

Dynamic Modeling and Optimization of Large-Scale Cryogenic Processes

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- Objective
- Natural Gas Processing Plants
- Methodology
- Mathematical Models
- Discussion of Results
- Conclusions and Current Work



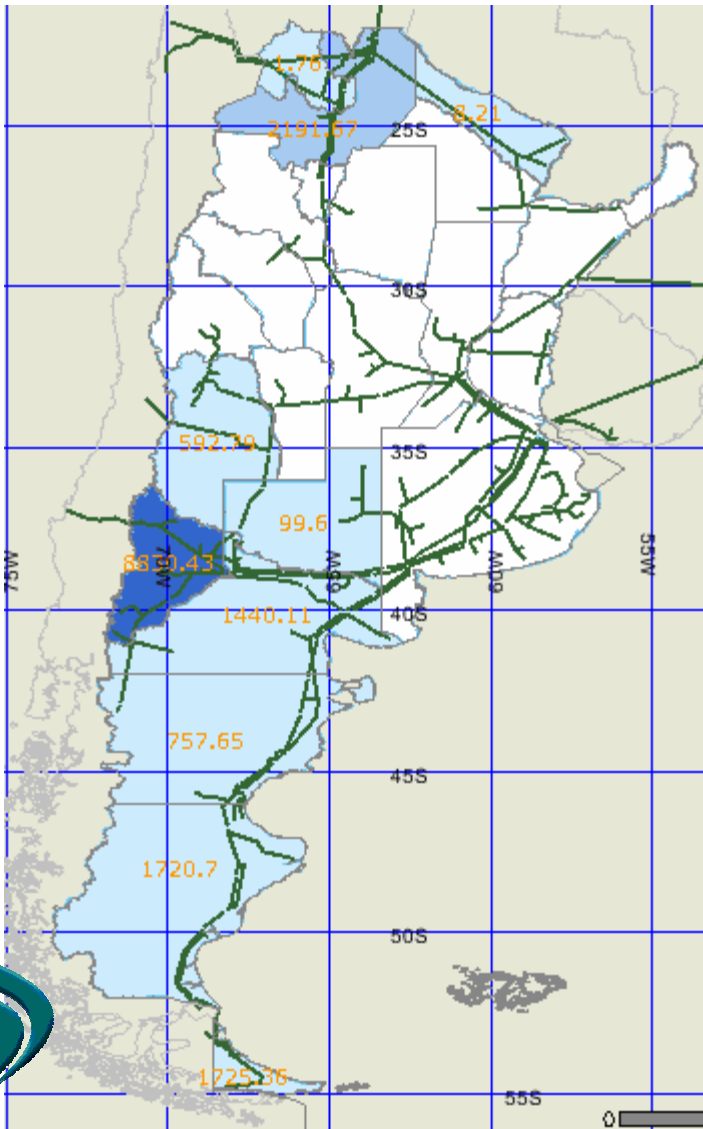
- Dynamic optimization model for natural gas processing plant units
 - Dynamic energy and mass balances
 - Phase equilibrium calculations with cubic equation of state
 - Hydraulic correlations
 - Phase change in countercurrent heat exchanger
 - Carbon dioxide precipitation in column
 - Phase existence in column

- Simultaneous dynamic optimization approach
 - Discretization of state and control variables
 - Resolution of large-scale Nonlinear Programming problem

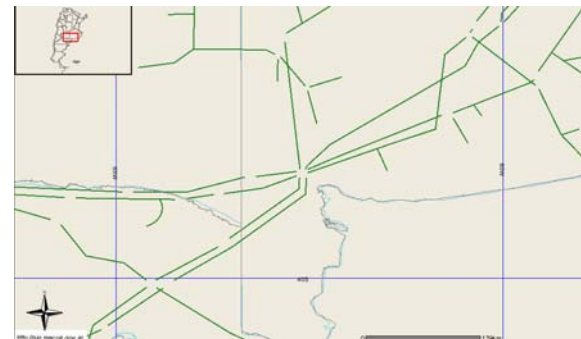
- Analysis of main variables temporal and spatial profiles



Natural Gas in Argentina

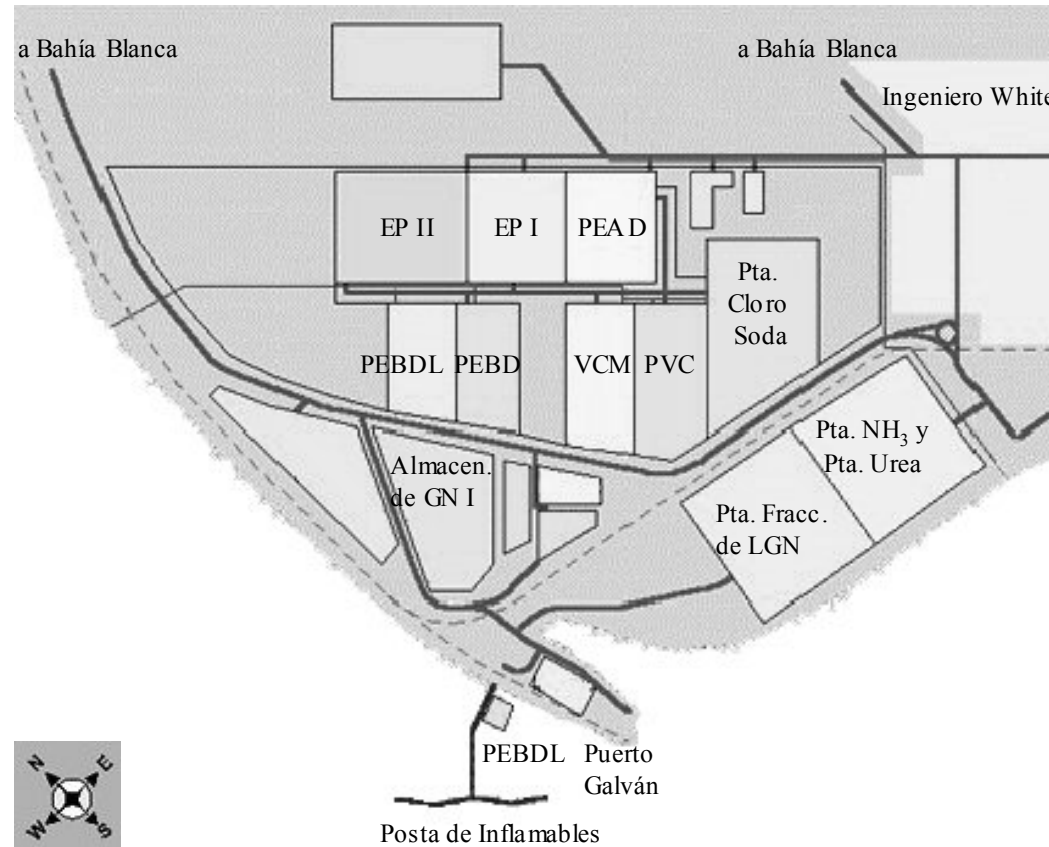
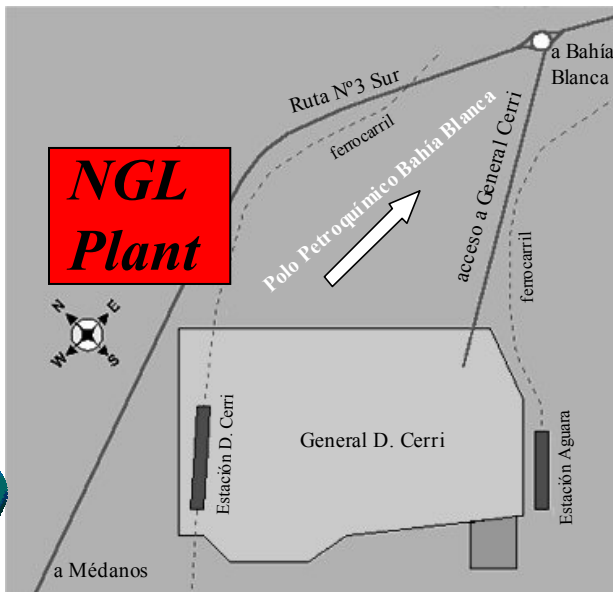


Component	Feed A	Feed B
Nitrogen	1.44	1.37
Carbon dioxide	0.65	1.30
Methane	90.43	89.40
Ethane	4.61	4.43
Propane	1.76	2.04
Butanes	0.77	0.96
Pentanes+	0.34	0.50

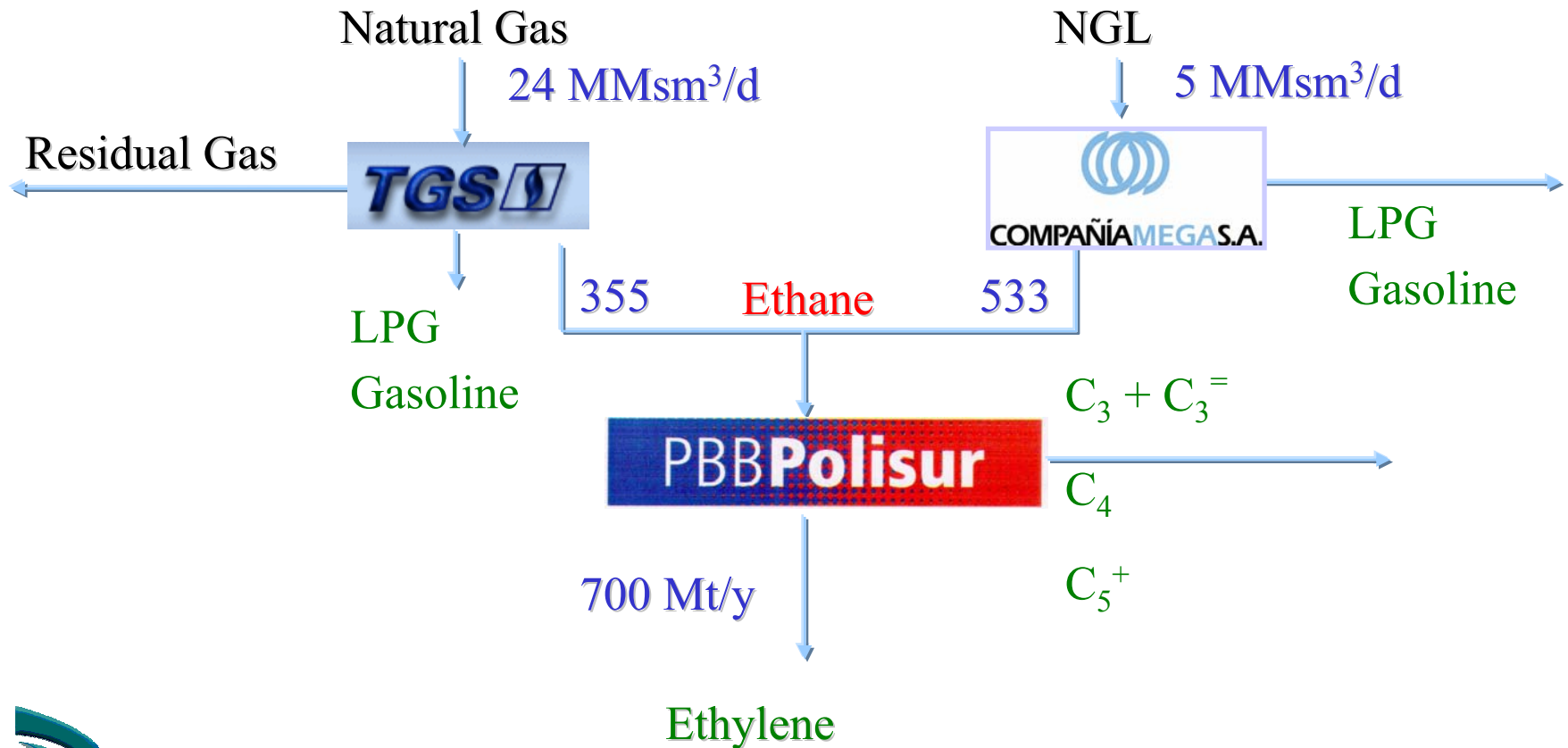


Natural Gas Processing Plants

- Provide ethane as raw material for olefin plants (ethylene) and petrochemical
- High ethylene yield
- Minimum by-products



Natural Gas Plants at Bahia Blanca

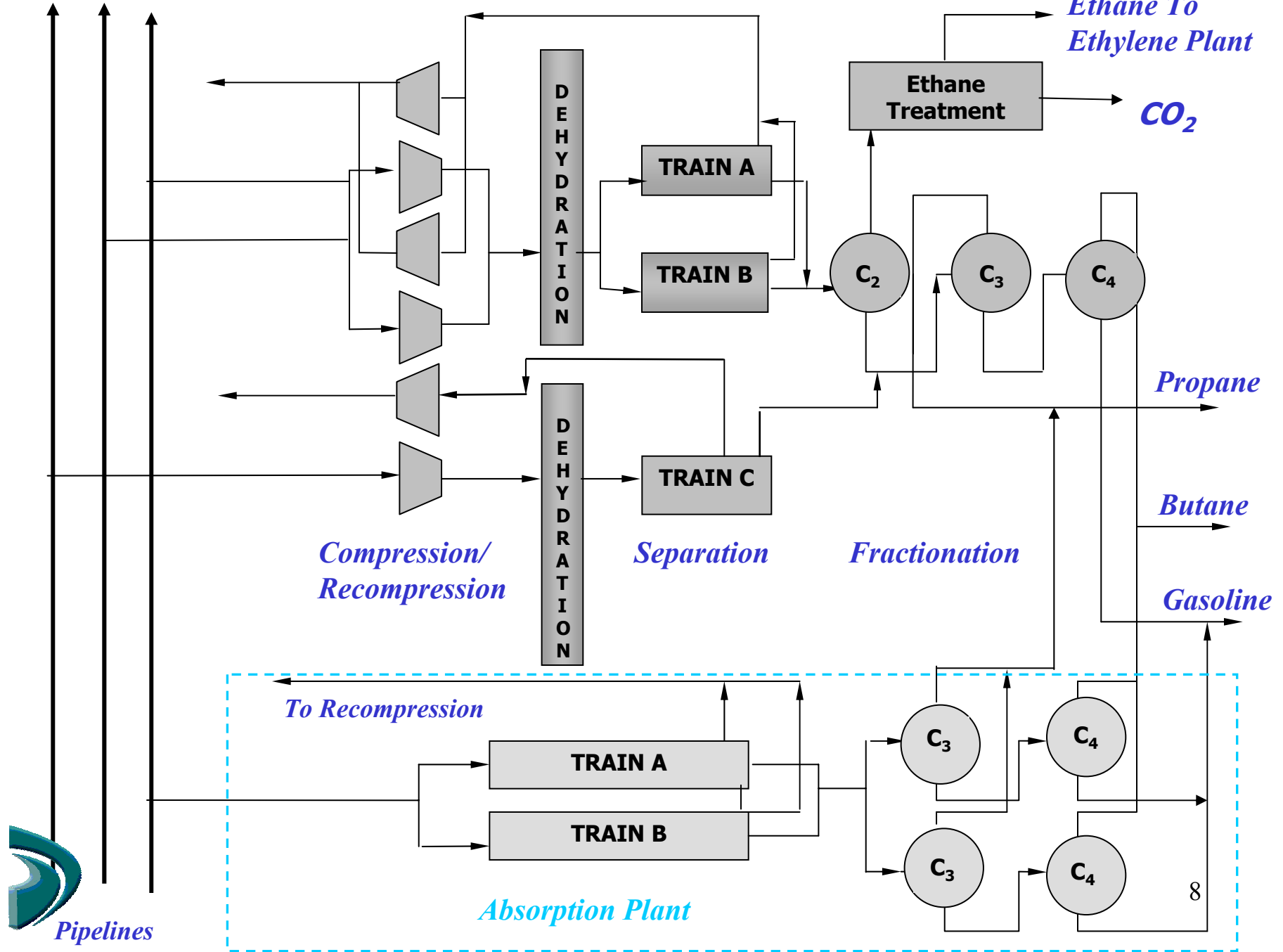


Natural Gas Processing Plants

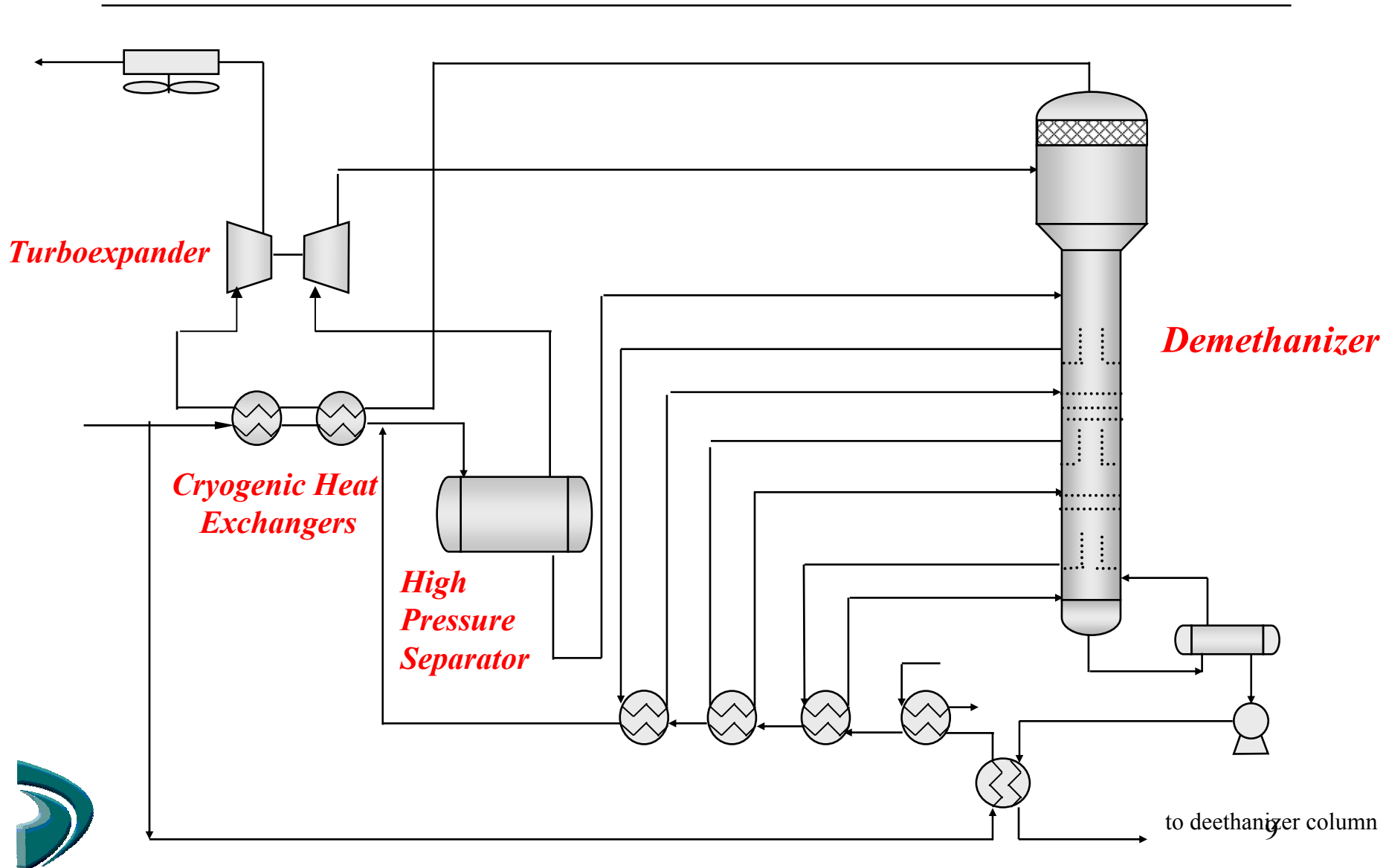
- Cryogenic sector (demethanizing), Conventional fractionation (deethanizing, depropanizing, debutanizing, gasoline stabilization), CO₂ removal
- Current technology: Turboexpansion (Old technology: Absorption)
- High pressure
- Cryogenic conditions



NATURAL GAS PLANT



Cryogenic Sector



- Operating conditions optimization in natural gas processing plants (NLP) (Diaz, Serrani, Be Deistegui, Brignole, 1995)
- Automatic design and debottlenecking of ethane extraction plants (MINLP) (Diaz, Serrani, Bandoni Brignole, 1997)
- Thermodynamic model effect on the design and optimization of natural gas plants (NLP) (Diaz, Zabaloy, Brignole, 1999)
- Flexibility of natural gas processing plants - Dual mode operation (Bilevel NLP) (Diaz, Bandoni, Brignole, 2002)



Dynamic Optimization Problem

General Formulation

$$\min \Phi(z(tf), y(tf), u(tf), tf, p)$$
$$z(t), y(t), u(t), tf, p$$

st

$$F(dz(t)/dt, (z(t), y(t), u(t), p, t) = 0$$
$$G(z(t), y(t), u(t), p, t) = 0$$

$$z_0 = z(0)$$

$$z(t) \in [z^l, z^u] \quad , \quad y(t) \in [x^l, x^u]$$
$$u(t) \in [u^l, u^u] \quad , \quad p \in [p^l, p^u]$$

t , time

z , differential variables

y , algebraic variables

tf , final time

u , control variables

p , time independent parameters



Dynamic Optimization Strategy

Simultaneous Approach Cervantes, Biegler (1998)

Nonlinear DAE optimization problem



Discretization of Control and State variables

Collocation on finite elements

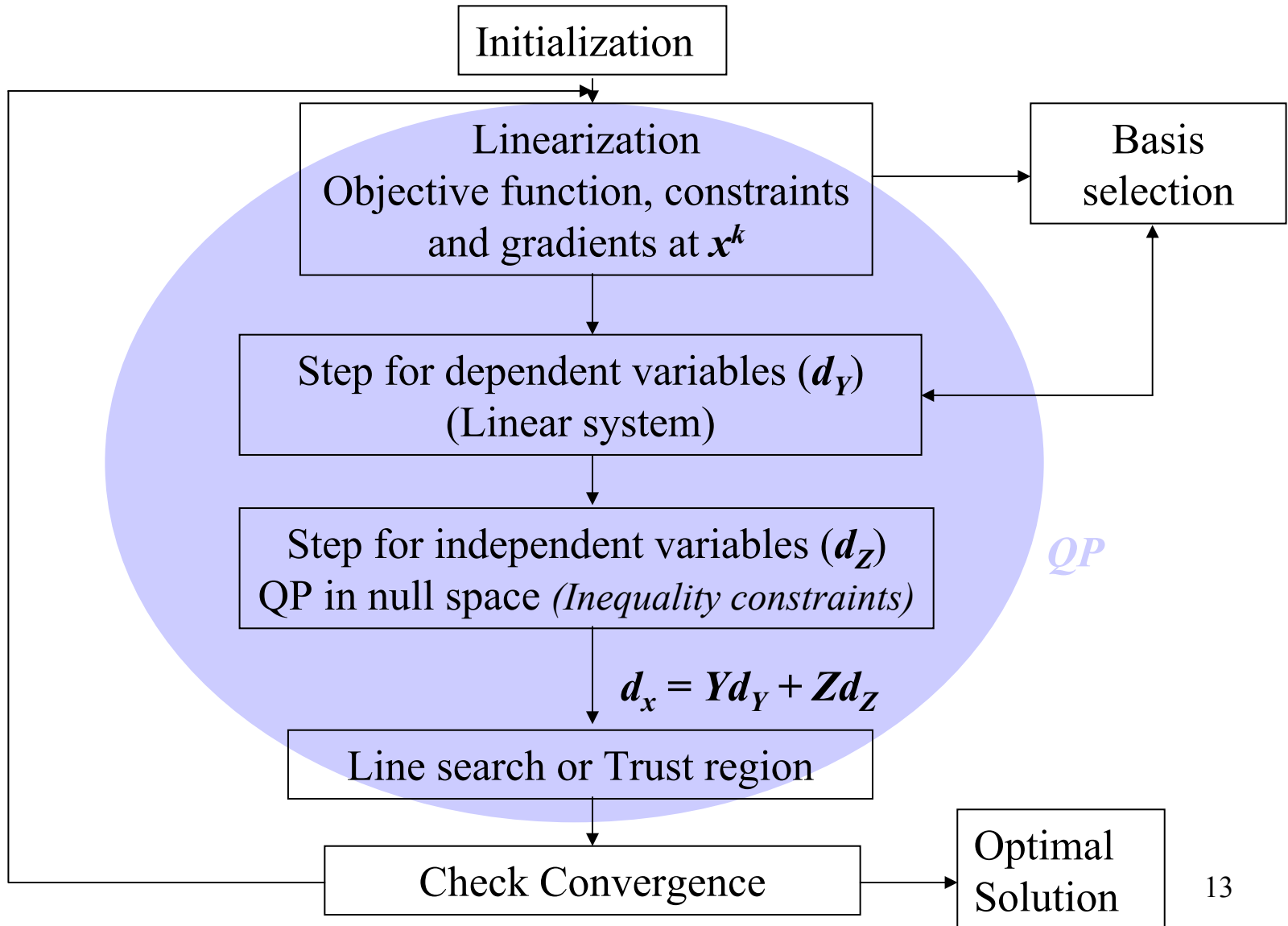


Large-Scale Nonlinear Programming Problem (NLP)

Interior Point Algorithm Biegler, Cervantes,
Waechter (2002)¹²



rSQP Algorithm



Nonlinear Programming Problem

Application of Barrier method

$$\min f(x)$$

$$\text{s.t } c(x) = 0$$

$$x \geq 0$$



$$\min f(x) - \mu \sum_{j=1}^n \ln x_j$$

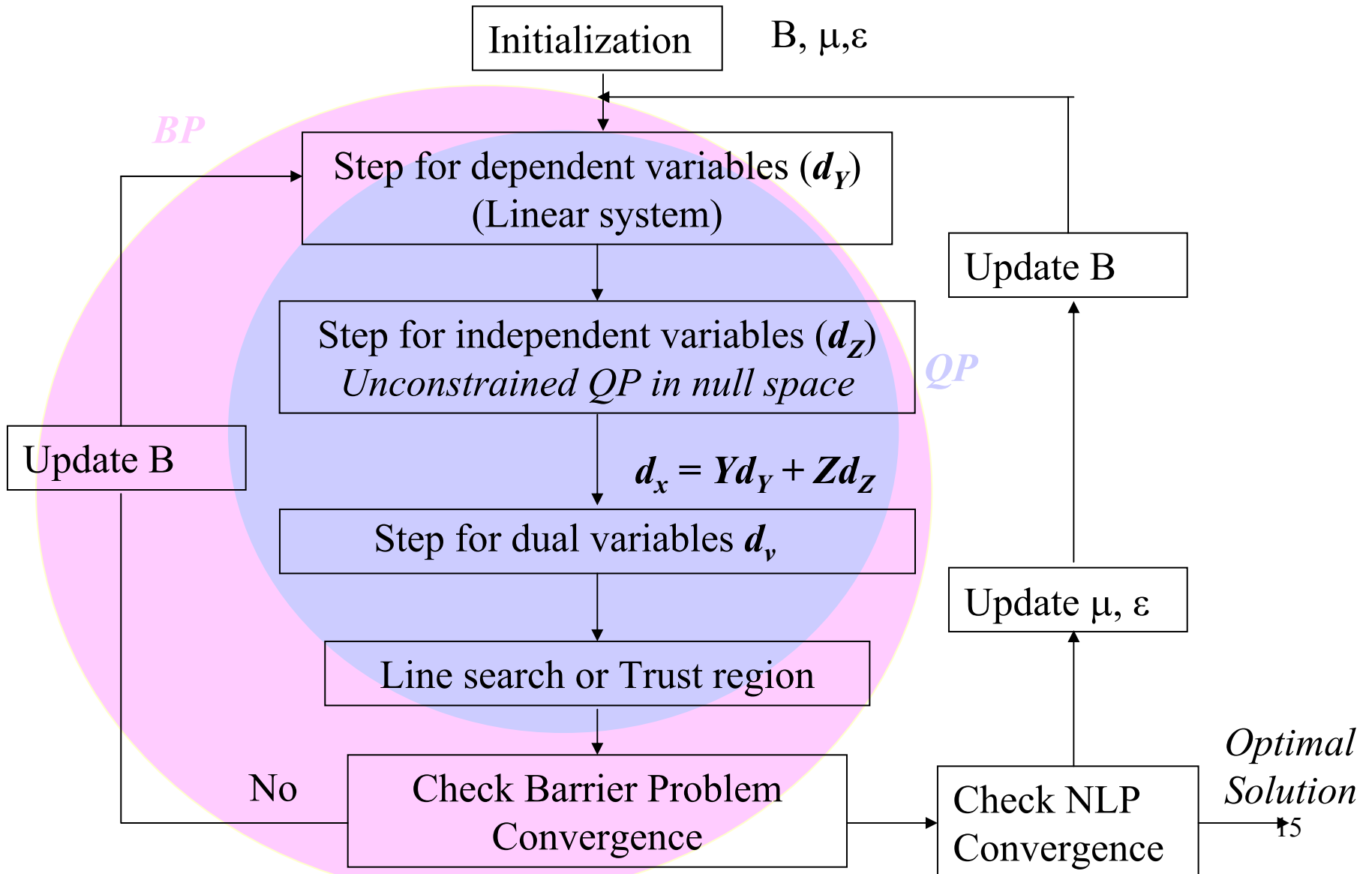
$$\text{s.t } c(x) = 0$$

As $\mu \rightarrow 0$, $x^*(\mu) \rightarrow x^*$

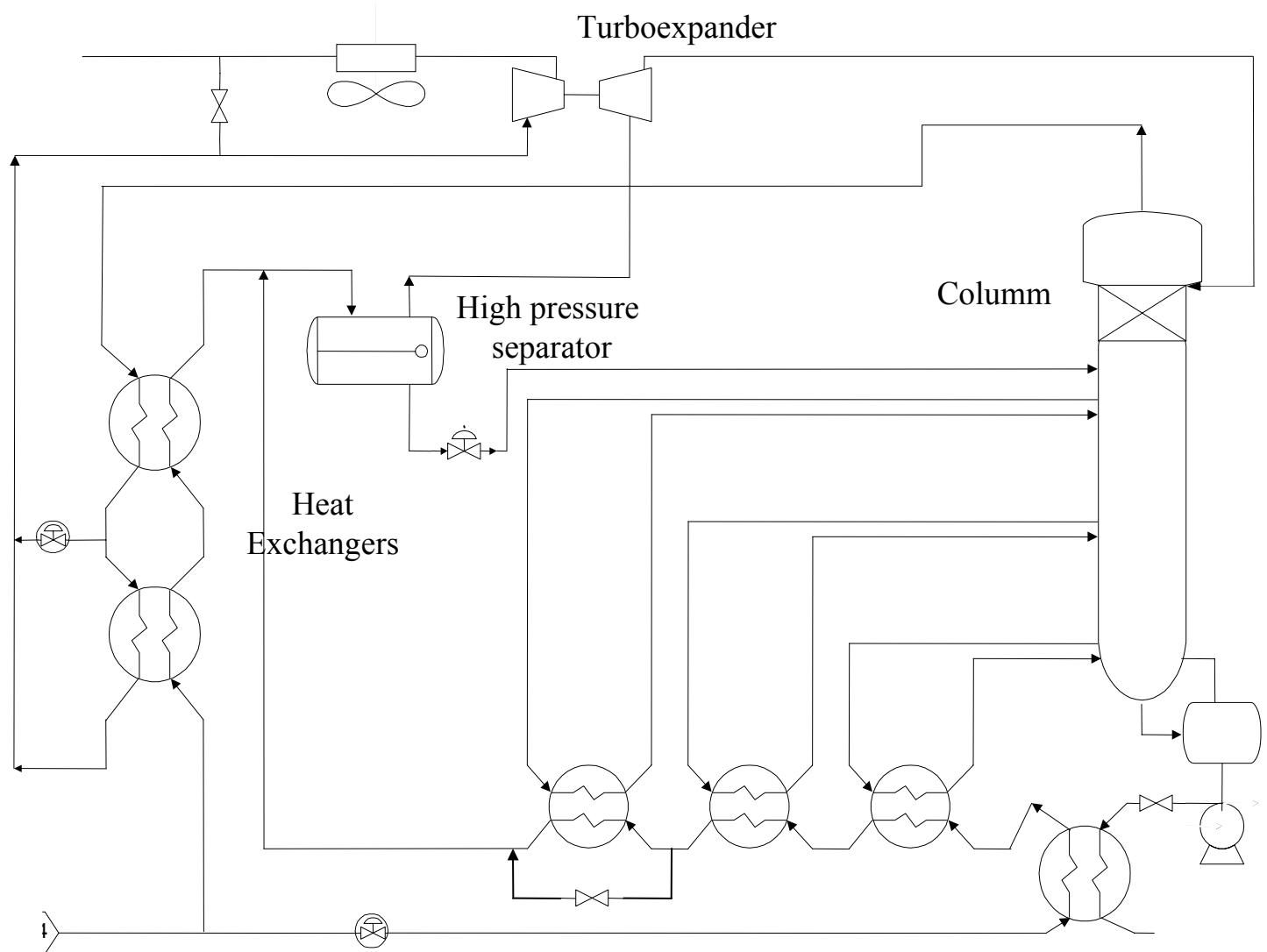


Sequence of barrier problems for decreasing μ values

Barrier Method Algorithm: Primal-Dual Approach

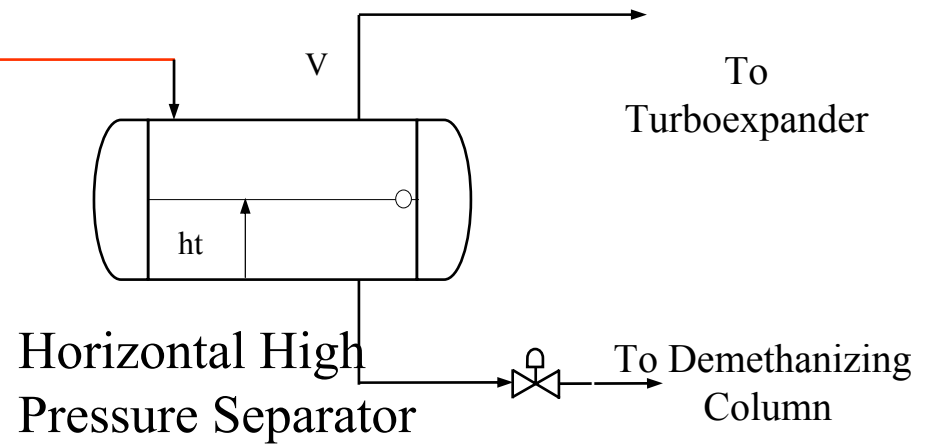
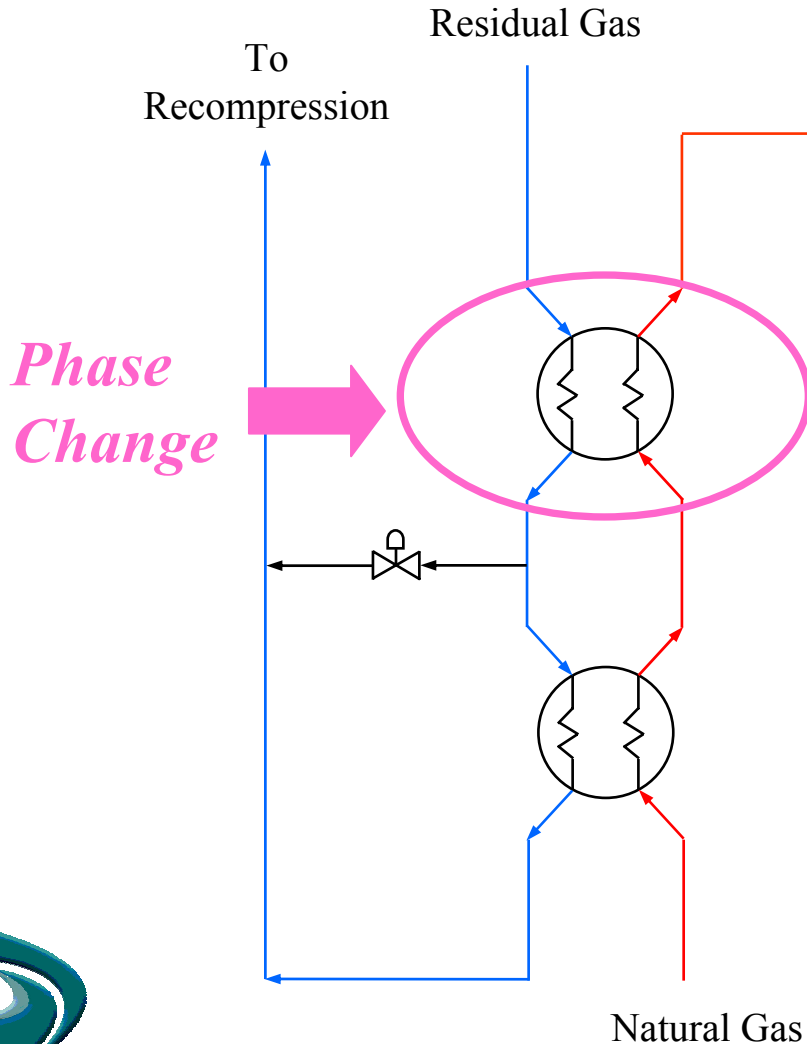


Cryogenic Sector



Cryogenic Heat Exchangers and HP Separator

Rodriguez, Bandoni, Diaz (2004)



Baffled shell-and-tube single-pass countercurrent heat exchangers

Tubes: residual gas

Shell: natural gas feed

Partial condensation: Second heat exchanger



HE Model (no Phase Change)

Energy Balances: Partial Differential Equations System

Tube side
$$\frac{\partial T_t(z,t)}{\partial t} + v_t \frac{\partial T_t(z,t)}{\partial z} = \frac{h_t^* A_{sup}}{\rho_t^* C_{p_t}^* A_t^* L} (T_s(z,t) - T_t(z,t))$$

Shell side
$$\frac{\partial T_s(z,t)}{\partial t} - v_s \frac{\partial T_s(z,t)}{\partial z} = \frac{h_s^* A_{sup}}{\rho_s^* C_{p_s}^* A_s^* L} (T_t(z,t) - T_s(z,t))$$

$$T_t(0,t) = T_{to}(t)$$

$$T_s(L,t) = T_{so}(t)$$

$$T_t(z,0) = T_t^*(z)$$

$$T_s(z,0) = T_s^*(z)$$

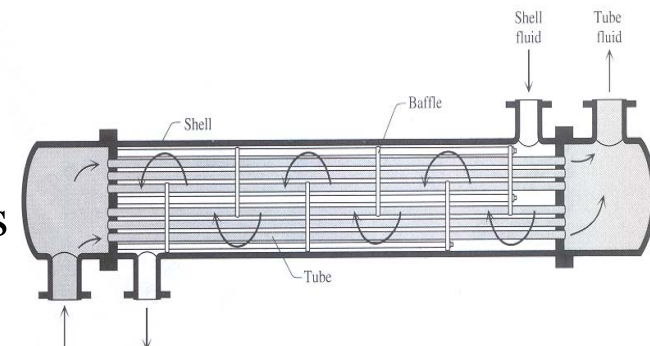
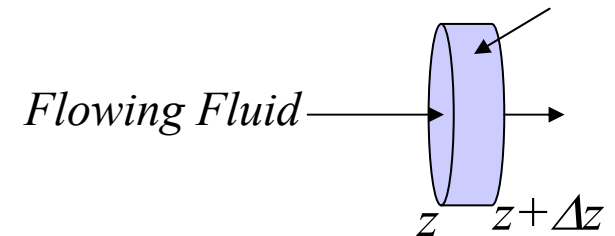
First-order hyperbolic PDE

Method of Lines



Spatial Discretization: Backward Finite Differences

for Convection Term \rightarrow ODE System

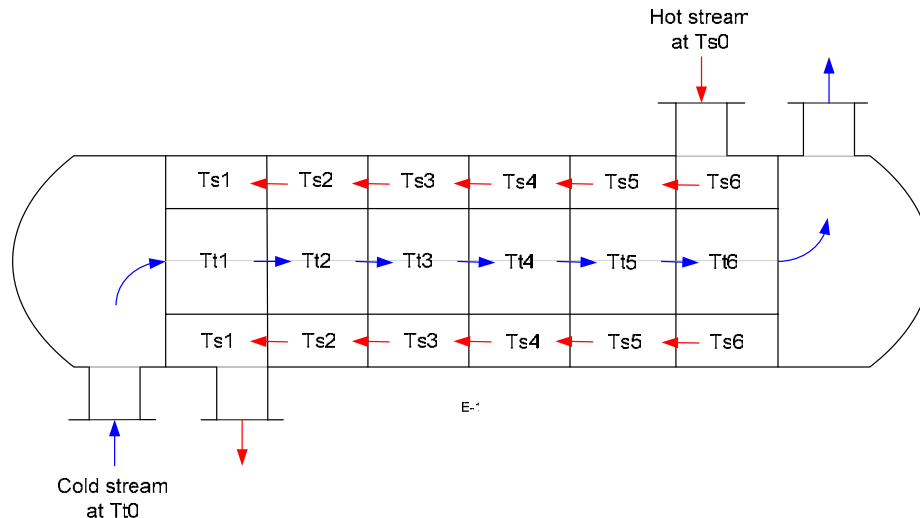


HE Model (no Phase Change)

Energy Balance at cell i

Tube side
$$\frac{dTt_i}{dt} = -\frac{vt_i}{\Delta z} (Tt_i - Tt_{i-1}) + \frac{h_t * A_{sup}}{\rho t_i * Cp_t * A_t * L} (Ts_i - Tt_i)$$

Shell side
$$\frac{dTs_i}{dt} = \frac{vs_i}{\Delta z} (Ts_i - Ts_{i-1}) + \frac{h_s * A_{sup}}{\rho s_i * Cp_s * A_s * L} (Tt_i - Ts_i)$$



Multicell model



Algebraic Equations (no Phase Change)

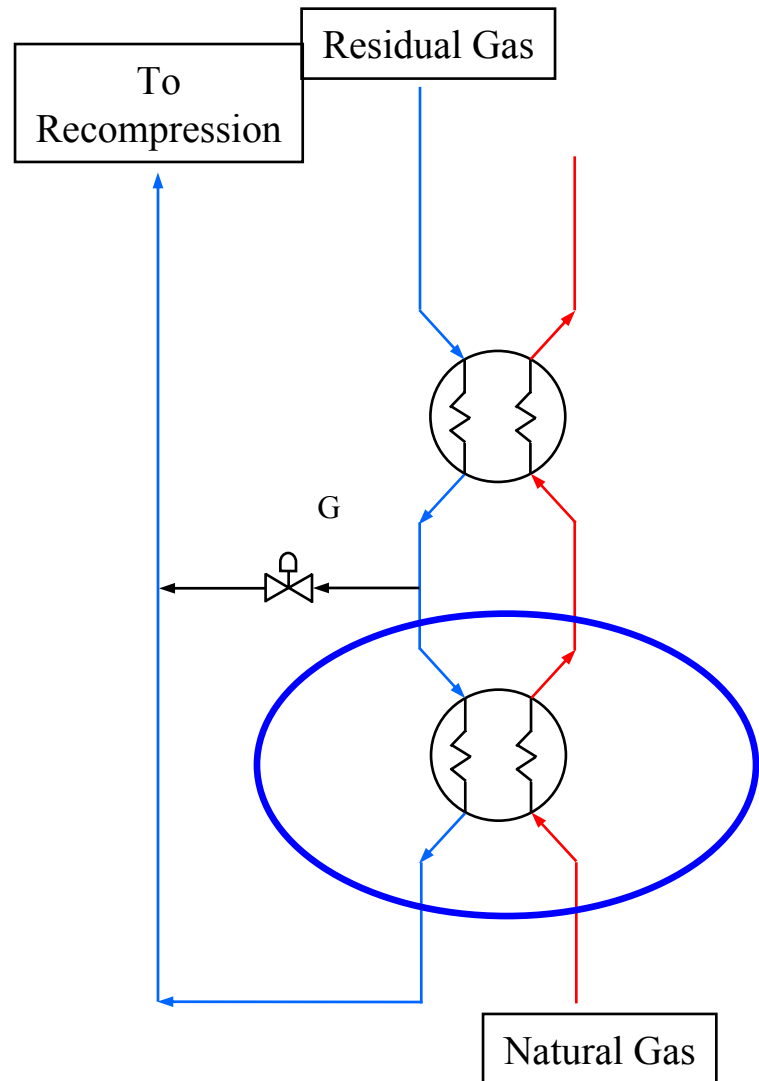
$$\rho_{j,i} = \frac{M * P}{z_{j,i} * R * T_{j,i}}$$

$$v_{j,i} = \frac{F_j}{\rho_{j,i} * A_j}$$

$i = 1, \dots, N$ (cells)

$j = \text{tube or shell side}$

HE model → **DAE System**



HE Model (Partial Phase Change on Shell Side)

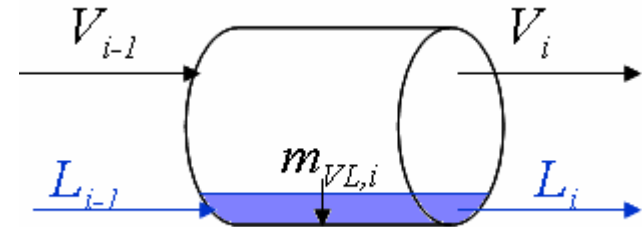
Rodriguez, Bandoni, Diaz (2005)

Mass Balance at cell i

Vapor Phase $\frac{dM_{V,i}}{dt} = V_{i-1} - V_i - m_{VL,i}$

Liquid Phase $\frac{dM_{L,i}}{dt} = L_{i-1} - L_i + m_{VL,i}$

Component $\frac{dm_{ij}}{dt} = V_{i-1}y_{i-1,j} + L_{i-1}x_{i-1,j} - V_i y_{i,j} - L_i x_{i,j}$



$m_{VL,i}$: interfacial mass-transport rate from vapor to liquid phase

Energy Balance at cell i

$$\frac{dE_i}{dt} = L_{i-1}^* h_{i-1} + V_{i-1}^* H_{i-1} - L_i^* h_i - V_i^* H_i + Q_i^t$$



HE Model (Partial Phase Change on Shell Side)

Momentum Balance at cell i

Vapor Phase $\frac{d(M_{V,i}v_{V,i})}{dt} = V_{i-1}v_{V,i-1} - V_i v_{V,i} + F_{P,i} - F_{inter,i} - m_{VL,i}v_{L,i}$

Liquid Phase $\frac{d(M_{L,i}v_{L,i})}{dt} = L_{i-1}v_{L,i-1} - L_i v_{L,i} + F_{P,i} + F_{inter,i} + m_{VL,i}v_{L,i} - F_{w,i}$

Algebraic Equations at cell i

$\theta_i = \frac{A_{V,i}}{A_T}$ Void fraction based on cross-section area

$M_{V,i} = A_T L e_i \theta_i \rho_{V,i}$

$$v_{V,i} = \frac{V_i}{A_{V,i} \rho_{V,i} L_i}$$

 $M_{L,i} = A_T L e_i (1 - \theta_i) \rho_{L,i}$

$$v_{L,i} = \frac{L_i}{(1 - A_{V,i}) \rho_{L,i}}$$

HE Model (Partial Phase Change on Shell Side)

Driving Force $F_{P,i} = (P_{s_{i-1}}\theta_{i-1} - P_{s_i}\theta_i)A_T$

Interfacial shear stress $F_{inter,i} = \frac{4\sqrt{\theta_i}}{D} f_{f,i} \frac{\rho_{L,i}}{2} (v_{V,i} - v_{L,i})^2$

Interfacial friction factor $f_{f,i} = f_v \left[1 + 24 \left(\frac{\rho_{L,i}}{\rho_{V,i}} \right)^{1/3} \frac{\delta}{D} \right]$

Friction resistance on wall surface $F_{w,i} = \frac{4}{D} \tau_w$

$$f_v = \begin{cases} 0.079 Re^{-0.25}, & 4000 \leq Re \leq 30000 \\ 0.046 Re^{-0.20}, & Re > 30000 \end{cases}$$



HE Model (Partial Phase Change on Shell Side)

Internal Energy $E_i = M_{V,i}H_i + M_{L,i}h_i$

Summation Eqns $\sum_j y_{i,j} - \sum_j x_{i,j} = 0$

Equilibrium ratio $K_{i,j} = \frac{\phi_{i,j}^L}{\phi_{i,j}^V} \quad y_{i,j} = K_{i,j}x_{i,j}$

$$h_i = h_i^{ideal} - \Delta h_i, \quad h_i^{ideal} = \sum_{j=1}^{nc} h_{i,j}^{ideal} (T_i)x_{i,j}$$

$$H_i = H_i^{ideal} - \Delta H_i, \quad H_i^{ideal} = \sum_{j=1}^{nc} H_{i,j}^{ideal} (T_i)y_{i,j}$$



HE Model (Partial Phase Change on Shell Side)

Compressibility factor

$$z_i^V = z^V (T_{s_i}, P_{s_i}, y_{i,j})$$

$$z_i^L = z^L (T_{s_i}, P_{s_i}, x_{i,j})$$

Residual enthalpies

$$\Delta H_i = \Delta H (T_{s_i}, P_{s_i}, y_{i,j})$$


$$\Delta h_i = \Delta h (T_{s_i}, P_{s_i}, x_{i,j})$$

Fugacity coefficients

$$\phi_{i,j}^V = \phi^V (T_{s_i}, P_{s_i}, y_{i,j})$$

$$\phi_{i,j}^L = \phi^L (T_{s_i}, P_{s_i}, x_{i,j})$$

Soave-Redlich-Kwong EoS


$$P_{s_i} = \frac{\rho_{L,i} * z_i^L * R * T_{s_i}}{P_{mols_i}^L}$$

$$P_{s_i} = \frac{\rho_{V,i} * z_i^V * R * T_{s_i}}{P_{mols_i}^V}$$

High Pressure Separator

$$\frac{dM}{dt} = Lp + Vp - L - V$$

Horizontal tank

$$Vol_V = \pi r^2 Long - \frac{M_L}{\rho_L}$$

$$L = C_v x \sqrt{\frac{(P_t - P_s) + g \rho_L (h_t + r)}{\rho_L / \rho_w}}$$

$$M_V = \rho_V Vol_V \quad M = M_V + M_L$$

$$M_L = \frac{\rho_L Long}{P mol_L} \left(\pi r^2 - r^2 \arccos\left(\frac{h_t}{r}\right) + h_t \sqrt{r^2 - h_t^2} \right)$$



Turboexpander

- Feed: vapor from High Pressure Separator
- Polytropic expansion ($\eta = 1.26$, natural gas)

$$\frac{T_s}{T_e} = \left(\frac{P_s}{P_e} \right)^{\frac{\eta-1}{\eta}}$$

- Assumption: Static mass and energy balances
- Thermodynamic predictions: Soave-Redlich-Kwong
- Algebraic equations: Same as Flash
- Outlet partially condensed stream: to Demethanizing column



Demethanizing Column

Diaz, Tonelli, Bandoni, Biegler (2003)

Differential Equations at stage i ($1 \leq i \leq N$)

$$\frac{dM_i}{dt} = F_i + V_{i+1} + L_{i-1} - V_i - L_i$$

$$\frac{dm_{ij}}{dt} = F_i z_{i,j} + V_{i+1} y_{i+1,j} + L_{i-1} x_{i-1,j} - V_i y_{i,j} - L_i x_{i,j} \quad j=1, \dots, ncomp$$

$$\frac{dE_i}{dt} = V_{i+1} H_{i+1} + L_{i-1} h_{i-1} - V_i H_i - L_i h_i - F_i [\varphi_i H_{Fi} + (1 - \varphi_i) h_{Fi}] + Qsr_i$$



$$N = 8; \quad ncomp = 10$$

Demethanizing Column

Algebraic Equations

Compressibility factor (z_i^V, z_i^L)

Residual enthalpies ($\Delta H_i, \Delta h_i$)

Fugacity coefficients ($\phi_{i,j}^V, \phi_{i,j}^L$)

SRK

$$K_{i,j} = \frac{\phi_{i,j}^L}{\phi_{i,j}^V} \quad y_{i,j} = K_{i,j} x_{i,j}$$

$$\sum_j y_{i,j} - \sum_j x_{i,j} = 0$$

$$H_i = H_i^{ideal} - \Delta H_i$$

$$h_i = h_i^{ideal} - \Delta h_i$$



Demethanizing Column

Algebraic Equations

Vapor volume

$$Vol_i^V = \pi \left(\frac{D_i}{2} \right)^2 H_{plate,i} - \frac{M_i^L}{\rho_i^L}$$

Vapor and liquid holdup

$$M_i^V = \rho_i^V Vol_i^V$$

$$M_i = M_i^V + M_i^L$$

Component holdup

$$m_{i,j} = M_i^V y_{i,j} + M_i^L x_{i,j}$$

Internal energy holdup

$$E_i = M_i^V \left(H_i - \frac{p_i}{\rho_i^V} \right) + M_i^L \left(h_i - \frac{p_i}{\rho_i^L} \right)$$



Demethanizing Column

Liquid holdup and hydraulic correlations

$$M_i^L = 0.896\pi \left(\frac{D_i}{2}\right)^2 (Hw_i + Hwo_i)\rho_i^L$$

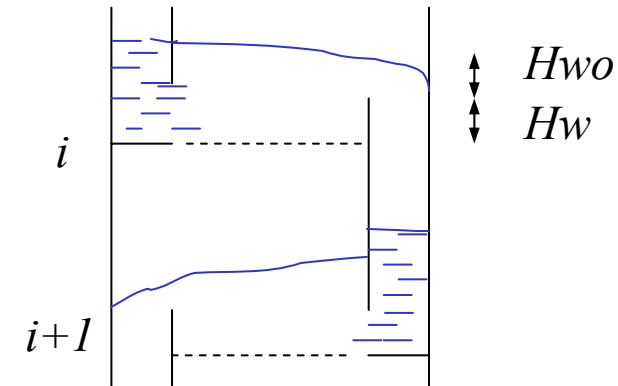
$$Hwo_i = 30(0.01495)^{2/3} Fw_i \left(\frac{L_i}{\rho_i^L Weir l_i}\right)^{2/3}$$

$$Hldrop_i = \beta(Hw_i + Hwo_i)$$

$$Ho_i = 5.56 \cdot 10^{-4} \left(\frac{V_i}{\rho_i^V A_{hole_i} Co_i}\right)^2 \left(\frac{\rho_i^V P_{mol_i}^V}{\rho_i^L P_{mol_i}^L}\right)$$

Pressure drop $P_1 = P_{TOP} - KK\rho_1^V V_1^2$

$$P_i = P_{i-1} + 0.24917 \cdot 10^{-5} (Ho_i + Hldrop_i)\rho_i^L P_{mol_i}^{31}$$



Equivalent height of vapor free liquid

Pressure drop through aerated liquid

Dry hole pressure drop



Demethanizing Column

Carbon dioxide solubility constraints

Phase Equilibrium $\mu_{i,CO_2}^V = \mu_{i,CO_2}^L = \mu_{i,CO_2}^S$

$$\overline{f_{i,CO_2}^V} = \overline{f_{i,CO_2}^L} = \overline{f_{i,CO_2}^S}$$

Assumption

No hydrocarbon in solid phase $\overline{f_{i,CO_2}^S} = f_{i,CO_2}^S$

To avoid CO₂ precipitation $\overline{f_{i,CO_2}^V} \leq f_{i,CO_2}^S$

$$\overline{f_{i,CO_2}^L} \leq f_{i,CO_2}^S$$

But $\overline{f_{i,CO_2}^L} = \overline{f_{i,CO_2}^V}$ from VLE calculations at each stage



Demethanizing Column

Carbon dioxide solubility constraints

$$\overline{f_{i,\text{CO}_2}^V} \leq f_{i,\text{CO}_2}^S$$

Solid phase

$$f_{i,\text{CO}_2}^S = P_{i,\text{CO}_2}^S \phi_{i,\text{CO}_2}^V$$

$$\ln\left(\frac{P_{i,\text{CO}_2}^S}{P_{\text{CO}_2}^t}\right) = 14.568\left(1 - \frac{T_{\text{CO}_2}^t}{T_i}\right) - 14.480 \ln\left(\frac{T_i}{T_{\text{CO}_2}^t}\right) + 65.356\left(\frac{T_i}{T_{\text{CO}_2}^t} - 1\right) - 47.146\left[\left(\frac{T_i}{T_{\text{CO}_2}^t}\right)^2 - 1\right] + 14.540\left[\left(\frac{T_i}{T_{\text{CO}_2}^t}\right)^3 - 1\right]$$

Vapor phase

$$\overline{f_{i,\text{CO}_2}^V} = y_{i,\text{CO}_2} \overline{P_i \phi_{i,\text{CO}_2}^V}$$



Demethanizing Column: Phase Existence

Raghunathan, Diaz, Biegler (2004)

Gibbs Free Energy minimization at stage i



Karush Kuhn Tucker conditions



Raghunathan, Biegler (2003)

MPECs (Mathematical Program with Equilibrium Constraints)

as additional constraints ($x.y = 0; x,y \geq 0$)



IPOPT-C under AMPL

$(x.y \leq \delta\mu)$



Demethanizing Column: Phase Existence

Relaxed Equilibrium Conditions

$$y_{i,j} = \gamma_i K_{i,j}(T_i, P_i, x_{i,j}, y_{i,j}) x_{i,j}$$

$$\sum_j y_{i,j} - \sum_j x_{i,j} = 0$$

$$\gamma_i - 1 = s_i^V - s_i^L$$

$$0 \leq M_i^L \perp s_i^L \geq 0$$

$$0 \leq M_i^V \perp s_i^V \geq 0$$

Complementarity
Constraints

$$\gamma = 1 \Rightarrow M_i^L > 0, M_i^V > 0$$

$$\gamma < 1 \Rightarrow M_i^L = 0, M_i^V > 0$$

$$\gamma > 1 \Rightarrow M_i^L > 0, M_i^V = 0,$$



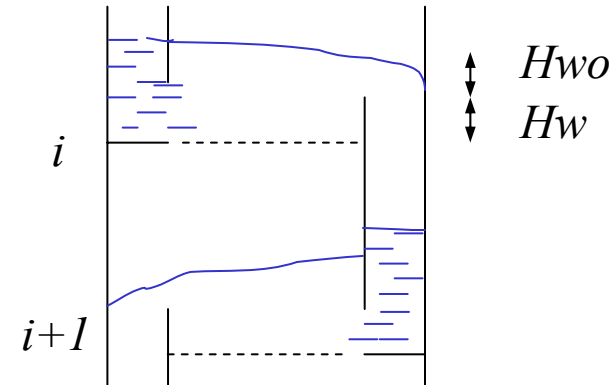
Demethanizing Column: Phase Existence

Liquid Holdup and Hydraulic correlations

$$M_i^L = 0.896\pi \left(\frac{D_i}{2}\right)^2 Hw_i \rho_i^L + M_i^{L+} - M_i^{L-}$$

$$M_i^{L+} = 0.896\pi \left(\frac{D_i}{2}\right)^2 Hw_{oi} \rho_i^L$$

$$Hw_{oi} = 30(0.01495)^{2/3} Fw_i \left(\frac{L_i}{\rho_i^L Weir l_i}\right)^{2/3}$$



$$\left. \begin{array}{l} M_i^{L+} \\ Hw_{oi} \end{array} \right\} \left(L_i = k_d (M_i^{L+})^{\frac{3}{2}} \right)$$

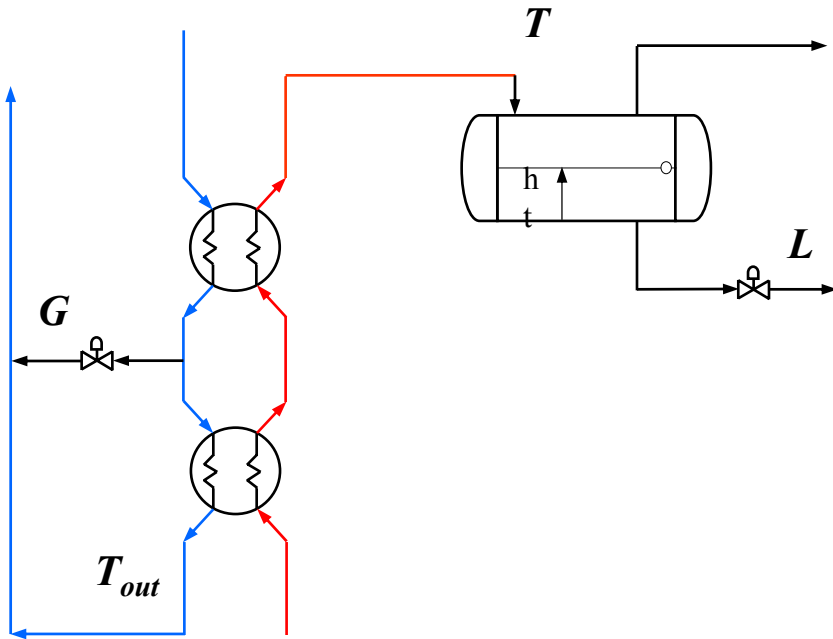
$$0 \leq M_i^{L+} \perp M_i^{L-} \geq 0$$



Numerical Results



Optimization Problem: HEs + HPS



$$\min \int_0^{t_f} (T - T_{SP})^2 dt$$

st.

{DAE System}

$$60 \leq L \leq 80 (\text{kmol} / \text{min})$$

$$5 \leq G \leq 80 (\text{kmol} / \text{min})$$

$$303 \leq T_{out} \leq 306 (\text{K})$$

(8+6 cells in HEs)



Numerical Results: HEs + HPS

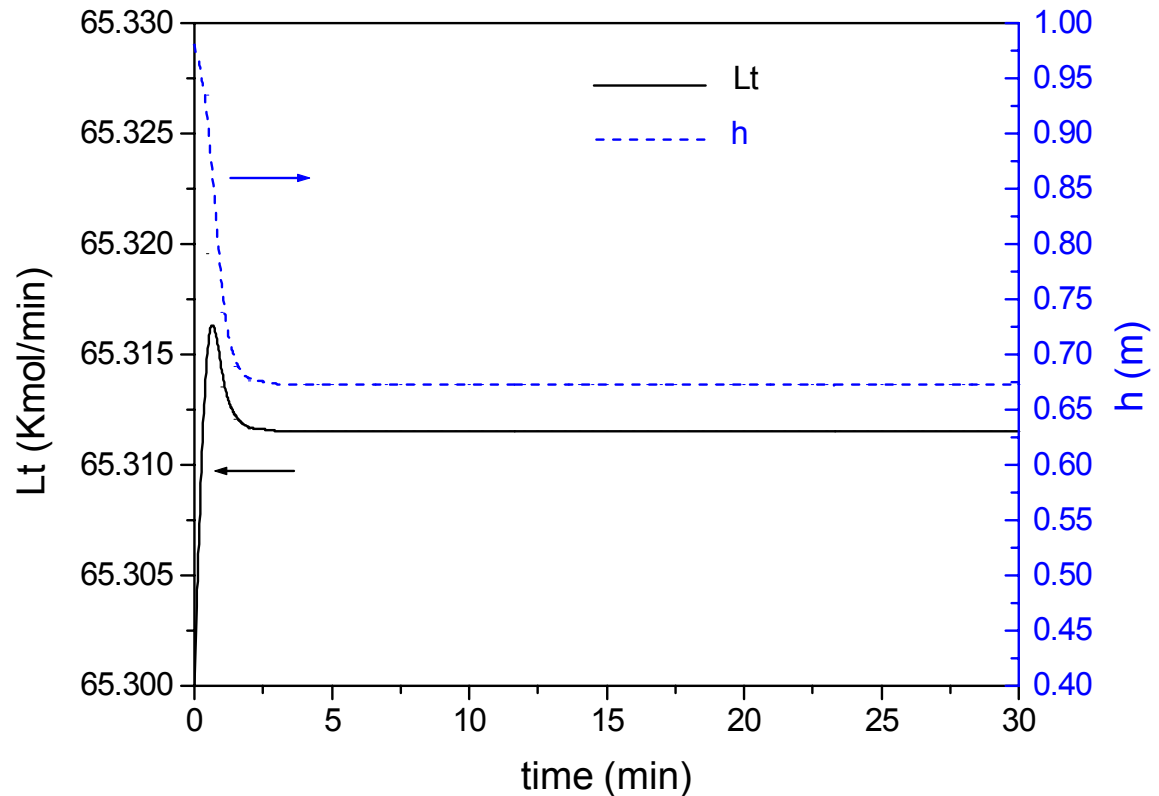
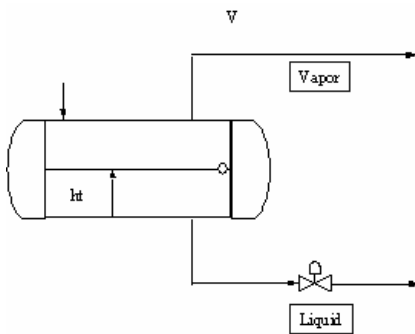
Optimization variables: G, L

20 elements

2 collocation points

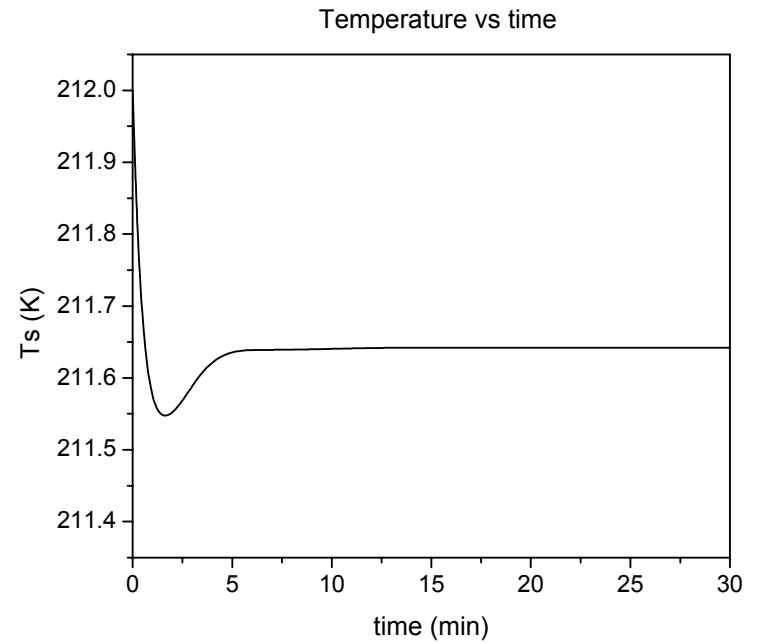
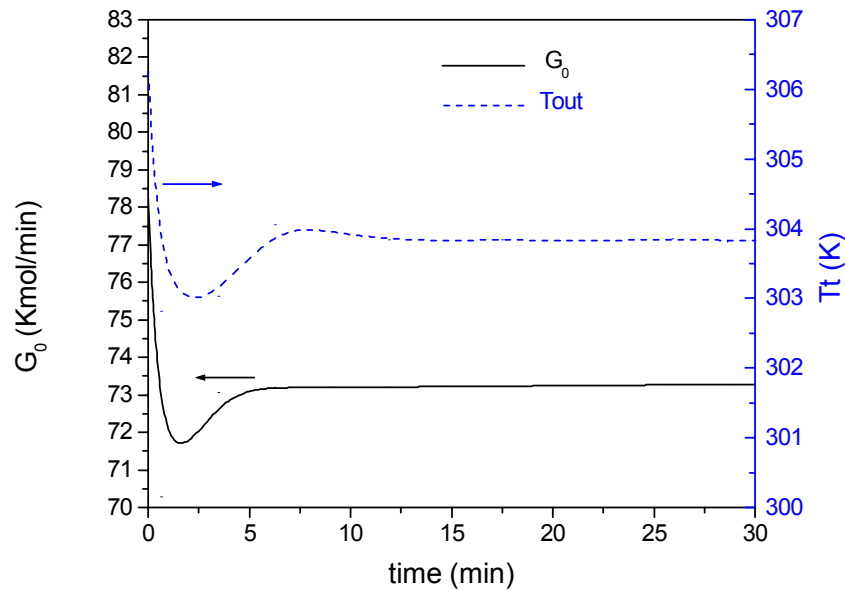
18274 disc. variables

18 iter. (3 barrier problems)

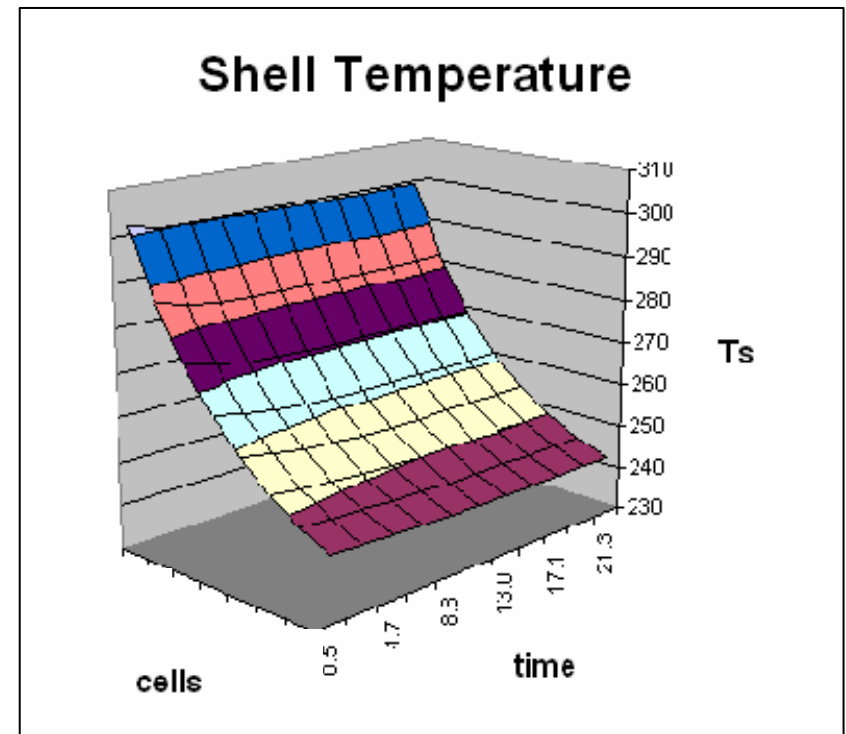
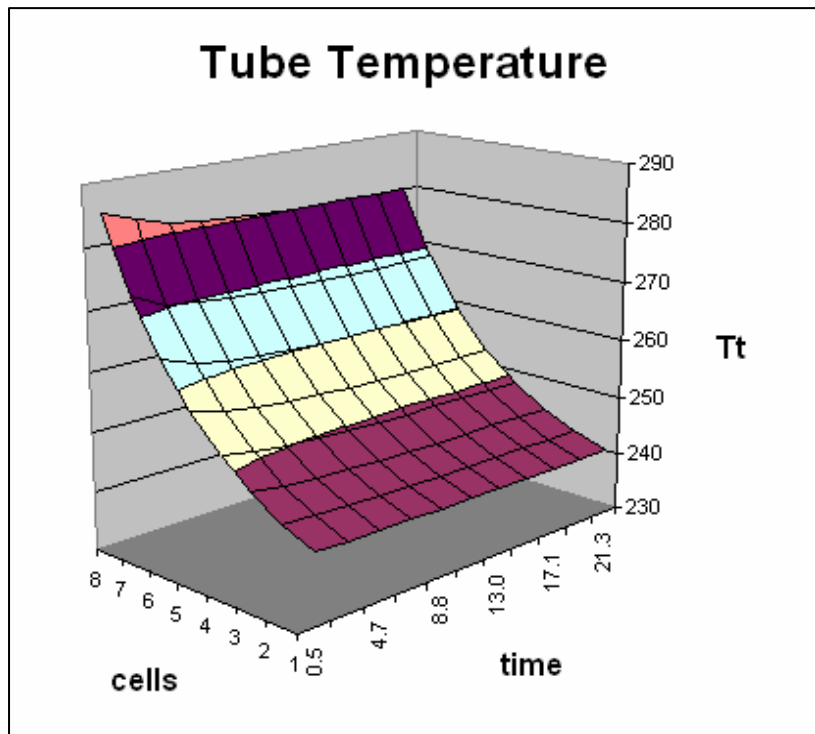


Numerical Results: HEs + HPS

Optimization variables: G , L



Numerical Results: HEs + HPS

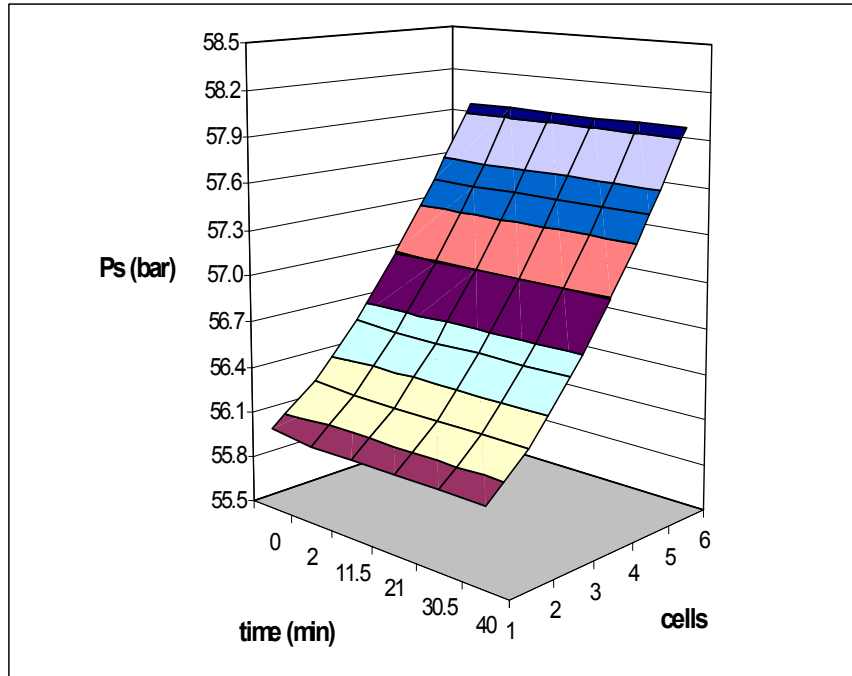


Temp. profile, tubes side

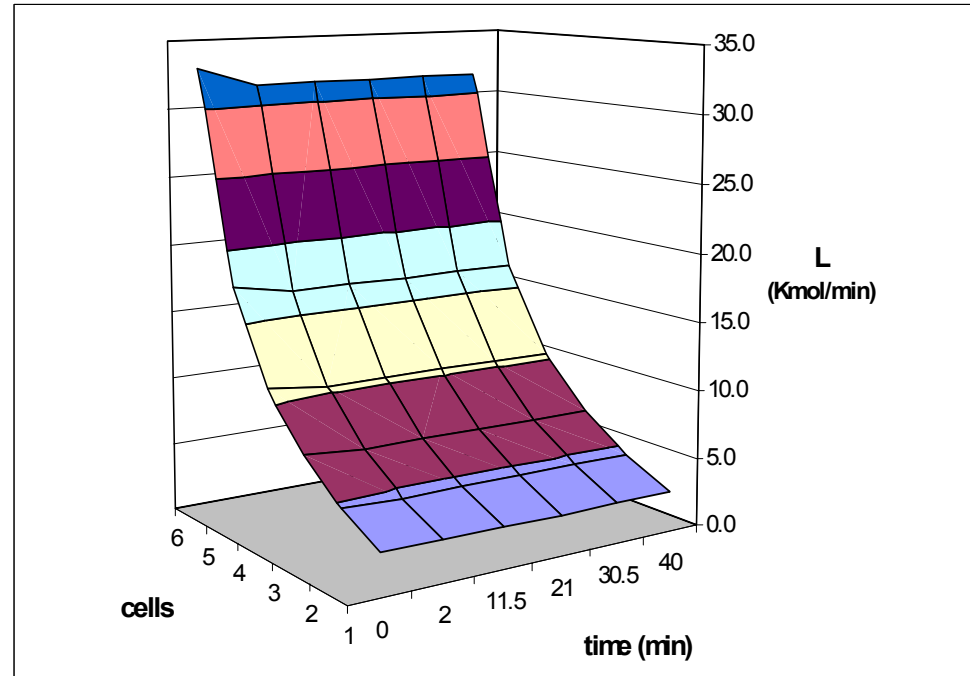
*Temp. profile, shell side
(no phase change)*



Numerical Results: HEs + HPS



Pressure profile, shell side



Liquid flowrate, shell side



Optimization Problem: Demethanizing Column

$$\min \int_0^t (\eta_{ethane} - \eta_{SP})^2 dt$$

st.

{DAE System}

$$15 \leq P_{TOP} \leq 22(\text{bar})$$

$$99 \leq Q_{REB} \leq 200(\text{kJ} / \text{min})$$

$$0 \leq x_{B,CH_4} \leq 0.008$$

$$\overline{f_{i,CO_2}^V} \leq 0.90 f_{i,CO_2}^S$$

Alg. Eqns = 385

Differ. Eqns = 97



Numerical Results: Demethanizer

*Feed A: Low CO₂
content (0.65%)*

Optimization variable: P_{TOP}

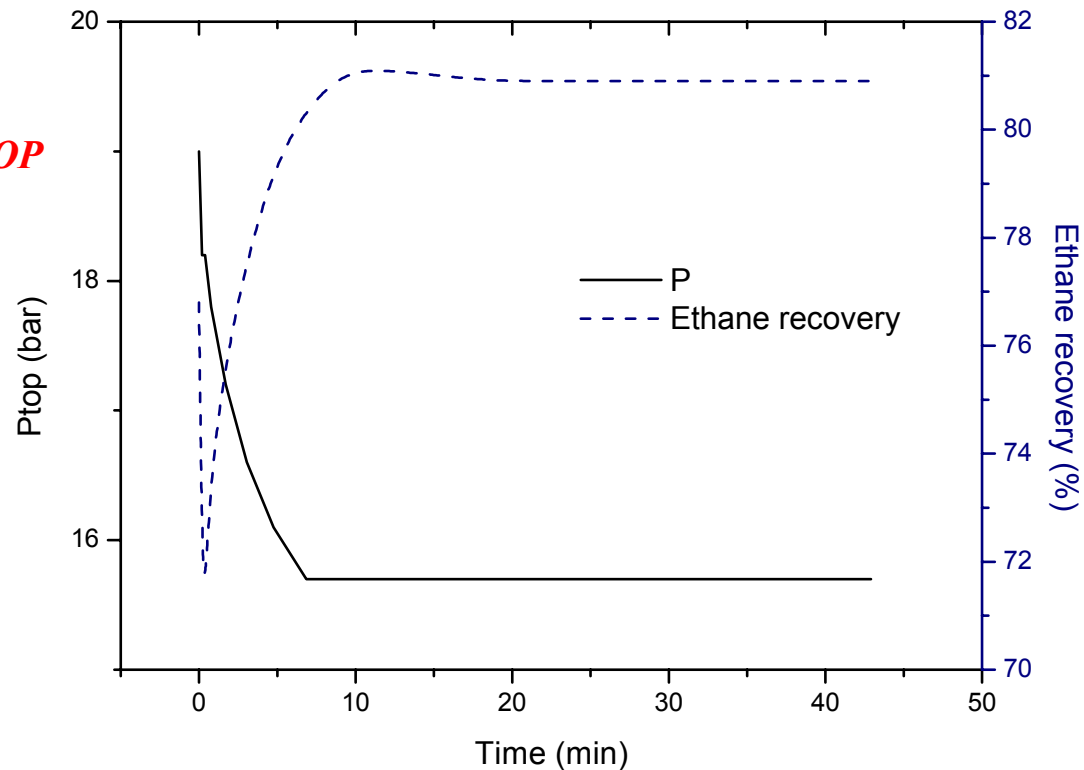
20 finite elements

2 collocation points

21357 discretized variables

40 iter. (3 barrier problems)

$\eta_{ss} = 80.9\%$



Numerical Results: Demethanizer

*Feed B: High CO₂ content
(2%)*

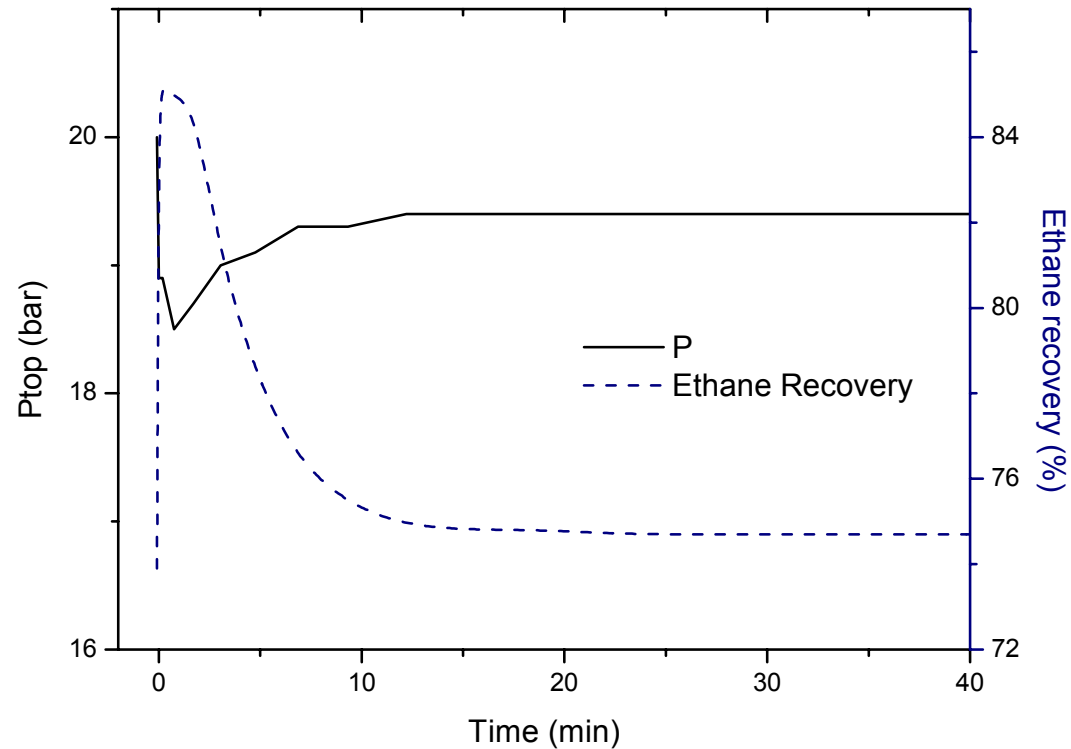
Optimization variable: P_{TOP}

20 finite elements

2 collocation points

43 iter. (3 barrier problems)

$\eta_{ss} = 74.5 \%$



Numerical Results: Demethanizer

Feed B (high CO₂ content)

No solubility constraints

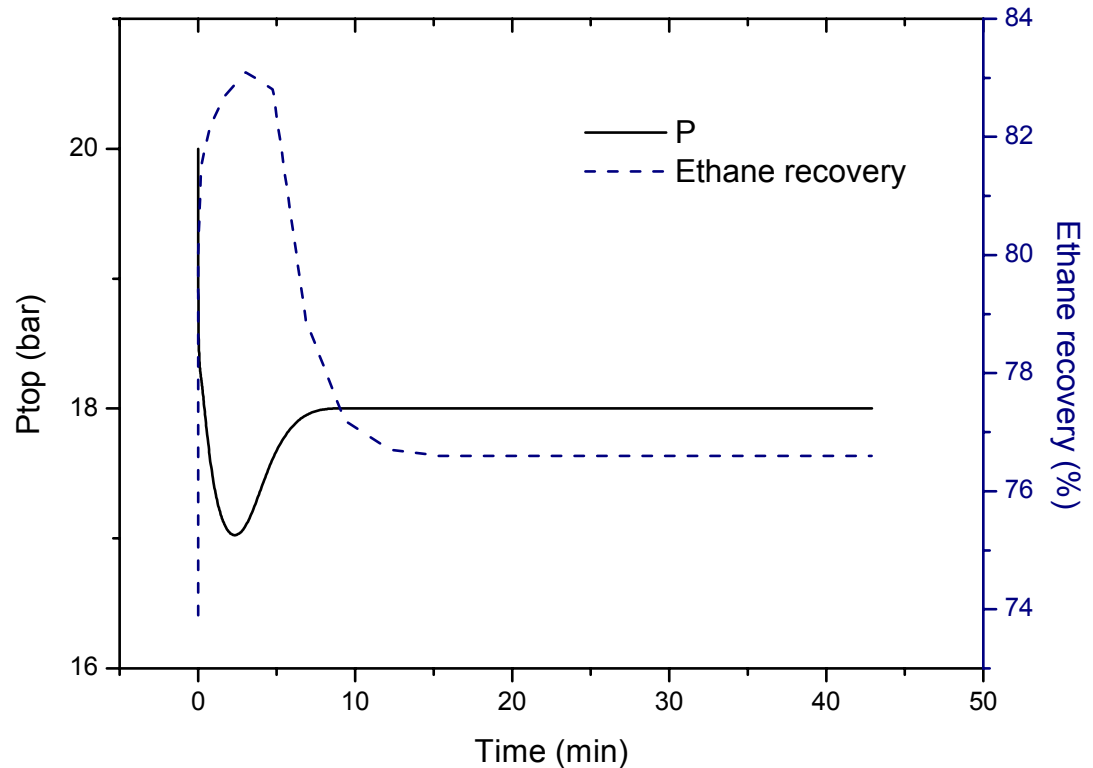
Optimization variable: P_{TOP}

20 finite elements

2 collocation points

59 iter. (4 barrier problems)

$\eta_{ss} = 76.6\%$



Numerical Results: Demethanizer

Feed A

Optimization variable:
Reboiler Heat Duty (Q_R)

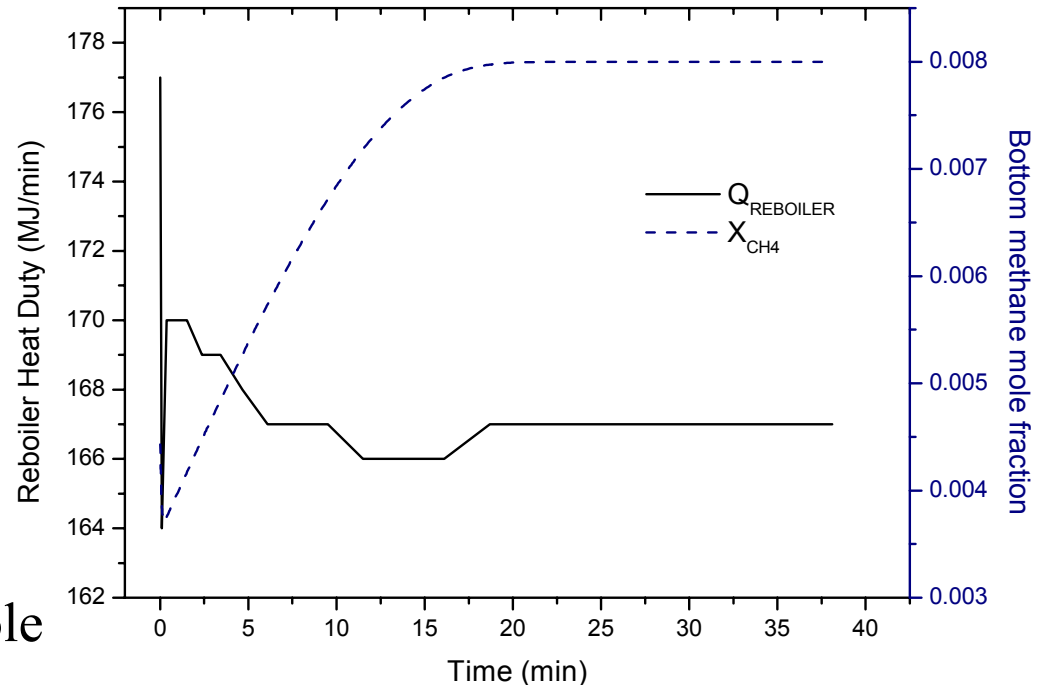
20 finite elements

2 collocation points

57 iter. (5 barrier problems)

$\eta_{ss} = 77.8\%$ ($P_{TOP}=19\text{bar}$)

Active constraint: Methane mole fraction in bottoms



Numerical Results: Demethanizer Start Up

MPECs

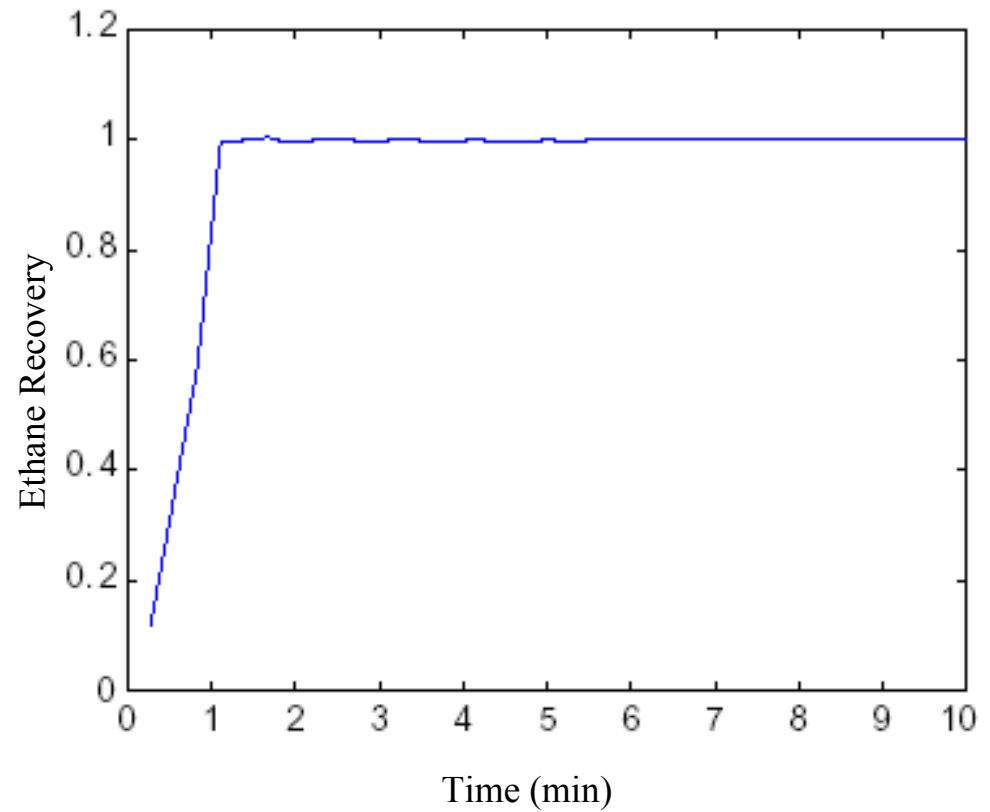
Optimization variable:

P_{TOP} , Q_R

Three time periods

Ideal Gas, Ideal solution

Solved in AMPL with
IPOPT-C (Raghuathan,
Biegler, 2003)



Numerical Results: Demethanizer Start Up

MPECs

Optimization variable:

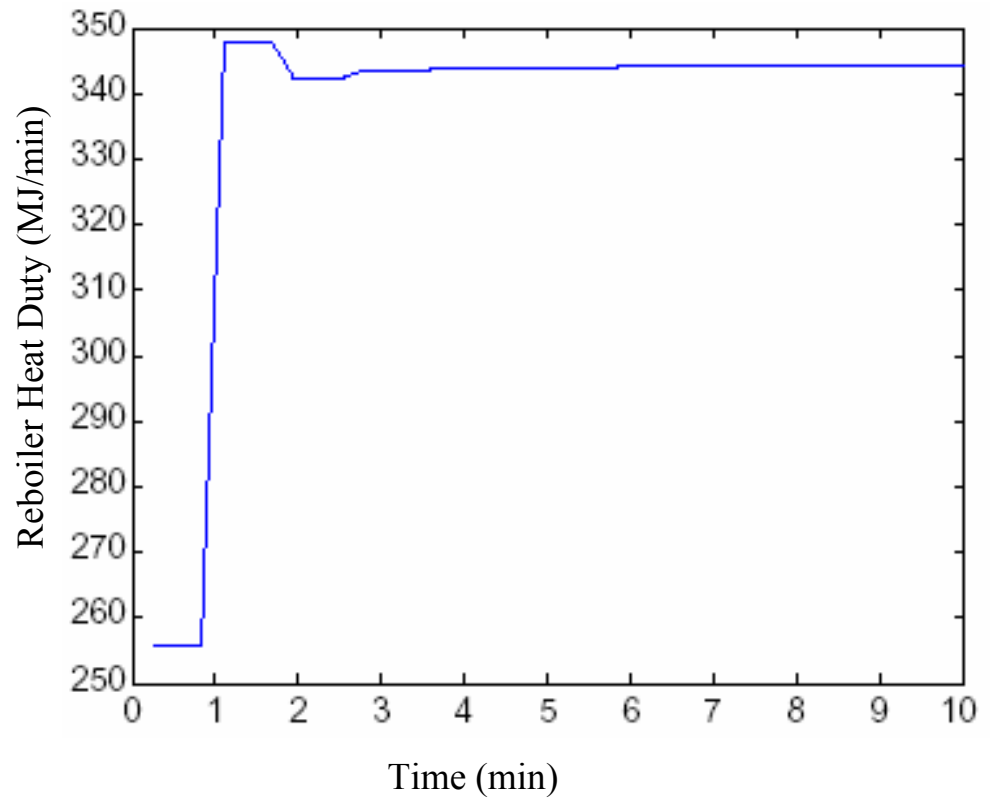
P_{TOP} , Q_R

3rd time period

17944 discretized
variables

576 complementarity
constraints

97 iterations



Boiler Dynamic Optimization

Rodriguez, Bandoni, Diaz (2005b)

Dynamic optimization model to determine controller parameters

Boiler

Combustion chamber

Risers

Superheater

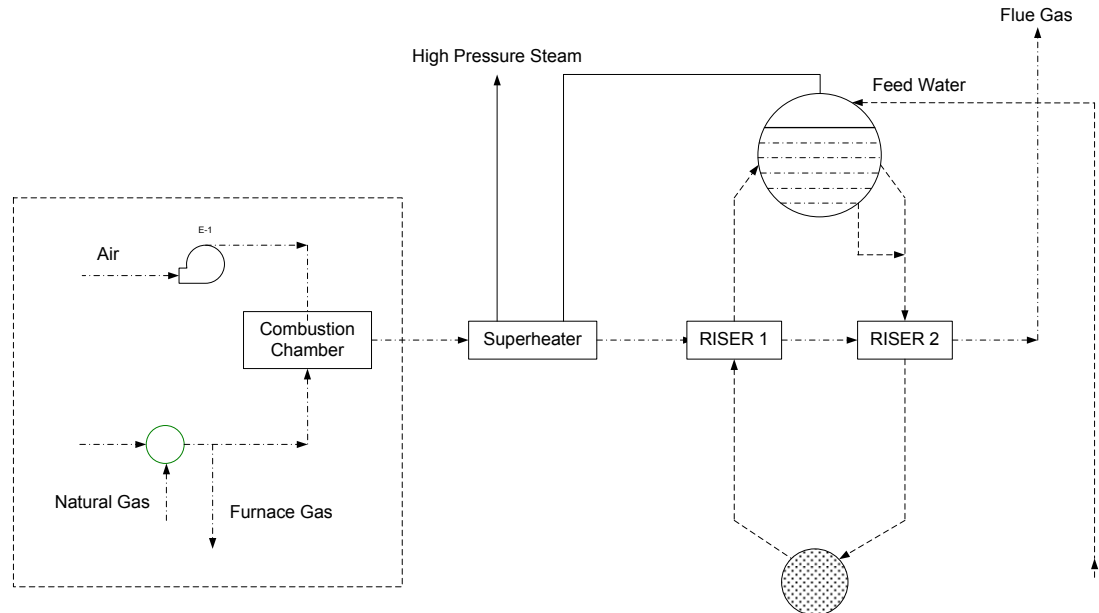
Steam drum

Gas path

PI Controller

(Liquid level in drum)

Manipulated: Feed water flowrate



Boiler Dynamic Optimization

DAE Optimization model

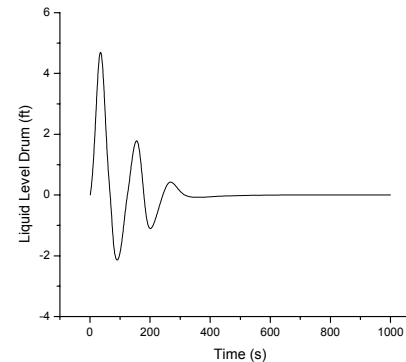
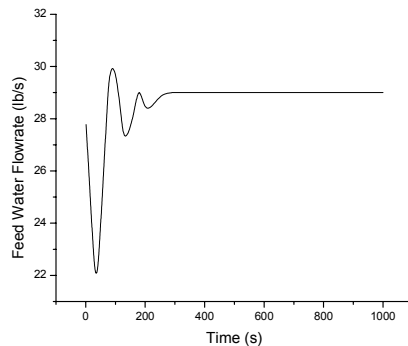
- Differential equations
 - Momentum and energy balances in risers and superheater
 - Mass and energy balances in drum
 - Integral part of controller and valve equation
- Algebraic equations
 - Friction loss
 - Mixture properties
 - Vapor holdup in drum
 - Gas – temperature distribution equations in furnace, risers and superheaters (6 eqns.)
 - Air / fuel ratio
 - Drum geometric relations (5 eqns.)
 - Steam tables correlations for saturated and high pressure steam density, pressure and enthalpy



Boiler Dynamic Optimization

DAE Optimization model

- Optimization variables
 - PI controller parameters
- Step change in high pressure steam demand



- Additional PI controllers (current work)
 - Outlet steam pressure (fuel gas flowrate)
 - Superheated steam temperature (air excess)



Conclusions

- Rigorous dynamic optimization models for cryogenic train in Natural Gas Processing plant within a simultaneous approach
- Thermodynamic models with cubic equation of state
- Carbon dioxide solubility addressed in both liquid and vapor phases as path constraints, at each column stage
- Phase detection through the first order optimality conditions for Gibbs free energy minimization (MPECs)
- Boiler dynamic model for controller parameters
- Simultaneous approach provides efficient framework for the formulation and resolution of DAE optimization problems for large-scale real plants



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Appendix: Soave Redlich Kwong Equation of State

Compressibility factor
for a mixture (z_i^L, z_i^V)


$$z = \frac{V}{V-b} - \frac{a\alpha}{R_g T(V+b)}$$

$$a\alpha = \sum x_j^* z_j^*; x_j^* = x_j a_j \alpha_j; z_j = \sum_k x_k^* (1 - k_{j,k})$$

$$b = \sum_j b_j; b_j = 0.08664 R_g T_{cj} / P_{cj}$$

$$a_j = \left(\frac{0.42747 R_g^2 T_{cj}^2}{P_{cj}} \right)^{0.5}; \alpha_j = 1 + m_j \left[1 - \left(\frac{T}{T_{cj}} \right)^{0.5} \right]$$

Soave modification
(Temperature
dependence)



$$m_j = 0.48 + 1.574\omega_j - 0.176\omega_j^2$$

Appendix: Soave Redlich Kwong Equation of State

Fugacity Coeff. for
component j in
mixture ($\phi_{i,j}^V, \phi_{i,j}^L$)

$$\ln \phi_j = \frac{b_j}{b}(z-1) - \ln(z-B) - \frac{A}{B} \left[\frac{2a_j \alpha_j z_j^*}{a\alpha} - \frac{b_j}{b} \right] \ln \left(1 + \frac{B}{z} \right)$$

Residual enthalpy
in mixture ($\Delta H_i, \Delta h_i$)

$$-\frac{\Delta H}{R_g T} = -1 - \frac{1}{b R_g T} \ln \left(1 + \frac{B}{z} \right) \sum_j x_j^* z_j^* \left(\frac{1+m_j}{\alpha_j} \right)$$

$$A = \frac{a\alpha P}{R_g^2 T^2}; B = \frac{bP}{R_g T}$$

