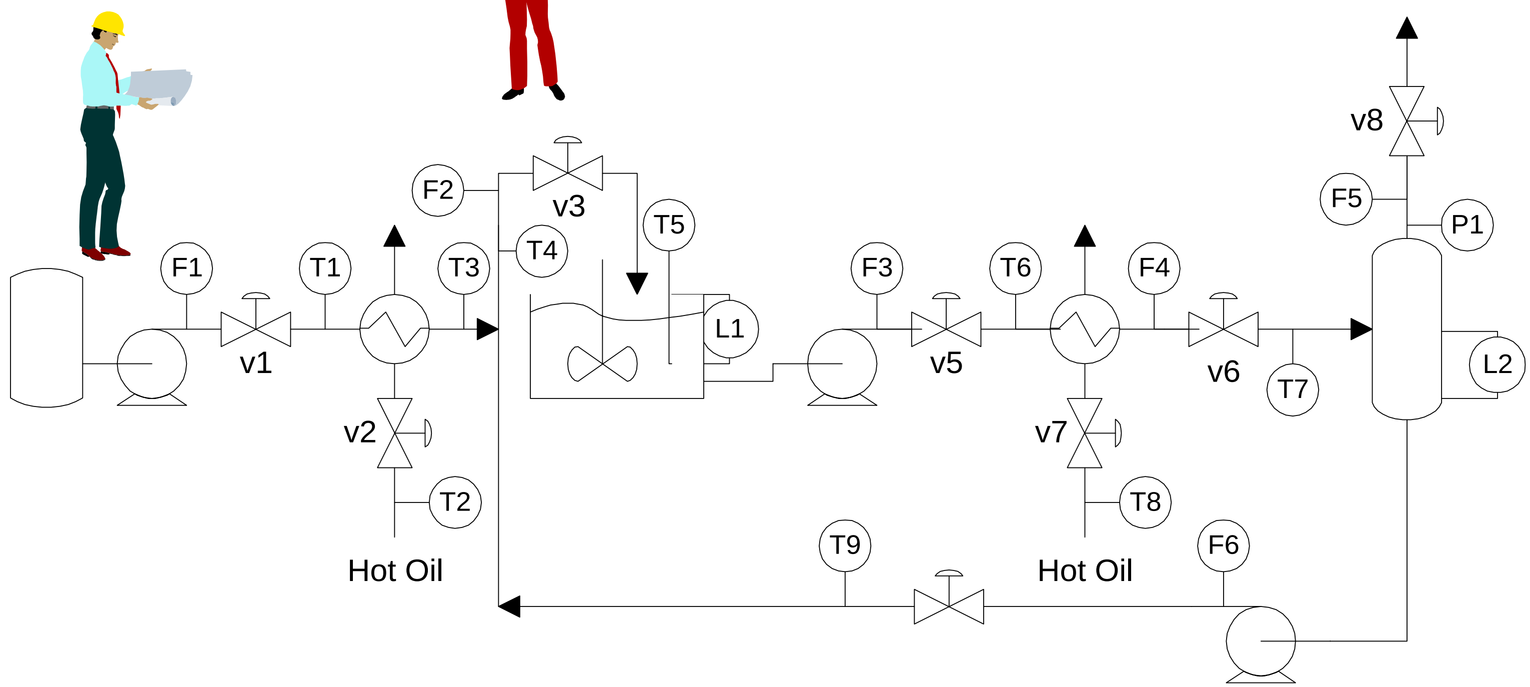
c. For the pairings with good integrity, is the interaction favorable or unfavorable?

d. Do you recommend decoupling for the disturbance response?

Structured, Short-cut Control Design

This is a conventional process plant

We should be able to design controls using our process insights and principles of multivariable processes



Structured, Short-cut Control Design

Short-cut Methods

- Several metrics, each addresses one objective
- Does not yield final design, but eliminates candidates
- Limited information and simple calculations
- Finds "conventional" designs
- Verify with simulation

Optimization-based Methods

- More general definition of desired behavior
- Address all criteria simultaneously
- Evaluates full transient behavior
- Calculations can be extensive, but must be computable
- Finds "unconventional"

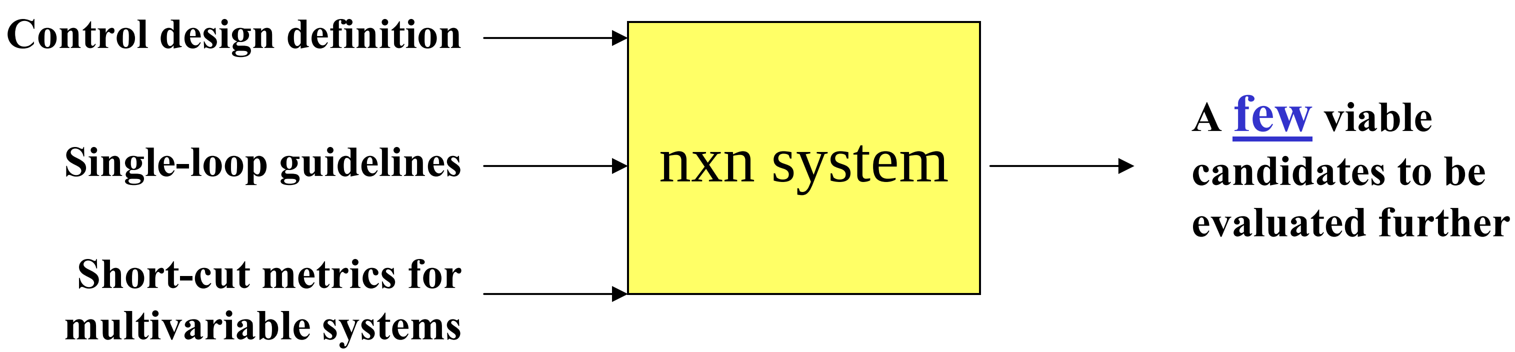
Structured, Short-cut Control Design

Before moving on to systematic use of dynamic simulation and optimization, we will develop some <u>guidelines</u> for a complete short-cut control design method. Why?

- **Most control design is done this way!**
- **Integrate short-cut metrics introduced to this point**
- **Identify challenges needing a more systematic and complete analysis. i.e, optimization-based**

Structured, Short-cut Control Design

For a square (nxn variable) process, there are **n!** potential candidate designs. We need to reduce this number!



Structured, Short-cut Control Design

Engineers need good guidelines based on principles and experience to solve the "easy" problems.

- **Provide good control performance for typical process systems**
- **Require limited information, e.g., process flowsheet, steady-state design, steady-state gains, qualitative dynamics**
- **Can be applied without dynamic simulation or plant tests**
- **Recognize that essentially every guideline will be violated for special conditions**

Structured, Short-cut Control Design

Class Workshop 1: Develop a comprehensive set of control design guidelines

Some hints:

- **Define the objectives first! Consider the seven categories of design objectives**
- **Insure that the goals are possible for the process!**
- **Integrate principles from single-loop and interaction topics**
- **Use all process insights!**

Structured, Short-cut Control Design

A.
- **1. Process analysis and control objectives**
- **2. Select measurements and sensors**

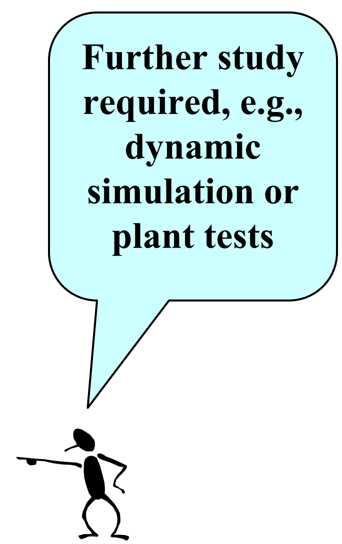
B.
- **3. Select manipulated variables and final elements**

C.
- **4. Check whether goals are achievable for the process**

D.
- **5. Eliminate clearly unacceptable loop pairings**

6. Define one or a few acceptable loop pairings

Short-cut Approach Completed



Structured, Short-cut Control Design

1. Process analysis and control objectives
2. Select measurements and sensors
3. Select manipulated variables and final elements
4. Check whether goals are achievable for the process
5. Eliminate clearly unacceptable loop pairings
6. Define one or a few acceptable loop pairings

Short-cut Approach Completed

Temporal decomposition for developing good candidate loop pairings

The control decisions are usually made in the following order, which roughly follows the speed of the feedback.

1) flow and inventory (level & pressure) for main process flows

2) process environment, find inferential or partial control variables. Good selections reduce feedback for quality and profit

3) product quality

4) efficiency and profit

5) monitoring and diagnosis

Structured, Short-cut Control Design

1. Process analysis and control objectives

2. Select measurements and sensors

3. Select manipulated variables and final elements

4. Check whether goals are achievable for the process

5. Eliminate clearly unacceptable loop pairings

6. Define one or a few acceptable loop pairings

Short-cut Approach Completed

Class Workshop

Now, let's apply this structured approach to some realistic control design problems.

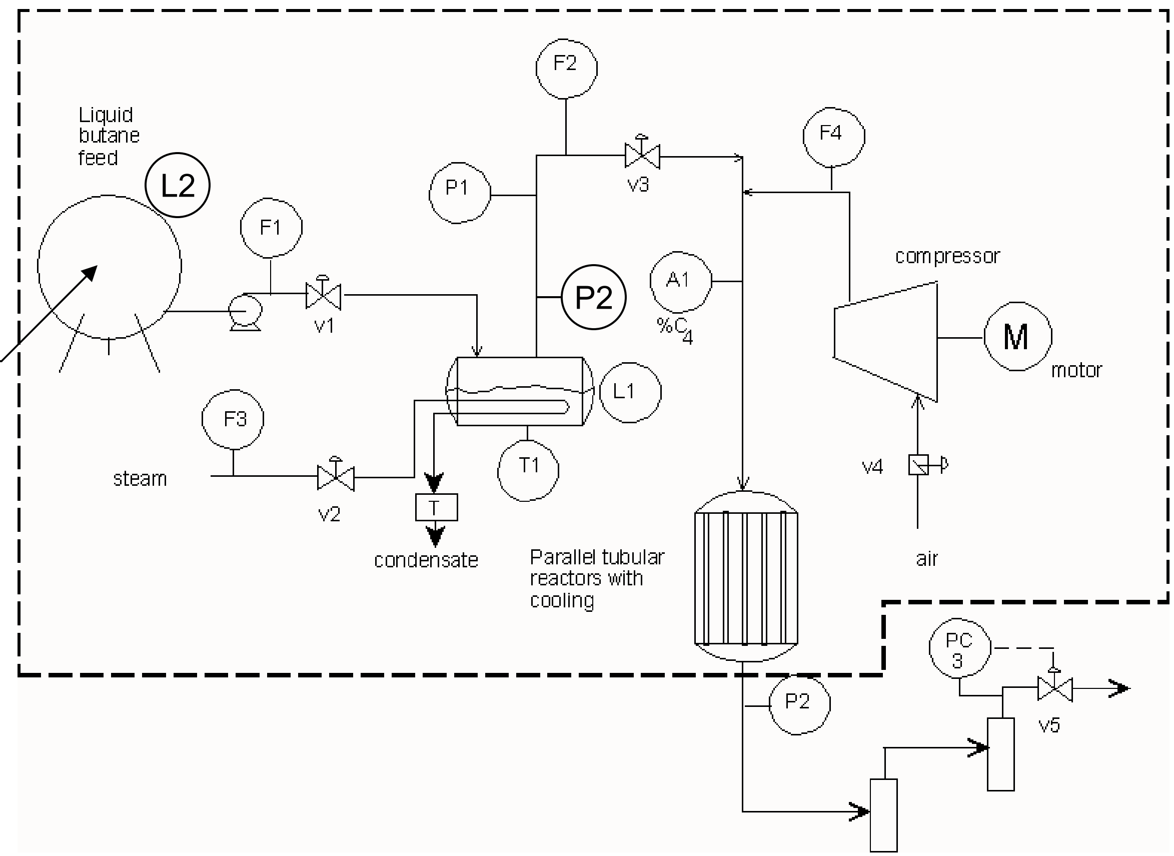
Processes are selected to have "obvious" steady-state and dynamic behavior to make them suitable for short, classroom exercises.

Your designs can be sketched on the figures.

Short-cut Control Design Workshop 2

Class Workshop: Design controls for the Butane vaporizer which is the first unit in a Maleic Anhydride process.

Periodic feed delivery



Short-cut Control Design Workshop 2

Some useful information about the plant.

1. Essentially pure butane is delivered to the plant periodically via rail car.
2. Butane is stored under pressure.
3. The "feed preparation" unit is highlighted in the figure. The goal is to vaporize the appropriate amount of butane and mix it with air. After the feed preparation, the mixed feed flows to a packed bed reactor; effluent from the reactor is processed in separation units, which are not shown in detail.
4. Heat is provided by condensing steam in the vaporizer.
5. Air is compressed by a compressor that is driven by a steam turbine.
6. There is an explosion limit for the air/C4 ratio. Normal is 1.6% butane, and the explosive range is 1.8% to 8.0%

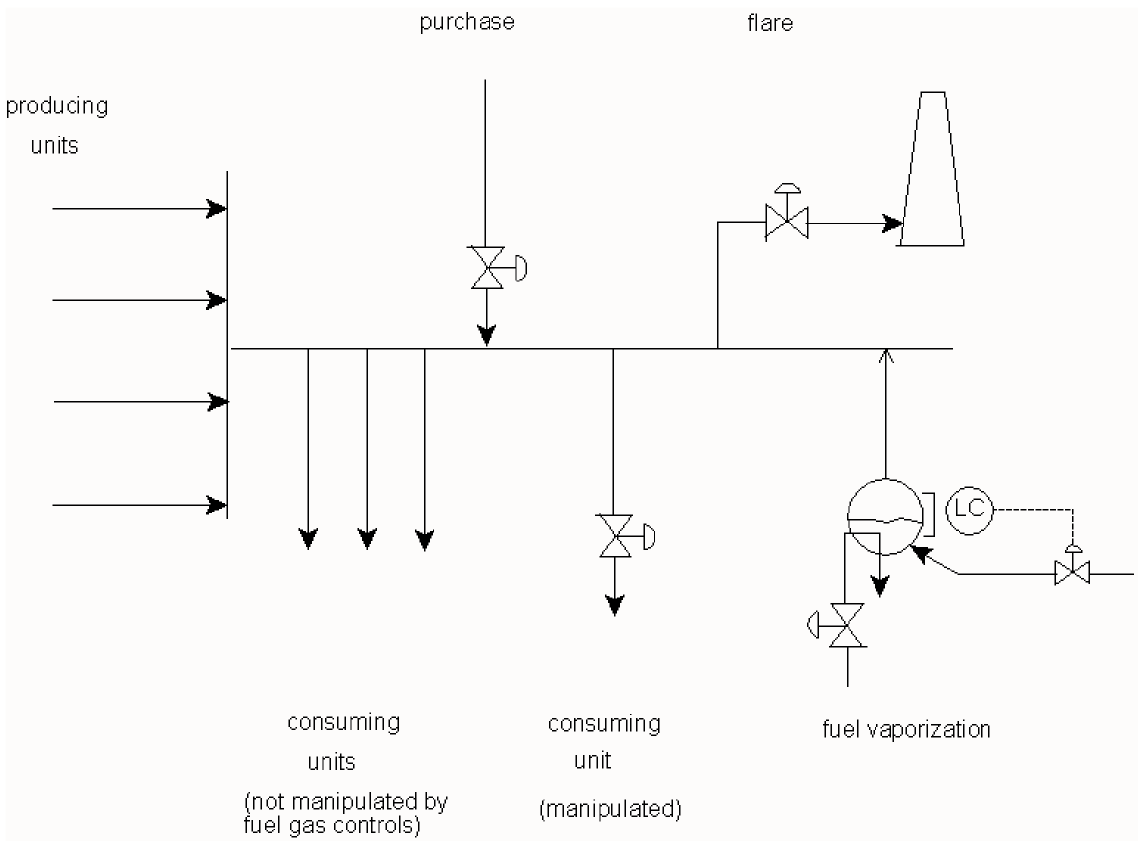
You are asked to design a control system for the process in the dashed box. You should

a. Briefly, list the control objectives for the seven categories.
b. Add sensors and valves needed for good control.
c. Sketch the loop pairing on the figure.
d. Provide a brief explanation for your design.

e. If you feel especially keen, include "control for safety" in your design. This would include the following items (among others).
   - alarms
   - safety shutdown systems
   - pressure relief
   - failure position for valves

Short-cut Control Design Workshop 3

Class Workshop: Design controls for the fuel gas distribution system.



Short-cut Control Design Workshop 3

The gas distribution process in the figure provides fuel to the process units. Several processes in the plant generate excess gas, and this control strategy is not allowed to interfere with these units. Also, several processes consume gas, and the rate of consumption of only one of the processes can be manipulated by the control system. The flows from producers and to consumers can change rapidly. Extra sources are provided by the purchase of fuel gas and vaporizer, and an extra consumer is provided by the flare. The relative dynamics, costs and range of manipulation are summarized in the following table.

flow	manipulated	dynamics	range (% of total flow)	cost
producing	no	fast	0-100%	n/a
consuming	only one flow	fast	0-20%	very low
generation	yes	?	0-100%	low
purchase	yes	?	0-100%	medium
disposal	yes	?	0-100%	high

a. Complete the blank entries in the table based on engineering judgement for the processes in the figure.

b. Complete a Control Design Form for the problem. Specifically, define the dynamic and economic requirements.

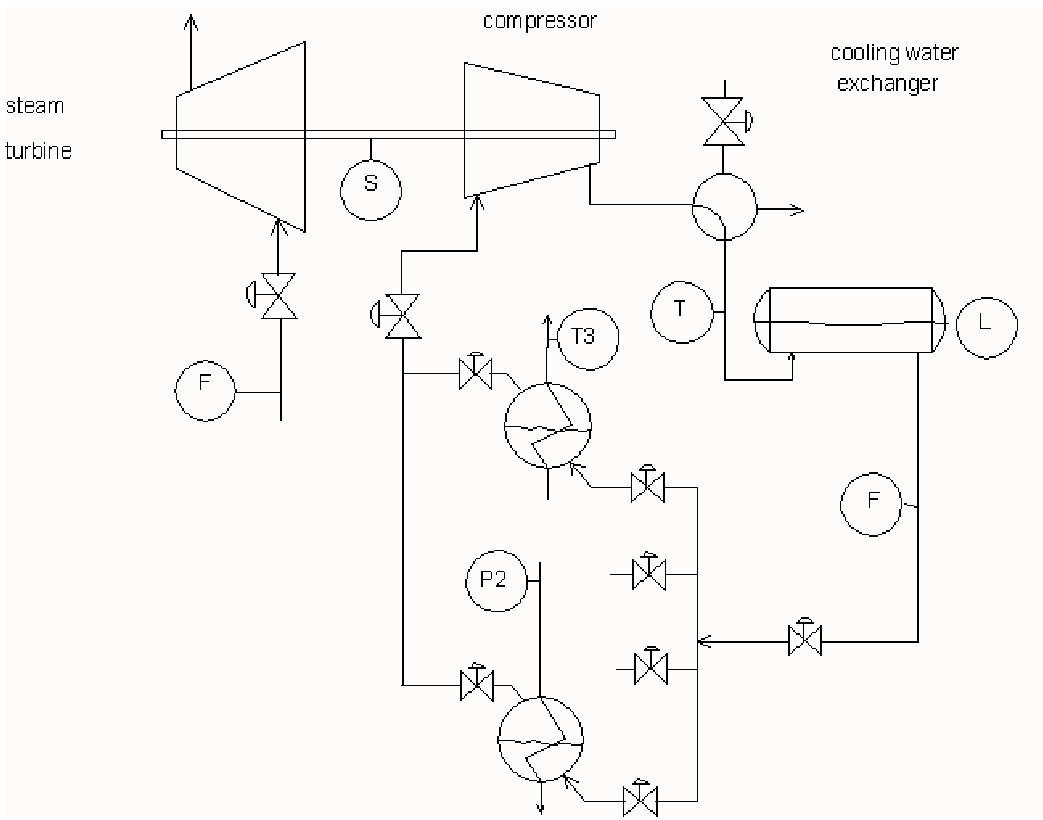
   Hint: To assist in defining the proper behavior, plot all fuel gas flows vs. (consumption - production) on the x-axis.

c. Design a multiloop control strategy to satisfy the objectives. You may add sensors as required but make no other changes.

d. Suggest process change(s) to improve the performance of the system.

Short-cut Control Design Workshop 4

Class Workshop: Design controls for the refrigeration system.



Short-cut Control Design Workshop 4

Refrigeration is very important for industrial processes and our daily comfort in the summer. In industry, it is used to provide cooling when the temperatures are below the temperature of cooling water. The controlled objective could be a temperature (heat exchanger), a pressure (condenser) or any other variable that could be influenced by heat transfer.

Refrigeration can consume large amounts of energy for the heat transfer, especially at low temperatures. Thus, the control system should provide the desired control performance at the lowest energy input possible.

Before designing the controllers for this exercise, you might need to quickly review the principles of vapor recompression refrigeration.

This exercise involves the simple, single stage refrigeration circuit in Figure 1.

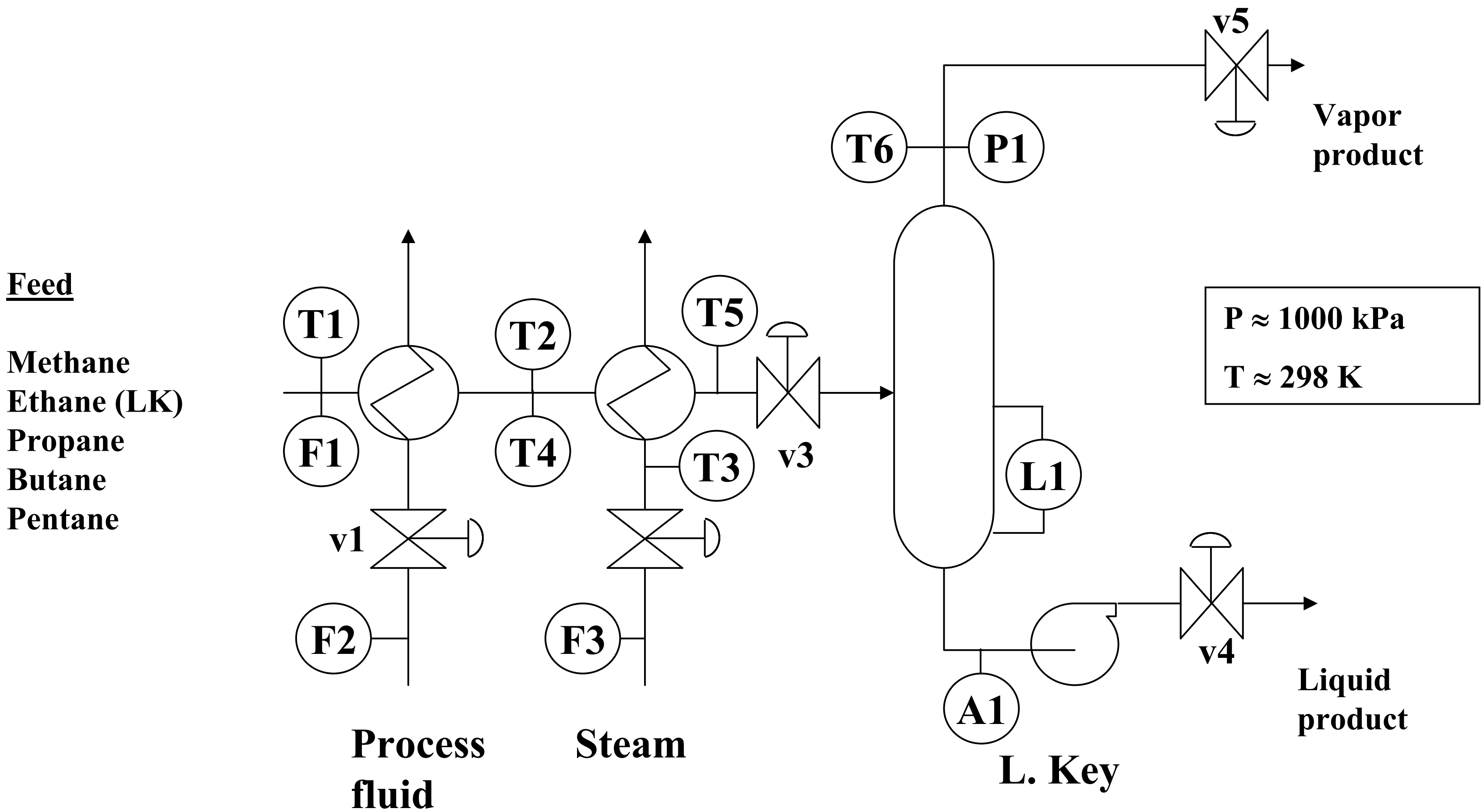
A. Develop a regulatory control design for this system which satisfies the demands of the consumers. Two consumers are shown as a heat exchanger (T3) and a condenser (P2); naturally, many others could exist. Part of your design should provide control for the two consumers shown in the figure. Provide a brief explanation for each controller.

B. Add necessary controls to minimize the energy consumption to the turbine while satisfying the consumers' demands. Explain your design.

In both parts of this question you may add sensors and add and delete valves.

Short-cut Control Design Workshop 5

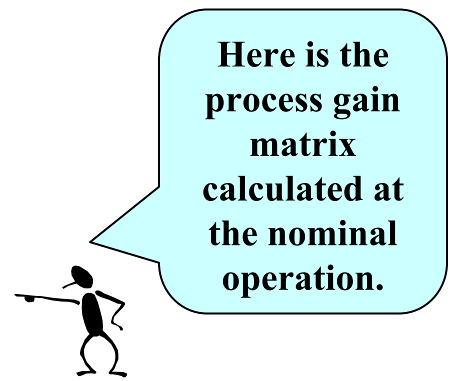
Class Workshop: Design controls for the flash process.



Short-cut Control Design Workshop 5

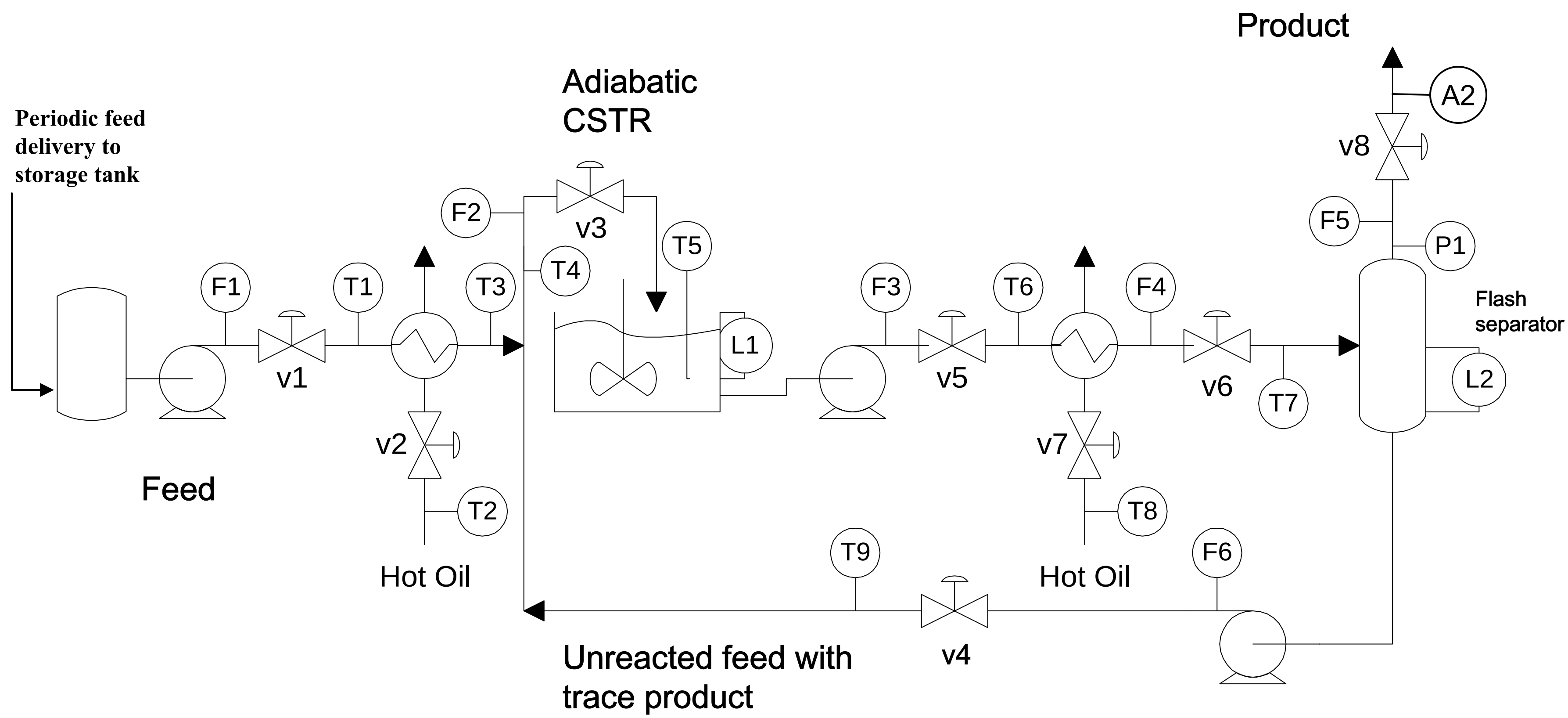
1. **Safety**
   - Maintain vessel pressure below 1200 kPa
2. **Environmental protection**
   - Prevent release of hydrocarbons to the atmosphere
3. **Equipment protection**
   - Ensure that liquid flows through the pump
4. **Smooth operation**
   - When possible, make slow adjustments to liquid product product flow rate
5. **Product quality**
   - Maintain the liquid product at 10 ± 1 mole% L. Key.
6. **Profit**
   - Minimize the use of the expensive steam for heating
7. **Monitoring and diagnosis**
   - Provide alarms for immediate attention by operating personnel

$$\begin{bmatrix} F1 \\ T6 \\ A1 \\ P1 \\ dL_1/dt \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2.0 & 0 & 0 \\ .0708 & .85 & -.44 & 0 & -.19 \\ -.00917 & -.11 & -.44 & 0 & .043 \\ .567 & 6.80 & 1.39 & 0 & -5.36 \\ -.0113 & -.136 & .31 & -.179 & -.0265 \end{bmatrix} \begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \\ v5 \end{bmatrix}$$



Short-cut Control Design Workshop 6

Class Workshop: Design controls for the CSTR with recycle.



Short-cut Control Design Workshop 6

TITLE: Chemical reactor	ORGANIZATION: McMaster Chemical Engineering
PROCESS UNIT: Hamilton chemical plant	DESIGNER: I. M. Learning
DRAWING : Figure 25-8	ORIGINAL DATE: January 1, 1994
	REVISION No. 1

CONTROL OBJECTIVES:

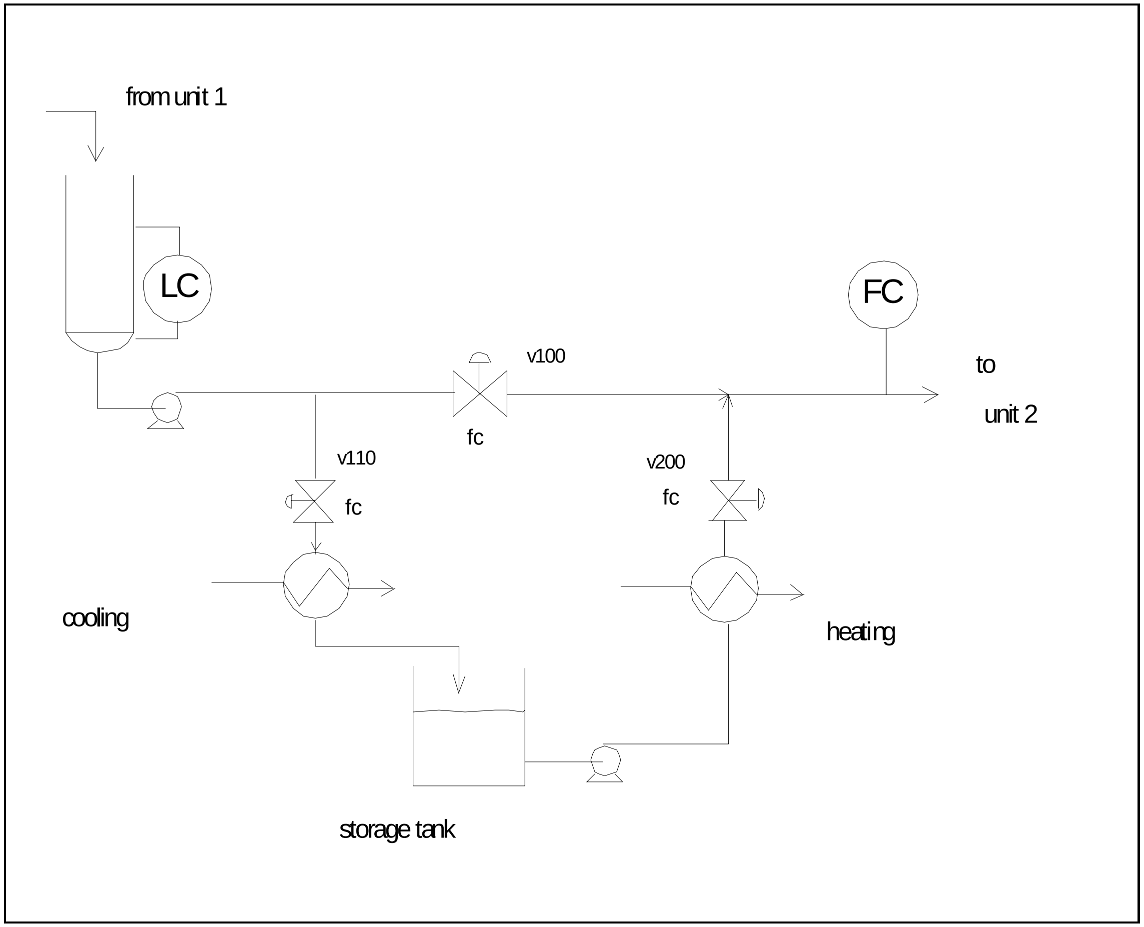
1) SAFETY OF PERSONNEL
   a) the maximum pressure in the flash drum must not be exceeded under any circumstances
   b) no material should overflow the reactor vessel
2) ENVIRONMENTAL PROTECTION
   a) none
3) EQUIPMENT PROTECTION
   a) none
4) SMOOTH, EASY OPERATION
   a) the production rate, F5, need not be controlled exactly constant; its instantaneous value may deviate by 1 unit from its desired value for periods of up to 20 minutes. Its hourly average should be close to its desired value, and the daily feed rate should be set to satisfy a daily total production target.
   b) the interaction of fresh and recycle feed should be minimized
5) PRODUCT QUALITY
   a) the vapor product should be controlled at 10 mole% A, with deviations of ±0.7% allowed for periods of up to 10 minutes.
6) EFFICIENCY AND OPTIMIZATION
   a) the required equipment capacities should not be excessive
7) MONITORING AND DIAGNOSIS
   a) sensors and displays needed to monitor the normal and upset conditions of the unit must be provided to the plant operator
   b) sensors and calculated variables required to monitor the product quality and thermal efficiency of the unit should be provided for longer term monitoring

DISTURBANCES:

SOURCE	MAGNITUDE	PERIOD	MEASURED?
1) impurity in feed (Influences the reaction rate, basically affecting the frequency factor, k_0.)		day	no
2) hot oil temperature	± 20°C	200+ min	yes (T2)
3) hot oil temperature	± 20°C	200+ min	yes (T8)
4) feed rate	±1, step	shift-day	yes (F1)

Short-cut Control Design Workshop 7

Class Workshop: Design controls tank with by-pass.



Short-cut Control Design Workshop 7

Control objectives:

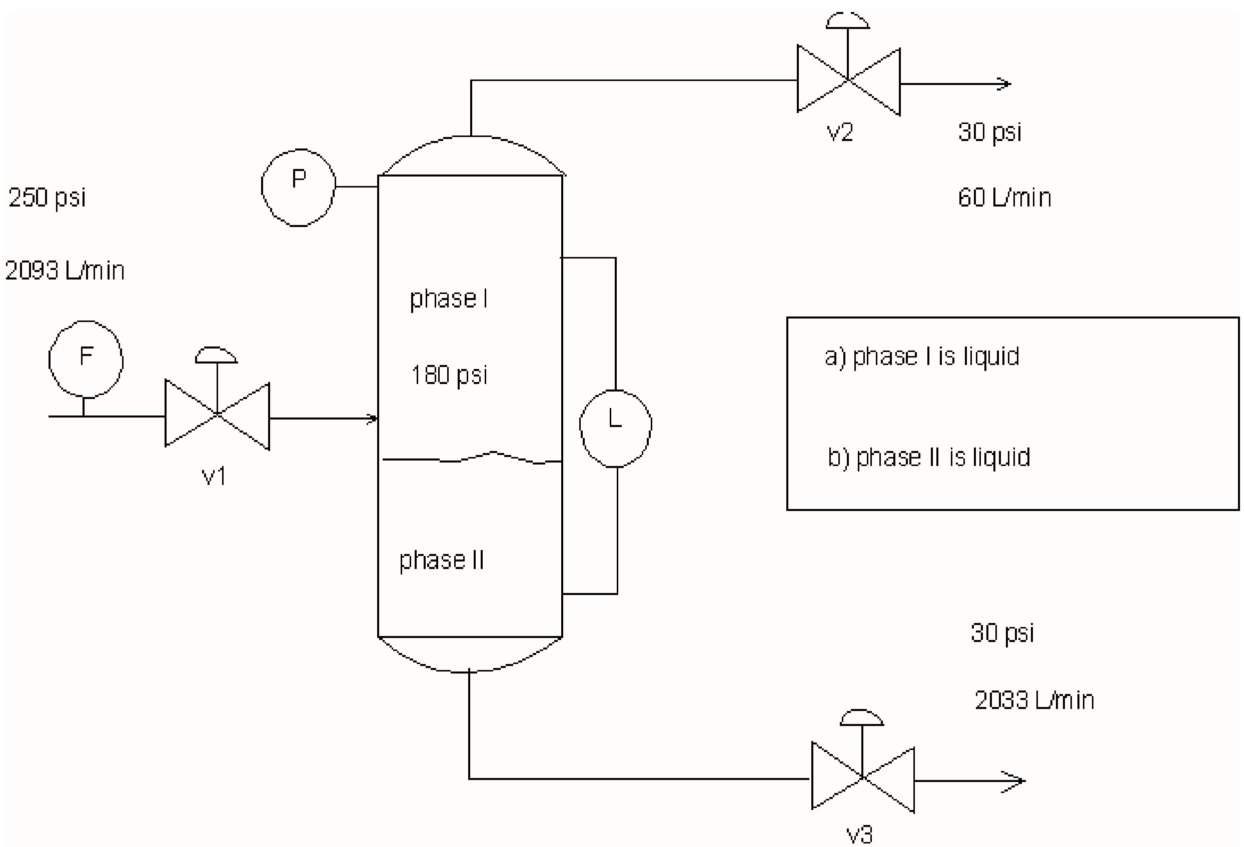
1. **Control the level in the bottom of the Unit 1 tower**
2. **Control the flow rate to Unit 2**
3. **Cool any flow to the tank, which has an upper limit for material stored**
4. **Reheat any material from the tank to Unit 2, which requires heated feed**
5. **Minimize the heating and cooling**

Disturbances:

The flows from Unit 1 and to Unit 2 cannot be adjusted by this control system. They are typically not equal, and either can be larger at a specific time.

Short-cut Control Design Workshop 8

Class Workshop: Design controls for a decanter.



Short-cut Control Design Workshop 8

Control Objectives:

1. **Pressure in the vessel**
2. **Interface level in the vessel**
3. **Flow rate(s) How many can be controlled independently?**

Disturbances:

The following additional information is provided about the variability of the process operation; the feed flow is 1400-2600, the percent overhead in feed is 1-5%, and the pressures are essentially constant.

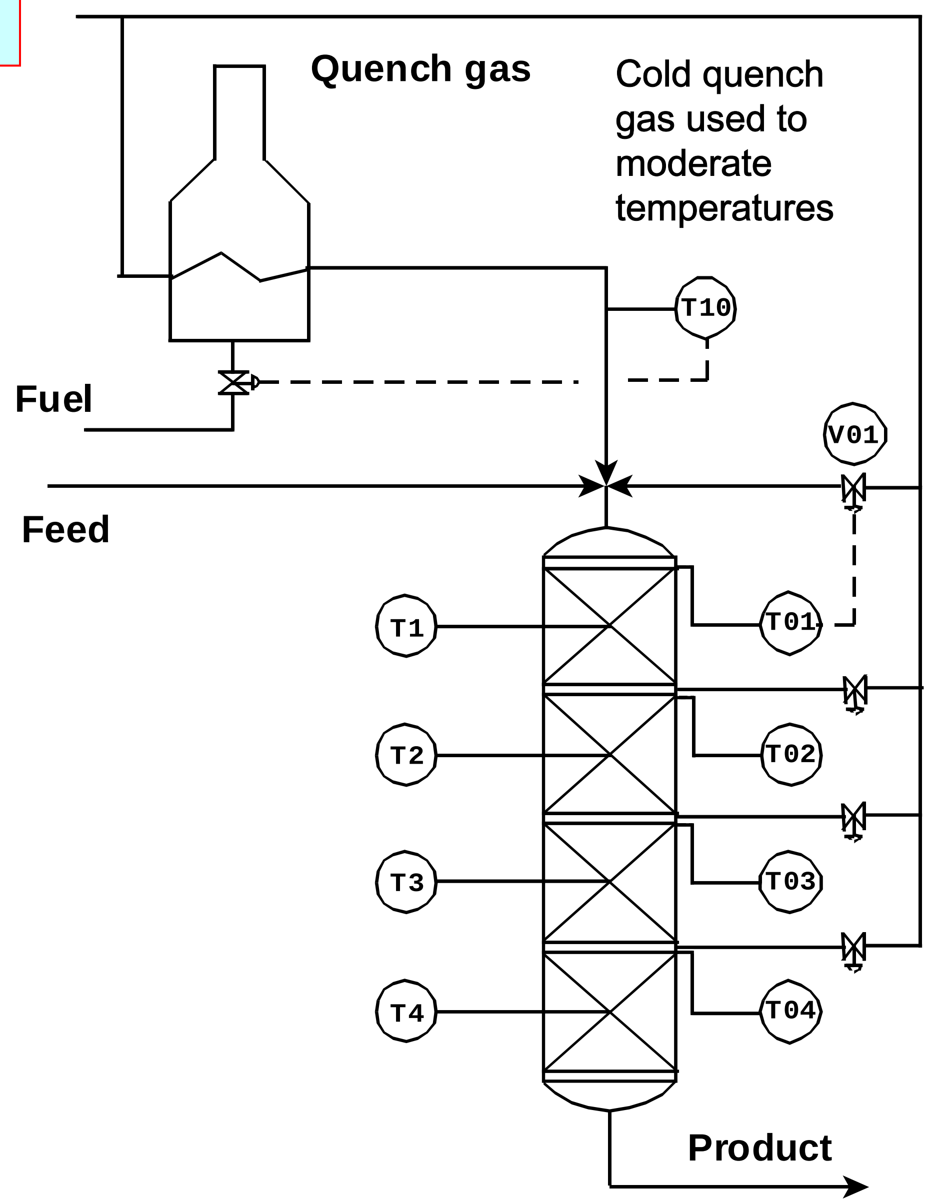
Process information:

You may assume that the flows are proportional to the square root of the pressure drop and the valve % open; the valves are all 50% open at the base case conditions.

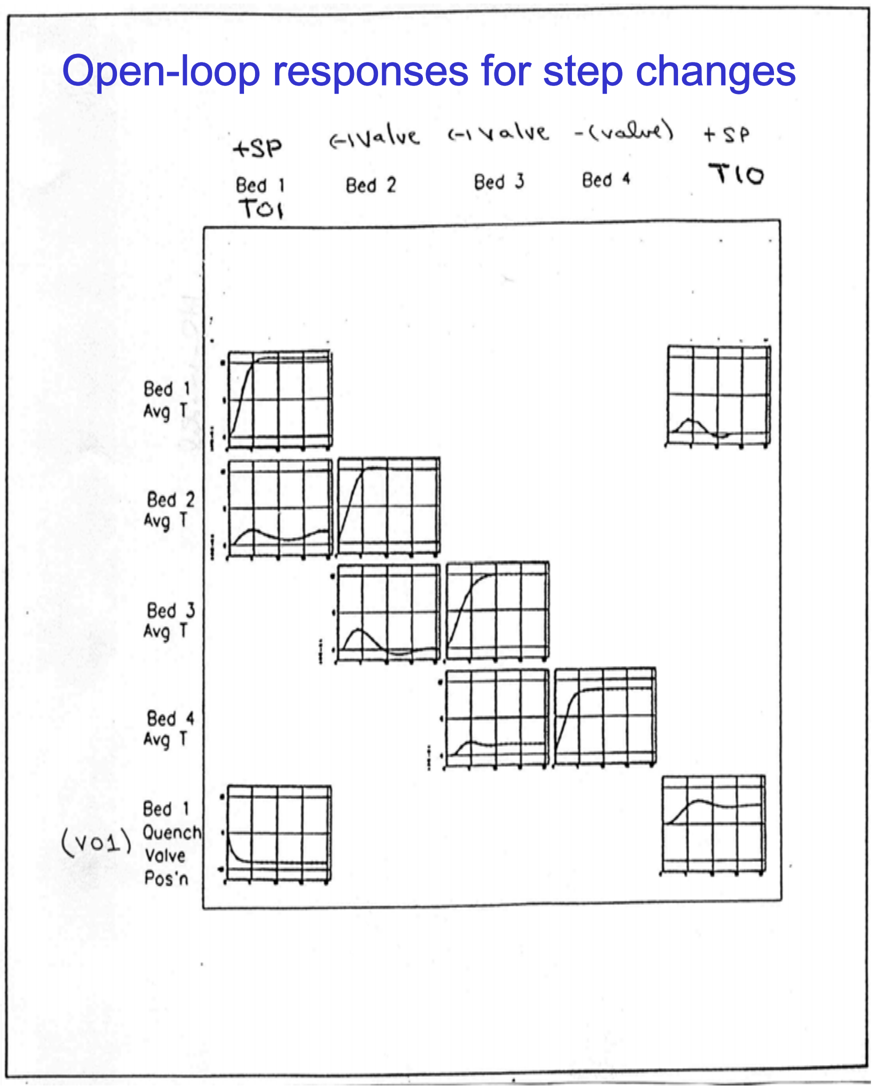
Short-cut Control Design Workshop 9

Class Workshop: Design controls for the series of packed bed reactors with highly exothermic reactions.

Hydrocracker reactor, preheat and quench process



Short-cut Control Design Workshop 9

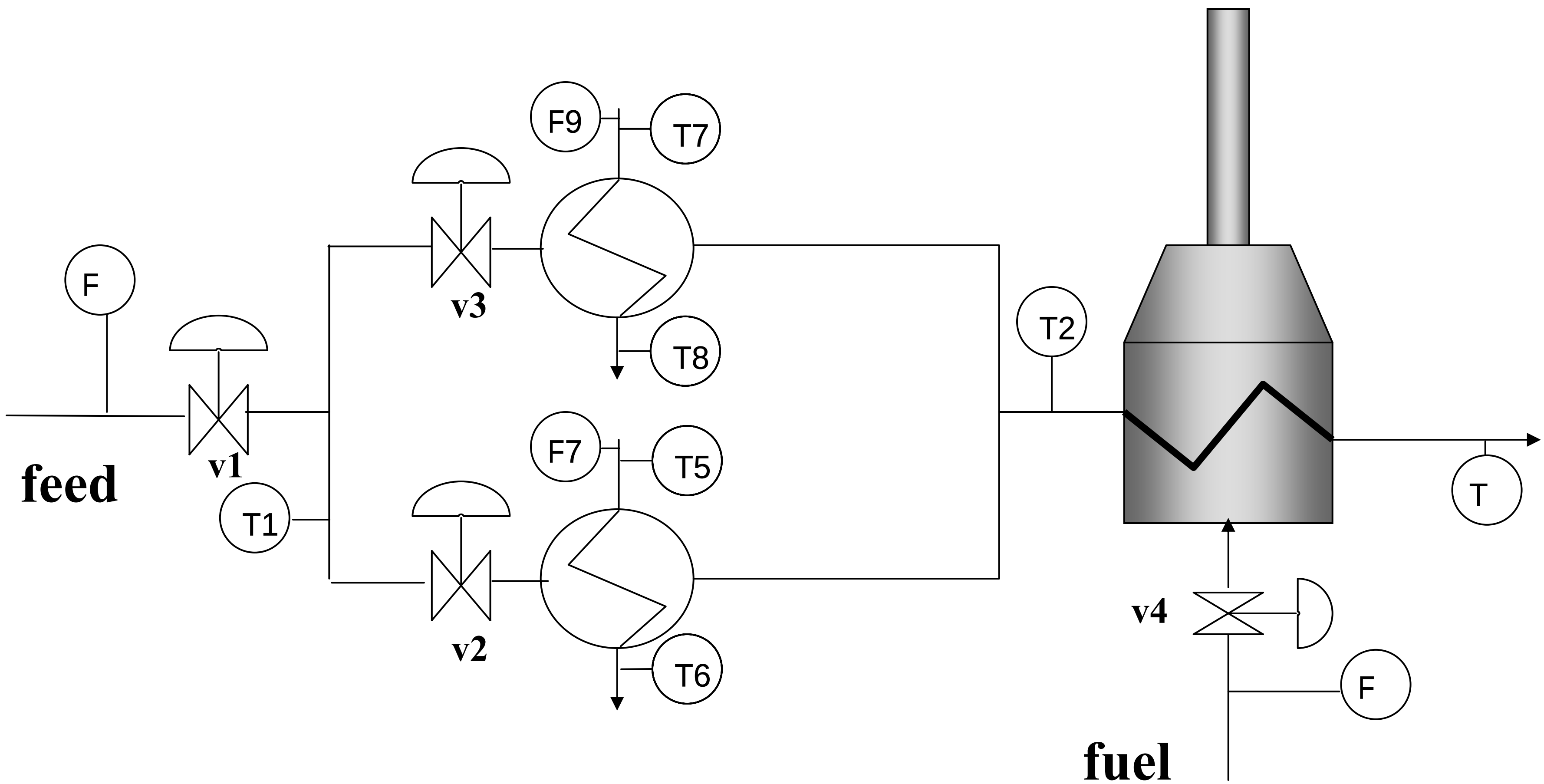


Control Objectives

1. Prevent runaway reaction
2. Control "total conversion", weighted average bed temperature (T1, T2, ..)
3. Prevent too high/low temperature in any bed
4. Minimize fuel to fired heater

Short-cut Control Design Workshop 10

Class Workshop: Design controls to minimize fuel consumption for a specified feed rate.



Short-cut Control Design Workshop 10

Control objectives:

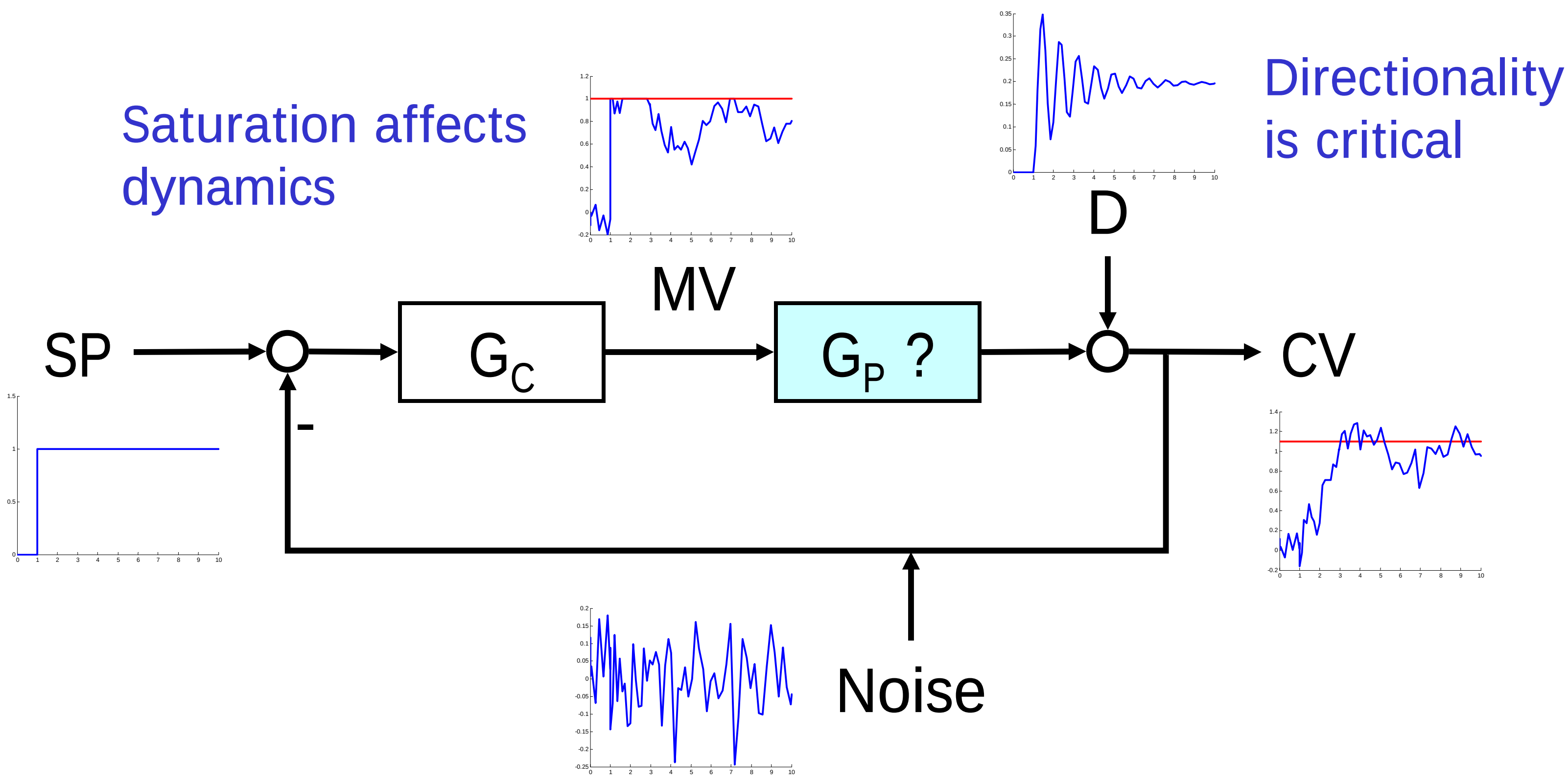
1. **Maintain TC at a desired value (set point)**
2. **Maintain feed flow at a desired value (set point)**
3. **Minimize the fuel to the fired heater**

Disturbances:

F9, F7, T7 and T5 change frequently and over large magnitudes

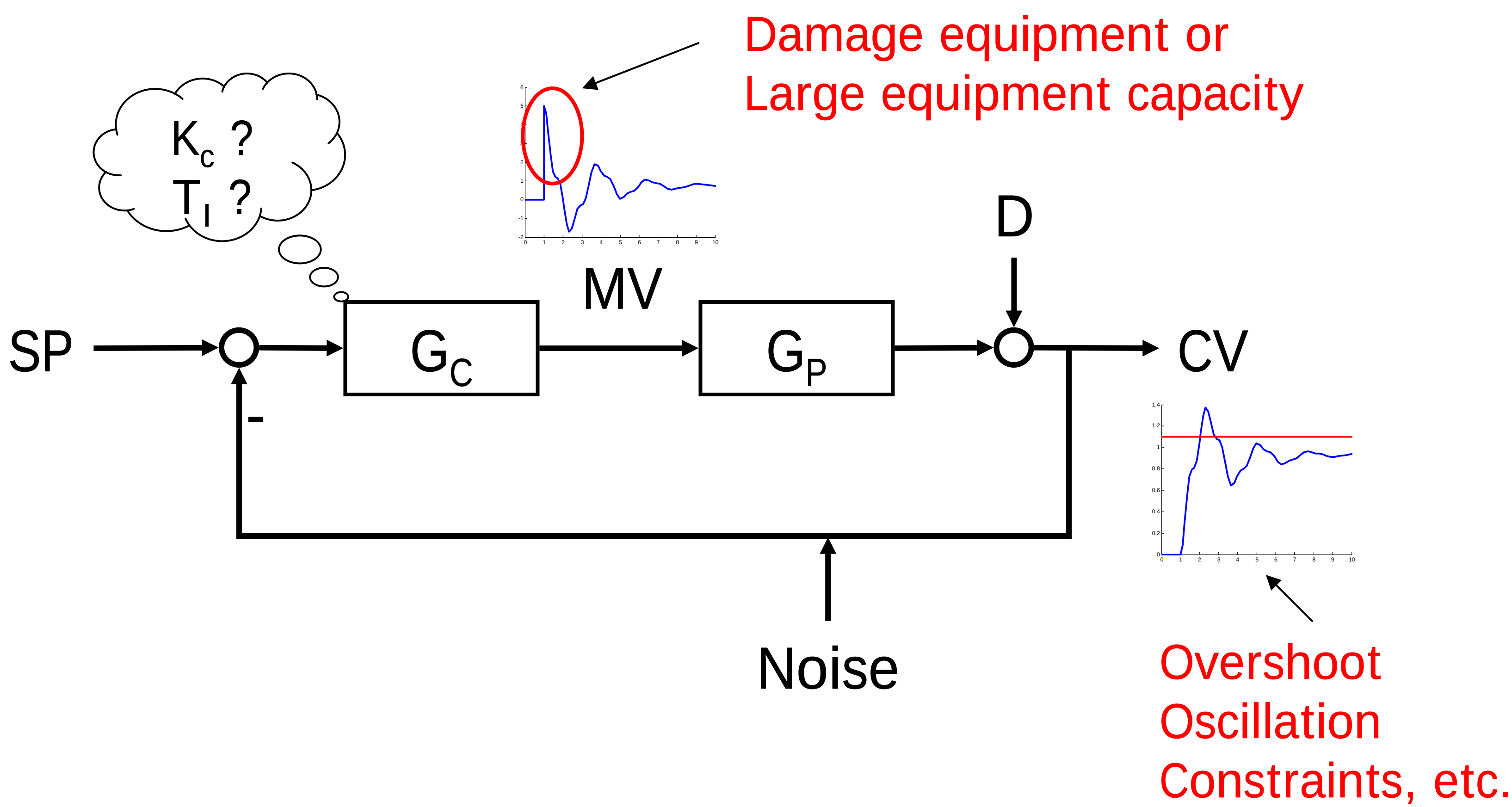
Optimization in Control Design: Problem Formulation

Systematic design requires a realistic problem definition.



Optimization in Control Design: Problem Formulation

Evaluation of candidate requires the full transient with good controller tuning



Optimization in Control Design: Problem Formulation

Evaluation of candidate requires the full transient with good controller tuning

$$\min_{K_C,\, T_I} \quad Q\Sigma|E|$$

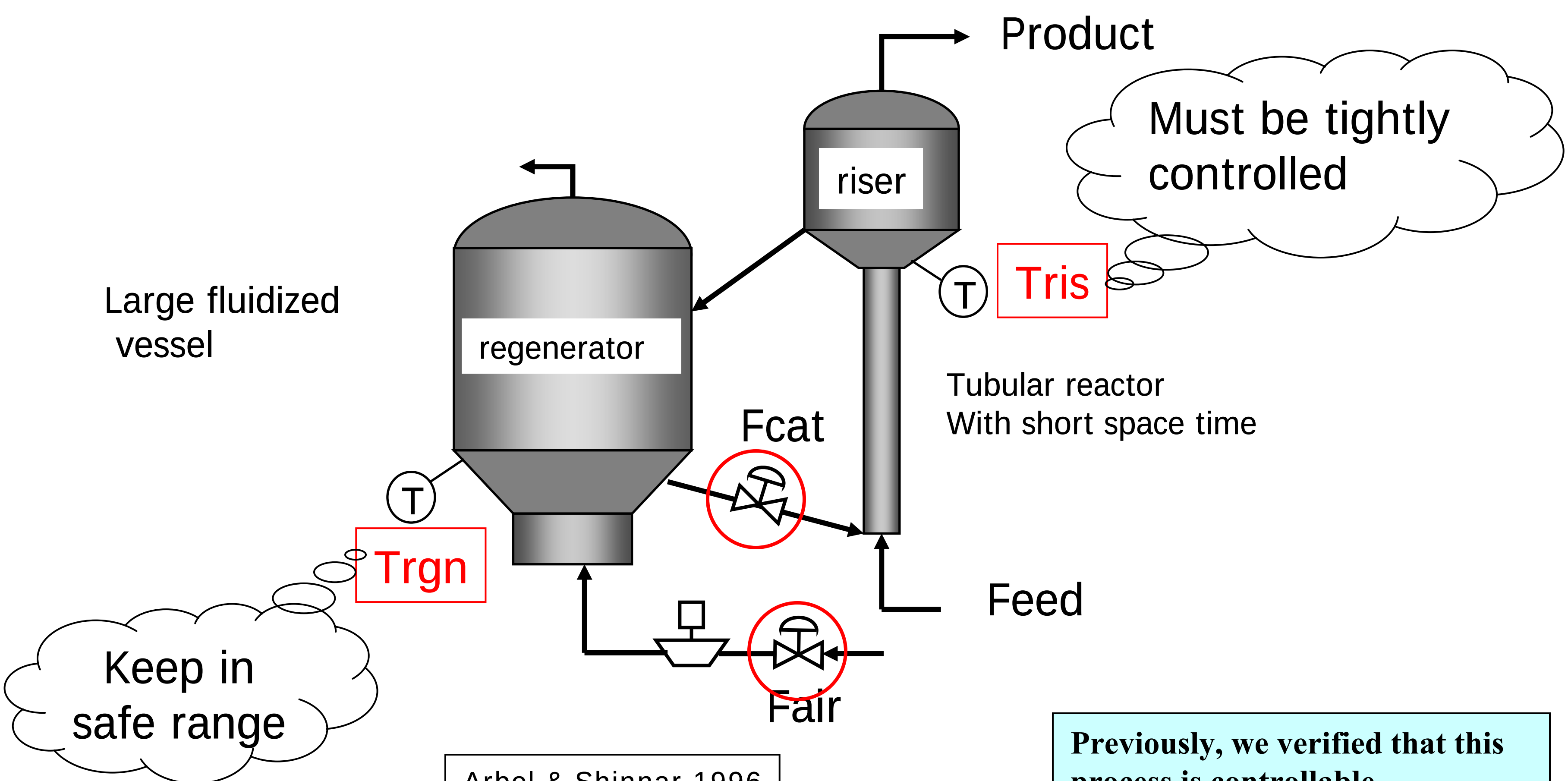
s.t.

Nominal plant model
Mismatch plant model
Disturbance and noise models
Controller equations
Saturation constraints
Constraints defining loop pairing

More details later

Optimization in Control Design: Problem Formulation

Example of Performance Evaluation for 2x2 Fluidized Catalytic Cracking Reactor

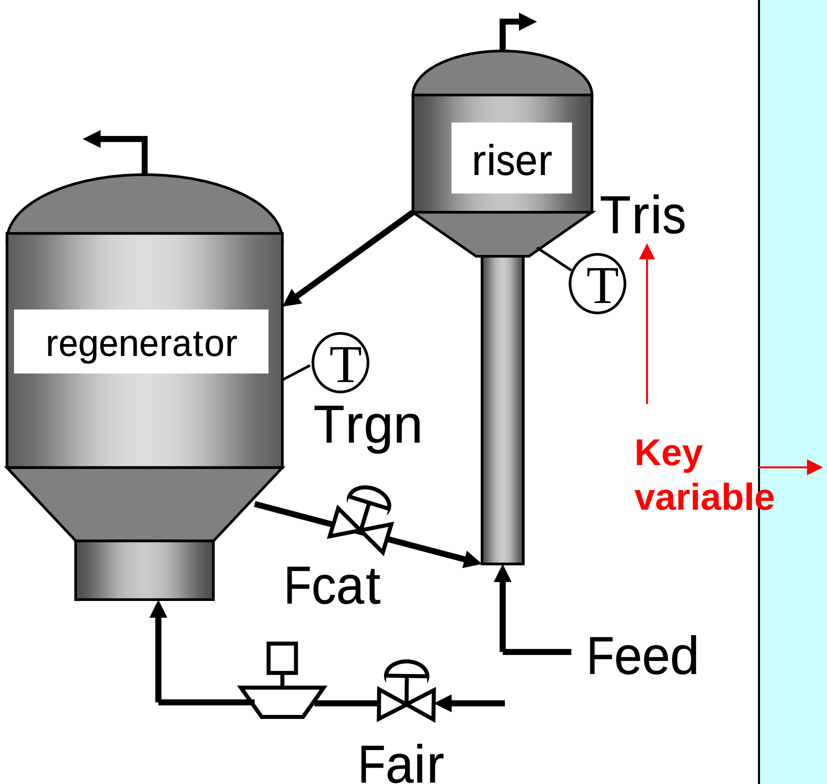


Arbel & Shinnar 1996

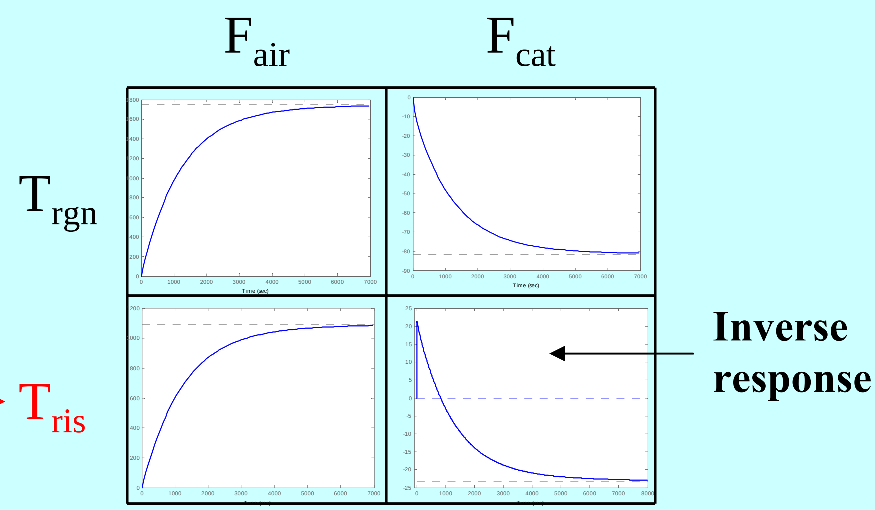
Previously, we verified that this process is controllable.

Optimization in Control Design: Problem Formulation

Fluidized Catalytic Cracking Reactor
Understanding the Process Dynamics



Plant model:

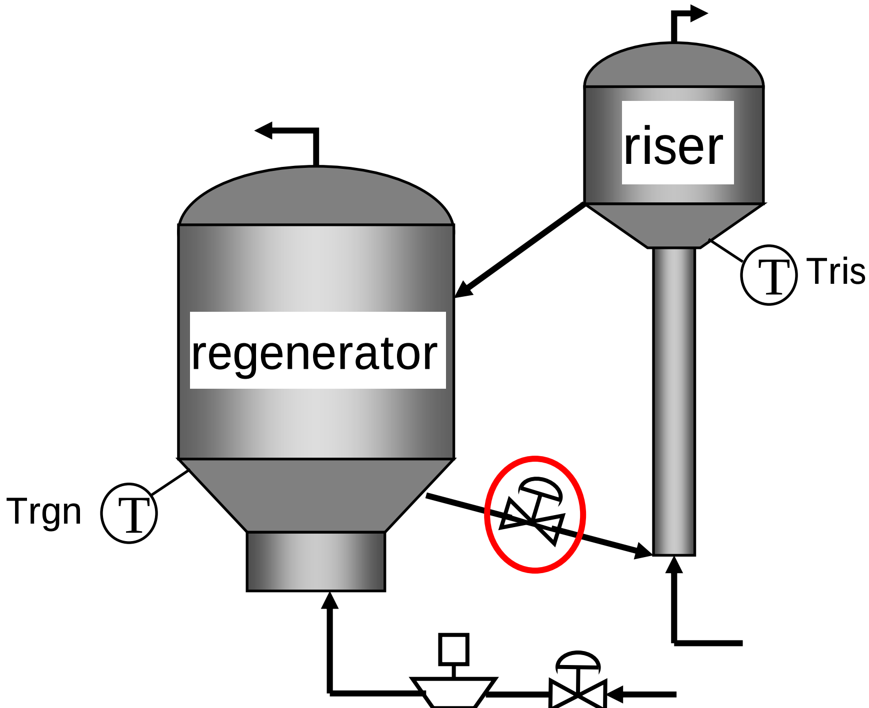


RGA:

	F_{air}	F_{cat}
T_{rgn}	-	+
T_{ris}	+	-

Optimization in Control Design: Problem Formulation

Fluidized Catalytic Cracking Reactor
Why does the inverse response occur?



Deficient air in the regenerator

$$\text{Coke} + O_2 = \begin{cases} \rightarrow CO \\ \rightarrow CO_2 \end{cases}$$

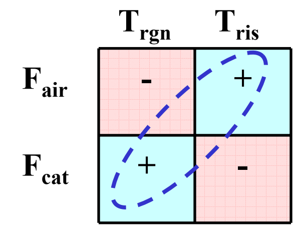
$F_{cat}\uparrow \rightarrow T_{ris}\uparrow \rightarrow \text{coke}\uparrow \rightarrow CO\uparrow \rightarrow CO_2\downarrow \rightarrow T_{rgn}\downarrow \rightarrow T_{ris}\downarrow$

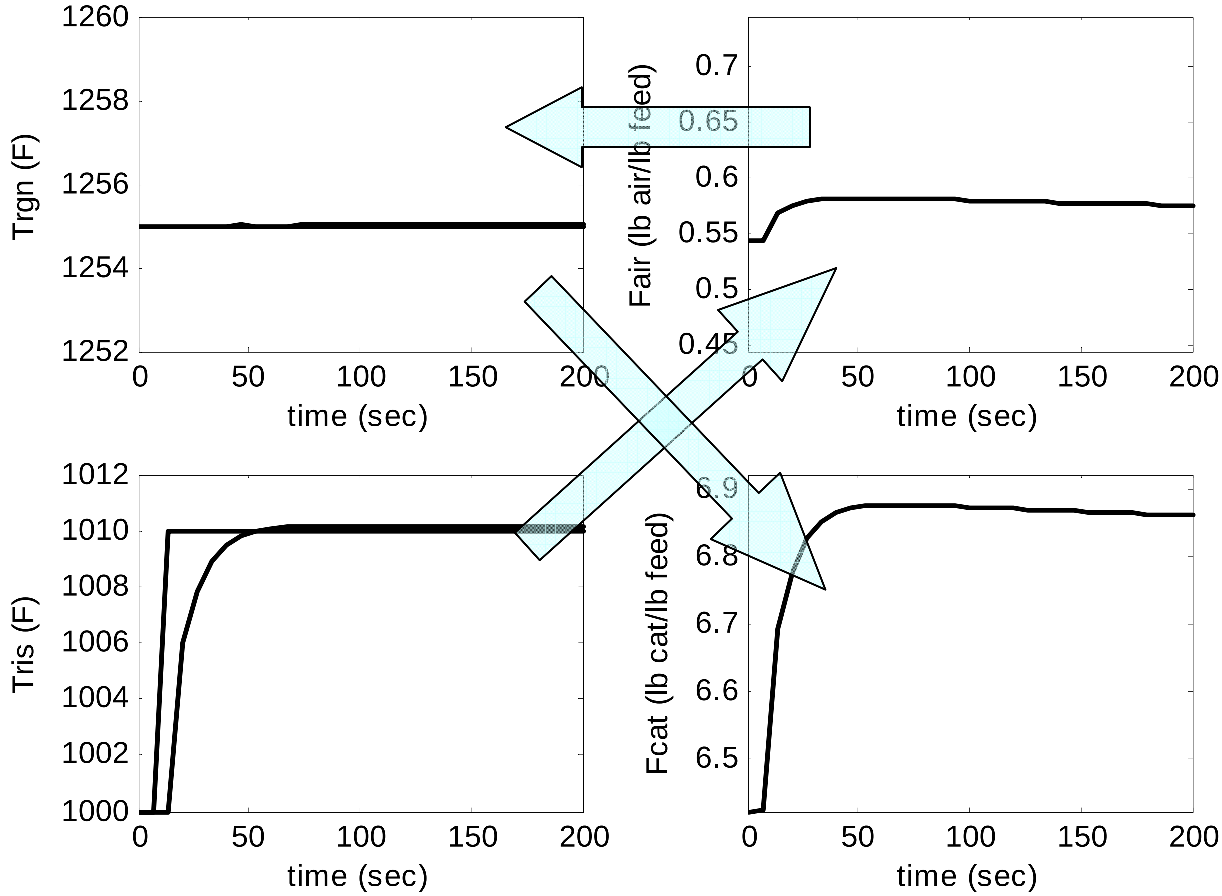
Fast (for the first $T_{ris}\uparrow$) — Slow (for the final $T_{ris}\downarrow$)

Optimization in Control Design: Problem Formulation

Fluidized Catalytic Cracking Reactor

Positive RGA: Transient response for perfect model





$$\int |err(T_{ris})| = 23.5$$

Very good performance

$$K_C = \begin{bmatrix} 0 & 2.5\times10^{-4} \\ -2\times10^{4} & 0 \end{bmatrix}$$

large controller gain

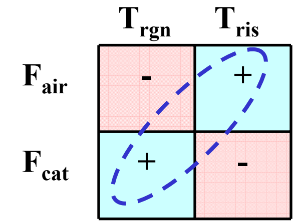
$K_C K_P \cong 10^6$

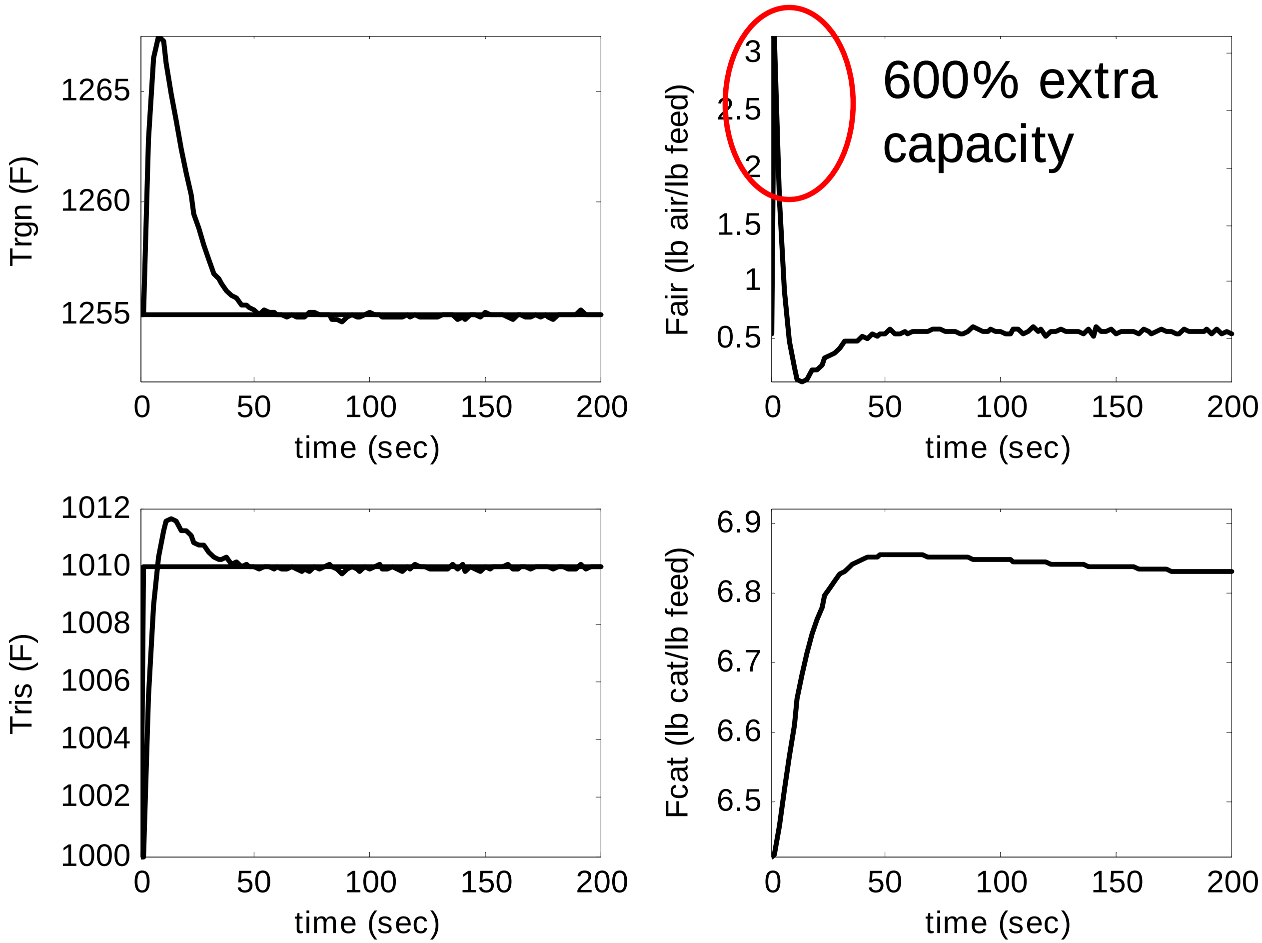
$$K_I = \begin{bmatrix} 0 & 0.8 \\ 0.8 & 0 \end{bmatrix}$$

Optimization in Control Design: Problem Formulation

Fluidized Catalytic Cracking Reactor

Positive RGA: perfect + mismatch models + noise





$$\int |err(T_{ris})| = 35.5$$

Good performance

$K_C K_P \cong 500$

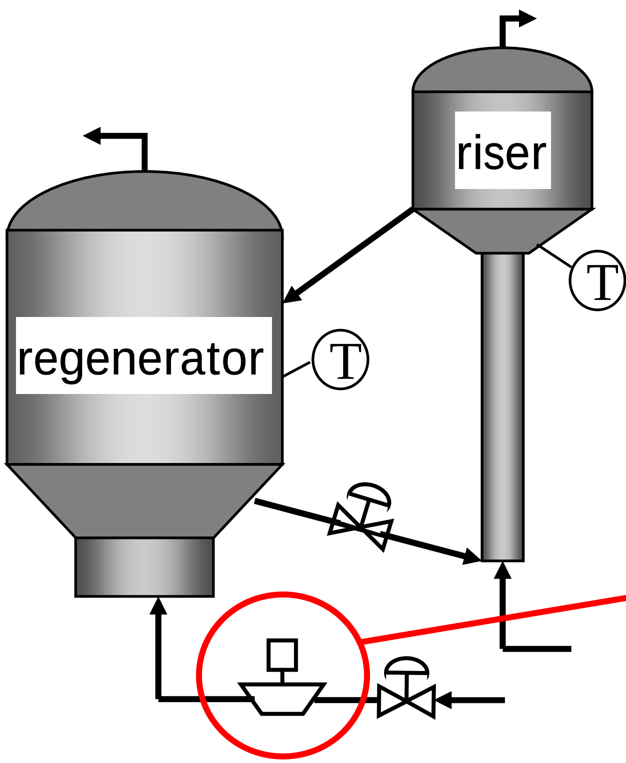
$$K_C = \begin{bmatrix} 0 & 0.26 \\ -0.0019 & 0 \end{bmatrix}$$

$K_C K_P \cong 1$

$$K_I = \begin{bmatrix} 0 & 0.0044 \\ 2.02 & 0 \end{bmatrix}$$

Optimization in Control Design: Problem Formulation

Fluidized Catalytic Cracking Reactor
Cost for the larger air blower capacity?



	Capacity (m^3/h)	Capital Cost (US$)
Steady State	48,000	997,000
Peak	288,000	2,950,000
Equipment cost for larger capacity air flow		**1,953,000***

* Plus additional cost for large anti-surge recycle at normal operation

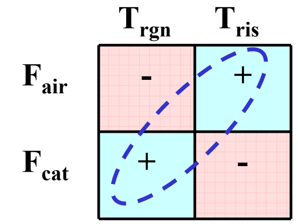
Can you think of another reason why this MV increase is not desired?

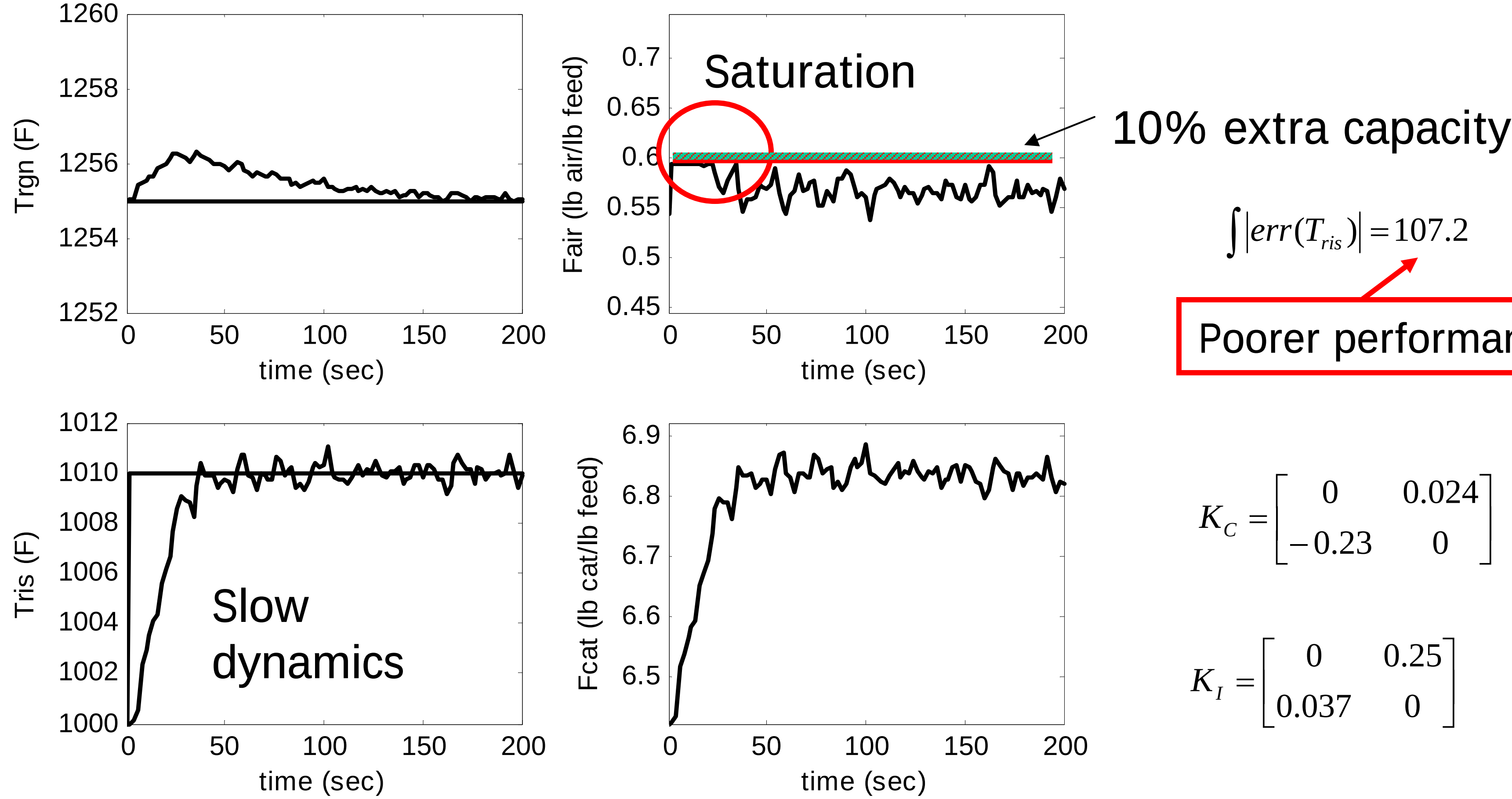
Optimization in Control Design: Problem Formulation

Fluidized Catalytic Cracking Reactor

Positive RGA: perfect + mismatch models + noise + MV limit

Reasonable air blower capacity





$$\int |err(T_{ris})| = 107.2$$

Poorer performance

$$K_C = \begin{bmatrix} 0 & 0.024 \\ -0.23 & 0 \end{bmatrix}$$

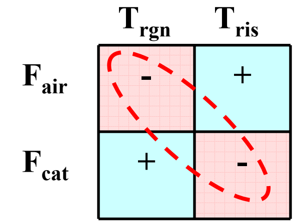
$$K_I = \begin{bmatrix} 0 & 0.25 \\ 0.037 & 0 \end{bmatrix}$$

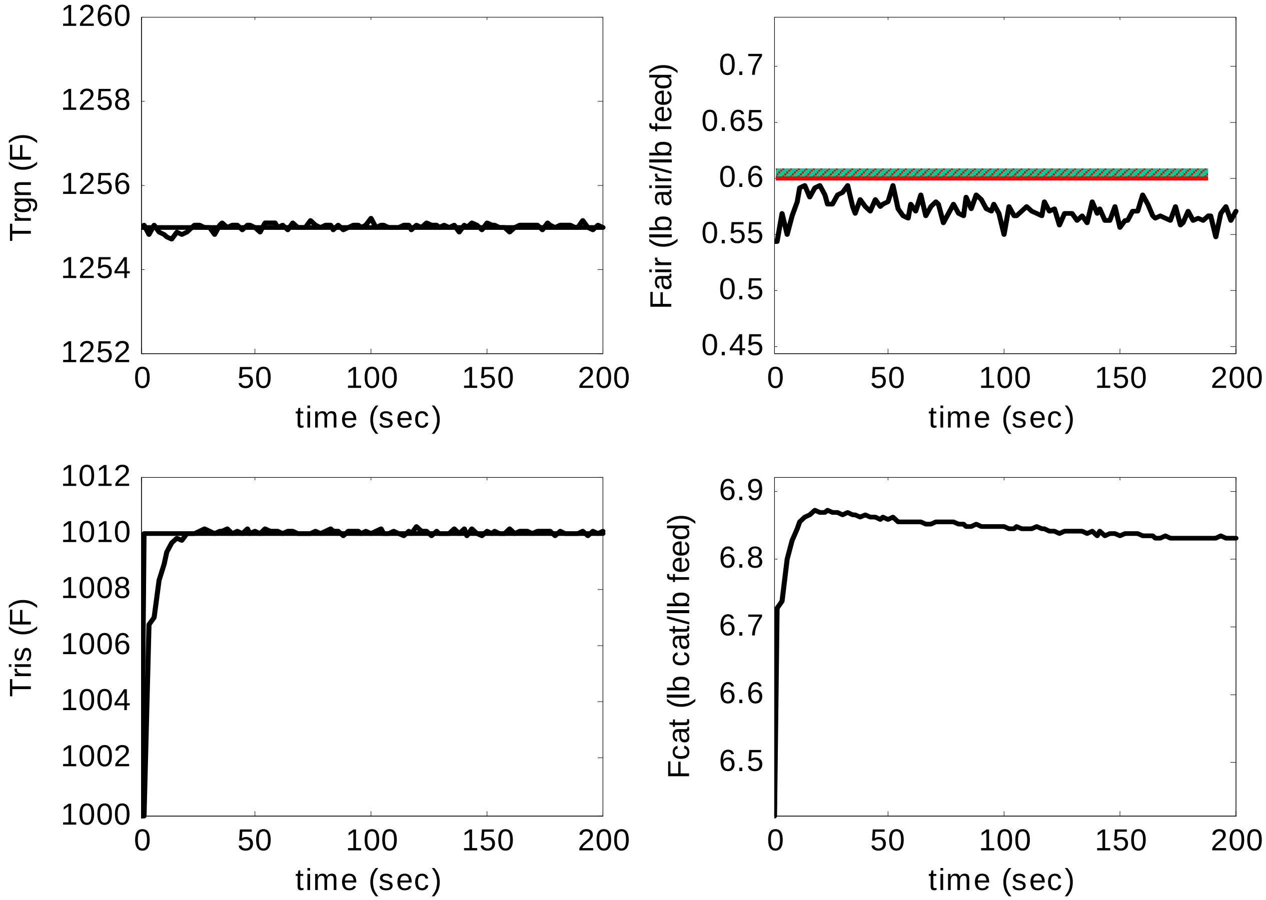
Optimization in Control Design: Problem Formulation

Fluidized Catalytic Cracking Reactor

Negative RGA: perfect + mismatch models + noise + MV limit

Reasonable air blower capacity





$$\int \left| err(T_{ris}) \right| = 25.6$$

Good performance

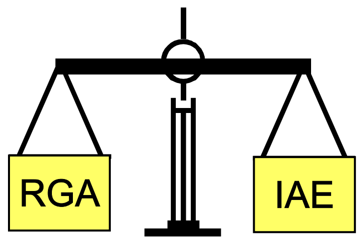
$$K_C = \begin{bmatrix} 0.097 & 0 \\ 0 & 0.009 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.34 & 0 \\ 0 & 2.4 \end{bmatrix}$$

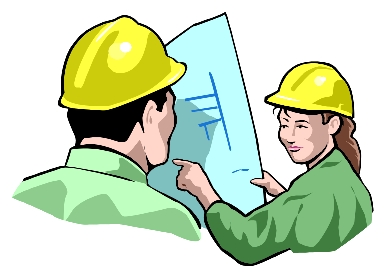
Optimization in Control Design: Problem Formulation

Fluidized Catalytic Cracking Reactor
The Engineer decides the proper balance

Integrity RGA		Dynamic performance of reactor temperature $\int err(T_{ris})$
Yes		107.2
No		25.6



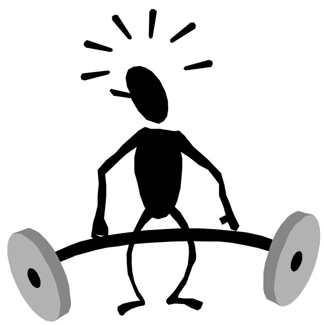
Industrial standard pairing (Shinnar, 1996)



Control Design is a Challenging Problem!

- **Multi-objective,** e.g.,

  - CV behavior (for specific input forcing)
  - MV Behavior (for specific input forcing)
  - Integrity
  - Robustness
  - Profit



- **Continuous variables (tuning)**
- **Discrete variables (CV_i-MV_j loop pairing)**
- **Tailored to specific application,** e.g.,

  - MV variation might be severely limited or allowed to be large
  - Some CVs might be of much greater importance

Optimization in Control Design: Tailored Solution

GOALS

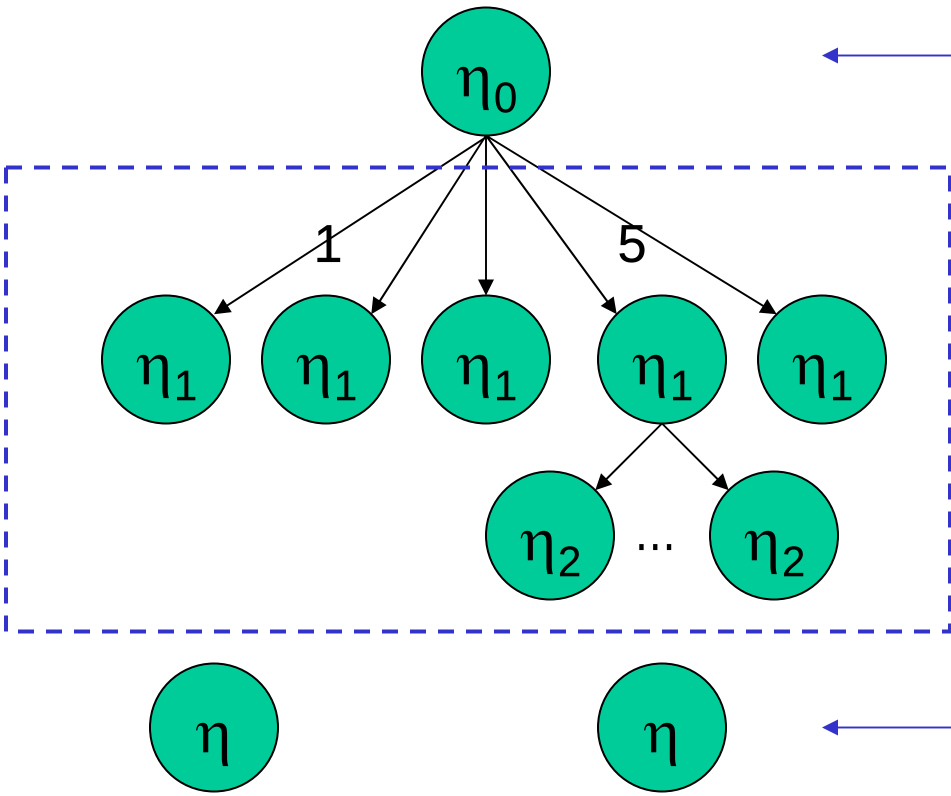
- Develop a systematic method for control design
- Provide generality to discover unconventional designs
- Complete computation within reasonable time

CHALLENGES

- Each tuning NLP is non-convex. The problem has "important" local optima because the controller signs are not known (when negative RGAs allowed).
- Straightforward B&B for MINLP computes for many days with little reduction in gap

Optimization in Control Design: Tailored Solution

Tailored B&B Approach for Optimization of Control Structure and Performance



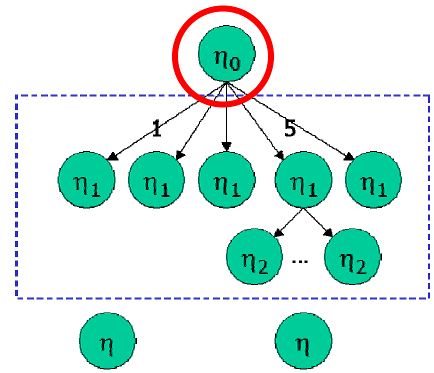
Check whether **feedback** solution is possible

- Combine short-cut & full transient
- **Simplify** transient analysis

Thorough transient analysis of few remaining all-integer

Optimization in Control Design: Tailored Solution

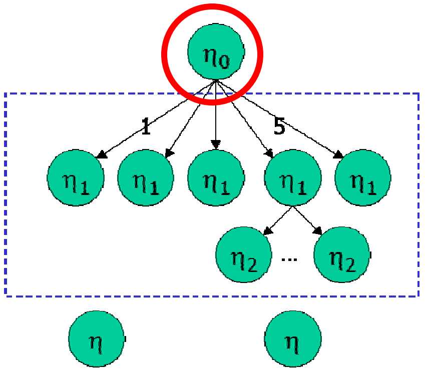
First Step Determines Whether a Feedback Controller Could Achieve the Desired Performance

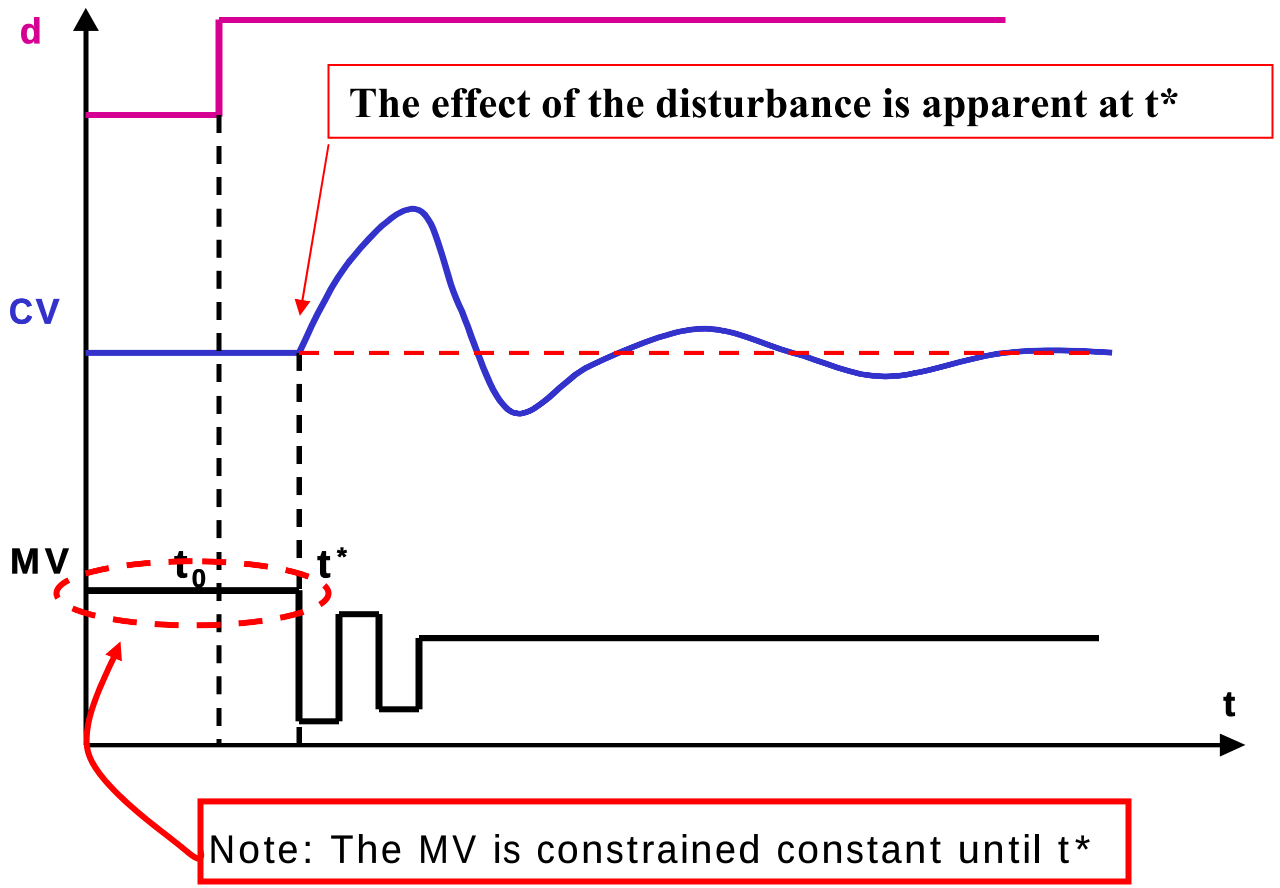


- **Controllability established that the desired dynamic performance can be achieved by adjusting the manipulated variables**
- **Controllability does not require causality, i.e., feedback control.**
- **We must determine whether feedback can achieve the performance before selecting a specific structure.**

Optimization in Control Design: Tailored Solution

First Step Determines Whether a Feedback Controller Could Achieve the Desired Performance





Provides a <u>lower bound</u> on performance for all control structures

Optimization in Control Design: Tailored Solution

The objective could be modified to be consistent with other performance specifications, as needed.

No control law is imposed at this step.

Bound $\Delta u_n = 0$ for appropriate number of time steps, to t*.

If the desired performance cannot be achieved, fix the process!

$$\min_{\mathbf{u}} \quad \mathbf{f} = \sum (s_{2n}^{+} + s_{2n}^{-})$$

$$s.t.$$

$$x_n = Ax_{n-1} + Bu_n + Dd_n$$

$$y_n = Cx_n$$

$$y_n - s_{1n}^{+} - s_{2n}^{+} \le (y_{\max})_n$$

$$y_n + s_{1n}^{-} + s_{2n}^{-} \ge (y_{\min})_n$$

$$(u_{\min})_n \le u_n \le (u_{\max})_n$$

$$(\Delta u_n)_{\min} \le u_n - u_{n-1} \le (\Delta u_n)_{\max}$$

$$0 \le s_{1n}^{+} \le (s_{1n}^{+})_{\max}$$

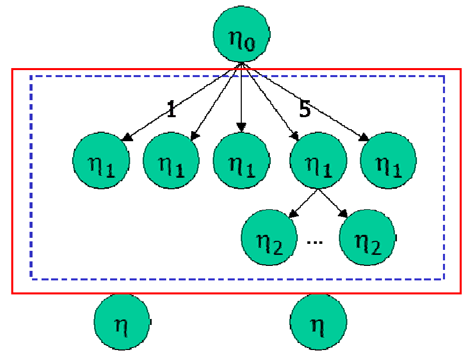
$$0 \le s_{1n}^{-} \le (s_{1n}^{-})_{\max}$$

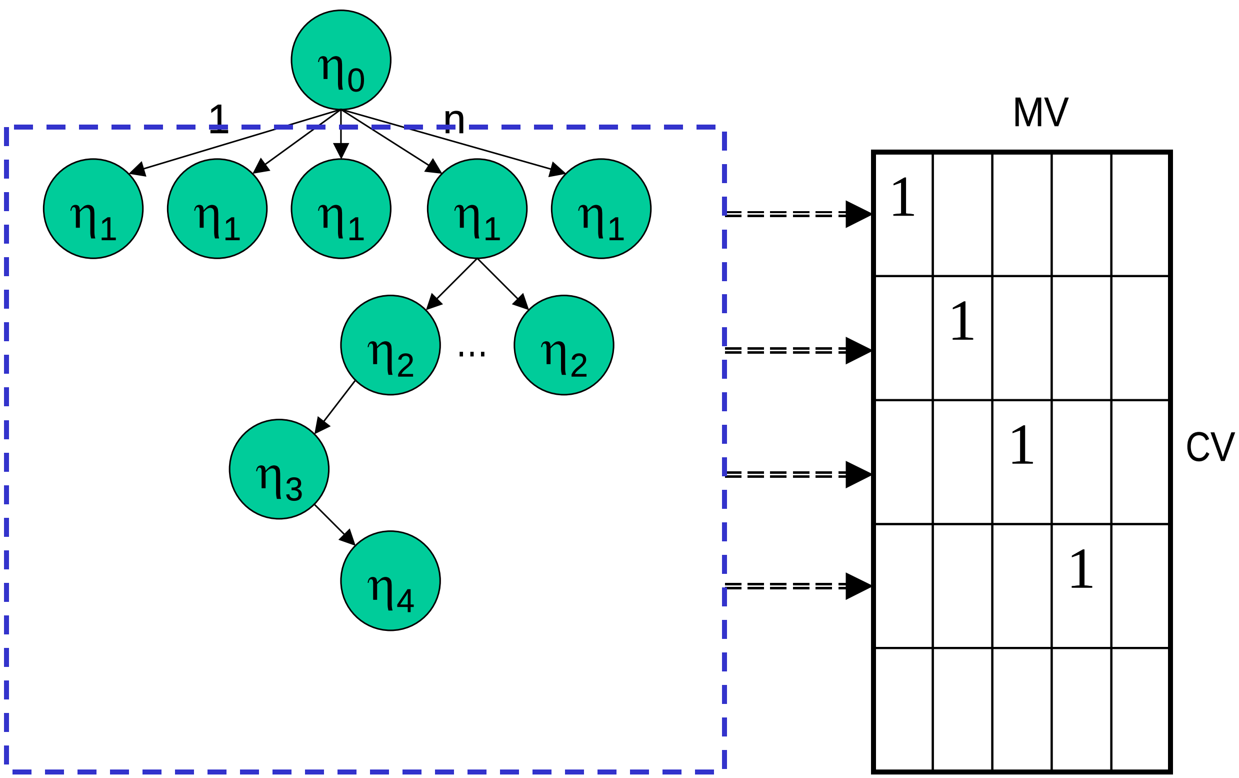
$$0 \le s_{2n}, \quad 0 \le s_{2n}$$

$$given \quad u_o, x_o, y_0, d_n$$

Optimization in Control Design: Tailored Solution

The Branch & Bound Evaluates only Feasible Pairings



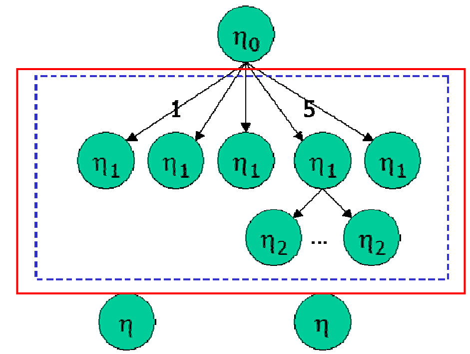


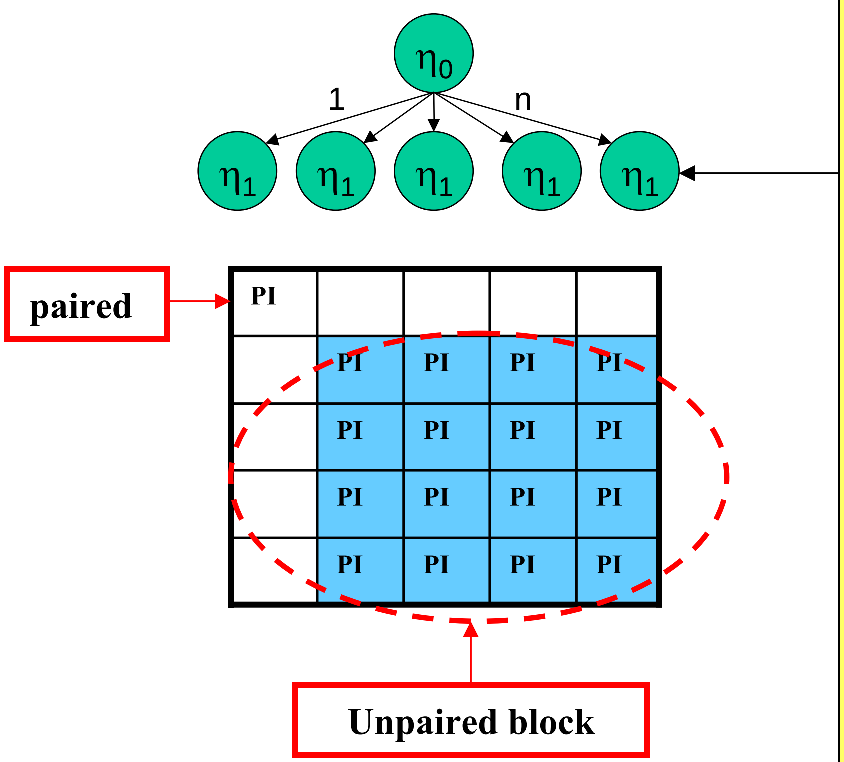
Integer (not binary) formulation

Each row is CV; each column is MV of a feasible possible pairing.

Optimization in Control Design: Tailored Solution

The Solution at each Node Provides a Lower Bound without Extensive Computations



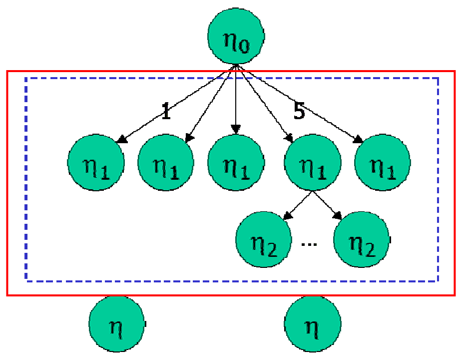


The solution of the transient at each node could involve

- <u>Non-convex</u> tuning problem
- <u>Complementarity</u> resulting from MV saturation with fixed control law
- <u>Many controllers</u> for lower bound of unpaired!
- We seek a simpler problem giving a valid lower bound

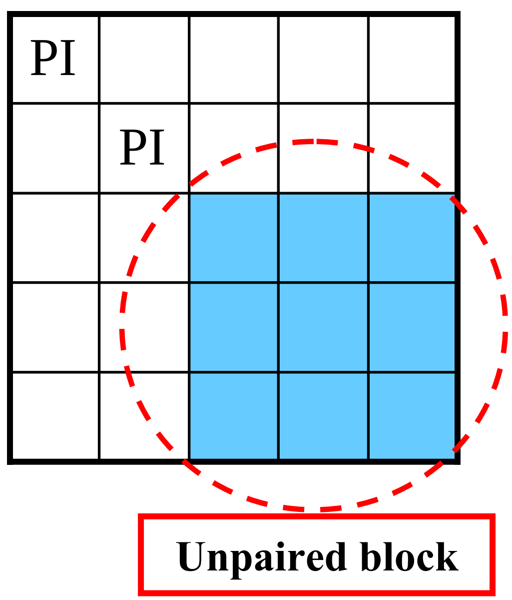
Optimization in Control Design: Tailored Solution

The Solution at each Node Provides a Lower Bound without Extensive Computations



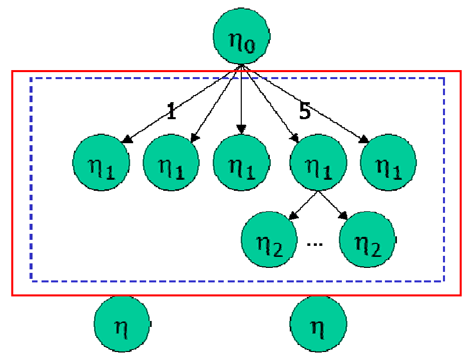
As we proceed, we will have some paired loops and a block of unpaired variables.

We will model the unpaired as an optimization problem enforcing causality, but without a specific controller structure (similar to the top, feasibility test).



Optimization in Control Design: Tailored Solution

The Solution at each Node Provides a Lower Bound by Solving a Convex QP



$$\min_{\Delta u(t)} \sum_{t=1}^{tf} [\mathbf{e}(t) \cdot \mathbf{Q} \cdot \mathbf{e}(t)]$$

$$\mathbf{x}(t+1) = \mathbf{Ax}(t) + \mathbf{Bu}(t) + \mathbf{Wd}(t)$$

$$\mathbf{y}(t) = \mathbf{Cx}(t) + \mathbf{Vd}(t) + N(t)$$

$$\Delta\mathbf{u}_i(t) = \mathbf{K}_c\left[\mathbf{e}_j(t) - \mathbf{e}_j(t-1) + \mathbf{K}_I\mathbf{e}_j(t)\right] \quad \forall \textit{ paired } [i, j]$$

$$\mathbf{e}(t) = \mathbf{sp}(t) - \mathbf{y}(t)$$

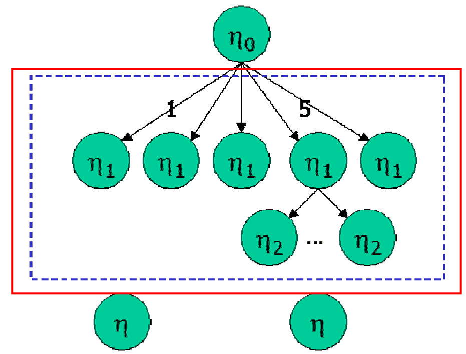
$$y_i \ (t)_{\min} \le y_i(t) \le y_i \ (t)_{\max}$$

$$\left.\begin{aligned} & u_j \ (t)_{\min} \le u_j(t) \le u_j \ (t)_{\max} \\ & \Delta u_{j\min} \le u_j(t) - u_j(t-1) \le \Delta u_{j\max} \\ & \Delta u_j(t) = 0 \text{ for } t<t^* \end{aligned}\right\} \quad \textit{For unpaired}$$

- **Paired loops modelled as PI controllers.**
- **Unpaired MVs (u's) are free variables, after disturbance is measured**
- **Relaxation of control law for "unpaired" variables**
- **Result is a lower bound on performance**

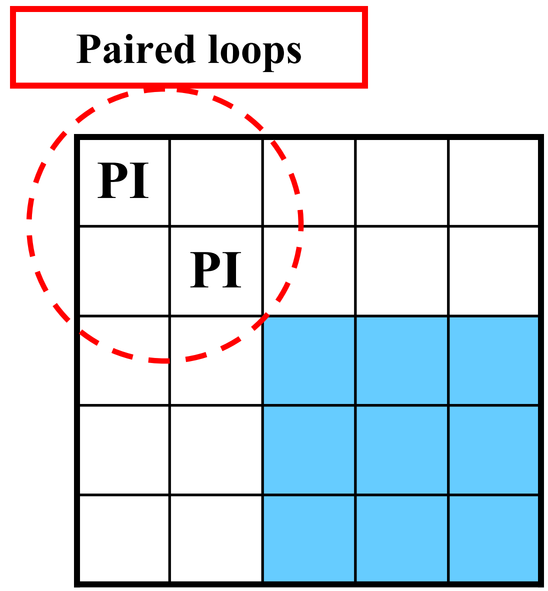
Optimization in Control Design: Tailored Solution

The Solution at each Node Provides a Lower Bound without Extensive Computations



Tuning of the paired Loops

- Tuning is non-convex
- Multiloop is much different from single-loop tuning
- If negative RGAs allowed, the sign of each Kc is not known



Tuning is determined by grid search on QP problem.

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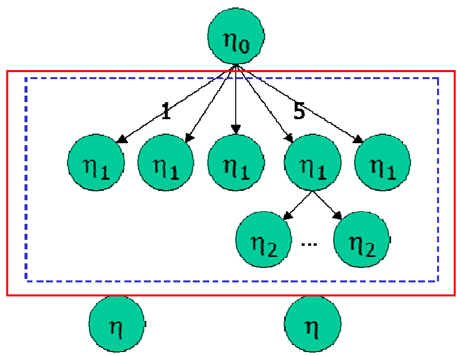
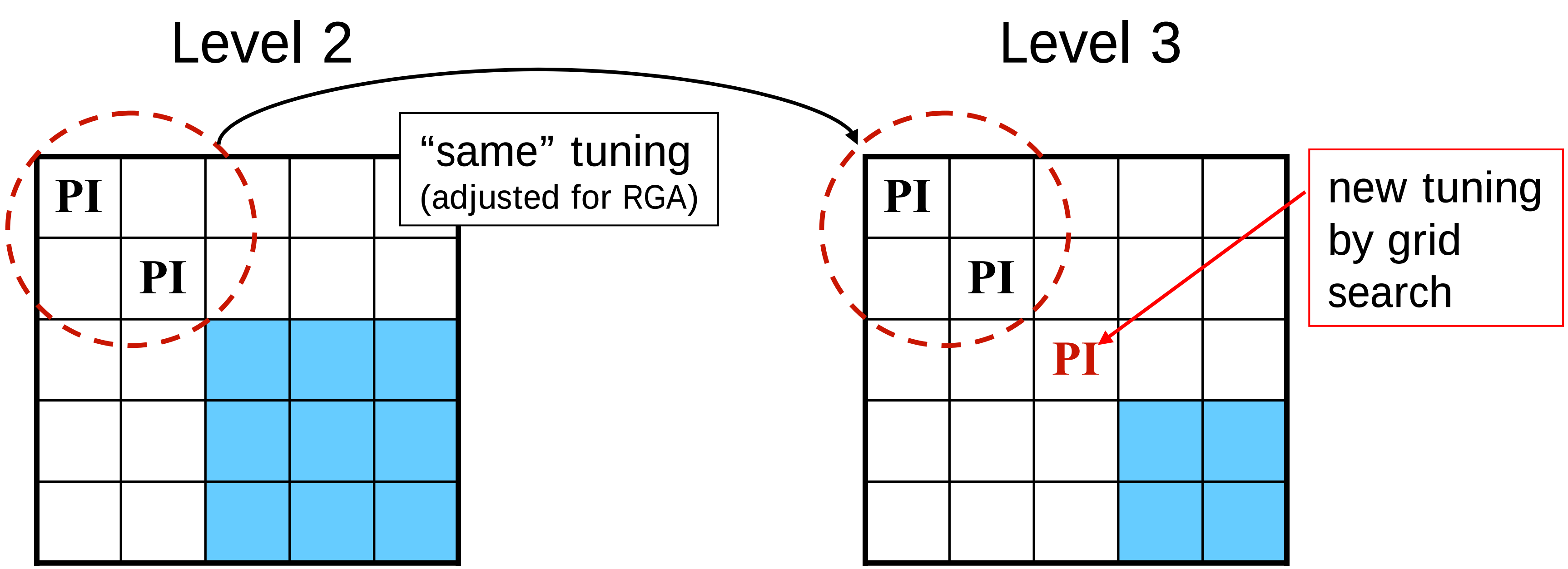
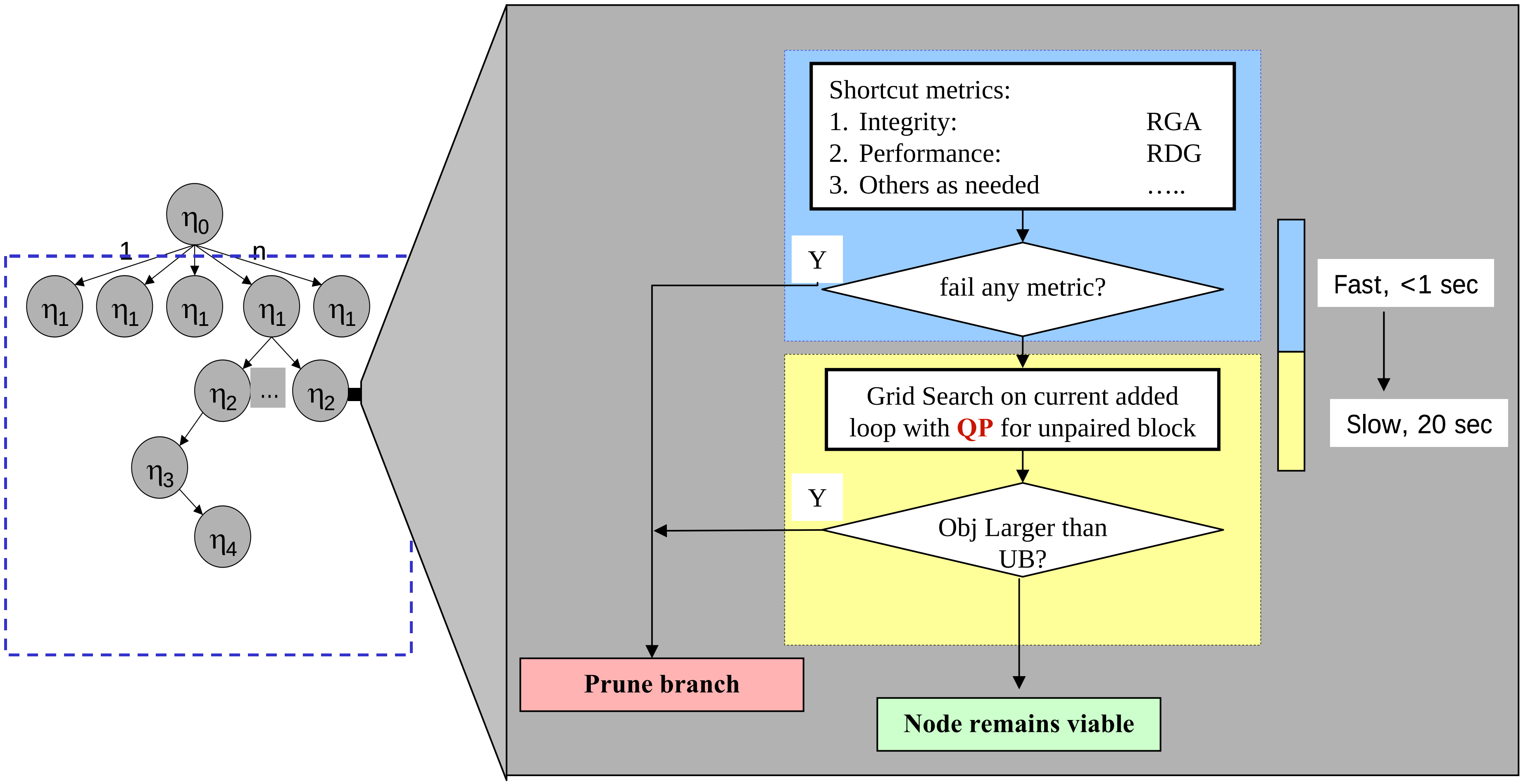


"Sequential Tuning" further reduces computation.



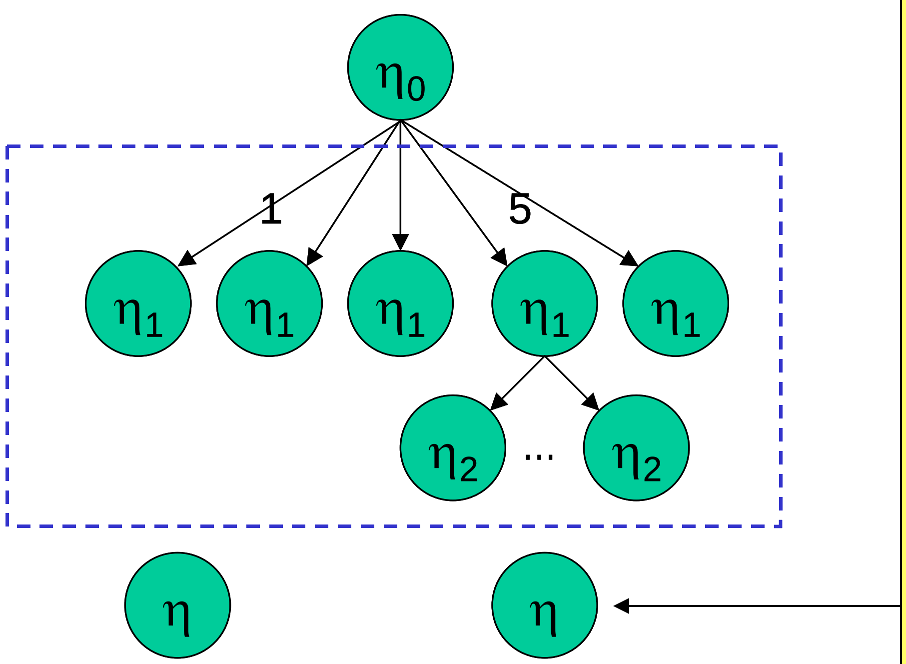
Optimization in Control Design: Tailored Solution

The Solution at each Node Combines Multiple Objectives, using Short-cut Metrics and Full Transient



Optimization in Control Design: Tailored Solution

The Remaining All-Integer Candidates are Evaluated using the Full Transient with <u>Mismatch, Noise & Saturation</u>



The evaluation is similar to the presentation on the FCC example.

The problem is an NLP with complementarity constraints

Used IPOPT-C from Raghunathan and Biegler

Computationally demanding

Optimization in Control Design: Tailored Solution

Tailored B&B Approach for Optimization of Control Structure and Performance

Strengths

- Integrates metrics (RGA, RDG, etc.) with full transient analysis
- Flexible performance specification, including multi-objective
- Unique relaxation of unpaired loops
- Prune many candidates with short computation
- Find few good candidates relatively rapidly

Limitations

- Cannot guarantee the global minimum has been found

  Reason: The optimal tuning at each node is not solved rigorously.

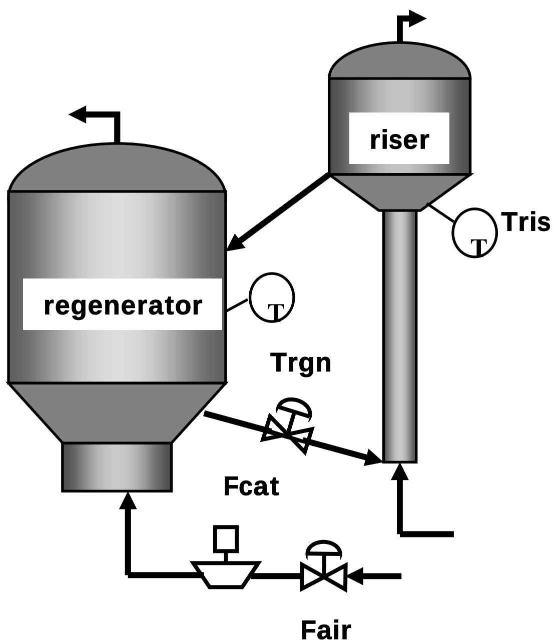
- Evaluation of final candidates computationally demanding

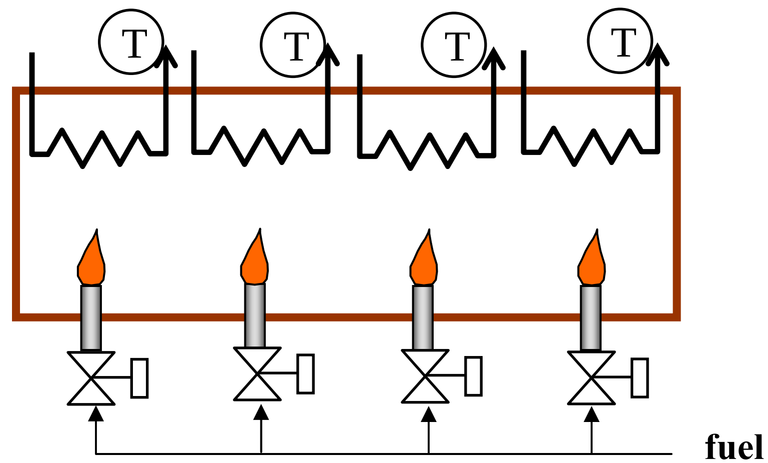
  Solution: Use tuning from B&B

- Using linearized process models

Optimization in Control Design: Tailored Solution

The optimization based design approach has been applied successfully to the following problems.





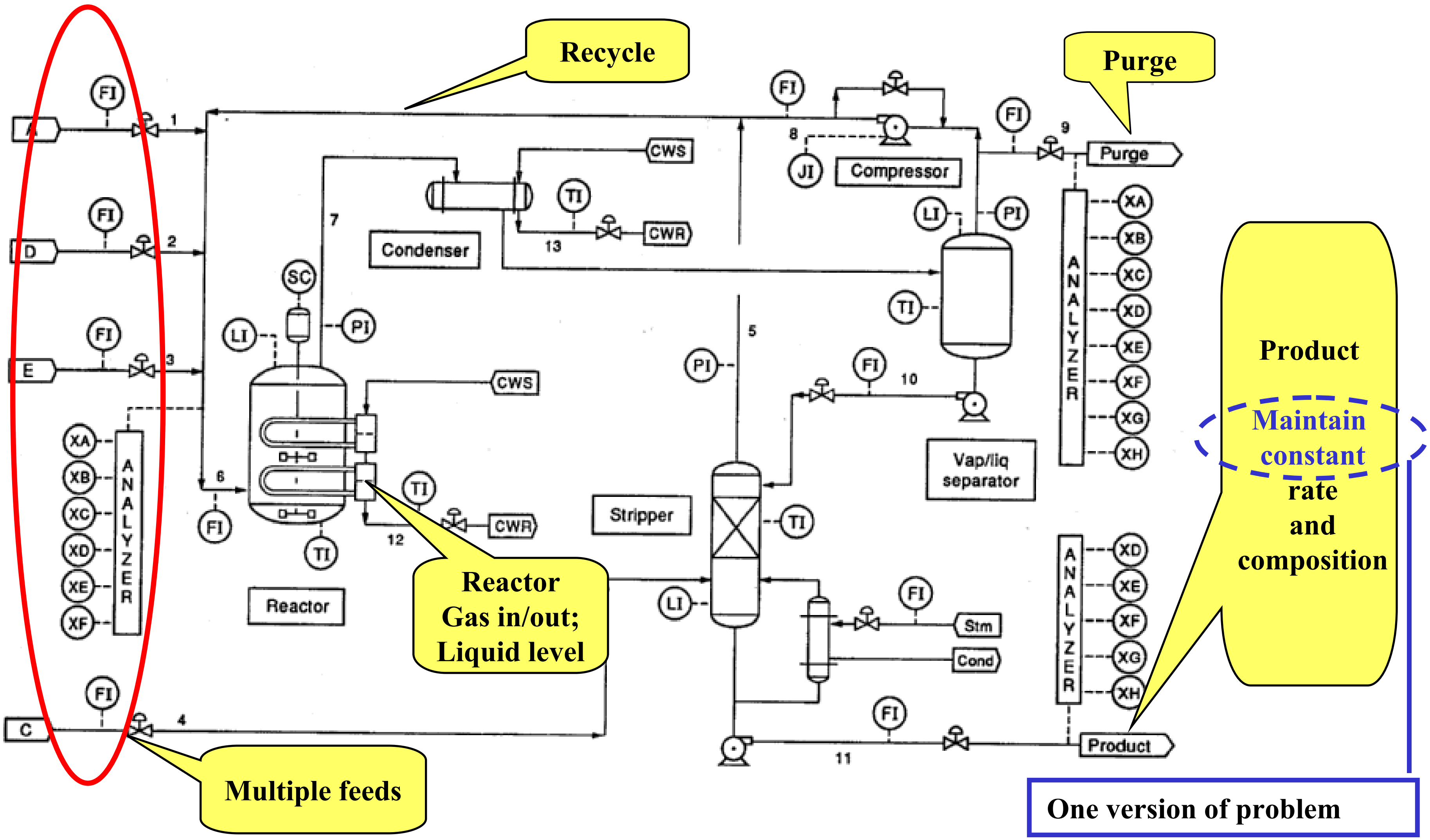
Shinnar FCC, 2x2

- **Pair on negative RGA**
- **One CV much more important**

Rosenbrock Heater, 4x4

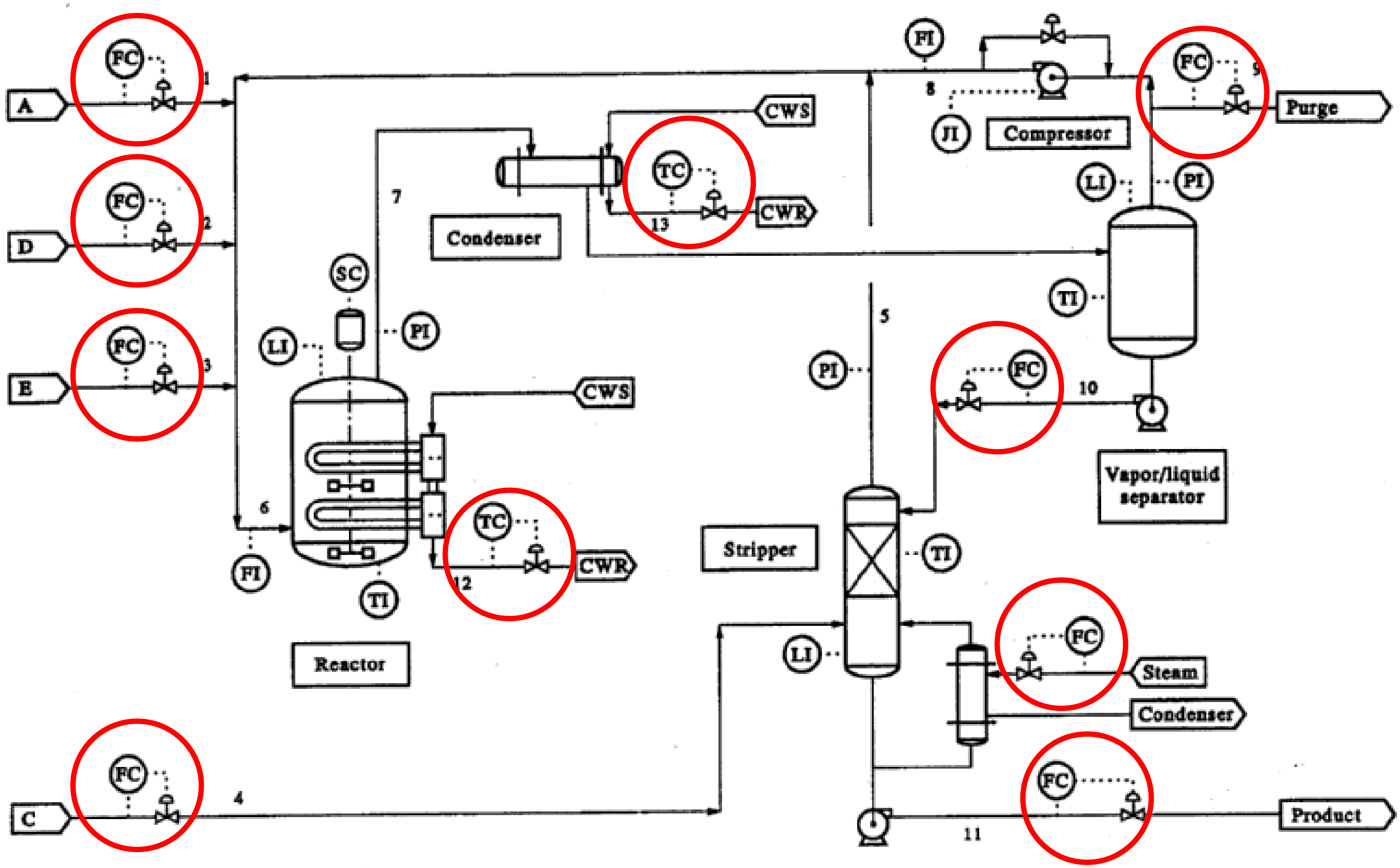
- **Pair on positive RGA**
- **Multiloop as good as centralized MPC for disturbance response**

Optimization Approach for the Tennessee Eastman Problem.



Process Environment

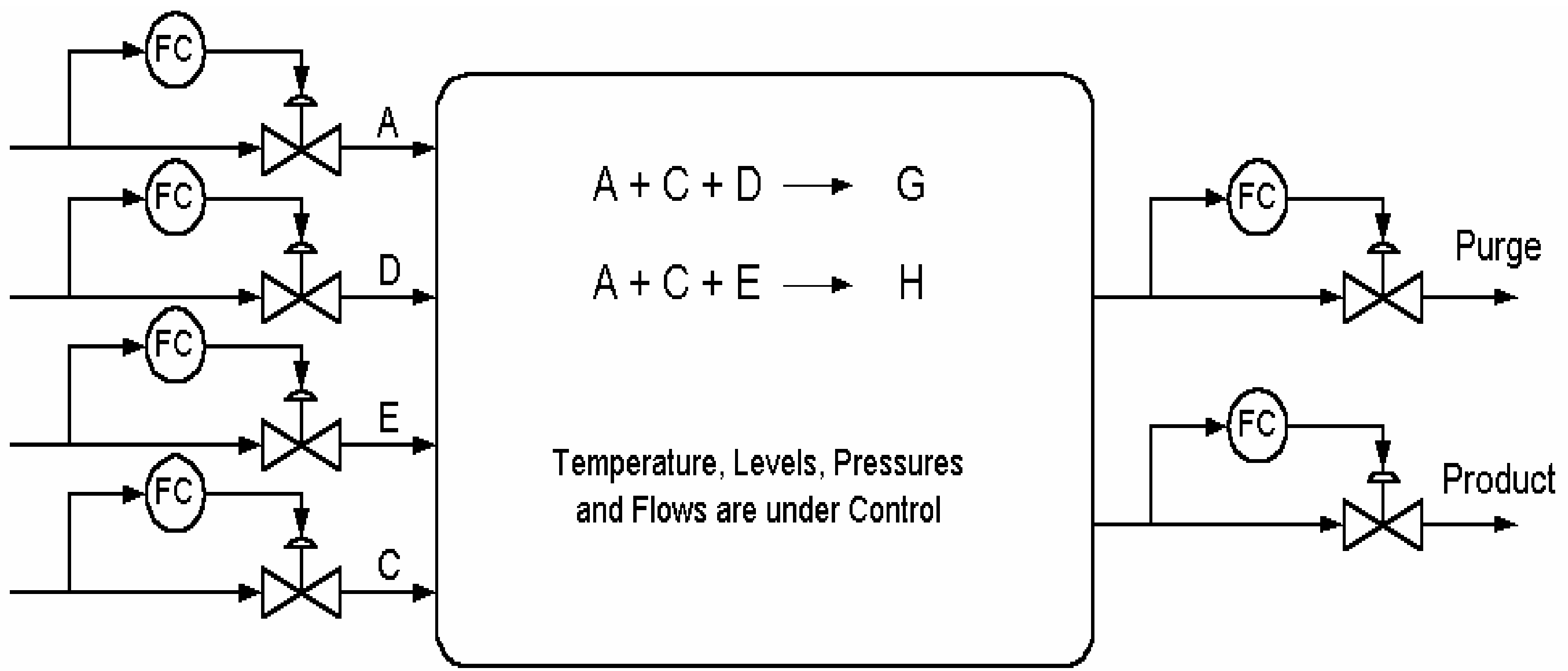
But first, use our control insights. Tight control by inner loops for cascades (McAvoy).



Partial Control or Inferential Variables

Next, use our chemical engineering insights. The reactions define a stoichiometry. Therefore, we should select ratios of feeds to manipulate.

Some MVs are = FD/FE, FE/FC, FA/FC, FC



Optimization Approach for the Tennessee Eastman Problem.

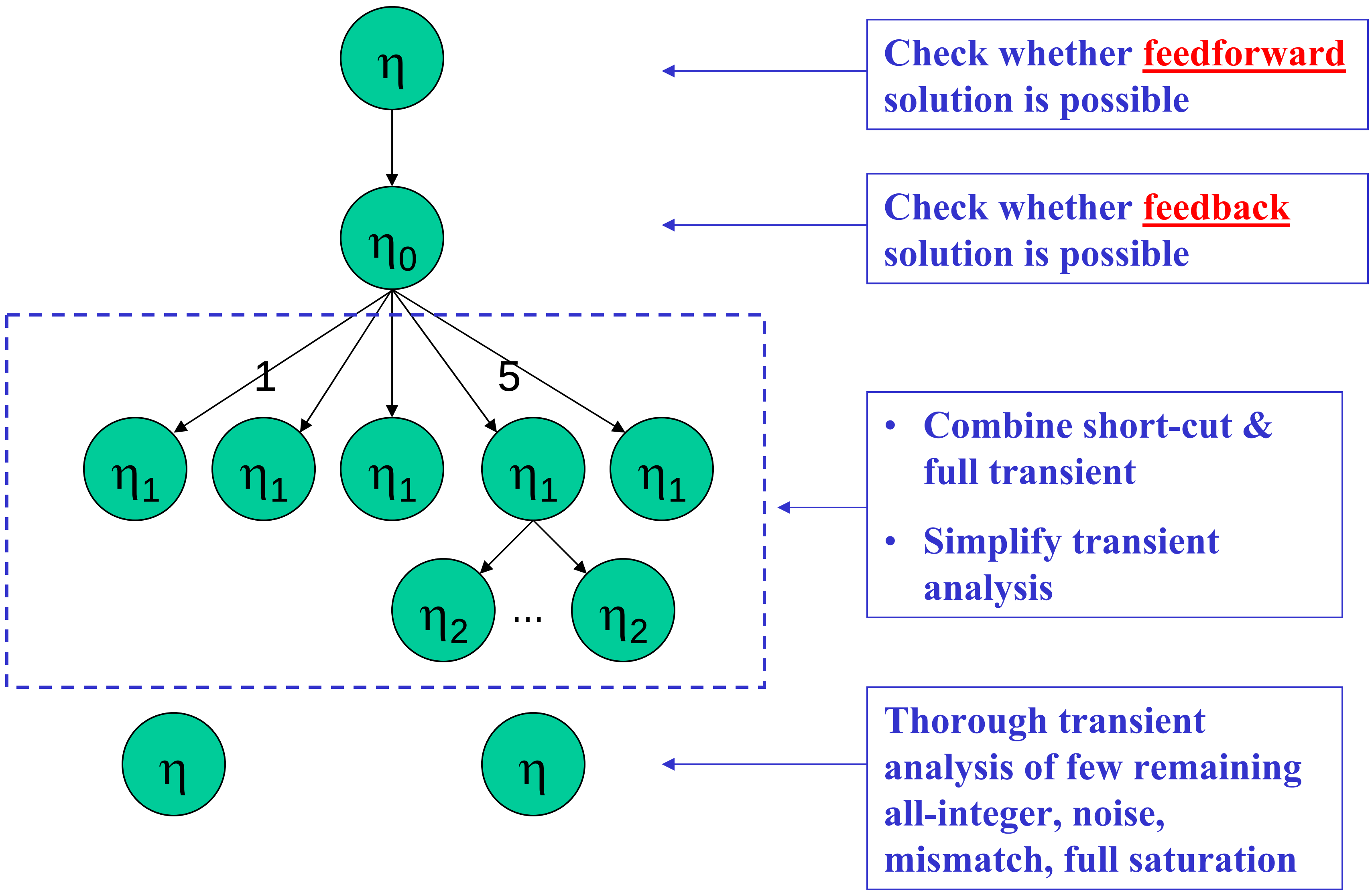
The remaining problem is 9x9

Controlled variables	Manipulated variables
Separator Level	Condenser CWT
Stripper Level	Liquid flow to separator
Pressure	Flow of FC
Product flow	Flow of product
Product G/H	FD/FE
Reactor Level	FE/FC
Mixed feed A/C	FA/FC
% B in Purge	Purge flow
Separator T	Reactor CWT

obvious

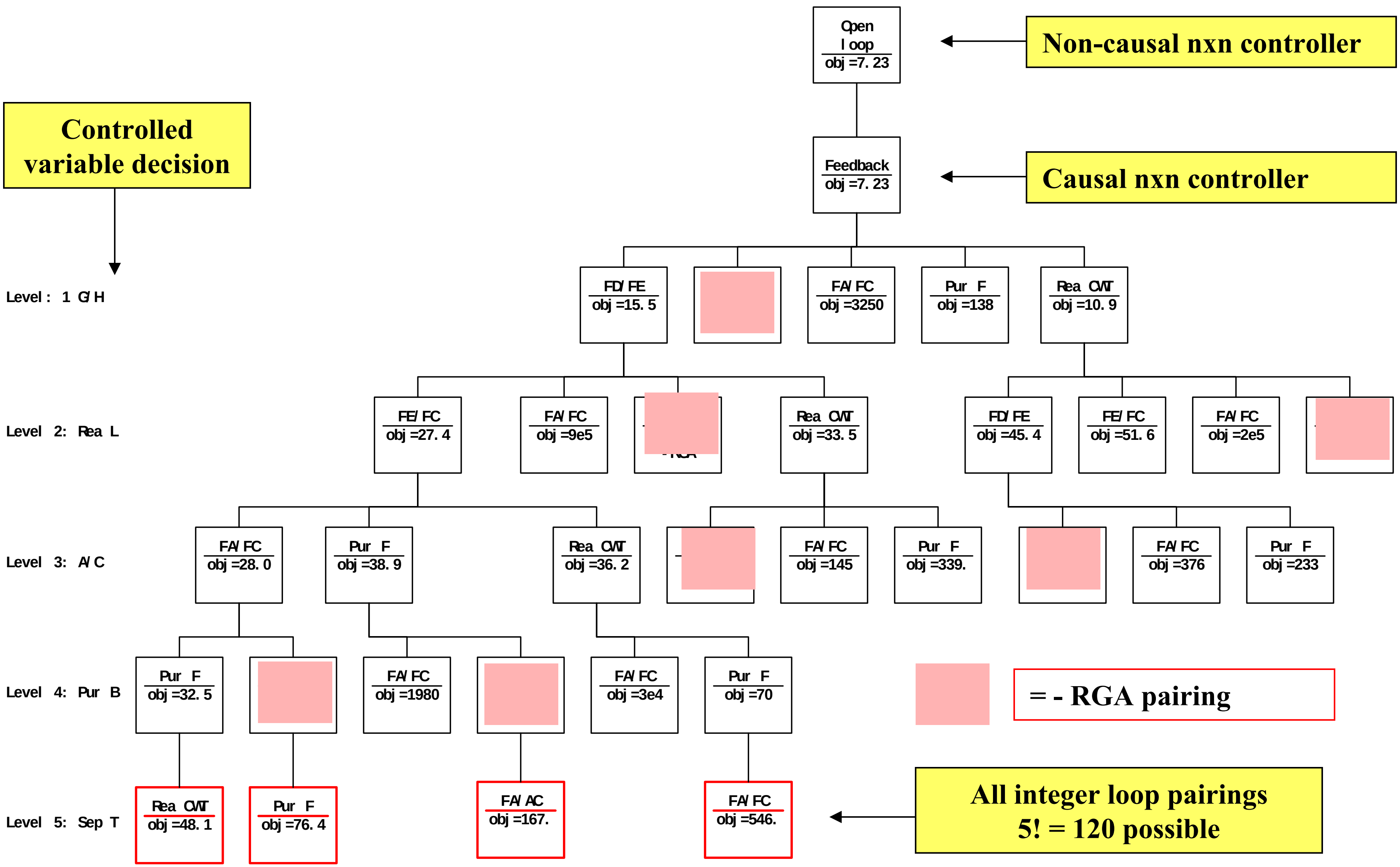
We chose to pair three variables based on <u>integrity and safety</u>.

Optimization Approach for the Tennessee Eastman Problem.



Optimization Approach for the Tennessee Eastman Problem.

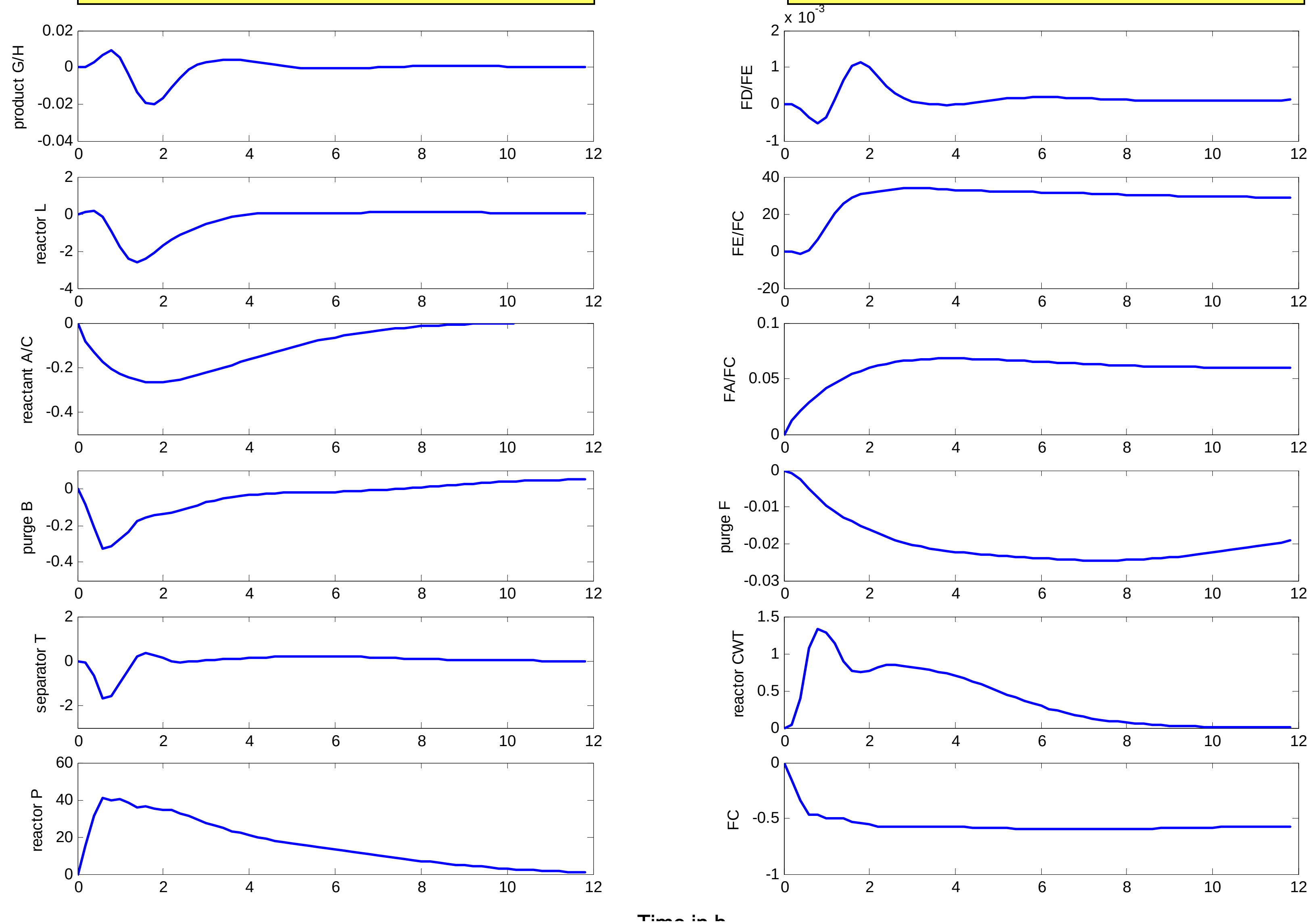
Decision Tree for A Tennessee Eastman Problem – Product Flow



Best Design - Transient response in deviation variables

Positive RGA, Objective = 48

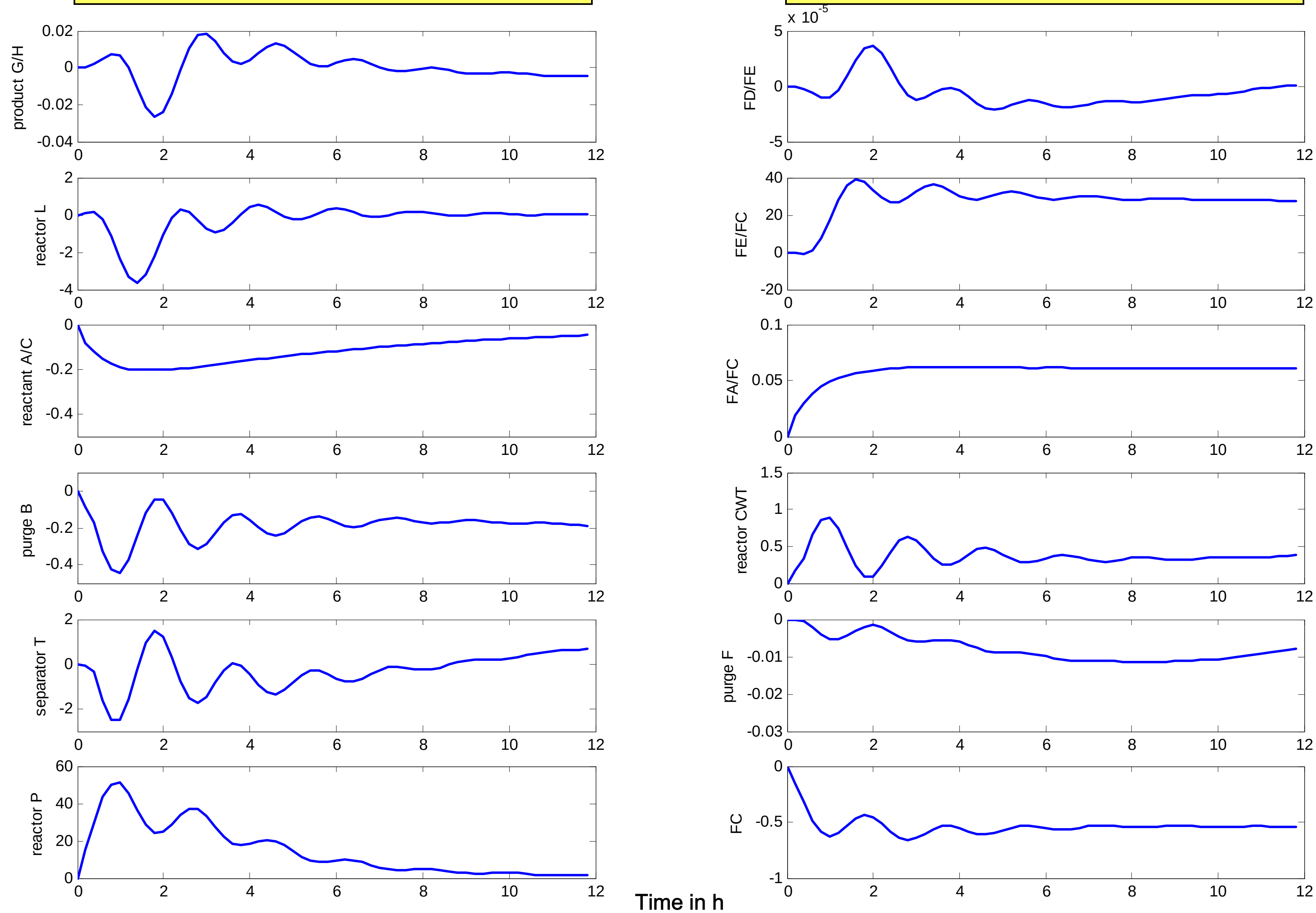
Controlled variable → **Paired manipulated variable**



Second Best Design - Transient response in deviation variables

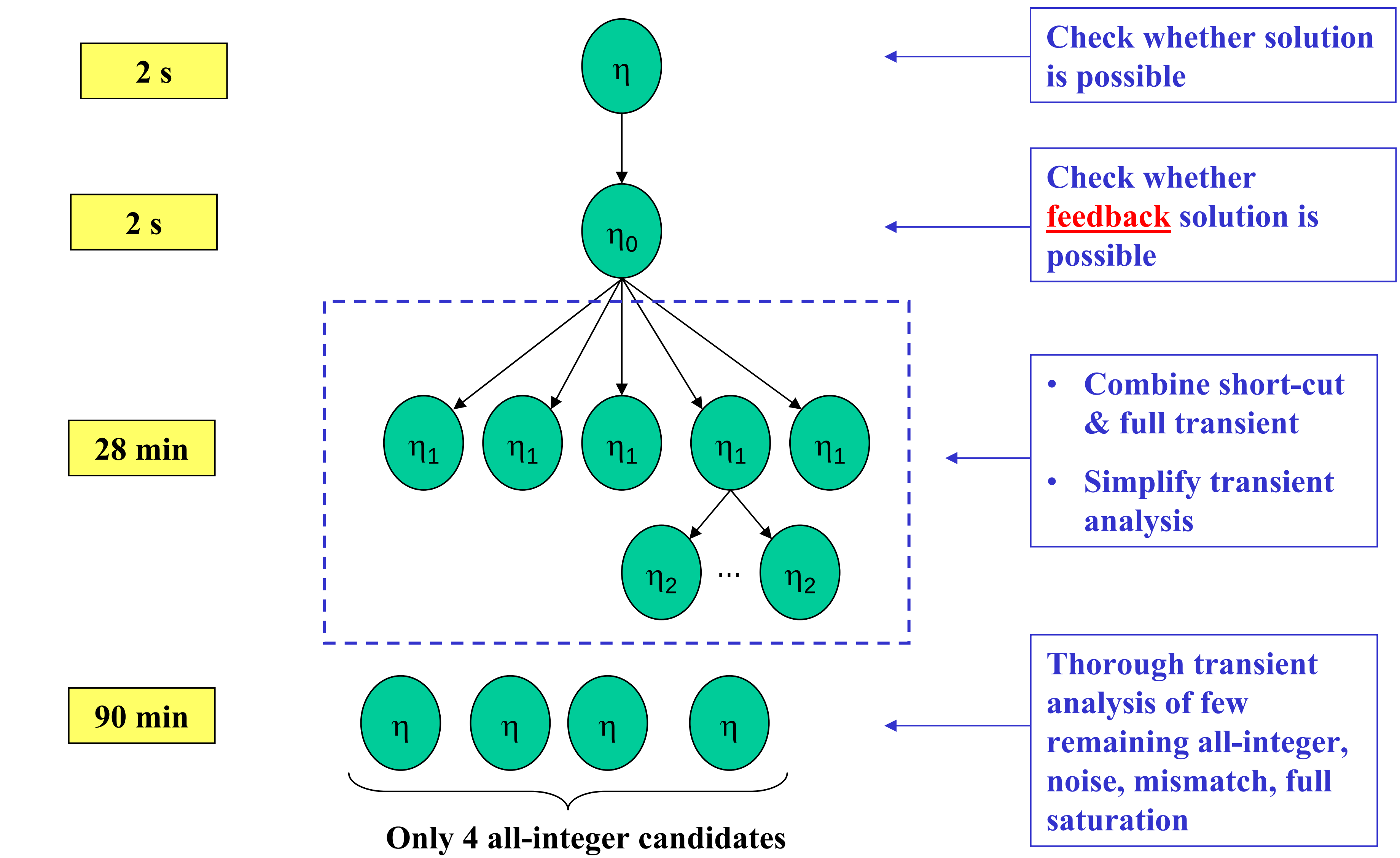
Negative RGA, Objective = 77

Controlled variable → **Paired manipulated variable**



Optimization Approach for the Tennessee Eastman Problem.

Computing times



Optimization in Control Design: Tailored Solution

Optimization Approach for the Tennessee Eastman Problem.

CONCLUSIONS

- **Always use process insights**
- **Always use control insights**
- **Early feasibility and tree pairing calculations are efficient**

  - Sequential tuning extends method to large problems
- **Short-cut metrics help prune tree (if relevant)**
- **Final Control Design for Challenging Problems requires evaluation of <u>transient behavior</u>**

PROCESS CONTROL DESIGN

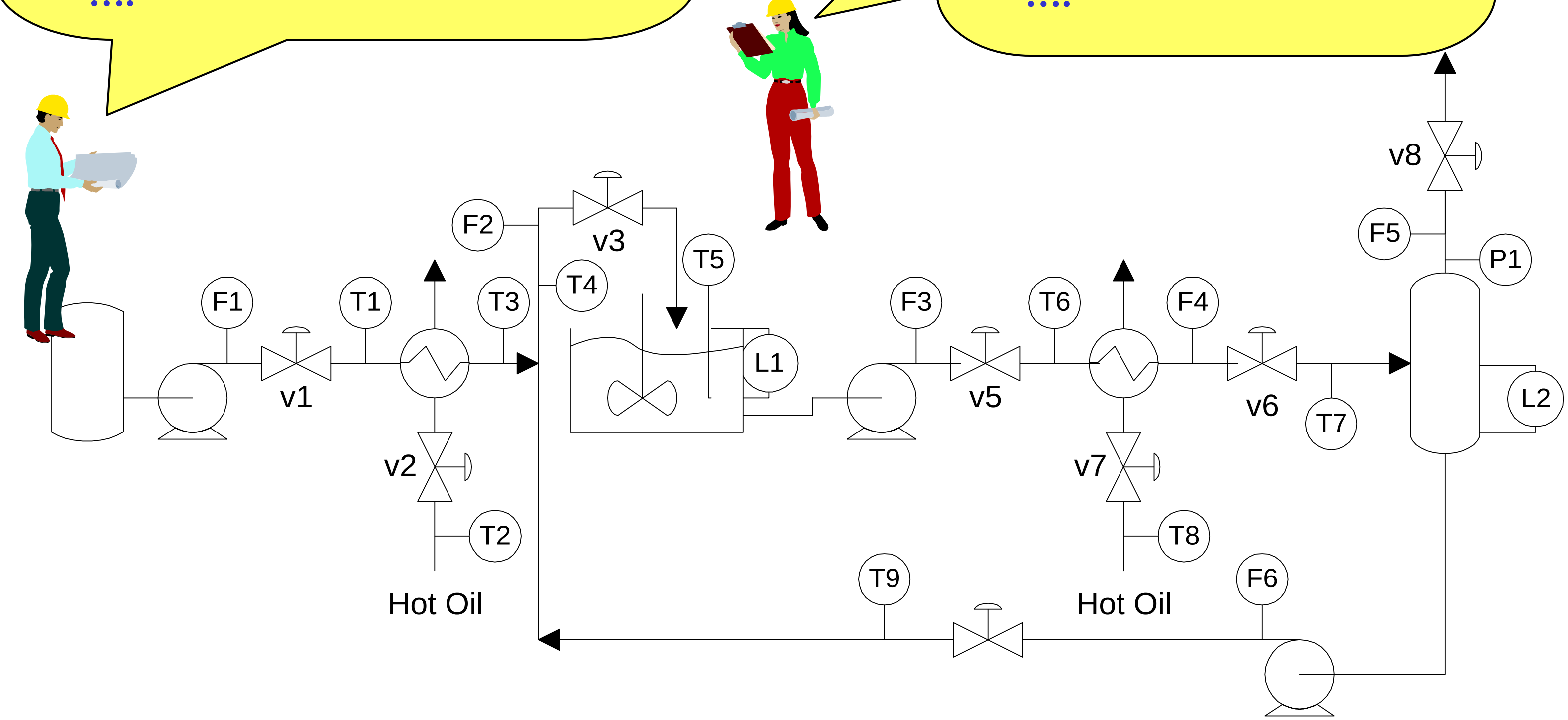
Lessons Learned - In Just Three Hours

Where do we start?

- Define objectives (7 categories)
- Define constraints, disturbances,…
- Rank multi-objectives
- Check controllability, feasibility
- Use short-cut metrics
- ….

When are we finished?

- Input-output pairings
- Dynamic performance predictions
- Integrity defined
- Initial tuning
- ….



PROCESS CONTROL DESIGN

Lessons Learned- In Just Three Hours

- **Defining Objectives is Essential**
- **Control Performance is Multi-Objective**
- **Short-Cut Metrics can Reduce the Candidates**
- **Full Transient Analysis is required for Challenging Problems**
- **The Key Decision in Control Design is Structure**
- **The Final Performance is an Estimate using Linear Models and Expected Disturbances**

PROCESS CONTROL DESIGN

Lessons Learned- In Just Three Hours

Hidden

- **Knowledge of Process Equipment is Essential**

  - Pumps, compressors, distillation, flash …

- **Application of Process Principles is Essential**

  - V/L Equilibrium, Stoichiometry, …

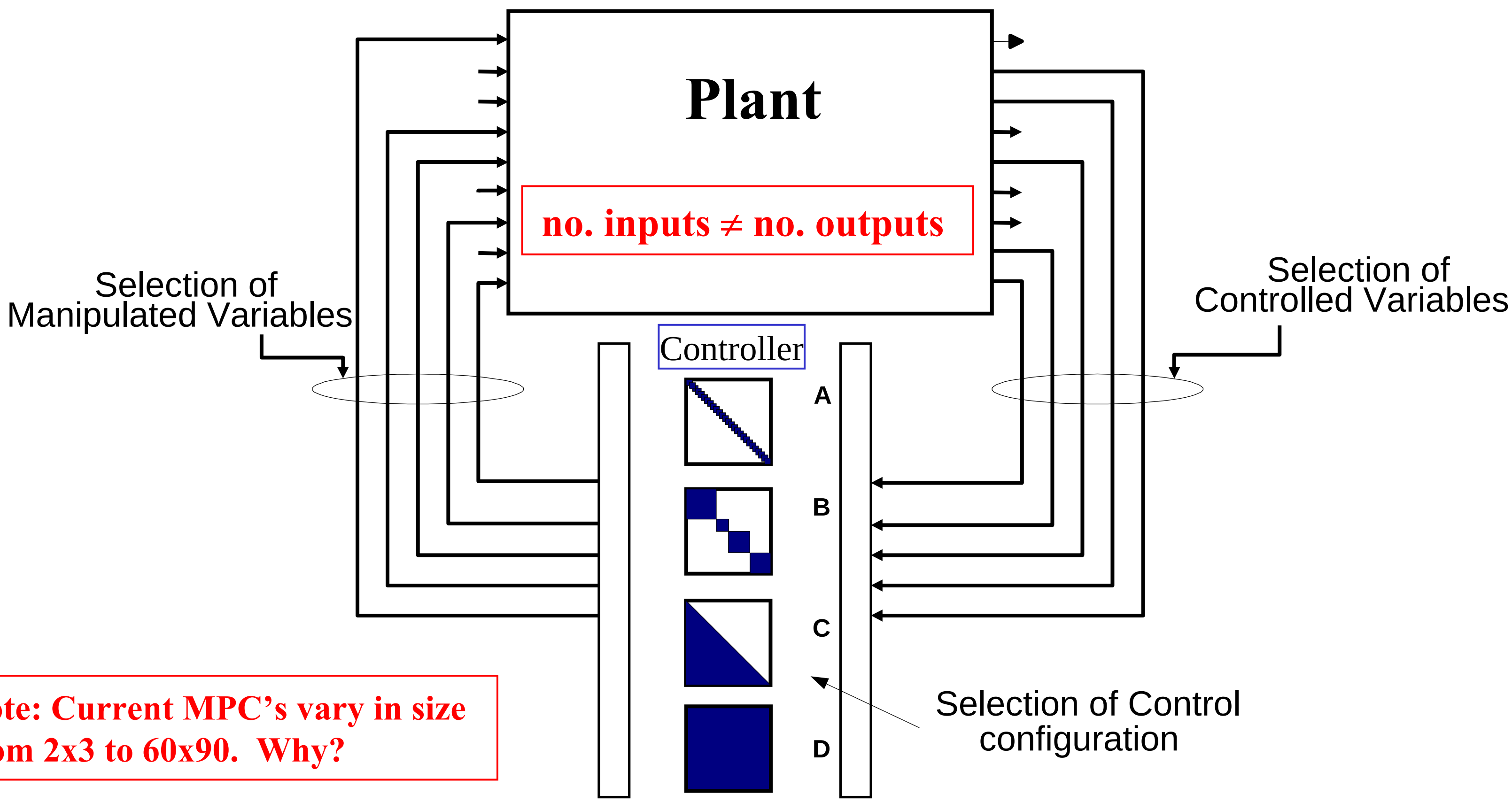
- **Many Designs can be completed Without Simulation**
- **Challenging designs require full Transient Behavior**
- **Continued Developments are Required for "Automatic Control Design"**

  - Better Robustness, Faster Candidate Elimination, Convex NLP,..

PROCESS CONTROL DESIGN

Lessons Learned- In Just Three Hours

An Outstanding Challenge – Block Centralized Design and Implementation



Note: Current MPC's vary in size from 2x3 to 60x90. Why?

PROCESS CONTROL DESIGN
Lessons Learned

I have learned many lessons too! Thanks to the following people (and many more).

- **Researchers who have published useful papers, especially Edgar Bristol**
- **Collaborators who provided insights, especially Tom McAvoy**
- **Students who did the hard work, especially Maria Marino (Un. Maryland) and Yongsong Cai (McMaster)**
- **Students attending Control Design courses for interesting questions and projects**

PROCESS CONTROL DESIGN

Lessons Learned- In Just Three Hours

You have a fast start in your life-long learning journey to expertise in Process Systems Engineering!